Remaining parts of the long-standing J/ψ polarization puzzle

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(Received 24 October 2018; published 30 January 2019)

Based on the nonrelativistic quantum chromodynamics factorization formalism, the polarization parameters $\lambda_{\theta\phi}$ and λ_ϕ of prompt J/ψ hadroproduction for CMS and LHCb are analyzed in the helicity frame and calculated at QCD next-to-leading order for the first time. For prompt J/ψ production, we take into account the feed-down contributions from χ_{cJ} and $\psi(2S)$ decays. The theoretical predictions for the polarization parameters $\lambda_{\theta\phi}$ and λ_ϕ of J/ψ are presented. Using the theoretical results, we perform a simultaneous fit to the experimental measurements of the yield and polarization of J/ψ hadroproduction, and find that the results describe the experimental measurements at the LHC quite well.

DOI: [10.1103/PhysRevD.99.014044](https://doi.org/10.1103/PhysRevD.99.014044)

I. INTRODUCTION

Nonrelativistic quantum chromodynamics (NRQCD) [1] is one of the most successful effective theories to describe the decay and production of heavy quarkonium (for a review see, e.g., Refs. [2,3]). By separating the processes related to heavy quarkonium into short-distance coefficients (SDCs) and the long-distance matrix elements (LDMEs), NRQCD allows one to organize perturbative calculations as double expansions in both the coupling constant α_s and the heavy quark relative velocity v . In the past decade, great improvements have been made in the next-to-leading-order (NLO) QCD correction calculation [4–10]. The NLO corrections to color-singlet J/ψ hadroproduction have been investigated in Refs. [4,5], and its transverse momentum (p_t) distribution was found to be enhanced by 2–3 orders of magnitude in the high- p_t region and the J/ψ polarization changes from transverse to longitudinal at NLO [5]. The NLO corrections to J/ψ production via S -wave color-octet (CO) states were first studied in Ref. [6], and the corrections to the p_t distributions of the J/ψ production yield and polarization are small. References [7–10] performed the complete NLO calculation for prompt

J/ψ hadroproduction and their results can fit the p_t distributions of the experimental measurements at the Tevatron and LHC.

Despite all of these successes, we cannot overlook the remaining challenges. In the J/ψ case, the determination of CO LDMEs suffers ambiguity due to the freedom of the fitting method. With different fitting strategies, quite different values of LDMEs are obtained, which lead to different descriptions of the polarization distribution. Three groups [11–13] have made great efforts to calculate the J/ψ polarization λ_θ at QCD NLO, but none of their CO LDMEs can reproduce the experimental measurements from the LHC [14,15] with good precision for the low- and high- p_t ranges of J/ψ simultaneously. The big uncertainty on the related LDMEs still remains, which is further complicated by the fact that only one [13] of the three groups included the χ_c and $\psi(2s)$ feed-downs from direct calculations, and the result can be compared with the experimental measurement of prompt J/ψ polarization. Thereafter, the η_c hadroproduction measured by the LHCb Collaboration [16] provided another laboratory for the study of NRQCD. Reference [17] considered it as a challenge to NRQCD, while Refs. [18,19] found that these data are consistent with the J/ψ hadroproduction measurements. Further, with the constraint on the LDMEs obtained in Refs. [19,20], the authors found a special way to reduce the uncertainty from LDMEs' for J/ψ hadroproduction.

J/ψ polarization, encoded in the angular distributions of the lepton pair, is described by

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$$\frac{d^2N}{d\cos\theta d\phi} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_{\theta\phi} \sin(2\theta) \cos\phi + \lambda_\phi \sin^2\theta \cos(2\phi), \quad (1)$$

where θ is the polar angle between the direction of the positive lepton and chosen polarization axis, and ϕ is the azimuthal angle measured with respect to the production plane. While all three coefficients provide independent information, almost all of the theoretical studies of J/ψ polarization have been restricted to λ_θ . The parameter λ_ϕ has only been studied at QCD NLO in Ref. [11] with a few experimental data points measured by the ALICE Collaboration [21]. For $\lambda_{\theta\phi}$, there is no theoretical prediction at all. On the other hand, there are experimental measurements of $\lambda_{\theta\phi}$ and λ_ϕ for J/ψ and $\psi(2S)$ polarization from LHCb [14,15] and CMS [22]. Based on the results from all of the theoretical studies, it is believed that a combined fit including p_t distributions of J/ψ production rate and the polarization parameter λ_θ can be achieved. But this does not mean that the J/ψ polarization puzzle is solved: the parameters $\lambda_{\theta\phi}$ and λ_ϕ from experimental measurements still lack comparable theoretical predictions. Therefore, a theoretical study of these two polarization parameters is certainly needed. Do the theoretical predictions of these two parameters coincide with the experimental measurements? Otherwise, could the uncertainty on the related LDMEs be reduced by fitting these measurements together with a previous data fit? These are very important issues to address if we wish to solve the long-standing J/ψ polarization puzzle.

In this paper, we perform a theoretical analysis of the properties of the polarization parameters, and finish the calculation of $\lambda_{\theta\phi}$ and λ_ϕ for J/ψ and $\psi(2S)$ polarizations in the helicity frame based on NRQCD at QCD NLO. The results are obtained for the first time. By performing a combined fit, they describe the experimental measurements at the LHC quite well. Therefore, the last two pieces of the J/ψ polarization are successfully explained.

II. CALCULATION

The three polarization parameters λ_θ , $\lambda_{\theta\phi}$, and λ_ϕ in Eq. (1) are defined as [23]

$$\lambda_\theta = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_{\theta\phi} = \frac{\sqrt{2} \text{Re} d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}, \quad \lambda_\phi = \frac{d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}}.$$

Here, $d\sigma_{\lambda\lambda'}(\lambda, \lambda' = 0, \pm 1)$ is the spin density matrix of J/ψ or $\psi(2S)$ hadroproduction, which—following the NRQCD factorization—can be expressed as

$$d\sigma_{\lambda\lambda'}(pp \rightarrow HX) = \sum_{a,b,n} \int dx_1 dx_2 f_{a/p}(x_1) f_{b/p}(x_2) \times d\hat{\sigma}_{\lambda\lambda'}(ab \rightarrow (c\bar{c})_n X) \langle \mathcal{O}_n^H \rangle, \quad (2)$$

where p is the proton, and the indices a, b run over the gluon (g) and light quarks (q) and antiquarks ($\bar{q}$). n denotes the color, spin, and angular momentum states of the $c\bar{c}$ intermediate states ($^3S_1^{[1]}$, $^1S_0^{[8]}$, $^3S_1^{[8]}$, $^3P_J^{[8]}$) for J/ψ and $\psi(2S)$, or ($^3P_J^{[1]}$, $^3S_1^{[8]}$) for χ_{cJ} . The functions $f_{a/p}(x_1)$ and $f_{b/p}(x_2)$ are the parton distribution functions for the incoming protons for parton types a and b . The SDC $d\hat{\sigma}$ can be perturbatively calculated and the LDMEs $\langle \mathcal{O}_n^H \rangle$ are governed by nonperturbative QCD effects.

For inclusive J/ψ production at the LHC, P parity is invariant for initial-state protons and it involves only QCD interactions which are also P -parity invariant. Based on the P -parity invariance of the production density matrix of J/ψ hadroproduction, together with the definition of the helicity frame, symmetry (asymmetry) relations can be deduced as (for details, see Ref. [24])

$$\left. \frac{d\sigma_{\lambda\lambda'}^H}{dy} \right|_{y=a} = n_{\lambda\lambda'} \left. \frac{d\sigma_{\lambda\lambda'}^H}{dy} \right|_{y=-a}, \quad n_{\lambda\lambda'} = \begin{cases} 1 & \lambda = \pm\lambda' \\ -1 & \lambda = \pm 1, \lambda' = 0 \end{cases}. \quad (3)$$

Then, we obtain that $\lambda_{\theta\phi} = 0$ for experiments with a symmetric rapidity range ($a < |y| < b$) (like that at CMS and ATLAS), $\lambda_{\theta\phi} \neq 0$ for the half-rapidity range ($y > b$) (like that at LHCb), $\lambda_{\theta\phi}$ and λ_ϕ are symmetric for positive and negative rapidity in the helicity frame.

To include the feed-down contributions from $\psi(2S)$ and χ_{cJ} to J/ψ , we follow the same treatment as in Ref. [13],

$$d\sigma_{\lambda\lambda'}^{J/\psi}|_{\psi(2S)} = \mathcal{B}[\psi(2S) \rightarrow J/\psi] d\sigma_{\lambda\lambda'}^{\psi(2S)}, \quad (4)$$

$$d\sigma_{\lambda\lambda'}^{J/\psi}|_{\chi_{cJ}} = \mathcal{B}[\chi_{cJ} \rightarrow J/\psi] \sum_{J_z, J'_z} \delta_{J_z - \lambda, J'_z - \lambda'} \times C_{J_z, J_z}^{\lambda, J_z - \lambda} C_{J_z, J'_z}^{*\lambda', J'_z - \lambda'} d\sigma_{J_z, J'_z}^{\chi_{cJ}}, \quad (5)$$

where $C_{J_z, J_z}^{\lambda, J_z - \lambda}$ is the Clebsch-Gordan coefficient, and λ and J_z are the quantum numbers of the angular momentum.

To calculate the NRQCD prediction for the transverse momentum p_t distribution of the yield and polarization for heavy quarkonium hadroproduction at QCD NLO we use the FDCHQHP package [25], which is based on the collection of FORTRAN codes generated for all 87 parton level subprocesses by using the FDC package [26] and an implementation tool for job submission and numerical precision control. It is a very powerful tool that saves us a lot of work and allows us to finish this study. However, we found a few areas for improvement regarding job submission and numerical precision control: the updated version of FDCHQHP [27] will be publicly available soon.

III. LDME STRATEGY

The CS LDMEs are estimated from the wave functions at the origin by

$$\begin{aligned}\langle \mathcal{O}^\psi(3S_1^{[1]}) \rangle &= \frac{3N_c}{2\pi} |R_\psi(0)|^2, \\ \langle \mathcal{O}^{\chi_{cJ}}(3P_J^{[1]}) \rangle &= \frac{3}{4\pi} (2J+1) |R'_{\chi_c}(0)|^2,\end{aligned}\quad (6)$$

where the wave functions at the origin can be calculated via the potential model [28], which gives $|R_{J/\psi}(0)|^2 = 0.81 \text{ GeV}^3$, $|R_{\psi(2S)}(0)|^2 = 0.53 \text{ GeV}^3$, and $|R'_{\chi_c}(0)|^2 = 0.075 \text{ GeV}^5$.

The CO LDMEs are extracted from the fit of the experimental data with the QCD NLO theoretical formula. However, different results for the LDMEs are obtained when different strategies are used in the fit. In the following we briefly discuss the different fit results and select a few of them to represent the uncertainty on the predictions from the LDMEs.

In the J/ψ case, several groups of LDMEs [8,9,12,13,18,19,29,30] can be found. They are extracted by fitting the data on the hadroproduction yield [8,13] or combining them with polarization data [12] for pp collisions. In the fits in Refs. [8,12,13], the data with $p_t < 7 \text{ GeV}$ were excluded. Reference [9] extracted the LDMEs with a wider set of data, including the lower- p_t region ($p_t > 3 \text{ GeV}$) of hadroproduction and the production at ep and $\gamma\gamma$ colliders with $p_t > 1 \text{ GeV}$. By assuming heavy quark spin symmetry, the fit in Refs. [18,19] took the η_c measurement [16] ($p_t \geq 6 \text{ GeV}$) into consideration, and they obtained consistent J/ψ LDMEs. In Refs. [29,30], the authors incorporated the leading-power fragmentation corrections together with the usual QCD NLO corrections, which involved different SDCs and resulted in different LDMEs. The authors of Ref. [8] later improved their analysis by taking the feed-down contributions into account [31], but no updated J/ψ LDMEs were presented. Among the LDME set mentioned above, only Ref. [13] fitted the prompt J/ψ hadroproduction by including the feed-down contributions from χ_{cJ} and $\psi(2S)$ which can be used to calculate these feed-down contributions to J/ψ polarization.

Following the same treatment as in Ref. [13], we refit the J/ψ LDMEs by using more experimental data where the data in the $p_t < 7 \text{ GeV}$ region are excluded, as usual.

In addition to the p_t distribution of the production yield data from CDF [32] and LHCb [33] in the old fit, the polarization data for λ_θ , $\lambda_{\theta\phi}$, and λ_ϕ from LHCb [14] and for λ_θ and λ_ϕ from CMS [22] are used. $\lambda_{\theta\phi}$ is exactly zero in our calculation for the CMS data, so it cannot be fitted. The nonzero data for $\lambda_{\theta\phi}$ from CMS could be from P -parity broken, i.e., from electroweak production for J/ψ . Therefore, we calculate the leading contributions from $gg \rightarrow (c\bar{c})_n + Z^0$, $ud \rightarrow (c\bar{c})_n + W$, and $uu \rightarrow (c\bar{c})_n + Z^0$ with $n = 3S_1^{[1]}$, $1S_0^{[8]}$, $3S_1^{[8]}$, $3P_J^{[8]}$, and find that their production rates are smaller than the QCD production processes by about 5 orders of magnitude. So we suggest that $\lambda_{\theta\phi}$ from CMS should be set to zero in the experimental measurement.

To deal with the feed-down contributions from $\psi(2S)$ and χ_{cJ} , we use the CO LDMEs from Ref. [13], namely, $\langle \mathcal{O}^{\psi(2S)}(1S_0^{[8]}) \rangle = -1.2 \times 10^{-4} \text{ GeV}^3$, $\langle \mathcal{O}^{\psi(2S)}(3S_1^{[8]}) \rangle = 3.4 \times 10^{-3} \text{ GeV}^3$, $\langle \mathcal{O}^{\psi(2S)}(3P_0^{[8]}) \rangle / m_Q^2 = 4.2 \times 10^{-3} \text{ GeV}^3$, and $\langle \mathcal{O}^{\chi_{c0}}(3S_1^{[8]}) \rangle = 2.21 \times 10^{-3} \text{ GeV}^3$.

By fitting all 83 data points for J/ψ and minimizing χ^2 (which is $\chi^2/\text{d.o.f.} = 80.78/83$), we obtain

$$\begin{aligned}\mathcal{O} &\equiv \left(\langle \mathcal{O}(1S_0^{[8]}) \rangle, \langle \mathcal{O}(3S_1^{[8]}) \rangle, \frac{\langle \mathcal{O}(3P_0^{[8]}) \rangle}{m_c^2} \right) \\ &= (56.6 \pm 4.7, 1.77 \pm 0.58, 3.42 \pm 1.02) \times 10^{-3} \text{ GeV}^3,\end{aligned}\quad (7)$$

where the uncertainties are correlated. To express the uncertainty from CO LDMEs independently, we use a covariance-matrix method as in Ref. [13], and the differential cross section $d\sigma$ is obtained as

$$\begin{aligned}d\sigma &= \sum \mathcal{O}_i d\hat{\sigma}_i = \sum \mathcal{O} V V^{-1} d\hat{\sigma} = \sum \Lambda V^{-1} d\hat{\sigma}, \\ \Lambda &= (5.43 \pm 0.48, 1.60 \pm 0.05, 0.054 \pm 0.005) \times 10^{-2} \text{ GeV}^3,\end{aligned}\quad (8)$$

where the corresponding rotated matrix is

$$V = \begin{pmatrix} 0.976 & 0.217 & 0.010 \\ -0.109 & 0.451 & 0.886 \\ -0.188 & 0.866 & -0.464 \end{pmatrix}. \quad (9)$$

TABLE I. The values of LDMEs for J/ψ (in units of GeV^3).

Reference	$\langle \mathcal{O}^{J/\psi}(3S_1^{[1]}) \rangle$	$\langle \mathcal{O}^{J/\psi}(1S_0^{[8]}) \rangle$	$\langle \mathcal{O}^{J/\psi}(3S_1^{[8]}) \rangle$	$\langle \mathcal{O}^{J/\psi}(3P_0^{[8]}) \rangle / m_Q^2$
Butenschoen <i>et al.</i> (2011) [9]	1.32	$(3.04 \pm 0.35) \times 10^{-2}$	$(1.68 \pm 0.46) \times 10^{-3}$	$(-4.04 \pm 0.72) \times 10^{-3}$
Chao <i>et al.</i> (2012) [12]	1.16	$(8.9 \pm 0.98) \times 10^{-2}$	$(3.0 \pm 1.2) \times 10^{-3}$	$(5.6 \pm 2.1) \times 10^{-3}$
Gong <i>et al.</i> (2013) [13]	1.16	$(9.7 \pm 0.9) \times 10^{-2}$	$(-4.6 \pm 1.3) \times 10^{-3}$	$(-9.5 \pm 2.5) \times 10^{-3}$
Zhang <i>et al.</i> (2015) [19]	0.645 ± 0.405	$(0.78 \pm 0.34) \times 10^{-2}$	$(1.0 \pm 0.3) \times 10^{-2}$	$(1.7 \pm 0.5) \times 10^{-2}$

FIG. 1. p_t distribution of prompt J/ψ hadroproduction. CDF and LHCb data are taken from Refs. [32,33].

FIG. 2. $\lambda_{\phi\phi}$ (left column) and λ_{ϕ} (right column) of J/ψ , with the rapidity region separated into positive (red dashed line) and negative parts (blue dash-dotted line).

FIG. 3. $\lambda_{0\phi}$ of J/ψ (left column) and $\psi(2S)$ (right column), with rapidity region separated into positive (red dashed line) and negative parts (blue dash-dotted line). The CMS data is from Ref. [22].

To investigate the uncertainties on the prediction from different LDME sets, the four sets of LDMEs in Table I are also used to present the final numerical results.

IV. NUMERICAL RESULTS

In our numerical calculation, we use the parton distribution function CTEQ6M [34] and the corresponding two-loop QCD coupling constant α_s . The charm-quark mass is chosen as approximately $m_c = M_H/2$, where the masses of the relevant quarkonia M_H are 3.0, 3.5, and 3.686 GeV for $H = J/\psi$, χ_{cJ} ($J = 0, 1, 2$), and $\psi(2S)$, respectively. The renormalization and factorization scales are chosen as

$\mu_r = \mu_f = \sqrt{4m_c^2 + p_t^2}$, while the NRQCD scale is $\mu_\Lambda = m_c$. The branching ratios are $\mathcal{B}[\psi(2S) \rightarrow J/\psi] = 0.61$ and $\mathcal{B}[\chi_{cJ} \rightarrow J/\psi] = 0.0127, 0.339, 0.192$ for $J = 0, 1, 2$ [35], respectively. Additionally, a shift $p_t^H \approx p_t^{H'} \times (M_H/M_{H'})$ is used when considering the kinematics effect in the feed-down from higher excited states.

In Fig. 1 we present the p_t spectrum of prompt J/ψ hadroproduction. In Fig. 2 we present $\lambda_{0\phi}$ for each ($^3S_1^{[1]}$, $^3S_1^{[8]}$, $^3P_J^{[8]}$) channel with the rapidity ranges $y > 0$, $y < 0$ and $|y| > 0$, and we can see that $\lambda_{0\phi}$ is asymmetric for these rapidity ranges, as in Eq. (3).

FIG. 4. λ_ϕ (left column), $\lambda_{0\phi}$ (middle column), and λ_ϕ (right column) for J/ψ with different sets of LDMEs in the middle rapidity region. The CMS data is from Ref. [22].

FIG. 5. λ_θ (left column), $\lambda_{\theta\phi}$ (middle column), and λ_ϕ (right column) for J/ψ with different sets of LDMEs in the forward rapidity region. The LHCb data is from Ref. [14].

In Fig. 3 we present $\lambda_{\theta\phi}$ at CMS using the default LDME set in Eq. (7). We can see that the final numerical results are asymmetric within very good numerical precision and the theoretical predictions describe the experimental measurements at CMS quite well for both J/ψ and $\psi(2S)$.

To investigate the uncertainties from different sets of LDMEs, λ_θ , $\lambda_{\theta\phi}$, and λ_ϕ are presented by using the LDME sets in Table I. In Fig. 4, the prediction of prompt J/ψ polarization can explain the CMS data in a wide p_t region ($10 \text{ GeV} < p_t < 70 \text{ GeV}$) in both the $|y| < 0.6$ and $0.6 < |y| < 1.2$ rapidity bins. The newly fitted LDMEs provide an excellent description of λ_θ in the CMS window in the transverse momentum range $14 \text{ GeV} < p_t < 70 \text{ GeV}$ [22]. All five fit schemes provide good descriptions of $\lambda_{\theta\phi}$ and λ_ϕ . As we mentioned above, the SDC $\text{Re}(d\sigma_{10})$ is exactly zero in the symmetric rapidity region (e.g., $a \leq |y| \leq b$) for all of the channels. So the values of the LDMEs would not change $\lambda_{\theta\phi}$ from zero for the CMS data. The CMS data for λ_ϕ are covered by the uncertainty from the five LDME fit schemes.

In Fig. 5, λ_θ , $\lambda_{\theta\phi}$, and λ_ϕ in the five rapidity bins at LHCb are presented, from top to bottom. For the different rapidity bins, the p_t distributions of λ_θ , $\lambda_{\theta\phi}$, and λ_ϕ behave in a similar way. In the low- p_t region, the different LDME sets provide large uncertainties for $\lambda_{\theta\phi}$ and λ_ϕ , while as p_t increase the results from the different LDME sets converge to zero. We can clearly see that our new fits describe the experimental measurements at LHCb quite well for J/ψ production.

There is not enough space to present the error bands in Figs. 4 and 5 for all of the other LDME sets in the fits from previous works. By plotting the error bands for each set of LDMEs separately, we find that the error bands are smaller for the LDME sets “Butenschoen *et al.* (2011)” and “Gong *et al.* (2013),” and they are larger for the LDME sets “Chao *et al.* (2012)” and “Zhang *et al.* (2015).” However, these observations are based on putting in the LDME errors directly without their rotated matrix, which can deduce the error bands.

V. SUMMARY AND CONCLUSION

In this paper, we finished the calculation of $\lambda_{\theta\phi}$ and λ_ϕ for J/ψ and $\psi(2S)$ polarization in the helicity frame based on NRQCD at QCD NLO. The results were obtained for the first time and they describe the experimental measurements at the LHC quite well. Therefore, the last two pieces of the J/ψ polarization puzzle have been successfully explained.

By applying a P -parity invariance analysis, we determined that in the helicity frame $\lambda_{\theta\phi} = 0$ for experiments with a symmetric rapidity range ($a < |y| < b$) (like that at CMS and ATLAS), and $\lambda_{\theta\phi} \neq 0$ for the half-rapidity range ($y > b$) (like that at LHCb) for the polarization of J/ψ and $\psi(2S)$ hadroproduction at the LHC. Although the electroweak production of J/ψ can break P parity, our calculations show that the contribution is negligible. This conclusion could be applied in CMS or ATLAS experimental measurements to keep $\lambda_{\theta\phi} = 0$ in each iteration of fitting to improve the measurements. It also indicates that the polarization measurements could be performed in the half-rapidity range with $y > 0$ or $y < 0$ separately, and thus $\lambda_{\theta\phi} \neq 0$ could be achieved.

ACKNOWLEDGMENTS

The results described here were achieved by using the HPC Cluster of ITP-CAS. Y. F. would like to thank the very kind invitation of CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics (ITP), CAS where part of this work was completed. This work was supported in part by the National Natural Science Foundation of China with Grants No. 11747037, No. 11275243, No. 11275036, No. 11447601, No. 11535002, No. 11675239, and No. 11475183. It was also supported by the Key Research Program of Frontier Sciences, CAS, Grant No. QYZDY-SSW-SYS006. Y. F. was also supported by the Army Medical University of PLA of China No. 2016XPY06.

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