Transverse densities and generalized parton distributions of the ρ meson in the light front quark model

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We have investigated the transverse charge density for the longitudinal and transversely polarized ρ meson in the light-front quark model (LFQM). Charge densities are obtained from the elastic form factors of the ρ meson calculated in LFQM including the zero-mode contributions. We have computed the different helicity matrix elements of the ρ meson. In addition to this, we have also presented the results for the generalized parton distributions (GPDs) and impact-parameter dependent parton distribution functions (ipdfds) of the ρ meson.

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I. INTRODUCTION

Electromagnetic form factors (FFs) are the key source to understand the internal structure of the hadrons. By taking the Fourier transform of the form factors of hadrons one can get the spatial distributions like charge and magnetization densities [1,2]. These densities provide the model independent information about the charge distribution. Basically, it gives the density of the hadron at the transverse distance from the center of momentum. It is to be noted that information about the charge density in terms of longitudinal coordinates is not obtained in a meaningful way. Principally, hadron states have explicit dependence on momentum while initial and final hadron states have different momentum which invalidates their interpretation as probability density. However, it is shown in Ref. [3] that charge density can be calculated from the hadronic form factor which directly involves the 3D charge density of partons. For the case of nucleons there have been considerable efforts made to understand these densities [4–9] in various phenomenological models [10–13]. Transverse densities in fixed light-front time has been studied in Refs. [14,15]. One of the most important works in Ref. [2] showed that neutron charge density is negative at the center of the core and also the spatial extent of the magnetization density for the proton is much larger than that of its charge density [16].

Except for nucleons, a lot of work has been done to obtain the charge densities of the spin-1 system like

deuteron [17–19]. For a complete overview over the theoretical and experimental studies on the deuteron form factors, see Refs. [20–23]. Despite the spin-1 system like deuteron, ρ meson is also an interesting subject to investigate. ρ meson is a short-living hadronic particle having three states denoted as ρ^+ , ρ^0 and ρ^- . Due to spin-1, there are three form factors: charge (G_C), magnetic (G_M), and quadrupole (G_Q), respectively, for ρ meson. ρ meson is the lightest strong interacting particle after pions and kaons. From an experimental point of view, diffractive photo production and electroproduction of the ρ meson is extensively studied at the HERA H1 experiment [24–30]. In addition to this, ρ meson light-front wave functions (LFWFs) are also extracted using HERA data on diffractive ρ photoproduction [31]. Spacelike and timelike $\gamma^*\pi \rightarrow \rho$ and $\gamma^*\rho \rightarrow \pi$ transition form factors are also studied in light-cone formalism [32]. Recently, Sun *et al.* have also discussed the generalized parton distributions and parton distributions in impact-parameter for ρ meson [33,34].

Recently, AdS/QCD formalism has been successfully applied to various hadronic properties like, generalized parton distributions (GPDs), parton distribution functions (PDFs), form factors and transverse densities [35–65]. Regarding spin-1 particles like ρ meson, a lot of work has been done in holographic QCD [66–70].

In the present work, we have studied the transverse charge densities for unpolarized and polarized ρ mesons in light-front quark model (LFQM) [71–78]. This model is quite successful in explaining the various electroweak properties of light and heavy mesons compared with experimental data. A very important calculation on the vector meson form factor (e.g., ρ meson) is discussed in Ref. [79]. Calculations on the structure of wave functions of mesons as bound states on relativistic quarks are discussed in Ref. [80]. Form factors of the ρ meson in

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the light-front constituent quark model are also discussed in Ref. [81]. Further, timelike form factors of ρ mesons are calculated in Ref. [82]. Distribution amplitudes, decay constants, and radiative decays for mesons have been also studied in Refs. [83,84], respectively. In addition to this, skewed parton distributions for pions have been also discussed in LFQM [85].

Transverse charge densities are obtained by taking the two-dimensional Fourier transform of the physical form factors of ρ meson. Transverse densities are also related with the generalized parton distributions (GPDs) with zero skewness [86,87], although GPDs reveal simultaneously information on both longitudinal and the transverse distribution of partons in a fast moving hadron. This physical picture becomes more intuitive when one take a Fourier transform from transverse momentum transfer to impact-parameter space. GPDs can be accessed by the deep virtual Compton scattering process (DVCS) and deep virtual meson production (DVMP) [88]. In this work, we have also calculated the GPDs and impact-parameter dependent parton distribution function (ipdpdf) for ρ meson [89,90].

To complete this work, we have used the results of physical electromagnetic form factors of ρ meson computed by Choi *et al.* in Ref. [91] by including the zero-mode contribution [92–95]. Authors take care of the zero-mode issue in calculating the form factors of ρ meson using different helicity components. The zero mode has a close relation with the off-diagonal elements in the Fock state expansion of the current matrix. The existence of the zero-mode contribution to the form factor is characterized by the contribution from off-diagonal elements in $q^+ \rightarrow 0$ limit. One can also represent the hadronic FFs without the presence of zero-mode contribution. However for the spin-1 system, FFs include the zero-mode contribution which arises in the matrix element of plus current.

The manuscript is arranged as follows. After the Introduction, we discuss the physical form factors and light-front quark model in Sec. II. In Sec. III, we discuss the transverse charge density for ρ mesons. After this, we discuss the GPDs for ρ mesons in Sec. IV followed by ipdpdfs in Sec. V. Conclusions are drawn in last section.

II. PHYSICAL FORM FACTORS AND LIGHT-FRONT QUARK MODEL

In the present work, we have used the LFQM which successfully explains various electromagnetic properties of the hadrons. For the spin-1 particle, Lorentz invariant electromagnetic form factors $[F_i (i = 1, 2, 3)]$ are defined by the matrix elements of the J^μ current sandwiched between initial $|P, \Lambda\rangle$ and final $\langle P', \Lambda' |$ states as follows [91]:

$$\begin{aligned} \langle P', \Lambda' | J^\mu | P, \Lambda \rangle = & -\epsilon_{\Lambda'}^* \cdot \epsilon_\Lambda (P + P')^\mu F_1(Q^2) \\ & + (\epsilon_{\Lambda'}^\mu q \cdot \epsilon_\Lambda^* - \epsilon_{\Lambda'}^{*\mu} q \cdot \epsilon_\Lambda) F_2(Q^2) \\ & + \frac{(\epsilon_{\Lambda'}^* \cdot q)(\epsilon_\Lambda \cdot q)}{2M_v^2} (P + P')^\mu F_3(Q^2), \end{aligned} \quad (1)$$

where $q = P' - P$ and $\epsilon_\Lambda(\epsilon_{\Lambda'})$ is the polarization vector of the initial (final) meson with the physical mass M_v . We employ the Breit frame [76,79,81] ($q^+ = 0, q_y = 0, q_x = Q, \mathbf{P}_\perp = -\mathbf{P}'_\perp$) and the momenta are defined by

$$\begin{aligned} q^\mu &= (0, 0, Q, 0), \\ P^\mu &= (M_v \sqrt{1 + \kappa}, M_v \sqrt{1 + \kappa}, -Q/2, 0), \\ P'^\mu &= (M_v \sqrt{1 + \kappa}, M_v \sqrt{1 + \kappa}, Q/2, 0), \end{aligned} \quad (2)$$

and $\kappa = Q^2/4M_v^2$, and the notation used is $p^\mu = (p^+, p^-, p^1, p^2)$. One can obtain the covariant form factors of the spin-1 particle by using the plus component of the current $I_{\Lambda'\Lambda}^+(0) = \langle P', \Lambda' | J^+ | P, \Lambda \rangle$. It is also clear from Eq. (1), that there are three form factors and nine total elements of $I_{\Lambda'\Lambda}^+(0)$ can be assigned to the current operator. Due to light-front parity and light-front time reversal invariance, we can reduce it to only four elements, $I_{++}^+, I_{+-}^+, I_{+0}^+$, and I_{00}^+ . Practically, the physical charge (G_C), magnetic (G_M), and quadrupole (G_Q) form factors are often used to describe electromagnetic properties of the hadron rather than $F_i(Q^2) (i = 1, 2, 3)$ form factors. The relation between these two form factors is given by:

$$\begin{aligned} G_C &= F_1 + \frac{2}{3}\kappa G_Q, \\ G_M &= -F_2, \\ G_Q &= F_1 + F_2 + (1 + \kappa)F_3. \end{aligned} \quad (3)$$

In addition to this, at zero momentum transfer these form factors are equal to static charge e , magnetic moment μ , and quadrupole moment Q , respectively:

$$\begin{aligned} eG_C(0) &= e, \\ eG_m(0) &= 2M_v\mu, \\ -eG_Q(0) &= M_v^2 Q. \end{aligned} \quad (4)$$

For the calculation of such types of form factors Grach and Kondratyuk (GK) [96] and Brodsky and Hiller (BH) [97] prescriptions have been used. However, we define the physical form factors in BH prescription which includes the zero-mode contributions and is given by

$$\begin{aligned}
G_C^{\text{BH}} &= \frac{1}{2P^+(1+2\kappa)} \left[\frac{3-2\kappa}{3} I_{00}^+ + \frac{16}{3} \kappa \frac{I_{+0}^+}{\sqrt{2\kappa}} \right. \\
&\quad \left. + \frac{2}{3} (2\kappa-1) I_{+-}^+ \right], \\
G_M^{\text{BH}} &= \frac{2}{2P^+(1+2\kappa)} \left[I_{00}^+ + \frac{(2\kappa-1)}{\sqrt{2\kappa}} I_{+0}^+ - I_{+-}^+ \right], \\
G_Q^{\text{BH}} &= \frac{-1}{2P^+(1+2\kappa)} \left[I_{00}^+ - 2 \frac{I_{+0}^+}{\sqrt{2\kappa}} + \frac{1+\kappa}{\kappa} I_{+-}^+ \right]. \quad (5)
\end{aligned}$$

They also satisfy the angular condition which is given by

$$\Delta(Q^2) = (1+2\kappa)I_{++}^+ + I_{+-}^+ - \sqrt{8\kappa} I_{+0}^+ - I_{00}^+. \quad (6)$$

This condition must be satisfied by the matrix elements so that physical form factors must be independent from the GK or BH prescriptions. We can also define the structure functions $A(Q^2)$, $B(Q^2)$, and tensor polarization T_{20} by relating physical form factors as:

$$\begin{aligned}
A(Q^2) &= G_C^2 + \frac{2}{3} \kappa G_M^2 + \frac{8}{9} \kappa^2 G_Q^2, \\
B(Q^2) &= \frac{4}{3} \kappa (1+\kappa) G_M^2, \\
T_{20}(Q^2, \theta) &= -\kappa \frac{\sqrt{2} \frac{4}{3} G_Q^2 + 4 G_Q G_C + [1/2 + (1+\kappa) \tan^2(\theta/2)] G_M^2}{A + B \tan^2(\theta/2)}. \quad (7)
\end{aligned}$$

In LFQM, the physical form factors are obtained from the matrix element $I_{\Lambda\Lambda}^+$ which is defined as

$$\begin{aligned}
I_{\Lambda\Lambda}^+ &= \int \frac{dx}{2(1-x)} \int d^2\mathbf{k}_\perp \sqrt{\frac{\partial k_z^+}{\partial x} \frac{\partial k_z^-}{\partial x}} \phi^*(x, \mathbf{k}_{\perp f}) \phi(x, \mathbf{k}_{\perp i}) \\
&\quad \times \frac{(S_{\Lambda\Lambda}^+)_{on}}{M_{oi} M_{of}}, \quad (8)
\end{aligned}$$

where $(S_{\Lambda\Lambda}^+)_{on}$ is defined in Ref. [91] and the radial wave function is defined as

$$\phi(x, \mathbf{k}^2) = \sqrt{\frac{1}{\pi^3/2\beta^3}} \exp(-\mathbf{k}^2/2\beta^2), \quad (9)$$

where $\mathbf{k}^2 = \mathbf{k}_\perp^2 + k_z^2$, $k_z = (x-1/2)M_o$ [98,99], and

$$M_{oi}^2 = M_{of}^2 = M_o^2 = \frac{\mathbf{k}_\perp^2 + m^2}{x(1-x)}. \quad (10)$$

However, it is shown in Ref. [91] that the zero-mode contribution arises from the S_{00}^+ component which is usually avoided by considering the GK prescription over the BH prescription, but it is to be noted that the (0,0) component is the longitudinal component, and it is the most dominant contributor in the high momentum transfer region or for the analysis in the high momentum perturbative QCD, and, therefore, it may be better to use the BH prescription, involving the (0,0), (+,0), and (+,-) amplitudes.

It is also observed that presence of zero mode contribution in (0,0) amplitude is quite significant in the

light-front quark model phenomenologically because the absence of zero mode in (+,0) amplitude can give a tremendous benefit on reliable predictions on the ρ meson when compared with the calculations done in the covariant formulation [100,101]. It is also interesting that transverse densities are obtained from the matrix elements which feel the zero mode but they do not as model do not contain the zero mode. It is probably due to the fact that Eq. (18) described the valence contribution of $I_{\Lambda\Lambda}^+$ and only on-shell trace terms. It is clear from Ref. [91] that zero mode contribution in LFQM arises from the $(S_{0,0}^+)_{off}$ term. Nevertheless, in literature GK prescription is preferred over BH prescription as it is free from zero mode by definition.

III. TRANSVERSE CHARGE DENSITIES FOR ρ MESON

The charge density in transverse plane as a standard interpretation can be obtained by two-dimensional Fourier transform of the form factor. In a true relativistic picture, form factors are Lorentz invariant and depend upon the Lorentz invariant quantities only. In the present work, we have extracted the transverse densities from the helicity matrix elements $G_{\Lambda\Lambda}^+$ which is further obtained from the certain combinations of charge G_C , magnetic G_M , and quadrupole G_Q form factors calculated in Breit frame within LFQM. It is to be noted that helicity matrix element is frame dependent which in turn reflects that transverse charge densities obtained from them are frame-dependent and they are not Lorentz invariant. This circumstance is also cleared from Ref. [18]. The transverse charge densities

FIG. 1. Plots of helicity-conserving matrix elements (a) $G_{00}^+(Q^2)$, (b) $G_{++}^+(Q^2)$, and (c) helicity-flip matrix element with one unit of helicity-flip $G_{0+}^+(Q^2)$, and (d) with two units of helicity-flip $G_{-+}^+(Q^2)$.

for ρ meson with light-front helicity state $\Lambda = \pm 1, 0$ are given as [1,2]

$$\begin{aligned} \rho_\Lambda^\rho(b) &= \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} G_{\Lambda\Lambda}^+(Q^2), \\ &= \int_0^\infty \frac{dQ Q}{2\pi} J_0(Qb) G_{\Lambda\Lambda}^+(Q^2), \end{aligned} \quad (11)$$

where $G_{\Lambda\Lambda}^+(Q^2)$ is the matrix element obtained from the electromagnetic current $J^+(0)$ sandwich between two ρ meson states [17]:

$$\left\langle P^+, \frac{\mathbf{q}_\perp}{2}, \Lambda' \left| J^+ \right| P^+, -\frac{\mathbf{q}_\perp}{2}, \Lambda \right\rangle = 2P^+ e^{i(\Lambda - \Lambda')\phi_q} G_{\Lambda'\Lambda}^+(Q^2), \quad (12)$$

where $\Lambda = \pm 1, 0$ ($\Lambda' = \pm 1, 0$) are the light-front helicity for initial (final) ρ meson state. We also define $\mathbf{q}_\perp = Q(\cos\phi\hat{x} + \sin\phi\hat{y})$, and impact-parameter $\mathbf{b}_\perp = b(\cos\phi\hat{x} + \sin\phi\hat{y})$ in the transverse plane gives the position of parton from center of position space for the ρ meson. Further, one can define the helicity-conserving matrix elements (G_{11}^+ , G_{00}^+) and helicity nonconserving matrix elements (G_{0+}^+ , G_{-+}^+), respectively, in terms of G_C , G_M and G_Q as [17]:

$$\begin{aligned} G_{++}^+ &= \frac{1}{1+\kappa} \left[G_C + G_M + \frac{\kappa}{3} G_Q \right], \\ G_{00}^+ &= \frac{1}{1+\kappa} \left[(1-\kappa)G_C + 2\kappa G_M - \frac{2\kappa}{3}(1+2\kappa)G_Q \right], \\ G_{0+}^+ &= -\frac{\sqrt{2\kappa}}{1+\kappa} \left[G_C - \frac{1}{2}(1-\kappa)G_M + \frac{\kappa}{3}G_Q \right], \\ G_{-+}^+ &= \frac{\kappa}{1+\kappa} \left[G_C - G_M - \left(1 + \frac{2\kappa}{3}\right)G_Q \right], \end{aligned} \quad (13)$$

where again $\kappa = \frac{Q^2}{4M_\rho^2}$. One should keep in mind that ρ meson helicity matrix element G_{0+}^+ will give the dipole pattern, whereas G_{-+}^+ will give the quadrupole pattern after getting transformed into impact-parameter space in terms of transverse density. The reason for the dipole and quadrupole pattern is due to angular dependencies which are associated with the one unit of helicity-flip (G_{0+}^+) matrix element and two units of helicity-flip (G_{-+}^+) matrix element which can be seen in Eq. (16), respectively. In Figs. 1(a) and 1(b), we present the results for helicity-conserving matrix elements G_{00}^+ and G_{++}^+ , respectively, whereas results for helicity nonconserving matrix elements (G_{0+}^+ and G_{-+}^+) are shown in Figs. 1(c) and 1(d). We present the results for the charge density for unpolarized ρ meson in Figs. 2(a) and 2(b). We observe that the distribution is axially symmetric, or one can say

FIG. 2. Transverse charge densities of the ρ meson evaluated in light-front quark model (a) and (c), and $\rho_0^\rho(b)$ and $\rho_1^\rho(b)$ for the unpolarized ρ meson in (b) and (d).

monopole in nature, for both the cases and peaks at the center of impact-parameter space. In Figs. 2(c) and 2(d), we present the results in 2D plots. We also consider the transversely polarized ρ meson state which provides information about dipole and quadrupole moments. Transverse charge density for transversely polarized ρ meson can be defined as

$$\rho_{s_\perp T}^\rho(\mathbf{b}_\perp) = \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \frac{1}{2P^+} \times \left\langle P^+, \frac{\mathbf{q}_\perp}{2}, s_\perp \left| J^+ \right| P^+, -\frac{\mathbf{q}_\perp}{2}, s_\perp \right\rangle, \quad (14)$$

where $s_\perp$ is the ρ meson transverse spin projection along the transverse polarization direction $S_\perp = \cos\phi \hat{x} + \sin\phi \hat{y}$, and for $s_\perp = 0, 1$, transverse charge densities can be written as [17]

$$\rho_{0T}(b) = \int_0^\infty \frac{dQ Q}{2\pi} [J_0(bQ) G_{++}^+ + \cos 2(\phi_b - \phi_s) J_2(bQ) G_{+-}^+], \quad (15)$$

$$\rho_{1T}(b) = \int_0^\infty \frac{dQ Q}{2\pi} \left[\frac{J_0(bQ)}{2} (G_{++}^+ + G_{00}^+) + \sin(\phi_b - \phi_s) J_1(bQ) \sqrt{2} G_{0+}^+ - \cos 2(\phi_b - \phi_s) J_2(bQ) \frac{G_{--}^+}{2} \right]. \quad (16)$$

Without loss of generality, we choose $\phi_s = 0$. In ρ_{0T} , we are getting contributions from two helicity matrix elements G_{++}^+ and G_{+-}^+ . The former contains no helicity flip but later includes a net shift of two units of helicity, i.e., $-1 \rightarrow +1$. However, in ρ_{1T} , contributions are received from G_{0+}^+ (total helicity flip $0 \rightarrow 1$) and again G_{+-}^+ . In Figs. 3(a) and 3(c), we present the results for ρ_{0T} which show that distribution gets stretched along $\hat{y}$ axis after getting contribution from G_{+-}^+ but there is no overall shift of the peak in the charge density. However, in Figs. 3(b) and 3(d) we consider the ρ meson to be polarized along $\hat{x}$ -direction. We found that ρ_{1T} received contributions from G_{0+}^+ and G_{+-}^+ . Due to significant contributions received from the dipole and quadrupole

FIG. 3. Plot for (a) monopole and (b) dipole contributions towards ρ_0^T and correspondingly two-dimensional plots in (c) and (d) for ρ meson in LFQM.

term the distribution gets distorted from the center of distribution. In Figs. 4(a) and 4(b), we show the results for the monopole and quadrupole contributions of ρ_{0T} . We also present the results for monopole, dipole, and quadrupole contributions for ρ_{1T} in Fig. 5. It is to be noted that dipole and quadrupole patterns in charge densities are due to the anomalous magnetic moment

coming from the second term of Eq. (16) which produced an electric dipole moment in $\hat{y}$ direction, and the anomalous quadrupole moment comes from the third term of Eq. (16). It is also noted that signs of quadrupole contributions are the same in ρ_{0T} and ρ_{1T} , which stretch the distribution for $\rho_T(0)$ and distort the distribution significantly for ρ_{1T} in $\hat{y}$ direction.

FIG. 4. Plot for (a) monopole and (b) quadrupole contributions towards ρ_0^T for ρ meson in LFQM.

FIG. 5. Plots for (a) monopole, (b) dipole, and (c) quadrupole contributions towards ρ_1^T for ρ meson in LFQM.

IV. GENERALIZED PARTON DISTRIBUTIONS FOR THE ρ MESON

Generalized parton distributions encode the three-dimensional structure of the hadrons. One can predict the physical form factors from the first moment of GPDs, i.e.,

$$G_z(Q^2) = \int dx H_i(x, Q^2), \quad (17)$$

where $z = C, M$, and Q and $i = 1, 2$, and 3 , respectively. The integrals over H_4 and H_5 vanish due to constraints of Lorentz invariance and time reversal. GPDs for the ρ meson can be defined by the correlator function as [102]

$$\begin{aligned} V_{\lambda'\lambda} &= \frac{1}{2} \int \frac{d\omega}{2\pi} e^{ix(Pz)} \langle P', \lambda' | \bar{q}(-z/2) \not{n} q(z/2) | P, \lambda \rangle |_{z=0n}, \\ &= \sum_i \epsilon^{J^* \nu} V_{\nu\mu}^{(i)} \epsilon^\mu H_i(x, \xi, t), \end{aligned} \quad (18)$$

where $\epsilon = \epsilon(p, \lambda)$ or $[\epsilon' = \epsilon(p', \lambda')]$ and $\lambda = 0$ ($\lambda' = \pm 1$) are the initial (final) polarization vector and helicity, respectively. GPDs of the ρ meson are defined as

$$\begin{aligned} V_{\lambda'\lambda} &= -(\epsilon^{*'} \cdot \epsilon) H_1 + \frac{(\epsilon \cdot n)(\epsilon^{*'} \cdot P) + (\epsilon^{*'} \cdot n)(\epsilon \cdot P)}{P \cdot n} H_2 \\ &\quad - \frac{2(\epsilon \cdot n)(\epsilon^{*'} \cdot n)}{M_v^2} H_3 \\ &\quad + \frac{(\epsilon \cdot n)(\epsilon^{*'} \cdot P) - (\epsilon^{*'} \cdot n)(\epsilon \cdot P)}{P \cdot n} H_4 \\ &\quad + \left[\frac{M_v^2(\epsilon \cdot n)(\epsilon^{*'} \cdot n)}{(P \cdot n)^2} + \frac{1}{3}(\epsilon^{*'} \cdot \epsilon) \right] H_5, \end{aligned} \quad (19)$$

where GPDs H_i ($i = 1$ to 5) are functions of x , ξ , and t . These three variables represent the fraction of plus components, skewness, and square of the momentum transfer during the process, respectively. In the forward limit, GPDs reduce to ordinary parton distribution functions (PDFs). In order to obtain the GPDs of the ρ meson, we have compared Eqs. (5) and (17). Corresponding to each physical FF, i.e., G_C , G_M , and G_Q , we can define the GPDs H_1 , H_2 , and H_3 , respectively. In Fig. 6, we have presented the results for ρ meson GPDs for fixed values of Q^2 with respect to x . We found that for low values of Q^2 , the magnitude of the peak is maximum at large values of x , but

FIG. 6. GPDs of the ρ meson evaluated in LFQM: (a) $H_1(x, Q^2)$, (b) $H_2(x, Q^2)$, and (c) $H_3(x, Q^2)$ with fixed values of Q^2 .

FIG. 7. 3D representation of GPDs for the ρ meson evaluated in LFQM: (a) $H_1(x, Q^2)$, (b) $H_2(x, Q^2)$, and (c) $H_3(x, Q^2)$.

FIG. 8. ipdpdf of the ρ meson evaluated in light-front quark model: (a) $H_1(x, b)$, (b) $H_2(x, b)$, and (c) $H_3(x, b)$ with fixed values of x .

as we increase the Q^2 , magnitude of the peak decreases and is also shifted towards lower values of x . This shows that the active quark in the ρ meson is dominating at lower values of x , i.e., most of the momentum carried by the active quark is at lower values of x . This fact is also evident from plots shown in Figs. 7(a)–7(c), respectively. However in Fig. 6(c), we have observed that GPDs appear to be negative which is due to presence of the -ve sign outside of the G_Q form factor.

V. IMPACT PARAMETER DEPENDENT PARTON DISTRIBUTION FUNCTION

In this section, we will discuss the ipdpdfs for the ρ meson. The FT with respect to the momentum transfer Q gives the GPDs in impact-parameter space. We introduce $b_\perp$ conjugate to Q giving

$$\begin{aligned} H(x, b) &= \int \frac{d^2 q}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} H(x, Q^2), \\ &= \int \frac{Q dQ}{2\pi} J_0(Qb) H(x, Q^2). \end{aligned} \quad (20)$$

We have shown the results for the ipdpdf for the ρ meson for fixed values of x with respect to b in Figs. 8(a)–8(c), respectively. It is clear from Fig. 8(a) that partons that have small impact parameters have maximum peaks at $b = 0$; however, we also noticed that as we increase the values of x , the magnitude of the peak decreases. This shows that the more longitudinal momentum fraction carried by the

parton, the less they are located near the center of the position space in the ρ meson. A similar observation has been made in Fig. 8(b). However, in Fig. 8(c) we have observed that for H_3 the distribution of partons is not exactly near the center of position space but somehow shifted near to $b = 1.0 \text{ GeV}^{-1}$.

VI. CONCLUSIONS

In the present work, we have discussed the transverse densities for unpolarized and transverse ρ mesons in LFQM. Using the results of the physical form factors, we have calculated the helicity matrix elements. We found that the charge density for longitudinally polarized ρ mesons is axially symmetric. Charge distributions for transversely polarized ρ mesons show a monopole pattern together with dipole and quadrupole patterns. The dipolar structure in the transversely polarized ρ meson causes the distortion in the charge distribution for ρ_{1T} , whereas quadrupole structure stretches the distribution for ρ_{0T} . In addition to this, we have also calculated the GPDs for ρ mesons obtained from the electromagnetic FFs. We found that in the ρ meson, the active quark appears to carry more momentum at lower values of x . Finally, we have investigated the ipdpdfs for ρ mesons. Although results are not Lorentz invariant, they can be examined by the experimental data, if available, and they still can be tested by the experiments with the appropriate choice of lab frames. Present work motivates one to calculate the GPDs for ρ mesons by considering the quark-antiquark-meson vertex

in LFQM, and such calculations are in progress. It will be also interesting to calculate the deuteron GPDs in LFQM.

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