Z-boson polarization as a model-discrimination analyzer

Seong Youl Choi*

Department of Physics and RIPC, Chonbuk National University, Jeonju 54896, Korea

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Determining the spin of any new particle is critical in identifying the true theory among various extensions of the standard model (SM). The degree of Z-boson polarization in any two-body decay process $A \rightarrow BZ$ is sensitive to the spin assignments of two new particles A and B . Considering all possible spin-0, 1/2, and 1 combinations in a renormalizable field theory, we demonstrate that Z-boson polarization can indeed play a role of a decisive and universal analyzer in distinguishing the different spin assignments.

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I. INTRODUCTION

As the electroweak (EW) scale ($v = 246$ GeV) and beyond are being explored at the Large Hadron Collider (LHC), it is highly anticipated that the true theory for the origin and stability of the EW scale [1–4] will be revealed.

One generic prediction in most of new models is the presence of new particles partnered with some or all of the SM particles. For instance, every SM particle in low-energy supersymmetry (SUSY) [5–10] has a heavier partner whose spin differs by 1/2. Alternatively, in universal extra dimension (UED) models [11], each SM particle is paired with a tower of Kaluza-Klein (KK) excitations with identical spin. Thus, model-independent spin measurements are crucial in discriminating among new scenarios.

Since the rest frame of the decaying particle is hardly reconstructible, the direct spin measurement at the LHC are performed through the Lorentz-invariant masses in sufficiently long decay chains [12–14]. Such spin-determination methods, however, rely heavily on the final state spins and the chiral structure of couplings [15].

In this paper we analyze the two-body decay of a new heavy state A into a new lighter state B and an on-shell neutral Z-boson as

$$A \rightarrow B + Z \rightarrow B + \ell^+ \ell^-, \quad (1)$$

where the Z-boson polarization can be measured through the lepton angular distributions in the decay $Z \rightarrow \ell^+ \ell^-$ with respect to the Z flight direction, reconstructed with great precision. If its branching fraction is sizable, the

leptonic decay (1) is highly expected to be a promising tool not only for diagnosing the properties of the new particles [16,17] but also for determining the A and B spins together.

If kinematically allowed, several two-body decays like the event topology as the process (1) can occur in each extension of the SM like SUSY, UED and little Higgs (LH) models (See Table I).

II. Z POLARIZATION IN THE REST FRAME

Before describing the two-body decays $A \rightarrow BZ$ in detail, we briefly summarize how to reconstruct the Z-polarization through the lepton-angle distributions of the leptonic Z-boson decays $Z \rightarrow \ell^+ \ell^-$, in particular, with $\ell = e$ and μ . In the rest frame of the decaying Z boson reconstructed with great precision by measuring the lepton momenta, the normalized ℓ^- polar-angle distributions with respect to the Z polarization axis defined to be the Z flight direction in the laboratory frame are given by

$$\frac{dD_{\pm}}{d \cos \theta_{\ell}} = \frac{3}{8} (1 + \cos^2 \theta_{\ell} \pm 2\xi_{\ell} \cos \theta_{\ell}), \quad (2)$$

$$\frac{dD_0}{d \cos \theta_{\ell}} = \frac{3}{4} (1 - \cos^2 \theta_{\ell}), \quad (3)$$

for transversely polarized (helicity = ± 1) and longitudinally polarized (helicity = 0) Z bosons, respectively, with $\xi_{\ell} = 2v_{\ell}a_{\ell}/(v_{\ell}^2 + a_{\ell}^2)$. (Although in principle the ℓ^- angular distributions for Z bosons with positive and negative helicity can be studied separately, but they are not easy to distinguish practically because of the very small LR asymmetry factor $\xi_{\ell} \simeq -0.147$ used for their distinction with $v_{\ell} = s_W^2 - 1/4 \simeq -0.02$ and $a_{\ell} = 1/4$.) Note that the polar-angle distribution can be determined even without knowing the full kinematics of the decay $A \rightarrow BZ$.

The general analysis of the Z-boson polarization developed in the two-body decay $A \rightarrow BZ$ is most transparent if

*choisy7@gmail.com

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TABLE I. Possible spin assignments to the states A and B in the decay $A \rightarrow BZ$ and typical processes in SUSY, UED and LH models, respectively, with the same event topology as the process (1), when kinematically accessible. The labels F, S and V in the left-most column denote a spin-1/2 fermion, a spin-0 scalar, and a spin-1 vector boson, respectively.

AB	SUSY	UED	LH
FF	$\tilde{\chi}_i^0 \rightarrow \tilde{\chi}_j^0 Z$ $\tilde{\chi}_i^\pm \rightarrow \tilde{\chi}_j^\pm Z$	$f_{1h} \rightarrow f_{1l} Z$ $\dots$	$T \rightarrow tZ$ $\dots$
SS	$\tilde{f}_2 \rightarrow \tilde{f}_1 Z$ $A \rightarrow hZ$	$\dots$ $\dots$	$\phi^P \rightarrow HZ$ $\dots$
VS	$\dots$	$W_1^\pm \rightarrow H_1^\pm Z$	$Z_H \rightarrow HZ$
	$\dots$	$\dots$	$\gamma_H \rightarrow HZ$
VV	$\dots$	$\dots$	$W_H^\pm \rightarrow W^\pm Z$

performed in the helicity formalism [18]. In the A rest frame the decay helicity amplitude can be decomposed in terms of the decay polar and azimuthal angles for the momentum direction of the Z boson produced in the A rest frame as

$$\mathcal{D}_{\sigma_A: \lambda \sigma_B}(\Theta, \Phi) = \mathcal{T}_{\lambda, \sigma_B} d_{\sigma_A, \lambda - \sigma_B}^{j_A}(\Theta) e^{i(\sigma_A - \lambda + \sigma_B)\Phi}$$

with $|\lambda - \sigma_B| \leq j_A$, (4)

where j_A , σ_A is the spin and helicity of the particle A and σ_B , λ are the helicities of the particle B and Z boson, respectively. (For the sake of discussion the Z momentum direction will be referred to as the production axis in the following.) Because of rotational invariance, the reduced matrix elements $\mathcal{T}_{\lambda, \sigma_B}$, which contains all the dynamical information on the decay process, is independent of the A helicity.

After integrating the absolute square of the amplitude over the angles and summing it over the A and B helicities, we can obtain the longitudinal and transverse polarizations, and the LR polarization asymmetry of the produced Z boson in the A rest frame,

$$P_L^A = \frac{\sum_{\sigma_B} |\mathcal{T}_{0, \sigma_B}|^2}{\sum_{\lambda} \sum_{\sigma_B} |\mathcal{T}_{\lambda, \sigma_B}|^2}, \quad (5)$$

$$P_T^A = 1 - P_L^A, \quad (6)$$

$$A_\pm^A = \frac{\sum_{\sigma_B} (|\mathcal{T}_{+1, \sigma_B}|^2 - |\mathcal{T}_{-1, \sigma_B}|^2)}{\sum_{\lambda} \sum_{\sigma_B} |\mathcal{T}_{\lambda, \sigma_B}|^2}, \quad (7)$$

where the subscript ± 1 or 0 denotes the Z helicity.

Combining the Z -boson production process (1) and the lepton angle distribution (3) coherently, ξ_ℓ and involving angular distribution linear in $\cos \theta_\ell$, we obtain the normalized and correlated lepton angle distribution

$$\frac{1}{C} \frac{dC}{d \cos \theta_\ell} = \frac{3}{8} [1 + \cos^2 \theta_\ell + P_L^A (1 - 3 \cos^2 \theta_\ell) + 2 \xi_\ell A_\pm^A \cos \theta_\ell], \quad (8)$$

uniquely determined by the degree of longitudinal Z polarization P_L^A and the LR asymmetry $A_\pm^A$.

The degree of longitudinal Z -polarization and the LR asymmetry from the two-body decay $A \rightarrow BZ$ depend crucially on the ABZ vertex structure dynamically, restricted by the spin assignments, and on the masses of the particles A and B kinematically, more specifically the ratios $z_{A,B} = m_{A,B}/m_Z$. Unless we are concerned about any overall numerical factors and higher-dimensional couplings such as those induced from loop corrections, then the ABZ vertex structure for a given spin assignment is *uniquely determined up to an overall factor*. For each spin assignment of the particles A and B , the general form of the ABZ vertex structure is given by

$$\boxed{\text{SS}}: (p + q)^\mu, \quad (9)$$

$$\boxed{\text{SV}}: g^{\mu\nu}, \quad (10)$$

$$\boxed{\text{VS}}: g^{\lambda\mu}, \quad (11)$$

$$\boxed{\text{VV}}: g^{\mu\nu}(k - q)^\lambda + (q + p)^\mu g^{\nu\lambda} - g^{\mu\lambda}(p + k)^\nu, \quad (12)$$

$$\boxed{\text{FF}}: \gamma^\mu(v + a\gamma_5), \quad (13)$$

where p , q , and k are the incoming A , outgoing B and Z four-momenta, the four-vector indices λ , ν , and μ correspond to the states A and B , and the Z boson, and v , a are the vector and axial-vector couplings to the Z -boson in the FF case.

An explicit calculation with the general vertex structures in Eqs. (9)–(13) yields the analytic expression for the longitudinal Z polarization for each spin assignment of the states A and B as

$$P_L^A[\text{SS}] = 1, \quad (14)$$

$$P_L^A[\text{SV}] = (z_A^2 - z_B^2 - 1)^2 / [(z_A^2 - z_B^2 - 1)^2 + 8z_B^2], \quad (15)$$

$$P_L^A[\text{VS}] = (z_A^2 - z_B^2 + 1)^2 / [(z_A^2 - z_B^2 + 1)^2 + 8z_A^2], \quad (16)$$

$$P_L^A[\text{VV}] = [9z_A^2 z_B^2 + 2(z_A^2 + z_B^2) + 1] / [9z_A^2 z_B^2 + 10(z_A^2 + z_B^2) + 1], \quad (17)$$

$$P_L^A[\text{FF}] = (e_F + \beta^2) / (3e_F + \beta^2), \quad (18)$$

and the LR asymmetry, nonvanishing only for the FF assignment, as

FIG. 1. The degree of longitudinal Z-boson polarization, P_L^A , as a function of $z_A = m_A/m_Z$ with $z_B = m_B/m_Z = 1$ fixed for each spin assignment of the particles A and B in the A rest frame. The asymmetries, ξ_F and A_N , in the FF case are taken to be 0.

$$A_{\pm}^A[FF] = \xi_F \frac{2\beta}{3e_F + \beta^2} \quad \text{with} \quad \xi_F = \frac{2va}{v^2 + a^2}, \quad (19)$$

where, for notational convenience, we have used the abbreviations

$$\beta = \lambda^{1/2}(z_A^2, z_B^2, 1), \quad (20)$$

$$e_F = z_A^2 + z_B^2 - 1 - 2z_A z_B A_N, \quad (21)$$

with $z_{A,B} = m_{A,B}/m_Z$ and the asymmetry $A_N = (v^2 - a^2)/(v^2 + a^2)$ defined in terms of the vector and axial-vector couplings v and a . In order for the two-body decay $A \rightarrow BZ$ to be kinematically allowed, the inequality $z_A \geq z_B + 1$ should be satisfied.

One aspect unique to the SS spin assignment is that the Z-boson is completely longitudinally polarized independently of the masses m_A and m_B . Another distinctive feature shared by all the spin assignments is that the Z boson becomes longitudinally polarized as $z_A \rightarrow \infty$, cf. Fig. 1.

III. Z POLARIZATION IN THE LABORATORY FRAME

Although the Z boson itself is fully reconstructed, the rest of the event, in particular, the rest frame of the decaying particle A, may not be reconstructed. In this situation, a natural axis for the Z-polarization is the Z-boson's momentum direction in the laboratory frame, which will be called the detection axis. The most natural experimental observable for Z decays is then the polar-angle θ' distribution of the charged lepton ℓ^- with respect to the Z-boson direction of motion in the Z-boson rest frame.

The degree of longitudinal Z polarization P_L^D and the LR asymmetry $A_{\pm}^D$ determined along the detection axis is related to the degree of longitudinal Z polarization P_L^A and the LR asymmetry $A_{\pm}^A$ computed in the rest frame of the decaying particle A by a Wigner rotation connecting the two spin bases [19]. The detected polarization P_L^D and the LR asymmetry $A_{\pm}^D$ are given by

$$P_L^D = \cos^2 \omega P_L^A + \frac{1}{2} \sin^2 \omega (1 - P_L^A), \quad (22)$$

$$A_{\pm}^D = \cos \omega A_{\pm}^A, \quad (23)$$

in terms of the computed polarization P_L^A and LR asymmetry $A_{\pm}^A$, respectively, and the Wigner angle ω between the production and detection axes in the Z rest frame. This angle is determined by the composition of boosts Λ_{ZL} from the Z rest frame to the laboratory frame, followed by Λ_{LP} from the laboratory frame to the A rest frame, and finally Λ_{PZ} from the A rest frame to the Z rest frame,

$$\mathcal{M}(\Lambda_{ZL})\mathcal{M}(\Lambda_{LP})\mathcal{M}(\Lambda_{PZ}) = \mathcal{R}(\omega), \quad (24)$$

where $\mathcal{M}(\Lambda)$ and $\mathcal{R}(\theta)$ are the representation matrices for the Lorentz transformations. Explicitly, the Wigner angle ω can be extracted from the expression

$$\tan \omega = \frac{m_Z \beta_A \sin \Theta}{p_Z + \beta_A E_Z \cos \Theta} = \frac{\beta_A \sin \Theta}{\gamma_Z (\beta_Z + \beta_A \cos \Theta)}, \quad (25)$$

where $E_Z = \gamma_Z m_Z$ and $p_Z = \gamma_Z \beta_Z m_Z$ are the energy and absolute momentum of the Z in the A rest frame, Θ is the polar angle of the Z boson in the A rest frame and β_A is the speed of the particle A in the laboratory frame.

The energy and absolute momentum of the Z boson are fixed with the masses of the particles in the decay $A \rightarrow BZ$ as

$$E_Z = \frac{m_Z}{2} \frac{(z_A^2 - z_B^2 + 1)}{z_A}, \quad (26)$$

$$p_Z = \frac{m_Z}{2} \lambda^{1/2}(z_A^2, z_B^2, 1), \quad (27)$$

The phase space integration over $\cos \Theta$ can be carried out as the matrix elements are independent of $\cos \Theta$ after averaging over the A and B spins.

The only nontrivial complication arises from the boost spectrum $\mathcal{D}(\beta_A)$ between the A rest frame and the laboratory frame which depends on the A production mechanism and the parton distribution functions. In principle the (normalized) distribution $\mathcal{D}$ can be computed for a given model. If so, the averages of the degree of observed longitudinal polarization and the observed LR asymmetry are given by

FIG. 2. The dependence of the averages $\langle \sin^2 \omega \rangle$ (left frame) and $\langle \cos \omega \rangle$ (right frame) as a polarization reduction factor on the boost β_A . For the purpose of illustration, three sets of new particle masses, $(m_A, m_B) = (0.5, 0.4), (0.6, 0.4)$, and $(0.9, 0.3)$ [TeV] with $m_Z = 91$ GeV are chosen.

$$\begin{aligned} \langle P_L^D \rangle &= P_L^A - \langle \sin^2 \omega \rangle \frac{1}{2} (3P_L^A - 1) \quad \text{and} \\ \langle A_{\pm}^D \rangle &= \langle \cos \omega \rangle A_{\pm}^A, \end{aligned} \quad (28)$$

with the double-integration expressions defined by

$$\begin{aligned} \langle \sin^2 \omega / \cos \omega \rangle &= \int_0^1 d\beta_A \mathcal{D}(\beta_A) \\ &\times \frac{1}{2} \int_{-1}^1 d\cos \Theta \sin^2 \omega / \cos \omega(\Theta, \beta_A), \end{aligned} \quad (29)$$

that allow fully detailed quantitative predictions for the spin assignments of the particles A and B .

There are two extreme kinematic limits for which we do not have to rely on any detailed information on the boost distributions in practice. First, if the particle A is produced near threshold with $\beta_A \rightarrow 0$, then $\cos \omega \rightarrow 1$ rendering the difference between the production and detection axes negligible. Second, if the mass splitting, $m_A - m_B$, of the particles A and B is much larger than m_Z . In this case the Z boson is highly boosted with E_Z and p_Z much larger than m_Z even in the A rest frame except for the far backward region with Θ very close to π .

In order to analyze the boost dependence quantitatively, we consider the averages of $\sin^2 \omega$ and $\cos \omega$ over the angle Θ , $\langle \sin^2 \omega / \cos \omega \rangle = \frac{1}{2} \int d\cos \Theta \sin^2 \omega / \cos \omega$, of which the analytic forms are

$$\begin{aligned} \langle \sin^2 \omega \rangle(\beta_A) &= \frac{1}{\gamma_Z^2 \beta_Z^3 \beta_A} \left[\frac{1}{\beta_Z \beta_A} \ln \left| \frac{\beta_Z + \beta_A}{\beta_Z - \beta_A} \right| \right. \\ &\quad \left. - \frac{(\gamma_Z^2 + \gamma_A^2)}{2\gamma_Z \beta_Z \gamma_A \beta_A} \ln \left| \frac{\gamma_Z \beta_Z + \gamma_A \beta_A}{\gamma_Z \beta_Z - \gamma_A \beta_A} \right| - 1 \right], \end{aligned} \quad (30)$$

$$\begin{aligned} \langle \cos \omega \rangle(\beta_A) &= \frac{1}{2\beta_Z^2 \beta_A} \left[\beta_Z + \beta_A - |\beta_Z - \beta_A| \right. \\ &\quad \left. - \frac{1}{\gamma_Z^2} \ln \left(\frac{1 + \beta_Z \beta_A + \beta_Z + \beta_A}{1 - \beta_Z \beta_A + |\beta_Z - \beta_A|} \right) \right], \end{aligned} \quad (31)$$

as a function of β_A , respectively, satisfying the boundary condition that $\langle \sin^2 \omega \rangle(0) = 0$ and $\langle \cos \omega \rangle(0) = 1$. Here, the Z -boson boost factors, $\gamma_Z = E_Z/m_Z$ and $\beta_Z = p_Z/E_Z$, with E_Z and p_Z in Eqs. (26) and (27) fixed with the masses m_A and m_B in the decay. As $\beta_A \rightarrow 1$, the two average functions approach asymptotically to

$$\langle \sin^2 \omega \rangle(1) = \frac{1}{\gamma_Z^2 \beta_Z^2} \left[\frac{1}{\beta_Z} \ln \left(\frac{1 + \beta_Z}{1 - \beta_Z} \right) - 2 \right] \rightarrow \frac{2}{3} \quad \text{as } \beta_Z \rightarrow 0, \quad (32)$$

$$\langle \cos \omega \rangle(1) = \frac{1}{\gamma_Z^2 \beta_Z^2} \left[\beta_Z - \frac{1}{2} \ln \left(\frac{1 + \beta_Z}{1 - \beta_Z} \right) \right] \rightarrow 0 \quad \text{as } \beta_Z \rightarrow 0. \quad (33)$$

As a numerical illustration, the dependence of the averages $\langle \sin^2 \omega \rangle$ and $\langle \cos \omega \rangle$ on the boost β_A is shown in Fig. 2 for three characteristic sets of new particle masses, $(m_A, m_B) = (0.5, 0.4), (0.6, 0.4)$, and $(0.9, 0.3)$ [TeV] with $m_Z = 91$ GeV.

IV. CONCLUSIONS

We have computed the expected Z polarization for various spin assignments from a well-motivated class of

models beyond the SM. In addition, we have provided a detailed analytic description about how to determine the degree of Z-boson polarization even if the decaying particle is not at rest. As demonstrated numerically, Z bosons produced in any two-body decay $A \rightarrow BZ$ involving two new particles A and B can provide us with an important and powerful handle for determining the spins of the new particles, if Z is not at rest nor boosted with extremely high energies.

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