Model-independent determination of $B_c^+ \rightarrow \eta_c \ell^+ \nu$ form factors

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We derive model-independent bounds on the form factors for the decay $B_c^+ \rightarrow \eta_c \ell^+ \nu$ including full mass effects, i.e., $\ell = e, \mu$, and τ . The bounds are obtained by using the Boyd-Grinstein-Lebed parametrization for the form factors, and fitting to the preliminary lattice data of the HPQCD Collaboration. Our main result after bounding the form factors is the Standard Model prediction for the ratio of branching fractions $R(\eta_c) = \mathcal{B}(B_c^+ \rightarrow \eta_c \tau^+ \nu_\tau) / \mathcal{B}(B_c^+ \rightarrow \eta_c \mu^+ \nu_\mu)$. We find $R(\eta_c)|_{\text{SM}} = 0.31^{+0.04}_{-0.02}$, and argue that a measurement of $R(\eta_c)$ is within the reach of LHCb during the high-luminosity run of the LHC. In addition, using the heavy-quark spin symmetry of the B_c meson we relate our results for $B_c^+ \rightarrow \eta_c \ell^+ \nu$ to those for $B_c^+ \rightarrow J/\psi \ell^+ \nu$ yielding the estimate $R(J/\psi)|_{\text{SM}} = 0.26 \pm 0.02$ in good agreement with other determinations.

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I. INTRODUCTION

Measurements by the BABAR [1,2], Belle [3–5], and LHCb [6–8] experiments are challenging the assumption of lepton universality in the decays of B mesons. In particular, the ratios of branching fractions

$$R(D^{(*)}) \equiv \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)} l \nu_l)} \quad (1)$$

are measured to be larger than predicted in the Standard Model (SM).¹ A global analysis [9] quantifies the discrepancy as a $\sim 3.8\sigma$ deviation from the SM predictions [10–16]. These measurements are complemented by tests of lepton universality by LHCb in the decays $B \rightarrow K^{(*)} l^+ l^-$, which show deficits with respect to the SM predictions [17,18]. If confirmed, either of the charged- or neutral-current measurements would be unambiguous signals of the breakdown of the SM. See Refs. [19–23] for some overviews of the situation.

If the enhancement of $R(D^{(*)})$ is due to physics beyond the SM it should be evident in other decays of B hadrons as well, such as $\Lambda_b \rightarrow \Lambda_c \ell \nu$ and $B_c \rightarrow (c\bar{c}) \ell \nu$, as the underlying weak matrix elements for $b \rightarrow c \ell \nu$ are independent of the hadronic physics. In fact, LHCb has also measured [24] a larger than expected value for the ratio

$$R(J/\psi) \equiv \frac{\mathcal{B}(B_c^+ \rightarrow J/\psi \tau^+ \nu_\tau)}{\mathcal{B}(B_c^+ \rightarrow J/\psi \mu^+ \nu_\mu)}. \quad (2)$$

However until very recently there was no model-independent prediction for $R(J/\psi)$. As this work was being finalized Ref. [25] appeared, which used the Boyd-Grinstein-Lebed (BGL) parametrization [26–28] to bound the form factors for $B_c^+ \rightarrow J/\psi \ell^+ \nu$ in a model-independent fashion. In light of this recent development we will concentrate on the complementary decay $B_c^+ \rightarrow \eta_c \ell^+ \nu$ in this work.

Currently there is no measurement of any decay $B_c^+ \rightarrow \eta_c X^+$, let alone a measurement of the analogous R ratio involving an η_c meson

$$R(\eta_c) \equiv \frac{\mathcal{B}(B_c^+ \rightarrow \eta_c \tau^+ \nu_\tau)}{\mathcal{B}(B_c^+ \rightarrow \eta_c \mu^+ \nu_\mu)}. \quad (3)$$

As no measurement currently exists, it is natural to wonder when a measurement of a $B_c^+ \rightarrow \eta_c X^+$ process might be made. In what follows we argue that a measurement of $R(\eta_c)$ is within the reach of LHCb during the planned high-luminosity run of the LHC (HL-LHC). LHCb has observed η_c in the decay $B_s^0 \rightarrow \eta_c \phi$, and has evidence for the decay $B_s^0 \rightarrow \eta_c \pi^+ \pi^-$ [29]. The branching fractions for these η_c modes are comparable to the corresponding decays involving a J/ψ [30]. Furthermore, the measured branching fraction for $B_c^+ \rightarrow J/\psi \ell^+ \nu_\ell$ is similar to the other measured charmonium mode $B_c \rightarrow \chi_{c0} \pi^+$ [31]. These similarities suggest the main difference in the rates for $B_c^+ \rightarrow \eta_c \ell^+ \nu_\ell$ and $B_c^+ \rightarrow J/\psi \ell^+ \nu_\ell$ is in the branching fractions for what the charmonia decay into. Reference [24] used the decay mode $J/\psi \rightarrow \mu^+ \mu^-$ to measure $R(J/\psi)$, which has a well-known branching fraction of slightly less

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¹In this work $l = e, \mu$, $\ell = e, \mu, \tau$, and charge conjugation is implied throughout.

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than 6%. On the other hand, the main decay modes LHCb used to observe the η_c in B_s^0 decays were $p\bar{p}$, $\pi^+\pi^-\pi^+\pi^-$, and $K^+K^-K^+K^-$, which combine to give a total branching fraction of approximately 1% [30]. All else being equal this implies that if these same modes were used by LHCb to measure $R(\eta_c)$ then roughly 100 fb^{-1} of integrated luminosity would be needed to match the statistics of $R(J/\psi)$ analysis, which used the full Run-1 data set of 3 fb^{-1} . This is within the 300 fb^{-1} target for LHCb at HL-LHC. Furthermore, it is possible that adding a cleaner, but rarer mode, such as $\eta_c \rightarrow \gamma\gamma$, could improve the efficiency and lower the amount of the data required.

Confident that a measurement of $R(\eta_c)$ will eventually be made, we derive model-independent bounds on the form factors for $B_c^+ \rightarrow \eta_c \ell^+ \nu$ using the BGL parametrization. This starts in Sec. II by reviewing the BGL formalism, and applying it to the case at hand, $B_c^+ \rightarrow \eta_c \ell^+ \nu$. Section III describes the lattice data from the HPQCD Collaboration that is used to fit the form factors. Our results for the form factors are given in Sec. IV. We find $R(\eta_c)|_{\text{SM}} = 0.31^{+0.04}_{-0.02}$. Then in Sec. V we use heavy-quark spin symmetry (HQSS) to relate the form factors for $B_c^+ \rightarrow \eta_c \ell^+ \nu$ to those for $B_c^+ \rightarrow J/\psi \ell^+ \nu$, yielding the estimate $R(J/\psi)|_{\text{SM}} = 0.26 \pm 0.02$. Concluding remarks follow in Sec. VI.

II. FORM FACTORS

The hadronic matrix element describing the decay $B_c^+ \rightarrow \eta_c \ell^+ \nu$ contains two factors. It can be written analogously to other form factors for pseudoscalar-to-pseudoscalar transitions

$$\begin{aligned} \langle \eta_c(p') | V^\mu | B_c(p) \rangle \\ = f_+(q^2)(p + p')^\mu \\ + (f_0(q^2) - f_+(q^2)) \frac{M_{B_c}^2 - M_{\eta_c}^2}{q^2} (p - p')^\mu, \end{aligned} \quad (4)$$

where $V^\mu = \bar{c}\gamma^\mu b$ and $q^2 = (p - p')^2$. With this definition we must have $f_0(0) = f_+(0)$ in order for the matrix element in Eq. (4) to be finite at $q^2 = 0$.² The differential decay rate is then given by

$$\begin{aligned} \frac{d\Gamma}{dq^2}(B_c^+ \rightarrow \eta_c \ell^+ \nu) &= \frac{\eta_{ew}^2 G_F^2 |V_{cb}|^2 M_{B_c} \sqrt{\lambda}}{192\pi^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \\ &\times [c_+ f_+(q^2)^2 + c_0 f_0(q^2)^2] \end{aligned} \quad (5)$$

with $\lambda \equiv \lambda(q^2, M_{B_c}^2, M_{\eta_c}^2)$, where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$, and

²In contrast zero recoil is defined by $q^2 = q_{\text{max}}^2$, where both the initial and final mesons are at rest.

$$c_+ = \frac{\lambda}{M_{B_c}^4} \left(1 + \frac{m_\ell^2}{2q^2}\right), \quad c_0 = (1 - r^2)^2 \frac{3m_\ell^2}{2q^2}, \quad r = \frac{M_{\eta_c}}{M_{B_c}}. \quad (6)$$

We use the BGL parametrization [26–28] in determining the form factors f_+ and f_0 . The BGL formalism follows from analyticity, crossing symmetry, and dispersion relations. This leads to unitarity bounds on the otherwise free parameters that characterize the form factors. These properties follow from QCD, and are independent of any model.³ For pedagogical introductions see Refs. [33,34]. The final result is that the form factors are written as a Taylor series

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{i,n=0}^{\infty} a_{i,n} z^n, \quad (7)$$

for $i = \{+, 0\}$, and the parameters $a_{i,n}$ are bounded by unitarity to satisfy

$$\sum_{n=0}^{\infty} a_{+,n}^2 < 1, \quad \sum_{m=0}^{\infty} a_{0,m}^2 < 1. \quad (8)$$

Having presented the final result, we now describe the ingredients of the BGL parametrization. We begin by defining $t = q^2$ and $t_\pm = (M_{B_c} \pm M_{\eta_c})^2$. A conformal transformation is made, $t \rightarrow z$, which maps the t plane to the unit disk. There is freedom to choose the point that is mapped to $z = -1$. We take this to be $t_* = (M_{B_c} + M_\eta)^2 \rightarrow -1$ such that there are no isospin-preserving branch cuts within the unit disk.⁴ There is further freedom in choosing the point that is mapped to the origin. Here we make the most common choice in the literature, $t_0 = t_- \rightarrow 0$, and do not optimize this value. With these choices the conformal parameter is given by

$$z(t; t_0) = \frac{t_0 - t}{(\sqrt{t_* - t} + \sqrt{t_* - t_0})^2}. \quad (9)$$

The minimal value of z is $z_{\text{min}} = z(t_-; t_0) = 0$, whereas the maximal value is given by $z_{\text{max}} = z(m_\ell^2; t_0)$ leading to $z_{\text{max},\tau} \approx 0.049$ and $z_{\text{max},\mu} \approx z_{\text{max},e} \approx 0.066$. See Table I for the numeric values employed in this work. Due to the small range of z , the series in Eq. (7) can be safely truncated after only a few terms.

³The numerical values for the masses of the B_c scalar and vector mesons are not yet measured experimentally. However they are well determined on the lattice [32].

⁴Note that our branch point involves the mass of the η and not the η_c . For an axial-vector current the first branch cut occurs for the three-body process $B_c + 2\pi$. Although branch cuts arising from B_c + light hadrons, and cuts with ≥ 3 particles, are dynamically suppressed with respect to those where each hadron contains a heavy quark, e.g., $B^{(*)} + D$ [33], the freedom exists to choose the branch point.

The pre-sum factors appearing in Eq. (7), P and ϕ , are known as Blaschke factors and outer functions, respectively. A Blaschke factor removes all the poles due to single-particle states appearing below the pair-production threshold t_*

$$P_i(z) = \prod_{p_i} \frac{z \cdot z_{p_i}}{1 - z \cdot z_{p_i}}, \quad (10)$$

where $z_{p_i} = z(M_{p_i}^2; t_0)$, again for $i = \{+, 0\}$. An advantage of our choice for t_* is each form factor only has one pole within the unit disk. The masses of the relevant B_c states, 0^+ and 1^- , are given in Table I. The numerical values for these masses were taken from Ref. [32]. The outer functions depend on kinematics and parameters χ that appear in the dispersion relations

$$\begin{aligned} \chi^L(q^2) &\equiv \frac{\partial \Pi^L}{\partial q^2} = \frac{1}{\pi} \int_0^\infty dt \frac{\text{Im} \Pi^L(t)}{(t - q^2)^2}, \\ \chi^T(q^2) &\equiv \frac{1}{2} \frac{\partial^2 \Pi^T}{\partial (q^2)^2} = \frac{1}{\pi} \int_0^\infty dt \frac{\text{Im} \Pi^T(t)}{(t - q^2)^3}, \end{aligned} \quad (11)$$

where

$$\begin{aligned} &\frac{1}{q^2} (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^T(q^2) + \frac{q^\mu q^\nu}{q^2} \Pi^L(q^2) \\ &= i \int d^4x e^{iq \cdot x} \langle 0 | T V^\mu(x) V^{\dagger\nu}(0) | 0 \rangle. \end{aligned} \quad (12)$$

Note that the integrands appearing on the right-hand sides of Eq. (11) must be analytic below the pair-production threshold, which in this case is $t_* = (M_{B_c} + M_\eta)^2$, if the dispersion

TABLE I. Numeric values for the masses employed in this work. The first six masses are experimentally measured. The masses of scalar and vector B_c mesons were determined on the lattice, and only enter the form factors through the Blaschke factors appearing in Eq. (10). The bottom- and charm-quark masses are given in the $\overline{\text{MS}}$ scheme at $\mu = m_b$ and $\mu = 3$ GeV, respectively. They only explicitly enter our calculations through the heavy-quark spin symmetry relations given in Sec. V, but are also implicitly used in the evaluations of the χ functions in Eq. (13).

Particle	Mass [GeV]	Reference
$B_c(0^-)$	6.2749	[30]
η_c	2.9839	[30]
η	0.547 862	[30]
J/ψ	3.096 90	[30]
τ	1.776 86	[30]
μ	0.105 658	[35]
$B_c(1^-)$	6.329(8)	[32]
$B_c(0^+)$	6.704(17)	[32]
b	4.163(16)	[36]
c	0.986(13)	[36]

relations are to be used to constrain the form factors in the semileptonic region. This is why it is necessary to include the Blaschke factors and judiciously choose t_* . The χ can be perturbatively computed in QCD for q^2 near 0. The state-of-the-art three-loop evaluations are [13]

$$\begin{aligned} \chi^T(0) &= 6.486(48) \times 10^{-4} \text{ GeV}^{-2}, \\ \chi^L(0) &= 6.204(81) \times 10^{-3}. \end{aligned} \quad (13)$$

With χ in hand, the outer functions are given by [28]

$$\begin{aligned} \phi_+(z) &= \frac{8}{M_{B_c}} \sqrt{\frac{8}{3\pi\chi^T(0)}} \frac{r^2 \sqrt{1-z}(1+z)^2}{((1+r)(1-z) + 2\sqrt{r}(1+z))^5}, \\ \phi_0(z) &= \sqrt{\frac{8}{\pi\chi^L(0)}} \frac{r(1-r)^2 \sqrt{1-z}(1-z^2)}{((1+r)(1-z) + 2\sqrt{r}(1+z))^4}. \end{aligned} \quad (14)$$

This completes the list of ingredients needed to parametrize the form factors.

III. LATTICE QCD CALCULATIONS

The HPQCD Collaboration made preliminary predictions for both form factors f_+ and f_0 at four and five values of q^2 , respectively, using lattice QCD [37]. In addition, in the same work preliminary predictions were made for two of four form factors describing the decay $B_c^+ \rightarrow J/\psi \ell^+ \nu$. The computations were done using a highly improved staggered quark action with $n_f = 2 + 1 + 1$ flavors and multiple lattice spacings. HPQCD only reported statistical errors in Ref. [37]. Following Ref. [25] we assign a 20% systematic uncertainty to these predictions to take into account discretization errors, finite-volume corrections, and quark-mass dependencies. This uncertainty is 20% with respect to the central value of the prediction, and is added in quadrature with the corresponding statistical uncertainty.

IV. RESULTS FOR $B_c^+ \rightarrow \eta_c \ell^+ \nu$

In this section we give our results for the form factors f_+ and f_0 by fitting to the preliminary lattice data using the BGL parametrization. We then proceed to calculate the SM value for the ratio $R(\eta_c)$. We find the best-fit values for the parameters, $a_{i,n}$, by minimizing a chi-squared function for each form factor

$$\begin{aligned} \chi_+^2 &= \sum_{j=1}^4 \frac{[f_+^{lat}(q_j^2) - f_+(q_j^2)]^2}{[\sigma_+^{lat}(q_j^2)]^2}, \\ \chi_0^2 &= \sum_{j=1}^5 \frac{[f_0^{lat}(q_j^2) - f_0(q_j^2)]^2}{[\sigma_0^{lat}(q_j^2)]^2}. \end{aligned} \quad (15)$$

The superscript *lat* indicates a lattice measurement of a form factor, f , and the corresponding uncertainty, σ , at a

FIG. 1. Result of our three-parameter fit of the BGL-parametrized form factor, f_+ , to the lattice data of Ref. [37], which are the blue points with errors bars. The blue curve is the best-fit value, and the shaded region is the 1σ allow range.

given value of q^2 . The f without a superscript is the form factor to be fit from Eq. (7), which is subject to the unitarity bound (8). The uncertainties on the form factors are found by propagating forward the correlated uncertainties on the a parameters

$$\delta f_i^2(q^2) = (\nabla_{a_i} f_i) \cdot \left(\frac{1}{2} \frac{\partial^2 \chi_i^2}{\partial a_{i,n} \partial a_{i,m}} \right)^{-1} \cdot (\nabla_{a_i} f_i). \quad (16)$$

Note that because the form factors only have a linear dependence on the parameters, a_i the uncertainties on the form factor are independent of the parameters themselves.

We retain only the first three terms in Eq. (7) for each form factor leading to maximum truncation errors of 1.0% and 0.4% for f_+ and f_0 , respectively, both of which are well below the size of the lattice uncertainties.⁵ In Figs. 1 and 2 we present the results of our three-parameter fits for the form factors f_+ and f_0 , respectively. The lattice data from Ref. [37] are the blue points with error bars. The blue curves are the best-fit values, and the shaded regions are the 1σ allow ranges. The minimal value of q^2 is set by the mass of the muon in Figs. 1 and 2. Comparing the results of our fits for $f_i(B_c \rightarrow \eta_c)$ against those for $f_i(B \rightarrow D)$ [13] and the preliminary results for $f_i(B_s \rightarrow D_s)$ [38] shows that the dependence of the form factors on the flavor of the spectator quark is mild.

The results of our fits can be found in Table II. We checked that the goodness of fit is better for these three-parameter fits than for one- or two-parameter fits even when the unitarity bounds are imposed. For f_0 a four-parameter fit is also possible, but it has a worse goodness of fit than the three-parameter fit as well. The reason is the unitarity bound becomes saturated once the third parameter

⁵The truncation error of an n -parameter fit is given by $(\Delta f_i)_{\text{trunc}} = z_{\text{max},\mu}^n / (P_i(z_{\text{max},\mu}) \phi_i(z_{\text{max},\mu}))$.

FIG. 2. The same as Fig. 1, but for the form factor f_0 .

is added (this is true in both the f_+ and f_0 cases). A slightly lower χ^2 is obtained by optimizing the value of t_0 . However the unitarity bounds are still saturated in the optimized case with three (or higher) parameter fits, and it does not affect our result for $R(\eta_c)|_{\text{SM}}$. We also checked that going beyond the BGL parametrization by imposing the additional constraints that f_+ and f_0 be maximal at zero recoil does not improve the bounds. The only downside of the three parameter fits versus one or two parameter fits is the wider uncertainty band for f_+ as the recoil becomes larger. The drawback is mild however as the differential partial width is kinematically suppressed at large recoil. Furthermore, it could be ameliorated by an additional lattice measurement near maximum recoil. This additional measurement could be synthesized using heavy-quark symmetry; see Sec. V.

To obtain a prediction for $R(\eta_c)$ in the SM we insert our fitted values for the form factors into Eq. (5), and integrate over q^2 . We find

$$R(\eta_c)|_{\text{SM}} = 0.31_{-0.02}^{+0.04}. \quad (17)$$

The uncertainties are determined by simultaneously shifting the form factors in the numerator and denominator by 1σ from their respective central values. Clearly, the uncertainty coming from the form factors mostly cancels in the

TABLE II. The results of our fits. The upper rows give the number of parameters and the χ^2 for a given fit to f_+ or f_0 . The bottom rows show the central values of the coefficients a_i for a given fit.

$n_{\text{params.}}$	f_+			f_0			
	1	2	3	1	2	3	4
χ^2	1.60	0.10	0.028	2.97	0.14	0.084	0.084
$a_{i,0}$	0.018	0.024	0.026	0.051	0.065	0.066	0.066
$a_{i,1}$		-0.13	-0.21		-0.35	-0.41	-0.41
$a_{i,2}$			0.98			0.91	0.90
$a_{i,3}$							0.10

ratio. For the sake of the comparison, the relative uncertainty on the partial width involving a muon is approximately 30%. Our model-independent prediction is in good agreement with recent model calculations: $R(\eta_c)|_{\text{SM}} = 0.31^{+0.12}_{-0.07}$ [39], 0.26 [40], and 0.26 ± 0.05 [41]. (For a calculation of the partial width for the decay $B_c \rightarrow \eta_c e \nu$ see Ref. [42].) Reference [43], which appeared on the arXiv shortly after our paper, also used the BGL parametrization to obtain a model-independent prediction $R(\eta_c)|_{\text{SM}} = 0.29 \pm 0.05$. Furthermore, our prediction agrees with those for $R(D)|_{\text{SM}}$ [11–14,16], again suggesting that the dependence of the form factors on the flavor of the spectator quark is mild.

V. HEAVY-QUARK SPIN SYMMETRY AND $B_c^+ \rightarrow J/\psi \ell^+ \nu$

The results derived in Sec. II relied only on analyticity, crossing symmetry, and unitarity. Form factors can generally be further constrained, or related to one another, if the system under consideration has additional symmetries. Heavy-quark effective theory (HQET) [44,45] accurately describes hadrons with one heavy quark by treating the heavy quark as static, and expanding in powers of the heavy-quark mass, m_Q . The resulting heavy-quark symmetry allows one to describe all the form factors at low recoil for the decays $B^{(*)} \rightarrow D^{(*)} \ell \nu$ in terms of a single function, called the Isgur-Wise function, with fixed normalization [46,47]. HQET is not well suited for hadrons containing more than one heavy quark. The heavy quarks can exchange low-energy gluons within the hadron leading to an IR divergence that is regulated by the kinetic energy of the hadron. The kinetic energy is formally suppressed in HQET by $1/m_Q$, and thus a new effective field theory (EFT) with a new power counting is needed. Nonrelativistic QCD (NRQCD) is the appropriate EFT to use, and its expansion parameter is (not surprisingly) the speed of light. Doubly heavy hadrons in NRQCD possess a residual heavy-quark spin symmetry [48–50] that, like heavy-quark symmetry, relates the form factors at zero recoil. However HQSS does not fix the normalization of the form factors.

We use HQSS to relate the form factors for $B_c^+ \rightarrow \eta_c \ell^+ \nu$ to $B_c^+ \rightarrow J/\psi \ell^+ \nu$. The differential partial width for $B_c^+ \rightarrow J/\psi \ell^+ \nu$ is parametrized as follows:

$$\begin{aligned} \frac{d\Gamma}{dq^2}(B_c^+ \rightarrow J/\psi \ell^+ \nu) &= \frac{\eta_{ew}^2 G_F^2 |V_{cb}|^2 M_{B_c} \sqrt{\lambda_\psi}}{192\pi^3} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \\ &\times [c_f f(q^2)^2 + c_1 \mathcal{F}_1(q^2)^2 + c_g g(q^2)^2 + c_2 \mathcal{F}_2(q^2)^2] \end{aligned} \quad (18)$$

with $\lambda_\psi = \lambda(q^2, M_{B_c}^2, M_{J/\psi}^2)$ and

$$\begin{aligned} c_f &= \frac{2q^2 + m_\ell^2}{M_{B_c}^4}, & c_1 &= \frac{c_f}{2q^2}, \\ c_g &= \frac{\lambda_\psi c_f}{4}, & c_2 &= \frac{3\lambda_\psi m_\ell^2}{8M_{B_c}^4 q^2}. \end{aligned} \quad (19)$$

The HQSS relations are [50]

$$\begin{aligned} f_0(t_-) &= \frac{r}{1+r} \frac{8}{3+r-\sigma+r\sigma} f_+(t_-), \\ f(t_{\psi-}) &= \frac{8M_{B_c} r}{3+r-\sigma+r\sigma} f_+(t_-), \\ g(t_{\psi-}) &= \frac{2r}{M_{B_c} r_\psi} \frac{3+\sigma}{3+r-\sigma+r\sigma} f_+(t_-), \\ \mathcal{F}_2(t_{\psi-}) &= \frac{2r}{r_\psi} \frac{3+r_\psi-\sigma+r_\psi\sigma}{3+r-\sigma+r\sigma} f_+(t_-), \end{aligned} \quad (20)$$

where $t_{\psi-} = (M_{B_c} - M_{J/\psi})^2$,

$$r_\psi = \frac{M_{J/\psi}}{M_{B_c}}, \quad \sigma = \frac{m_c}{m_b}, \quad (21)$$

and as previously defined $r = M_{\eta_c}/M_{B_c}$, $t_- = (M_{B_c} - M_{\eta_c})^2$. In addition, by construction we have [28]

$$\mathcal{F}_1(t_{\psi-}) = M_{B_c} (1 - r_\psi) f(t_{\psi-}). \quad (22)$$

Our key assumption to make an estimate of $R(J/\psi)$ is that the last three relations in Eqs. (20) and (22) hold beyond zero recoil. We then integrate Eq. (18) with respect to q^2 . A second estimate is obtained through the relation in the first line of Eq. (20), whereby $R(J/\psi)$ becomes a function of f_0 via Eq. (20), and these relations are again assumed to hold beyond zero recoil. This gives us a range of predictions

$$R(J/\psi)|_{\text{SM}} = 0.26 \pm 0.02, \quad (23)$$

where the central value is the average of the two estimates, one using f_0 and one with f_+ . The uncertainty is taken to be the difference of the two estimates. Our estimate is in good agreement with other recent determinations: $R(J/\psi)|_{\text{SM}} = 0.29 \pm 0.07$ [39], 0.24 ± 0.07 [41] and, 0.283 ± 0.048 [51]. It is also in agreement with Ref. [25], which quoted the 95% C.L. range 0.20 to 0.39.

VI. CONCLUSIONS

The hints of lepton flavor universality violation in the semileptonic decays of B mesons should be explored as thoroughly as possible. This includes studying the process $B_c^+ \rightarrow \eta_c \ell^+ \nu$, which should be within the reach of LHCb at HL-LHC. In this work we derived model-independent

bounds on the form factors for $B_c^+ \rightarrow \eta_c \ell^+ \nu$ using the BGL parametrization. The bounds were obtained by fitting the form factors to the preliminary lattice data of the HPQCD Collaboration. We then used the fitted form factors to compute the SM prediction for the ratio of branching fractions $R(\eta_c)$, yielding the result $R(\eta_c)|_{\text{SM}} = 0.31^{+0.04}_{-0.02}$. Last, we related our fitted form factors for $B_c^+ \rightarrow \eta_c \ell^+ \nu$ to those for $B_c^+ \rightarrow J/\psi \ell^+ \nu$ using heavy-quark spin symmetry. This allowed us to estimate the SM prediction for

$R(J/\psi)$. We found $R(J/\psi)|_{\text{SM}} = 0.26 \pm 0.02$ in good agreement with other determinations.

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