Gluon jet function at three loops in QCD

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We present here the first result on the three-loop gluon jet function in perturbative QCD. Using the three-loop coefficient functions [J. A. M. Vermaseren, A. Vogt, and S. Moch, *Nucl. Phys.* **B724**, 3 (2005)., G. Soar, S. Moch, J. A. M. Vermaseren, and A. Vogt, *Nucl. Phys.* **B832**, 152 (2010).] for deep-inelastic scattering via the exchange of a virtual photon that couples to quarks or a scalar that couples to gluons and employing the KG equation, renormalization group invariance and factorization theorem, we obtain both the quark and the gluon jet functions up to the three-loop level. The former agrees with the recent result [R. Brüser, Z. L. Liu, and M. Stahlhofen, *Phys. Rev. Lett.* **121**, 072003 (2018).]. These jet functions being universal ingredients in the SCET framework, will play an important role in the phenomenological studies at the Large Hadron Collider, such as resummation of jet observables and also in N-jettiness subtraction method.

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I. INTRODUCTION

The Standard Model (SM) is currently being tested to unprecedented accuracy at the Large Hadron Collider (LHC). In order to achieve such feats, it is important to accurately measure various observables and compare them with the precise theoretical predictions that uses state-of-the-art modern techniques. In addition such studies are of paramount importance to understand the SM background to constrain any physics beyond the SM. In particular, processes such as production of lepton pairs, vector bosons and multi-jets at the LHC play an important role in these studies. In addition, understanding of the jets will shed light on the underlying structure of QCD dynamics. The hadronic cross sections in high-momentum transfer processes such as the lepton-nucleon deep-inelastic scattering (DIS) or in the Drell-Yan processes factorize into hard and soft virtual (SV) parts. The hard part is associated with the physics of large momentum transfer Q ; the soft part of the SV cross section describes the dynamics associated with emission of soft and collinear partons. Soft-collinear effective theory (SCET) [1–6] captures the physics of soft and collinear dynamics of these processes at high energies

through the soft and jet functions. The jet functions explains the propagation of collinear partons inside jets and the soft functions mediate low-energy interaction between jets. They are important components of various observables that can be measured at the colliders. In SCET, the quark and gluon soft as well as jet functions have been computed to higher orders in perturbation theory. The quark and the gluon soft functions are already known to the three-loop level; see [7] and references therein. For the quark jet function, results were known up to the two-loop level for some time (see [8–10]), and the results at three loops were computed recently in [11]. On the other hand, only one- [12] and two-loop [13] results are known for the gluon jet function. Precise measurements of various observables at high-energy experiments demand accurate theoretical predictions from the SM. The latter obtained using sophisticated methods in turn shed light on the theoretical structure of the underlying dynamics. In the past, such computations have led to better understanding of the perturbative structure of various observables in terms of process-independent functions such as soft and jet functions and their anomalous dimensions. Owing to the fact that these quantities are universal, they form the building blocks for many interesting observables that probe physics at short distances. Our goal in this article is to demonstrate an intriguing connection between jet functions in SCET and coefficient functions of DIS cross sections computed in perturbative QCD. We exploit this novel connection to obtain quark and gluon jet functions up to the three-loop level using the known state-of-the-art three-loop coefficient functions in the context of DIS. In particular, we use the coefficient functions computed in [14] which uses a scalar

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that couples to gluons to probe the short distance structure of the hadron and obtain the three-loop gluon jet function. To achieve this, we exploit the KG equation, renormalization group invariance, factorization theorem and use various three-loop results. We use the framework developed in [15–17] which describes resummation of soft gluons to all orders in QCD perturbation theory. For the recent results on the soft gluon resummation of the rapidity distribution of Higgs and lepton pair production in Drell-Yan, see [18,19]. In [16], it was shown that DIS cross section in the threshold limit factorizes into the square of ultraviolet (UV) renormalized form factor, soft plus jet function and the mass factorization kernels. The soft plus jet function has an universal structure and it depends only on the nature of external states namely quark or gluon states. In addition, it satisfies KG type differential equation similar to the form factors. Factorization properties of the cross section and renormalization group invariance can be used to unravel the structure of these soft plus jet function to all orders in perturbation theory. Upon identifying the finite part of the square of the form factor with the matching hard coefficient in the SCET framework, the finite part of soft plus jet function can be shown to coincide with the corresponding jet function. The latter identification is simply due to the process-independent nature of both the soft plus jet function and the jet function. We first demonstrate that the quark jet function known up to the three-loop level agrees with the finite part of quark soft plus jet function obtained from the coefficient function [20] and proceed to obtain the corresponding gluon jet function from [14].

II. THEORETICAL FRAMEWORK AND RESULTS

The inclusive cross section for the scattering of a lepton with a hadron in the DIS is given by

$$\sigma^I(x, Q^2) = \sigma_B^I(\mu_R^2) \sum_{a=q,\bar{q},g} \int_x^1 \frac{dz}{z} f_a\left(\frac{x}{z}, \mu_F^2\right) \times \Delta_a^I(a_s, z, Q^2, \mu_R^2, \mu_F^2). \quad (1)$$

The scaling variable at the hadronic level is given by $x = \frac{-q^2}{2P \cdot q}$. P and q are the momenta of the hadron and the intermediate off-shell particle. For DIS we have $Q^2 = -q^2$. The parton distribution function is given by $f_a(y, \mu_F^2)$ with the momentum fraction y of the hadron at the factorization scale μ_F . The particle probing the structure of the hadron can be a photon ($I = q$) or the scalar particle ($I = g$). The scaling variable at the partonic level is $z = \frac{-q^2}{2p \cdot q}$ where p is the fraction of the parent hadron momentum. The perturbatively computed UV and infrared (IR) finite part of the partonic cross section, called the coefficient function, is given by $\Delta_a^I(a_s, z, Q^2, \mu_R^2, \mu_F^2)$, where μ_R denotes the UV renormalization scale. The Born contribution σ_B^I is chosen in such a way that Δ_a^I to lowest order in perturbation theory

is equal to $\delta(1 - z)$. In dimensional regularization, the UV renormalized strong coupling constant is $a_s = g_s^2(\mu_R^2)/16\pi^2$ is written in terms of bare coupling constant $\hat{a}_s = \hat{g}_s^2/16\pi^2$ as $\hat{a}_s(\mu^2)S_\epsilon = a_s(\mu_R^2)(\mu^2/\mu_R^2)^{\epsilon/2}Z(a_s(\mu_R^2))$. The scale μ keeps $\hat{g}_s$ dimensionless in $n = 4 + \epsilon$ space-time dimensions. S_ϵ is the spherical function defined as $S_\epsilon = \exp[(\gamma_E - \ln 4\pi)\frac{\epsilon}{2}]$, where γ_E is the Euler-Mascheroni constant. $Z(a_s(\mu_R^2))$ is the coupling constant renormalization term.

The infrared safe coefficient functions Δ_a^I get contributions from both soft gluons as well as from hard partons. We write them as sum of contributions from soft plus virtual and the remaining hard part:

$$\Delta_a^I(a_s, z, Q^2, \mu_F^2, \mu_R^2) = \Delta_a^{I,\text{hard}}(a_s, z, Q^2, \mu_F^2, \mu_R^2) + \Delta^{I,\text{SV}}(a_s, z, Q^2, \mu_F^2, \mu_R^2). \quad (2)$$

The SV part of the cross section consists only of plus distributions $\mathcal{D}_i(z) = [\frac{\ln^i(1-z)}{1-z}]_+$ and $\delta(1 - z)$. In the soft limit $z \rightarrow 1$ these distributions give dominant contributions to the hadronic cross section after they are convolved with the parton distribution functions as expressed through (1). As discussed in [15] the SV part of the coefficient function can be shown to factorize in terms of the square of the UV renormalized virtual contributions, soft plus jet distribution function appropriately convoluted with mass factorization kernels as

$$\Delta^{I,\text{SV}}(z, Q^2) = (Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon))^2 |\hat{\mathcal{F}}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)|^2 \times \delta(1 - z) \otimes \mathcal{C} e^{2\Phi_{\text{SJ}}^I(\hat{a}_s, Q^2, \mu^2, z, \epsilon)} \otimes \Gamma_{II}^{-1}(\hat{a}_s, \mu^2, \mu_F^2, z, \epsilon). \quad (3)$$

The symbol $\mathcal{C}$ denotes convolution and its operation on the exponential of a function $f(z)$ can be found in [15]. and $\otimes$ indicates the Mellin convolution, which convolutes with respect to the variables z . As we are interested in evaluating the SV part of the cross sections, we neglect all the regular functions that come from different convolutions. In (3), the overall renormalization constant, $Z^I(\hat{a}_s, \mu_R^2, \mu^2, \epsilon)$, for $I = g$ can be obtained from that of Higgs-gluon effective operator obtained in the large top quark mass limit and the exact form can be written in term of the anomalous dimension γ_g , which is known to the three-loop level [21]. For $I = q$, one finds $Z^q(\hat{a}_s, \mu_R^2, \mu^2, \epsilon) = 1$ to all orders. $\Gamma_{II}(\hat{a}_s, \mu^2, \mu_F^2, z, \epsilon)$ is the mass factorization kernel, known fully up to the three-loop level [22,23] and in the large n_f limit at the four-loop level [24]. The quantity $\hat{\mathcal{F}}^I(\hat{a}_s, Q^2, \mu^2, \epsilon)$ is the bare form factor which satisfies the KG equation, where the latter is a consequence of factorization, gauge and renormalization group invariances [25–28]. Its general solution up to four loops can be found in [15,29] in terms of the universal three-loop cusp (A^I)

[22,23,30–32], collinear (B^I) [22,23], soft (f^I) [23,33] anomalous dimensions known to the three-loop level and some form factor-dependent constants. The quantity $\Phi_{\text{SJ}}^I(\hat{a}_s, Q^2, \mu^2, z, \epsilon)$ in (3) is the soft plus jet distribution function which contains singular as well as finite parts due to soft gluons and collinear parton emissions. In [15,16], it was shown that by demanding finiteness of $\Delta^{I,\text{SV}}$, $\Phi_{\text{SJ}}^I(\hat{a}_s, Q^2, \mu^2, z, \epsilon)$ can also be shown to satisfy a Sudakov-type differential equation, namely (we suppress the arguments of Φ_{SJ}^I for brevity),

$$Q^2 \frac{d}{dQ^2} \Phi_{\text{SJ}}^I = \frac{1}{2} \left[\bar{K}^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \epsilon \right) + \bar{G}_{\text{SJ}}^I \left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \epsilon \right) \right]. \quad (4)$$

The terms $\bar{K}^I$ and $\bar{G}^I$ are such that the former contains poles as $\epsilon \rightarrow 0$ while the latter is finite in the same limit. Their expressions have similar forms as those of the corresponding ones that appear in form factors. The solution to the above equation is found to be

$$\Phi_{\text{SJ}}^I = \sum_{i=1}^{\infty} \hat{a}_s^i S_{\epsilon}^i \left(\frac{Q^2(1-z)}{\mu^2} \right)^{\frac{i}{2}} \frac{i\epsilon}{2(1-z)} \hat{\phi}_{\text{SJ}}^{I,(i)}(\epsilon) \quad (5)$$

where $\hat{\phi}_{\text{SJ}}^{I,(i)}(\epsilon) = [\bar{K}^{I,(i)}(\epsilon) + \bar{G}_{\text{SJ}}^{I,(i)}(\epsilon)]/i\epsilon$. Expressing $\bar{K}^I = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{\mu_R^2}{\mu^2} \right)^{\frac{i}{2}} S_{\epsilon}^i \bar{K}^{I,(i)}(\epsilon)$, the coefficients $\bar{K}^{I,(i)}(\epsilon)$ can be expressed in terms of A_i^I , beta function of QCD β_i [34] and $\bar{G}_{\text{SJ}}^{I,(i)}(\epsilon)$ given by

$$\sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q_z^2}{\mu^2} \right)^{\frac{i}{2}} S_{\epsilon}^i \bar{G}_{\text{SJ}}^{I,(i)}(\epsilon) = \sum_{i=1}^{\infty} a_i^I(Q_z^2) \bar{G}_{i,\text{SJ}}^I(\epsilon), \quad (6)$$

with $Q_z^2 = Q^2(1-z)$, can be expressed in terms of the B^I -, f^I - and ϵ -dependent part from lower order in the following way: $\bar{G}_{i,\text{SJ}}^I = -(B_i^I + f_i^I) + C_i^I + \sum_{k=1}^{\infty} \epsilon^k \bar{G}_{i,\text{SJ}}^{I,k}$, where the constants C_i^I up to three-loop order are given by $C_1^I = 0$, $C_2^I = -2\beta_0 \bar{G}_{2,\text{SJ}}^{I,1}$, $C_3^I = -2\beta_1 \bar{G}_{1,\text{SJ}}^{I,1} - 2\beta_0 (\bar{G}_{2,\text{SJ}}^{I,1} + 2\beta_0 \bar{G}_{1,\text{SJ}}^{I,2})$. The z -independent constants $\bar{G}_{i,\text{SJ}}^{I,k}$ are determined from the explicit computation of the SV coefficient functions $\Delta^{I,\text{SV}}(z, Q^2)$.

Computation of the coefficient functions Δ_a^I in perturbative QCD plays an important role in understanding the structure of hadrons. In the DIS process, the cross section factorizes into hadronic and the leptonic parts and the former can be computed by using operator product expansion in the Björken limit. Using various symmetries, the hadronic part can be expressed in terms of the structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$. These functions factorize into calculable coefficient functions $c_i(x, Q^2, \mu_F^2)$, $i = 1, 2$ and nonperturbative parton distribution functions

$f_a(x, \mu_F^2)$, $a = q, \bar{q}, g$. Applying the optical theorem, one relates the DIS cross section to the imaginary part of the forward scattering amplitude, where a virtual photon scatters off a nucleon. This forward scattering amplitude can be written in terms of coefficient functions c_i , where the latter can be computed by expanding in a perturbative series of the strong coupling constant. Computation of the higher-order coefficient functions [35–39] along with the higher-order splitting functions [22,23] and the precise measurements at DIS experiments were used to extract F_1 and F_2 accurately. Using the nonsinglet part of the quark coefficient function $c_{2,q}$ computed up to three loops [20] and $c_{\phi,g}^3$ computed using the off-shell scalar DIS process in [14], we can extract $\Delta^{q,\text{SV}}(z, Q^2)$ and $\Delta^{g,\text{SV}}$, respectively, up to three loops. Using these results and the known three-loop results for A , B , f and the form factor-dependent constants, we can determine $\bar{G}_{i,\text{SJ}}^{I,k}$ to desired accuracy in ϵ .

At the hadron colliders, the jets of quarks and gluons [40–42] capture the properties of QCD and provide insight into the IR structure of QCD processes. Jets are useful to study both SM as well as BSM processes. The definition of jets and its various properties play an important role in various new physics search scenarios. SCET provides a suitable framework to study the scattering or decay processes involving jet final states with invariant masses having large hierarchy with the center-of-mass energy of the process. The corresponding observables can be factorized in terms of certain process-dependent functions such as hard functions and process-independent soft and jet functions. Jet functions can result from quark or gluon radiating jet of collinear partons. The jet functions are important ingredients of SCET factorization for many processes with quark and gluon initiated states. The massless quark jet function in SCET at one loop was computed in [8,9]; for gluon see [12]. The two-loop results for the quark can be found in [10]; for the gluon, see [13]. The three-loop quark jet function have been recently computed in [11].

If we apply SCET formalism to the DIS process for the cases with $I = q, g$, we can identify the UV and IR finite parts of the form factor and soft plus jet function with the process-dependent matching coefficient and the jet function of SCET, respectively. Note that only one massless parton initiates the hard process in each case, (for $I = q$, quark/antiquark scatters of the virtual photon in the hard process and for $I = g$, gluon scatters of off-shell scalar), there will be only one jet function in each case and hence it is straightforward to identify it with the finite part of soft plus jet function.

The soft plus jet function Φ_{SJ}^I can be factorized into part containing IR poles in ϵ and a part containing the finite terms in the limit $\epsilon \rightarrow 0$, that is

$$C e^{2\Phi_{\text{SJ}}^I} = Z^I \otimes C e^{2\Phi_{\text{SJ}}^{I,\text{fin}}} \quad (7)$$

where $\mathcal{Z}^I$ contains only IR poles in ϵ and can be expanded as

$$\mathcal{Z}^I = \delta(1-z) + \sum_{i=1}^n \sum_{j=1}^{2i} a_s^i \frac{\mathcal{Z}_{ij}^I}{\epsilon^j}. \quad (8)$$

The coefficients $\mathcal{Z}_{ij}^I$ contain A^I , B^I , f^I , $\delta(1-z)$ and $\mathcal{D}_i(z)$ and it reads as

$$\begin{aligned} \mathcal{Z}_{11}^I|_{\mathcal{D}_0} &= 2A_1^I, \mathcal{Z}_{11}^I|_{\delta} = -2(B_1^I + f_1^I), \mathcal{Z}_{12}^I|_{\delta} = 4A_1^I, \\ \mathcal{Z}_{21}^I|_{\mathcal{D}_0} &= A_2^I, \mathcal{Z}_{21}^I|_{\delta} = -(B_2^I + f_2^I), \mathcal{Z}_{22}^I|_{\mathcal{D}_1} = 4A_1^{I^2}, \\ \mathcal{Z}_{22}^I|_{\mathcal{D}_0} &= -4A_1^I(B_1^I + f_1^I) + 2\beta_0 A_1^I, \mathcal{Z}_{22}^I|_{\delta} = 2(B_1^I + f_1^I)^2 + A_2^I - 2\beta_0(B_1^I + f_1^I) - \frac{1}{3}\pi^2 A_1^{I^2}, \\ \mathcal{Z}_{23}^I|_{\mathcal{D}_0} &= 8A_1^{I^2}, \mathcal{Z}_{23}^I|_{\delta} = -8A_1^I(B_1^I + f_1^I) + 6\beta_0 A_1^I, \\ \mathcal{Z}_{24}^I|_{\delta} &= 8A_1^{I^2}, \mathcal{Z}_{31}^I|_{\mathcal{D}_0} = \frac{2}{3}A_3^I, \mathcal{Z}_{31}^I|_{\delta} = -\frac{2}{3}(B_3^I + f_3^I), \\ \mathcal{Z}_{32}^I|_{\mathcal{D}_1} &= 4A_1^I A_2^I, \mathcal{Z}_{32}^I|_{\mathcal{D}_0} = -2A_2^I(B_1^I + f_1^I) - 2A_1^I(B_2^I + f_2^I) + \frac{4}{3}\beta_1 A_1^I + \frac{4}{3}\beta_0 A_2^I, \\ \mathcal{Z}_{32}^I|_{\delta} &= 2(B_1^I + f_1^I)(B_2^I + f_2^I) + \frac{4}{9}A_3^I - \frac{4}{3}\beta_1(B_1^I + f_1^I) - \frac{4}{3}\beta_0(B_2^I + f_2^I) - \frac{1}{3}\pi^2 A_1^I A_2^I, \mathcal{Z}_{33}^I|_{\mathcal{D}_2} = 4A_1^{I^3}, \\ \mathcal{Z}_{33}^I|_{\mathcal{D}_1} &= -8A_1^{I^2}(B_1^I + f_1^I) + 8\beta_0 A_1^{I^2}, \\ \mathcal{Z}_{33}^I|_{\mathcal{D}_0} &= 4A_1^I(B_1^I + f_1^I)^2 + 6A_1^I A_2^I - 8\beta_0 A_1^I(B_1^I + f_1^I) + \frac{8}{3}\beta_0^2 A_1^I - \frac{2}{3}\pi^2 A_1^{I^3}, \\ \mathcal{Z}_{33}^I|_{\delta} &= -\frac{4}{3}(B_1^I + f_1^I)^3 - 2A_2^I(B_1^I + f_1^I) - 4A_1^I(B_2^I + f_2^I) + \frac{32}{9}\beta_1 A_1^I \\ &\quad + 4\beta_0(B_1^I + f_1^I)^2 + \frac{20}{9}\beta_0 A_2^I - \frac{8}{3}\beta_0^2(B_1^I + f_1^I) + \frac{8}{3}\zeta_3 A_1^{I^3} + \frac{2}{3}\pi^2 A_1^{I^2}(B_1^I + f_1^I) - \frac{2}{3}\pi^2 \beta_0 A_1^{I^2}, \\ \mathcal{Z}_{34}^I|_{\mathcal{D}_1} &= 16A_1^{I^3}, \mathcal{Z}_{34}^I|_{\mathcal{D}_0} = -16A_1^{I^2}(B_1^I + f_1^I) + 20\beta_0 A_1^{I^2}, \\ \mathcal{Z}_{34}^I|_{\delta} &= 8A_1^I(B_1^I + f_1^I)^2 + 4A_1^I A_2^I - 20\beta_0 A_1^I(B_1^I + f_1^I) + \frac{88}{9}\beta_0^2 A_1^I - \frac{4}{3}\pi^2 A_1^{I^3}, \mathcal{Z}_{35}^I|_{\mathcal{D}_0} = 16A_1^{I^3}, \\ \mathcal{Z}_{35}^I|_{\delta} &= -16A_1^{I^2}(B_1^I + f_1^I) + 24\beta_0 A_1^{I^2}, \mathcal{Z}_{36}^I|_{\delta} = \frac{32}{3}A_1^{I^3}. \end{aligned} \quad (9)$$

We now present the general expression for the jet function $\mathcal{C}e^{2\Phi_{\text{SJ}}^{I,\text{fin}}} = \delta(1-z) + \sum_{i=1}^{\infty} a_s^i J_i^I|_k$ up to three loops, where $J_i^I|_k$ represent the coefficients of $\mathcal{D}_j(z)$, δ for $j \leq (2i-1)$. We have set $Q^2 = \mu_R^2 = \mu_F^2$ in our computation:

$$\begin{aligned} J_1^I|_{\mathcal{D}_1} &= A_1^I, J_1^I|_{\mathcal{D}_0} = -(B_1^I + f_1^I), J_1^I|_{\delta} = 2\bar{\mathcal{G}}_{1,\text{SJ}}^{I,1}, \\ J_2^I|_{\mathcal{D}_3} &= \frac{1}{2}A_1^{I^2}, J_2^I|_{\mathcal{D}_2} = -\frac{3}{2}A_1^I(B_1^I + f_1^I) - \frac{1}{2}\beta_0 A_1^I, \\ J_2^I|_{\mathcal{D}_1} &= (B_1^I + f_1^I)^2 + A_2^I + 2A_1^I \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} + \beta_0(B_1^I + f_1^I) - \frac{1}{6}\pi^2 A_1^{I^2}, \\ J_2^I|_{\mathcal{D}_0} &= -(B_2^I + f_2^I) - 2\bar{\mathcal{G}}_{1,\text{SJ}}^{I,1}(B_1^I + f_1^I) - 2\beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} + \zeta_3 A_1^{I^2} + \frac{1}{6}\pi^2 A_1^I(B_1^I + f_1^I), \\ J_2^I|_{\delta} &= \bar{\mathcal{G}}_{2,\text{SJ}}^{I,1} + 2(\bar{\mathcal{G}}_{1,\text{SJ}}^{I,1})^2 + 2\beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,2} - \zeta_3 A_1^I(B_1^I + f_1^I) - \frac{1}{12}\pi^2(B_1^I + f_1^I)^2 - \frac{1}{720}\pi^4 A_1^{I^2}, J_3^I|_{\mathcal{D}_5} = \frac{1}{8}A_1^{I^3}, \\ J_3^I|_{\mathcal{D}_4} &= -\frac{5}{8}A_1^{I^2}(B_1^I + f_1^I) - \frac{5}{12}\beta_0 A_1^{I^2}, \end{aligned}$$

$$\begin{aligned}
J'_3|_{\mathcal{D}_3} &= A_1^I(B_1^I + f_1^I)^2 + A_1^I A_2^I + \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^{I^2} + \frac{5}{3} \beta_0 A_1^I (B_1^I + f_1^I) + \frac{1}{3} \beta_0^2 A_1^I - \frac{1}{6} \pi^2 A_1^{I^3}, \\
J'_3|_{\mathcal{D}_2} &= -\frac{1}{2} (B_1^I + f_1^I)^3 - \frac{3}{2} A_2^I (B_1^I + f_1^I) - \frac{3}{2} A_1^I (B_2^I + f_2^I) - 3 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^I (B_1^I + f_1^I) - \frac{1}{2} \beta_1 A_1^I \\
&\quad - \frac{3}{2} \beta_0 (B_1^I + f_1^I)^2 - \beta_0 A_2^I - 4 \beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^I - \beta_0^2 (B_1^I + f_1^I) + \frac{5}{2} \zeta_3 A_1^{I^3} + \frac{1}{2} \pi^2 A_1^{I^2} (B_1^I + f_1^I) + \frac{1}{4} \pi^2 \beta_0 A_1^{I^2}, \\
J'_3|_{\mathcal{D}_1} &= 2(B_1^I + f_1^I)(B_2^I + f_2^I) + A_3^I + \bar{\mathcal{G}}_{2,\text{SJ}}^{I,1} A_1^I + 2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} (B_1^I + f_1^I)^2 + 2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} (A_2^I + \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^I) \\
&\quad + \beta_1 (B_1^I + f_1^I) + 2 \beta_0 (B_2^I + f_2^I) + 2 \beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,2} A_1^I + 6 \beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} (B_1^I + f_1^I) + 4 \beta_0^2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} - 5 \zeta_3 A_1^{I^2} (B_1^I + f_1^I) \\
&\quad - 3 \zeta_3 \beta_0 A_1^{I^2} - \frac{5}{12} \pi^2 A_1^I (B_1^I + f_1^I)^2 - \frac{1}{3} \pi^2 A_1^I A_2^I - \frac{1}{3} \pi^2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^{I^2} - \frac{1}{2} \pi^2 \beta_0 A_1^I (B_1^I + f_1^I) - \frac{1}{144} \pi^4 A_1^{I^3}, \\
J'_3|_{\mathcal{D}_0} &= -(B_3^I + f_3^I) - \bar{\mathcal{G}}_{2,\text{SJ}}^{I,1} (B_1^I + f_1^I) - 2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} (B_2^I + f_2^I) - 2 (\bar{\mathcal{G}}_{1,\text{SJ}}^{I,1})^2 (B_1^I + f_1^I) - 2 \beta_1 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} - 2 \beta_0 \bar{\mathcal{G}}_{2,\text{SJ}}^{I,1} \\
&\quad - 2 \beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,2} (B_1^I + f_1^I) - 4 \beta_0 (\bar{\mathcal{G}}_{1,\text{SJ}}^{I,1})^2 - 4 \beta_0^2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,2} + 3 \zeta_3 A_1^{I^3} + 2 \zeta_3 A_1^I (B_1^I + f_1^I)^2 + 2 \zeta_3 A_1^I A_2^I \\
&\quad + 2 \zeta_3 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^{I^2} + 3 \zeta_3 \beta_0 A_1^I (B_1^I + f_1^I) + \frac{1}{12} \pi^2 (B_1^I + f_1^I)^3 + \frac{1}{6} \pi^2 A_2^I (B_1^I + f_1^I) + \frac{1}{6} \pi^2 A_1^I (B_2^I + f_2^I) \\
&\quad + \frac{1}{3} \pi^2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^I (B_1^I + f_1^I) + \frac{1}{6} \pi^2 \beta_0 (B_1^I + f_1^I)^2 + \frac{1}{3} \pi^2 \beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^I - \frac{1}{3} \pi^2 \zeta_3 A_1^{I^3} + \frac{1}{144} \pi^4 A_1^{I^2} (B_1^I + f_1^I) + \frac{1}{72} \pi^4 \beta_0 A_1^{I^2}, \\
J'_3|_{\delta} &= \frac{2}{3} \bar{\mathcal{G}}_{3,\text{SJ}}^{I,1} + 2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} \bar{\mathcal{G}}_{2,\text{SJ}}^{I,1} + \frac{4}{3} (\bar{\mathcal{G}}_{1,\text{SJ}}^{I,1})^3 + \frac{4}{3} \beta_1 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,2} + \frac{4}{3} \beta_0 \bar{\mathcal{G}}_{2,\text{SJ}}^{I,2} + 4 \beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} \bar{\mathcal{G}}_{1,\text{SJ}}^{I,2} + \frac{8}{3} \beta_0^2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,3} \\
&\quad - 3 \zeta_3 A_1^{I^2} (B_1^I + f_1^I) - 2 \zeta_3 \beta_0 A_1^{I^2} - \frac{1}{3} \zeta_3 (B_1^I + f_1^I)^3 - \zeta_3 A_2^I (B_1^I + f_1^I) - \zeta_3 A_1^I (B_2^I + f_2^I) \\
&\quad - 2 \zeta_3 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^I (B_1^I + f_1^I) - \zeta_3 \beta_0 (B_1^I + f_1^I)^2 - 2 \zeta_3 \beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^I + \frac{5}{6} \zeta_3^2 A_1^{I^3} - \frac{1}{6} \pi^2 (B_1^I + f_1^I) (B_2^I + f_2^I) \\
&\quad - \frac{1}{6} \pi^2 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} (B_1^I + f_1^I)^2 - \frac{1}{3} \pi^2 \beta_0 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} (B_1^I + f_1^I) + \frac{1}{3} \pi^2 \zeta_3 A_1^{I^2} (B_1^I + f_1^I) + \frac{1}{6} \pi^2 \zeta_3 \beta_0 A_1^{I^2} - \frac{1}{360} \pi^4 A_1^I (B_1^I + f_1^I)^2 \\
&\quad - \frac{1}{360} \pi^4 A_1^I A_2^I - \frac{1}{360} \pi^4 \bar{\mathcal{G}}_{1,\text{SJ}}^{I,1} A_1^{I^2} - \frac{1}{72} \pi^4 \beta_0 A_1^I (B_1^I + f_1^I) - \frac{29}{45360} \pi^6 A_1^{I^3}. \tag{10}
\end{aligned}$$

Note that the coefficients $J_{i,n}^{(m)}$ given in [11] are nothing but our $J'_i|_k$, $k = \delta, \mathcal{D}_j(z)$ if we identify m, i, n with i, I, k in $J_{i,n}^{(m)}$ and $\mathcal{L}_n(s/\mu_R^2)$ with $\mathcal{D}_j$; the anomalous dimension of the jet function can be written in terms of collinear (B^I) and soft (f^I) anomalous dimension, that is $\gamma_J^I = B^I + f^I$ where $I = q, g$. The renormalization group equation satisfied by soft plus jet function is given by

$$\mu_R^2 \frac{d}{d\mu_R^2} J^I = \Gamma_J^I \otimes J^I \tag{11}$$

where $\Gamma_J^I = \{B^I + f^I - A^I \ln(\frac{Q^2}{\mu_R^2})\} \delta(1-z) - A^I \mathcal{D}_0$. We can get the logarithmic-dependent parts of J^I through the above equation. Rescaling $(1-z)$ by $Q^2/z\mu_R^2$, we can easily relate our Γ_J^I with $\gamma_J^I(s, \mu_R^2)$ given [11].

For $I = q$, our result $\mathcal{C}e^{2\Phi_{\text{SJ}}^{q,\text{fin}}}$ matches with the three-loop quark jet function given in [11] after identifying s in the latter reference without z through $s = (p+q)^2 = Q^2(1-z)/z$ and taking the soft limit $z \rightarrow 1$. Below, we present the gluon jet function up to three loops:

$$\begin{aligned}
J_1^g|_{\mathcal{D}_1} &= C_A[4], & J_1^g|_{\mathcal{D}_0} &= C_A \left[-\frac{11}{3} \right] + n_f \left[\frac{2}{3} \right], & J_1^g|_{\delta} &= C_A \left[\frac{67}{9} - \pi^2 \right] + n_f \left[-\frac{10}{9} \right], \\
J_2^g|_{\mathcal{D}_3} &= C_A^2[8], & J_2^g|_{\mathcal{D}_2} &= C_A^2 \left[-\frac{88}{3} \right] + C_A n_f \left[\frac{16}{3} \right], & J_2^g|_{\mathcal{D}_1} &= C_A^2 \left[\frac{778}{9} - 8\pi^2 \right] + C_A n_f \left[-\frac{56}{3} \right] + n_f^2 \left[\frac{8}{9} \right], \\
J_2^g|_{\mathcal{D}_0} &= C_A^2 \left[-\frac{2570}{27} + 32\zeta_3 + 11\pi^2 \right] + C_A n_f \left[\frac{224}{9} - 2\pi^2 \right] + C_F n_f[2] + n_f^2 \left[-\frac{40}{27} \right],
\end{aligned}$$

$$\begin{aligned}
J_2^g|_\delta &= C_A^2 \left[\frac{20215}{162} - \frac{440}{9} \zeta_3 - \frac{371}{18} \pi^2 + \frac{151}{180} \pi^4 \right] + C_A n_f \left[-\frac{760}{27} + \frac{8}{9} \zeta_3 + \frac{109}{27} \pi^2 \right] + C_F n_f \left[-\frac{55}{6} + 8 \zeta_3 \right] + n_f^2 \left[\frac{100}{81} - \frac{4}{27} \pi^2 \right], \\
J_3^g|_{D_3} &= C_A^3 [8], \quad J_3^g|_{D_4} = C_A^3 \left[-\frac{550}{9} \right] + C_A^2 n_f \left[\frac{100}{9} \right], \\
J_3^g|_{D_3} &= C_A^3 [340 - 24 \pi^2] + C_A^2 n_f \left[-\frac{256}{3} \right] + C_A n_f^2 \left[\frac{16}{3} \right], \\
J_3^g|_{D_2} &= C_A^3 \left[-\frac{9623}{9} + 256 \zeta_3 + \frac{1034}{9} \pi^2 \right] + C_A^2 n_f \left[\frac{3106}{9} - \frac{188}{9} \pi^2 \right] + C_A C_F n_f [16] + C_A n_f^2 \left[-\frac{292}{9} \right] + n_f^3 \left[\frac{8}{9} \right], \\
J_3^g|_{D_1} &= C_A^3 \left[\frac{171848}{81} - \frac{7304}{9} \zeta_3 - \frac{9314}{27} \pi^2 + \frac{439}{45} \pi^4 \right] + C_A^2 n_f \left[-\frac{20134}{27} + \frac{752}{9} \zeta_3 + \frac{2324}{27} \pi^2 \right] + C_A C_F n_f [-110 + 64 \zeta_3] \\
&\quad + C_A n_f^2 \left[\frac{6652}{81} - \frac{16}{3} \pi^2 \right] + C_F n_f^2 \left[\frac{20}{3} \right] + n_f^3 \left[-\frac{80}{27} \right], \\
J_3^g|_{D_0} &= C_A^3 \left[-\frac{1448021}{729} + 80 \zeta_5 + \frac{12436}{9} \zeta_3 + \frac{191963}{486} \pi^2 - \frac{736}{9} \pi^2 \zeta_3 - \frac{363}{20} \pi^4 \right] \\
&\quad + C_A^2 n_f \left[\frac{1052135}{1458} - \frac{5888}{27} \zeta_3 - \frac{30808}{243} \pi^2 + \frac{313}{90} \pi^4 \right] + C_A C_F n_f \left[\frac{5599}{27} - \frac{1096}{9} \zeta_3 - 6 \pi^2 - \frac{8}{45} \pi^4 \right] + C_F^2 n_f [-1] \\
&\quad + C_A n_f^2 \left[-\frac{116509}{1458} + \frac{32}{27} \zeta_3 + \frac{946}{81} \pi^2 \right] + C_F n_f^2 [-24 + 16 \zeta_3] + n_f^3 \left[\frac{200}{81} - \frac{8}{27} \pi^2 \right], \\
J_3^g|_\delta &= C_A^3 \left[\frac{55853711}{26244} - 44 \zeta_5 - \frac{452770}{243} \zeta_3 + \frac{1600}{9} \zeta_3^2 - \frac{2055109}{4374} \pi^2 + \frac{1364}{9} \pi^2 \zeta_3 + \frac{53633}{1620} \pi^4 - \frac{16309}{20412} \pi^6 \right] \\
&\quad + C_A^2 n_f \left[-\frac{17323633}{26244} + \frac{208}{9} \zeta_5 + \frac{2734}{9} \zeta_3 + \frac{330062}{2187} \pi^2 - \frac{88}{9} \pi^2 \zeta_3 - \frac{18727}{2430} \pi^4 \right] \\
&\quad + C_A C_F n_f \left[-\frac{389369}{972} + \frac{584}{9} \zeta_5 + \frac{21200}{81} \zeta_3 + \frac{712}{27} \pi^2 - \frac{160}{9} \pi^2 \zeta_3 + \frac{76}{405} \pi^4 \right] + C_F^2 n_f \left[\frac{143}{9} - 80 \zeta_5 + \frac{148}{3} \zeta_3 \right] \\
&\quad + C_A n_f^2 \left[\frac{1613639}{26244} - \frac{1004}{243} \zeta_3 - \frac{3656}{243} \pi^2 + \frac{506}{1215} \pi^4 \right] + C_F n_f^2 \left[\frac{7001}{162} - \frac{104}{3} \zeta_3 - \frac{10}{9} \pi^2 \right] + n_f^3 \left[-\frac{1000}{729} + \frac{40}{81} \pi^2 \right].
\end{aligned} \tag{12}$$

C_A and C_F are the quadratic Casimir in adjoint and fundamental representations, respectively, and n_f is the number of light flavors.

III. CONCLUSION

In this article, we have shown how one of the building blocks of SCET, namely, the jet function, can be related to the well-known coefficient function of the DIS cross section. This novel connection provides an alternate and elegant way to obtain both quark and gluon jet functions order by order in pQCD from the known coefficient functions. While we confirm the three-loop quark jet function reported recently in [11], the three-loop gluon jet function presented in the article is new. The important ingredient to obtain this result is the gluon coefficient function [14] of DIS process up to the three-loop level in QCD, computed using off-shell scalar particle scattering deep-inelastically of the massless gluons. We have used the factorization properties of scattering cross section and exploited universal structure of soft and collinear dynamics to relate soft plus jet function of DIS against the jet function in SCET. Using quark coefficient function [20]

known up to three loops in QCD, we confirm the recently computed three-loop quark jet function in [11] and present our result for the gluon jet function up to the three-loop level. The three-loop quark and gluon jet functions are the important ingredients to the N-jettiness IR subtraction method [43,44] at N³LO and to threshold resummation up to N³LL' in the SCET framework to study processes involving final state jets. Thanks to the wealth of precise predictions in perturbative QCD for various important observables, one can unfold the underlying universal infrared structure QCD amplitudes and determine process-independent building blocks that capture infrared dynamics of high-energy scattering processes.

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