Higgs boson ZZ couplings in Higgs-strahlung at the ILC

Taras Zagoskin^{*} and Alexander Korchin[†]

*NSC “Kharkov Institute of Physics and Technology,” 61108 Kharkov, Ukraine
and V. N. Karazin Kharkov National University, 61022 Kharkov, Ukraine*

(Received 25 May 2018; revised manuscript received 4 September 2018; published 30 November 2018)

We derive the fully differential cross section of the Higgs-strahlung process $f\bar{f} \rightarrow Z \rightarrow Z(\rightarrow f_Z\bar{f}_Z)X(\rightarrow f_X\bar{f}_X)$, where f , f_Z , and f_X are arbitrary fermions and X is a spin-zero particle with arbitrary couplings to Z bosons and fermions. This process with $f = e$ and $X = h$ (h denotes the Higgs boson) is planned to be measured at the International Linear Collider to put constraints on the couplings g_1, g_2, g_3 of the Higgs boson to Z bosons. Using the obtained fully differential cross section, we define the observables, measurement of which yields the tightest constraints on the couplings. Explicit dependences of these observables on g_i are derived.

DOI: 10.1103/PhysRevD.98.093008

I. INTRODUCTION

Since the discovery [1,2] of the Higgs boson (denoted by h in this paper) in 2012, it has been important to measure its couplings and CP properties. These measurements can prove or disprove the Higgs mechanism [3] and can let us describe processes involving the Higgs boson more precisely.

The couplings of the Higgs boson are measured at the LHC by the CMS and ATLAS Collaborations (see, e.g., [4–6]). These couplings are also planned [7] to be measured at the International Linear Collider (ILC). The latter measurements are expected to have lower backgrounds and to yield stricter constraints on a number of the Higgs boson parameters. Many papers have addressed the prospects of measurements at the ILC [8–17].

At the ILC, the Higgs couplings to a pair of Z bosons (hZZ couplings) will be measured in the Higgs-strahlung process $e^-e^+ \rightarrow Z \rightarrow Z(\rightarrow f_Z\bar{f}_Z)h(\rightarrow f_h\bar{f}_h)$, where $f_Z = e, \mu, u, d, s, c, b$ and $f_h = \tau, b$. A lot of papers (see, e.g., Refs. [18–21]) concern the process $e^-e^+ \rightarrow Z \rightarrow Z(\rightarrow f_Z\bar{f}_Z)h$, not considering a decay of the Higgs boson. Consideration of such a decay allows for the fact that the Higgs boson is off shell, thereby allowing us to obtain more precise total and differential cross sections and observables of the Higgs-strahlung. Moreover, the Higgs-strahlung with a decay of the Higgs boson is studied in Ref. [8]

[see Eq. (A2) there]. We generalize Eq. (A2) to the case of an arbitrary polarization of the positron beam.

In Sec. II we derive the fully differential cross section of this process for a spin-zero Higgs boson, accounting for the beyond the Standard Model (SM) hZZ and hff couplings and for arbitrary electron and positron polarizations. In Sec. III we use the obtained formula and Refs. [22,23] to define observables yielding the tightest constraints on the hZZ couplings. The dependence of these observables on the couplings is derived and analyzed.

II. FULLY DIFFERENTIAL CROSS SECTION

We consider the process

$$f\bar{f} \rightarrow Z \rightarrow ZX \rightarrow f_Z\bar{f}_Z f_X\bar{f}_X \quad (1)$$

(see Fig. 1), where f , f_Z , and f_X are some fermions, $f_Z \neq f_X$, and X is a particle with zero spin, arbitrary couplings to a pair of Z bosons and arbitrary couplings to a fermion-antifermion pair. This scattering is going to be measured at the ILC in the case $f = e$, $X = h$, $f_Z = e, \mu, u, d, s, c, b$, and $f_h = \tau, b$.

Because of the energy-momentum conservation in process (1),

$$a_1 = s, \quad a_2 \in (4m_{f_Z}^2, (\sqrt{a_1} - \sqrt{a_X})^2), \\ a_X \in (4m_{f_X}^2, (\sqrt{a_1} - 2m_{f_Z})^2), \quad (2)$$

where a_1, a_2 , and a_X are the squared invariant masses of the Z boson (called Z_1) produced by the fermion-antifermion pair $f\bar{f}$, of the Z boson (called Z_2) produced together with the boson X , and of the boson X itself, respectively, s is the squared invariant energy of $f\bar{f}$, m_{f_Z} and m_{f_X} are the masses of the fermions f_Z and f_X , respectively.

^{*}taras.zagoskin@gmail.com

[†]korchin@kipt.kharkov.ua

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

The amplitude $A(\lambda_1, \lambda_2)$ of the transition $Z \rightarrow ZX$ is analogous to the HVV amplitude in Ref. [4] [see Eq. (1) there] and to the XZZ amplitude in Ref. [24] [Eq. (7)] and in [25] [Eq. (3)]:

$$A(\lambda_1, \lambda_2) = 2\sqrt{\sqrt{2}G_F m_Z^2} \left(\left(g_1(a_1, a_2) - \frac{a_1 + a_2 - a_X}{m_Z^2} g_2(a_1, a_2) \right) (e_1 \cdot e_2^*) + 2 \frac{g_2(a_1, a_2)}{m_Z^2} (e_1 \cdot q_2)(e_2^* \cdot q_1) + 2 \frac{g_3(a_1, a_2)}{m_Z^2} \varepsilon_{\mu\nu\rho\sigma} q_1^\mu q_2^\nu e_1^\rho (e_2^\sigma)^* \right), \quad (3)$$

where λ_j , e_j , and q_j are the helicity, polarization four-vector, and 4-momentum of the boson Z_j , respectively, ($j = 1, 2$), G_F is the Fermi constant, m_Z is the pole mass of the Z boson, and $g_1(a_1, a_2)$, $g_2(a_1, a_2)$, and $g_3(a_1, a_2)$ are some complex-valued functions on a_1 and a_2 —we call these functions XZZ couplings; $\varepsilon_{\mu\nu\rho\sigma}$ is the Levi-Civita symbol ($\varepsilon_{0123} = 1$). In Ref. [4] the XZZ couplings are denoted as a_1 , a_2 , and a_3 —we denote them as g_1 , g_2 , and g_3 , respectively to avoid confusion.

At the tree level, the XZZ couplings are connected with the CP parity of the boson X , as shown in Table I. In the SM $g_1 = 1$ and $g_2 = g_3 = 0$.

In the Z_1 rest frame, or the center-of-mass frame of process (1), the polarization vectors read

$$e_1^\mu|_{\lambda_1=0} = (0, 0, 0, 1), \quad e_1^\mu|_{\lambda_1=\pm 1} = \left(0, \mp \frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}, 0\right), \quad (4)$$

$$e_2^\mu|_{\lambda_2=0} = \left(\frac{|\mathbf{q}_2|}{\sqrt{a_2}}, 0, 0, -\frac{\sqrt{|\mathbf{q}_2|^2 + a_2}}{\sqrt{a_2}}\right),$$

$$e_2^\mu|_{\lambda_2=\pm 1} = \left(0, \pm \frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}, 0\right). \quad (5)$$

FIG. 1. A Feynman diagram of the Higgs-strahlung.

Using Eqs. (3)–(5), we derive that

$$A(-1, 1) = -g_1 - \frac{a_X - a_1 - a_2}{m_Z^2} g_2 + 2i \frac{\sqrt{a_1} |\mathbf{q}_2|}{m_Z^2} g_3,$$

$$A(1, -1) = -g_1 - \frac{a_X - a_1 - a_2}{m_Z^2} g_2 - 2i \frac{\sqrt{a_1} |\mathbf{q}_2|}{m_Z^2} g_3,$$

$$A(0, 0) = \left(g_1 + \frac{a_X - a_1 - a_2}{m_Z^2} g_2 \right) \frac{\sqrt{|\mathbf{q}_2|^2 + a_2}}{\sqrt{a_2}} + 2 \frac{\sqrt{a_1} |\mathbf{q}_2|^2}{m_Z^2 \sqrt{a_2}} g_2,$$

$$A(\lambda_1, \lambda_2) = 0 \quad \text{for } \lambda_1 \neq -\lambda_2. \quad (6)$$

The amplitude of the decay $X \rightarrow f_X \bar{f}_X$ is [26]

$$A_{X \rightarrow f_X \bar{f}_X}(\lambda_{f_X}, \lambda_{\bar{f}_X}) = -\sqrt{\sqrt{2}G_F m_{f_X}} \bar{u}_{f_X}(\rho_1 + \rho_2 \gamma^5) v_{\bar{f}_X}, \quad (7)$$

where u and v are the Dirac spinors, λ_{f_X} ($\lambda_{\bar{f}_X}$) is the helicity of the fermion (antifermion), and ρ_1 and ρ_2 are some complex numbers which we call the Xff couplings. In the SM $\rho_1 = 1$ and $\rho_2 = 0$.

Calculation of $A_{X \rightarrow f_X \bar{f}_X}(\lambda_{f_X}, \lambda_{\bar{f}_X})$ in the X rest frame yields

$$A_{X \rightarrow f_X \bar{f}_X} \left(-\frac{1}{2}, -\frac{1}{2} \right) = \sqrt{\sqrt{2}G_F m_{f_X}} (\rho_1 + \rho_2) \sqrt{a_X},$$

$$A_{X \rightarrow f_X \bar{f}_X} \left(\frac{1}{2}, \frac{1}{2} \right) = \sqrt{\sqrt{2}G_F m_{f_X}} (\rho_1 - \rho_2) \sqrt{a_X},$$

$$A_{X \rightarrow f_X \bar{f}_X} \left(-\frac{1}{2}, \frac{1}{2} \right) = A_{X \rightarrow f_X \bar{f}_X} \left(\frac{1}{2}, -\frac{1}{2} \right) = 0. \quad (8)$$

TABLE I. The CP parity of the particle X for various values of g_1 , g_2 , and g_3 .

g_1	g_2	g_3	CP_X
Any	Any	0	1
0	0	$\neq 0$	-1
$\neq 0$	Any	$\neq 0$	Indefinite
Any	$\neq 0$	$\neq 0$	Indefinite

Using the helicity formalism and neglecting m_f , m_{f_Z} , and m_{f_X} everywhere save the m_{f_X} factor in Eq. (8), we derive the fully differential cross section of process (1):

$$\begin{aligned} \frac{d^7\sigma}{da_2 da_X d\theta_Z d\theta_{f_Z} d\phi d\theta_{f_X} d\phi_{f_X}} &= \frac{G_F^4 m_Z^8 m_{f_X}^2}{(4\pi)^7} \cdot \frac{a_2 a_X \lambda^{1/2}(s, a_2, a_X)}{s D(s) D(a_2) D_X(a_X)} (a_f^2 + v_f^2)(a_{f_Z}^2 + v_{f_Z}^2) \\ &\times (|\rho_1|^2 + |\rho_2|^2) \sin\theta_Z \sin\theta_{f_Z} \sin\theta_{f_X} [(|A_{\parallel}|^2 + |A_{\perp}|^2)(P_1(1 + \cos^2\theta_Z)(1 + \cos^2\theta_{f_Z}) \\ &+ 4P_2 A_{f_Z} \cos\theta_Z \cos\theta_{f_Z}) + \text{Re}(A_{\parallel}^* A_{\perp}) \cdot 4(P_1 A_{f_Z} \cos\theta_{f_Z}(1 + \cos^2\theta_Z) \\ &+ P_2 \cos\theta_Z(1 + \cos^2\theta_{f_Z})) + |A_0|^2 \cdot 4P_1 \sin^2\theta_Z \sin^2\theta_{f_Z} - 4\sqrt{2} \sin\theta_Z \sin\theta_{f_Z} \\ &\times ((\text{Re}(A_0^* A_{\parallel}) \cos\phi + \text{Im}(A_0^* A_{\perp}) \sin\phi)(P_2 A_{f_Z} + P_1 \cos\theta_Z \cos\theta_{f_Z}) \\ &+ (\text{Im}(A_0^* A_{\parallel}) \sin\phi + \text{Re}(A_0^* A_{\perp}) \cos\phi)(P_1 A_{f_Z} \cos\theta_Z + P_2 \cos\theta_{f_Z})) + P_1 \sin^2\theta_Z \sin^2\theta_{f_Z} \\ &\times ((|A_{\parallel}|^2 - |A_{\perp}|^2) \cos 2\phi + \text{Im}(A_{\parallel}^* A_{\perp}) \cdot 2 \sin 2\phi)], \end{aligned} \quad (9)$$

where (see Fig. 2)

- (i) θ_Z is the angle between the momentum $\mathbf{p}_f$ of the fermion f in the $f\bar{f}$ rest frame and the momentum $\mathbf{q}_2$ of the Z boson Z_2 in the same frame;
- (ii) θ_{f_Z} is the angle between $\mathbf{q}_2$ and the f_Z momentum $\mathbf{p}_{f_Z}$ in the Z_2 rest frame;
- (iii) ϕ is the azimuthal angle between the plane spanned by the vectors $\mathbf{p}_f$ and $\mathbf{q}_2$ and the plane spanned by $\mathbf{q}_2$ and $\mathbf{p}_{f_Z}$;
- (iv) θ_{f_X} is the angle between the X momentum in the $f\bar{f}$ rest frame and the f_X momentum in the X rest frame;

- (v) ϕ_{f_X} is the azimuthal angle of the fermion f_X ;
- (vi) $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2(xy + xz + yz)$;
- (vii) $D(x) \equiv (x - m_Z^2)^2 + (m_Z \Gamma_Z)^2$,
 $D_X(x) \equiv (x - m_X^2)^2 + (m_X \Gamma_X)^2$;
- (viii) Γ_Z (Γ_X) is the total width of the Z (X) boson;
- (ix) m_X is the pole mass of the X boson;
- (x) a_f is the projection of the weak isospin of a fermion f , $v_f \equiv a_f - 2\frac{q_f}{e} \sin^2\theta_W$, q_f is the electric charge of the fermion f , e is the electric charge of the positron, and θ_W is the weak mixing angle,
 $A_f \equiv \frac{2a_f v_f}{a_f^2 + v_f^2}$;

FIG. 2. Kinematics of process (1). The momenta of f , $\bar{f}$, Z_1 , X , and Z_2 are shown in the $f\bar{f}$ rest frame; the momenta of f_Z and $\bar{f}_Z$ are displayed in the Z_2 rest frame; and the momenta of f_X and $\bar{f}_X$ are described in the X rest frame. The z axis is codirectional with the X momentum while the x and y axes are arbitrary axes forming a right-handed system with the z axis.

- (xi) $A_0 \equiv \frac{s+a_2-a_X}{2\sqrt{s a_2}} g_1 - \frac{2\sqrt{s a_2}}{m_Z^2} g_2$,
 $A_{\parallel} \equiv -\frac{A(-1,1)+A(1,-1)}{\sqrt{2}} = \sqrt{2}(g_1 - \frac{s+a_2-a_X}{m_Z^2} g_2)$,
 $A_{\perp} \equiv -\frac{A(-1,1)-A(1,-1)}{\sqrt{2}} = i\sqrt{2}\frac{\lambda^{1/2}(s,a_2,a_X)}{m_Z^2} g_3$;
 (xii) $P_1 \equiv 1 - P_f P_{\bar{f}} - (P_f - P_{\bar{f}}) A_f$, $P_2 \equiv (1 - P_f P_{\bar{f}}) A_f - P_f + P_{\bar{f}}$, where P_f ($P_{\bar{f}}$) is the fermion (anti-fermion) beam longitudinal polarization.

According to Eq. (9), all the possible directions of the f_X momentum in the X rest frame are equiprobable. Having measured an observable describing Higgs-strahlung (1), one will be able to use Eq. (9) and to put some constraints on the functions A_0 , $A_{\parallel}$, and $A_{\perp}$, thus getting possible intervals for the XZZ couplings g_1 , g_2 , and g_3 .

The differential cross section of the process $e^-e^+ \rightarrow f\bar{f}h$ is obtained in Ref. [27]. However, considering process (1), we take into account that the boson X decays, which happens in reality. Moreover, integration of Eq. (9) yields more precise total and differential cross sections and observables than those which can be derived by integration of the fully differential cross section of $e^-e^+ \rightarrow f\bar{f}h$. The reason is that we can integrate (9) with respect to a_X without the narrow-width approximation and with any desired accuracy.

Integration of Eq. (9) with the narrow-width approximation for both Z and X bosons yields the total cross section of the Higgs-strahlung:

$$\begin{aligned} \sigma &= \frac{G_F^4 m_Z^9 m_X m_{f_X}^2}{288\pi^3 \Gamma_Z \Gamma_X} \cdot \frac{\lambda^{1/2}(s, m_Z^2, m_X^2)}{s D(s)} \\ &\times (a_f^2 + v_f^2)(a_{\bar{f}}^2 + v_{\bar{f}}^2)(|\rho_1|^2 + |\rho_2|^2) P_1 \\ &\times \sum_{p=0,\parallel,\perp} |A_p|^2|_{a_2=m_Z^2, a_X=m_X^2}. \end{aligned} \quad (10)$$

Since $\sigma \sim P_1$, the total cross section has its largest value at $P_f = -1$ and $P_{\bar{f}} = 1$ if $A_f > 0$.

III. OPTIMAL OBSERVABLES

In this section we demonstrate how differential cross section (9) can be used to determine the couplings g_1 , g_2 , g_3 after kinematic information on a number of events (1) is obtained. For this purpose, the method of Refs. [22,23] is applied. According to [22,23], if the fully differential cross section of a process can be presented in the form

$$\frac{d\sigma}{dk} = S_0(k) + \sum_{i=1}^n h_i S_i(k), \quad (11)$$

where k denotes all the kinematic variables of this process, $S_0(k), S_1(k), \dots, S_n(k)$ are functions of k , and h_i ($i = 1, \dots, n$) are real-valued small (i.e., $h_i \rightarrow 0$) dimensionless parameters independent of k , then the measurement of an observable

$$O_i \equiv \frac{1}{\sigma} \int dk \frac{d\sigma}{dk} \cdot \frac{S_i(k)}{S_0(k)} \quad (12)$$

yields the tightest constraint on the parameter h_i . “The tightest constraint on a parameter” means an experimental value of the parameter with the least standard deviation that is possible for measured data for the studied process. In (12) σ is the total cross section of the process:

$$\sigma \equiv \int dk \frac{d\sigma}{dk}. \quad (13)$$

The integration in Eqs. (12) and (13) is performed over all the possible values of the variables k . In Ref. [23] observables (12) are called optimal.

In our case, differential cross section (9) can be rewritten as

$$\begin{aligned} \frac{d\sigma}{dk} &= (|\rho_1|^2 + |\rho_2|^2)[f_{\text{SM}}(k) + (|g_1|^2 - 1)f_1(k) + |g_2|^2 f_2(k) + |g_3|^2 f_3(k) + \text{Re}(g_1^* g_2) f_4(k) + \text{Re}(g_1^* g_3) f_5(k) \\ &+ \text{Re}(g_2^* g_3) f_6(k) + \text{Im}(g_1^* g_2) f_7(k) + \text{Im}(g_1^* g_3) f_8(k) + \text{Im}(g_2^* g_3) f_9(k)], \end{aligned} \quad (14)$$

where $dk \equiv da_2 da_X d\theta_Z d\theta_{f_Z} d\phi d\theta_{f_X} d\phi_{f_X}$ and $f_{\text{SM}}(k), f_1(k) = f_{\text{SM}}(k), f_2(k), f_3(k), \dots, f_9(k)$ are some particular functions of the invariant masses squared a_2 , a_X , of the angular variables θ_Z , θ_{f_Z} , ϕ , θ_{f_X} , of the polarizations P_f , $P_{\bar{f}}$, and of the squared invariant energy s .

In the SM

$$\frac{d\sigma}{dk} = (|\rho_1|^2 + |\rho_2|^2) f_{\text{SM}}(k). \quad (15)$$

We suppose that the couplings g_1 , g_2 , and g_3 in (3) are close to their SM values and do not depend on a_1 or a_2 , i.e., $g_1(a_1, a_2) \approx 1$, $g_2(a_1, a_2) \approx 0$, $g_3(a_1, a_2) \approx 0$. Then Eq. (14) has the form of Eq. (11) with $n = 9$ and

$$\begin{aligned}
S_0(k) &= S_1(k) = (|\rho_1|^2 + |\rho_2|^2)f_{\text{SM}}(k), \\
S_2(k) &= (|\rho_1|^2 + |\rho_2|^2)f_2(k), \\
&\dots \\
S_9(k) &= (|\rho_1|^2 + |\rho_2|^2)f_9(k), \\
h_1 &= |g_1|^2 - 1, h_2 = |g_2|^2, h_3 = |g_3|^2, \\
h_4 &= \text{Re}(g_1^* g_2), \dots, h_9 = \text{Im}(g_2^* g_3). \quad (16)
\end{aligned}$$

Therefore, the observable O_1 for the Higgs-strahlung is

$$O_1 \equiv \frac{1}{\sigma} \int dk \frac{d\sigma}{dk} = 1. \quad (17)$$

Thus, this observable provides no information on the quantity $|g_1|^2 - 1$, i.e., no constraint on $|g_1|$. If we measure the fully differential distribution $\frac{1}{\sigma} \frac{d\sigma}{dk}$ of the Higgs-strahlung, we will gain no constraint on $|g_1|$ as well. The reason is that according to Eq. (14), the dependence of $\frac{1}{\sigma} \frac{d\sigma}{dk}$ on the couplings g_1 , g_2 , and g_3 reduces only to a dependence on the fractions $\frac{g_2}{g_1}$ and $\frac{g_3}{g_1}$. Thus, the measurement of the

TABLE II. The parameters used to calculate optimal observables (18) for the Higgs-strahlung. These parameters are taken from Refs. [7,28]. For Γ_h we use its SM value.

$P_{e^-} = -0.8$, $P_{e^+} = 0.3$, $\sqrt{s} = 250$ GeV
$A_e = 0.1515$
$m_h = 125.09$ GeV, $\Gamma_h = 4.15$ MeV
$m_Z = 91.1876$ GeV, $\Gamma_Z = 2.4952$ GeV

fully differential distribution yields constraints only on the latter fractions. To constrain $|g_1|$, one can measure σ .

We calculate numerically the observables

$$O_i \equiv \frac{1}{\sigma} \int dk \frac{d\sigma}{dk} \frac{f_i(k)}{f_{\text{SM}}(k)}, \quad i = 2, 3, \dots, 9 \quad (18)$$

for process (1) with $f = e$, using the values listed in Table II. In this table we show the $\sqrt{s}$, P_{e^-} , and P_{e^+} values planned for the first stage of the ILC (see [7]).

Observables (18) calculated for process (1) with $f = f_Z = e$ and $f_X = e, \mu, \tau, u, d, s, c, b$ are

$$\begin{aligned}
O_2 &= G^{-1} \left(34.1 + 1281.8 \left| \frac{g_2}{g_1} \right|^2 + 311.2 \left| \frac{g_3}{g_1} \right|^2 - 395.4 \text{Re} \left(\frac{g_2}{g_1} \right) \right), \\
O_3 &= G^{-1} \left(8.0 + 311.2 \left| \frac{g_2}{g_1} \right|^2 + 82.7 \left| \frac{g_3}{g_1} \right|^2 - 94.0 \text{Re} \left(\frac{g_2}{g_1} \right) \right), \\
O_4 &= G^{-1} \left(-11.5 - 395.4 \left| \frac{g_2}{g_1} \right|^2 - 94.0 \left| \frac{g_3}{g_1} \right|^2 + 132.9 \text{Re} \left(\frac{g_2}{g_1} \right) \right), \\
O_5 &= G^{-1} \left(1.1 \text{Re} \left(\frac{g_3}{g_1} \right) - 6.6 \text{Re} \left(\left(\frac{g_2}{g_1} \right)^* \frac{g_3}{g_1} \right) \right), \\
O_6 &= G^{-1} \left(-6.6 \text{Re} \left(\frac{g_3}{g_1} \right) + 40.1 \text{Re} \left(\left(\frac{g_2}{g_1} \right)^* \frac{g_3}{g_1} \right) \right), \\
O_7 &= G^{-1} \cdot 0.24 \text{Im} \left(\frac{g_2}{g_1} \right), \\
O_8 &= G^{-1} \left(13.6 \text{Im} \left(\frac{g_3}{g_1} \right) - 88.7 \text{Im} \left(\left(\frac{g_2}{g_1} \right)^* \frac{g_3}{g_1} \right) \right), \\
O_9 &= G^{-1} \left(-88.7 \text{Im} \left(\frac{g_3}{g_1} \right) + 583.4 \text{Im} \left(\left(\frac{g_2}{g_1} \right)^* \frac{g_3}{g_1} \right) \right), \quad (19)
\end{aligned}$$

where

$$G \equiv 1 + 34.1 \left| \frac{g_2}{g_1} \right|^2 + 8.0 \left| \frac{g_3}{g_1} \right|^2 - 11.5 \text{Re} \left(\frac{g_2}{g_1} \right). \quad (20)$$

Observables (19) depend only on the ratios $\frac{g_2}{g_1}$ and $\frac{g_3}{g_1}$. Thus, their measurement gives constraints only on the real quantities $\text{Re}(\frac{g_2}{g_1})$, $\text{Im}(\frac{g_2}{g_1})$, $\text{Re}(\frac{g_3}{g_1})$, and $\text{Im}(\frac{g_3}{g_1})$.

If the couplings g_1 , g_2 , and g_3 are real, the Lagrangian describing the interaction of the Higgs boson with two Z bosons is Hermitian. Considering only real values of $\frac{g_2}{g_1}$ and $\frac{g_3}{g_1}$, we present plots of the observables $O_2, \dots, O_6$ in Fig. 3. The observables O_7 , O_8 , and O_9 are all zero if $\frac{g_2}{g_1}$ and $\frac{g_3}{g_1}$ are real [see Eq. (19)].

The observables O_2 , O_3 , and O_4 are approximately proportional to each other. Moreover, we have analytically

FIG. 3. Plots of the observables $O_2, \dots, O_6$ for $\frac{g_2}{g_1}, \frac{g_3}{g_1} \in [-0.5, 0.5]$.

investigated the critical points of $O_2, \dots, O_6$. Our calculation shows that each of O_2 , O_3 , and O_4 has a local maximum at $\frac{g_2}{g_1} \approx 0.17$ and $\frac{g_3}{g_1} = 0$. Both O_5 and O_6 have a saddle point at the same point $\frac{g_2}{g_1} \approx 0.17$, $\frac{g_3}{g_1} = 0$. The closer the values of $O_2, \dots, O_6$ are to their values at the maximum or saddle points, the faster these observables change and the tighter constraints on $\frac{g_2}{g_1}$ and $\frac{g_3}{g_1}$ these observables provide.

If one calculates observables (18) for process (1) with $f = e$, $f_Z = e$, μ and $f_X = e, \mu, \tau, u, d, s, c, b$, then in the approximation $A_e = A_\mu$ the result still will be Eqs. (19). In order to take into account the difference between A_e and A_μ ($A_e = 0.1515$ and $A_\mu = 0.142$ [28]), one can edit Supplemental Material [29]. Moreover, in an experiment, events (1) cannot be measured with all the possible values

of the kinematic variables k . To account for that, one can change the limits of integration for observables (18) in [29].

IV. CONCLUSIONS

We have derived the fully differential cross section of the Higgs-strahlung process $f\bar{f} \rightarrow Z \rightarrow ZX \rightarrow f_Z\bar{f}_Z f_X\bar{f}_X$, where f, f_Z , and f_X are arbitrary fermions; X is a particle with zero spin and arbitrary ZZ and ff couplings. This process with $f = e$, $X = h$, $f_Z = e, \mu, u, d, s, c, b$, and $f_h = \tau, b$ is going to be measured at the ILC.

We have defined the observables, measurement of which gives the tightest constraints on the Higgs boson couplings g_1, g_2, g_3 to Z bosons. These observables are called optimal. Their explicit dependences on g_i are presented.

The sensitivity of the observable to the couplings is analyzed. Therefore, after the optimal observables are measured at the ILC, one can constrain the Higgs boson couplings g_i . Using [29], one can account for experimental constraints on the kinematic variables or derive explicit expressions of optimal observables for other similar processes.

ACKNOWLEDGMENTS

This research was partially supported by the National Academy of Sciences of Ukraine (Project No. Ts-3/53-2018) and the Ministry of Education and Science of Ukraine (Project No. 0117U004866).

-
- [1] G. Aad *et al.* (ATLAS Collaboration), *Phys. Lett. B* **716**, 1 (2012).
 - [2] S. Chatrchyan *et al.* (CMS Collaboration), *Phys. Lett. B* **716**, 30 (2012).
 - [3] P. W. Higgs, *Phys. Rev. Lett.* **13**, 508 (1964).
 - [4] A. M. Sirunyan *et al.* (CMS Collaboration), *Phys. Lett. B* **775**, 1 (2017).
 - [5] M. Aaboud *et al.* (ATLAS Collaboration), *J. High Energy Phys.* **03** (2018) 095.
 - [6] G. Aad *et al.* (ATLAS and CMS Collaborations), *J. High Energy Phys.* **08** (2016) 045.
 - [7] D. M. Asner *et al.*, [arXiv:1310.0763](#).
 - [8] I. Anderson *et al.*, *Phys. Rev. D* **89**, 035007 (2014).
 - [9] M. Hashemi, [arXiv:1805.10513](#).
 - [10] P. Drechsel, G. Moortgat-Pick, and G. Weiglein, [arXiv:1801.09662](#).
 - [11] D. Barducci and A. J. Helmboldt, *J. High Energy Phys.* **12** (2017) 105.
 - [12] T. Kamon, P. Ko, and J. Li, *Eur. Phys. J. C* **77**, 652 (2017).
 - [13] A. Angelescu, G. Moreau, and F. Richard, *Phys. Rev. D* **96**, 015019 (2017).
 - [14] G. L. Liu, F. Wang, K. Xie, and X. F. Guo, *Phys. Rev. D* **96**, 035005 (2017).
 - [15] M. Frank, K. Huitu, U. Maitra, and M. Patra, *Phys. Rev. D* **94**, 055016 (2016).
 - [16] I. Chakraborty, A. Datta, and A. Kundu, *J. Phys. G* **43**, 125001 (2016).
 - [17] S. Banerjee, B. Bhattacharjee, M. Mitra, and M. Spannowsky, *J. High Energy Phys.* **07** (2016) 059.
 - [18] J. P. Ma and B. H. J. McKellar, *Phys. Rev. D* **52**, 22 (1995).
 - [19] T. Han and J. Jiang, *Phys. Rev. D* **63**, 096007 (2001).
 - [20] D. Chang, W. Y. Keung, and I. Phillips, *Phys. Rev. D* **48**, 3225 (1993).
 - [21] K. Rao and S. D. Rindani, *Phys. Lett. B* **642**, 85 (2006).
 - [22] D. Atwood and A. Soni, *Phys. Rev. D* **45**, 2405 (1992).
 - [23] M. Diehl, O. Nachtmann, and F. Nagel, *Eur. Phys. J. C* **27**, 375 (2003).
 - [24] T. V. Zagoskin and A. Y. Korchin, *Int. J. Mod. Phys. A* **32**, 1750166 (2017).
 - [25] T. V. Zagoskin and A. Y. Korchin, *J. Exp. Theor. Phys.* **122**, 663 (2016).
 - [26] Y. Gao, A. V. Gritsan, Z. Guo, K. Melnikov, M. Schulze, and N. V. Tran, *Phys. Rev. D* **81**, 075022 (2010).
 - [27] K. Hagiwara and M. L. Stong, *Z. Phys. C* **62**, 99 (1994).
 - [28] C. Patrignani *et al.* (Particle Data Group), *Chin. Phys. C* **40**, 100001 (2016) and 2017 update.
 - [29] A Mathematica notebook where the optimal observables for the Higgs-strahlung are worked out is available from the authors upon request.