Probing the Weizsäcker-Williams gluon Wigner distribution in pp collisions

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We show that the diffractive forward production of two quarkonia, especially the χ_{cJ} and η_c states, in proton-proton or proton-ion collisions can access the Weizsäcker-Williams gluon Wigner distribution of the proton. We use the hybrid factorization approach where the collinear, double gluon PDF is applied for one of the protons and the k_T -dependent (Wigner or GTMD) distribution for the other. The production of quarkonia is treated in the NRQCD framework. A particularly concise formula is obtained for double $\chi_{J=1}$ production.

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I. INTRODUCTION

In hadron physics research, one of the ultimate goals is to depict the colorful and kaleidoscopic multidimensional landscape of the internal structure of hadrons including nucleons and nucleus. In particular, in addition to the longitudinal momentum distribution of partons inside hadrons as given by the Feynman parton distributions, we also intend to learn about the transverse spatial [generalized parton distributions (GPD)] and transverse momentum [transverse momentum dependent (TMD)] distributions. The so-called quantum phase space Wigner distributions [1–3] encode all the above important information and are viewed as the mother distributions of all. In practice, since it is rather difficult to directly measure the spatial transverse coordinates in high energy scatterings, we also normally define the generalized transverse momentum distribution (GTMD) [4–6] as the Fourier transform of the corresponding Wigner distribution for quarks and gluons.

Interestingly, as pointed out in Refs. [7–11], quark and gluon TMDs are not unique due to different possible structures of gauge links representing the initial and final state interactions. As far as the gluon TMDs are concerned, there are at least two nontrivial gluon distributions whose difference becomes crucial especially in the small- x region. One is known as the dipole gluon distribution which

contains the dipole-shaped configuration of gauge links, while the other is called the Weizsäcker-Williams (WW) gluon distribution which features either a future-pointing or past-pointing gauge link, but not both. Naturally, one expects that this kind of complication should also persist in the case of Wigner distributions and GTMDs.

Generally speaking, up to now, the HERA experiment has provided us with the most precise knowledge of Feynman parton distributions for a large range of x region, through the inclusive deep inelastic scattering (DIS) process. Furthermore, less inclusive processes, such as the semi-inclusive DIS (SIDIS) and the deeply virtual Compton scattering (DVCS) can help us gain insights on the TMDs and GPDs, respectively. The important question here is whether one can probe the Wigner distributions or GTMDs experimentally. It is rather challenging to achieve such a goal because, obviously, one has to consider more exclusive/complicated processes than SIDIS and DVCS.

Recently, we have found that the goal of measuring the gluon Wigner distributions can be achieved at the future electron-hadron colliders [12], such as the planned electron-ion colliders (EIC) and the large hadron electron collider (LHeC) [13–15]. Specifically, the *dipole* gluon Wigner or GTMD can be probed in diffractive dijet process in the small- x region at EIC [16]. Subsequently, there have been a lot of progress on this topic in the last two years [17–28]. Especially, one can access and study the extremely evasive gluon orbital angular momentum with the help of the spin-dependent gluon GTMD as discussed in Refs. [20,21,23,27,29–31].

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On the other hand, it has been realized already in Ref. [12] that the Weizsäcker-Williams Wigner (WW) distribution is more difficult to access. One possibility, as suggested in [12], was to look at a process involving two incoming photons and four outgoing jets at EIC. However, the transition rate would be too small to make it a realistic measurement. The objective of this paper is to show that it is possible to rather directly probe the WW distribution and WW type GTMD for small- x gluons in the production of two heavy scalar quarkonia such as χ_c and η_c in the forward rapidity region in diffractive pp and pA collisions $pp, pA \rightarrow \chi_c \chi_c pX$.

The rest of the paper is organized as follows. We first introduce the WW gluon GTMD and briefly describe the process we will consider in this paper, i.e., double quarkonia production in the hard diffractive processes in pp and pA collisions. In Sec. III, we derived the amplitude for the double quarkonia production through double gluon scattering on the nucleon/nucleus targets. These amplitudes will be converted into pp/pA collisions cross sections in Sec. IV by applying the double parton scattering framework for the incoming two gluons from the projectile. In Sec. III and IV, we derive the results for both η and χ . In particular, we will present the explicit differential cross sections on different combinations of two quarkonia states of $\eta, \chi_{0,1,2}$. Finally, we summarize our paper in Sec. VI.

II. WEIZSÄCKER-WILLIAMS GLUON GTMD

We start by writing down the Weizsäcker-Williams (WW) gluon GTMD. For the “target” proton fast-moving in the negative z -direction we define

$$\begin{aligned} x\mathcal{G}^{ij}(\mathbf{K}, \Delta) &\equiv 2 \int \frac{d^3z}{P^-(2\pi)^3} e^{ixP^-z^+ - i\mathbf{K}\cdot\mathbf{z}} \\ &\times \left\langle P - \frac{\Delta}{2} \left| \text{Tr}[U_\pm^\dagger F^{-i}(z/2) U_\pm] \right. \right. \\ &\times \left. \left. F^{-j}(-z/2) \right| P + \frac{\Delta}{2} \right\rangle, \end{aligned} \quad (1)$$

where $U_\pm$ is the staple-shaped fundamental Wilson line connecting the points $z/2$ and $-z/2$ via light-like Wilson lines encircling $x^\pm = \pm\infty$. Boldface letters denote two-dimensional vectors. We have kept the transverse indices $i, j = 1, 2$ open because this is what we shall need in later calculations. In the small- x region, we may approximate $e^{ixP^-z^+} \approx 1$ and find (cf., [11])

$$\begin{aligned} x\mathcal{G}^{ij}(\mathbf{K}, \Delta) &\approx -\frac{2}{\alpha_s} \int \frac{d^2b_1 d^2b_2}{(2\pi)^4} e^{-i\Delta \cdot \frac{b_1+b_2}{2} - i\mathbf{K}\cdot(\mathbf{b}_1-\mathbf{b}_2)} \\ &\times \frac{\langle P - \frac{\Delta}{2} \left| \text{Tr}[(\partial^i U_{b_1}^\dagger) U_{b_1} (\partial^j U_{b_2}^\dagger) U_{b_2}] \right| P + \frac{\Delta}{2} \rangle}{\langle P|P \rangle}, \end{aligned} \quad (2)$$

where U_b is the lightlike Wilson line from $x^+ = -\infty$ to $x^+ = +\infty$ at fixed transverse position $\mathbf{b}$. It is easy

to check from (2) the relations $\mathcal{G}^{ij}(\mathbf{K}, \Delta) = \mathcal{G}^{ji}(-\mathbf{K}, \Delta) = (\mathcal{G}^{ij}(-\mathbf{K}, -\Delta))^*$. The general parametrization is thus¹

$$\begin{aligned} \mathcal{G}^{ij}(\mathbf{K}, \Delta) &\equiv \delta^{ij} \mathcal{G}_1 + \left(\frac{\mathbf{K}^i \mathbf{K}^j}{\mathbf{K}^2} - \frac{\delta^{ij}}{2} \right) \frac{\mathbf{K}^2}{M^2} \mathcal{G}_2 \\ &+ \left(\frac{\Delta^i \Delta^j}{\Delta^2} - \frac{\delta^{ij}}{2} \right) \frac{\Delta^2}{M^2} \mathcal{G}_3 + \left(\frac{\mathbf{K}^i \Delta^j - \Delta^i \mathbf{K}^j}{M^2} \right) \mathcal{G}_4, \end{aligned} \quad (3)$$

where M is the nucleon mass. $\mathcal{G}_{1,2,3,4}$ are all real and depend on $\mathbf{K}^2, \Delta^2$ and $(\mathbf{K} \cdot \Delta)^2$. In the forward limit $\Delta = 0$, $\mathcal{G}_1$ and $\mathcal{G}_2$ reduce to the unpolarized and linearly polarized WW gluon TMDs, respectively. The $\mathbf{K}$ -integral of $\mathcal{G}_3$ is proportional to the gluon transversity GPD which in turn is related to the so-called elliptic gluon Wigner distribution [24]. From what we know about the latter distribution [12,18,22], we presume that $\mathcal{G}_3$ is numerically small, on the order of a few percent effect compared to $\mathcal{G}_{1,2}$.

Various experimental processes have been identified to probe the WW gluon TMD [11]. On the other hand, the measurement of the GTMDs is more challenging, and most of the proposals so far concern the dipole gluon GTMD whose treatment is somewhat simpler because of its relation to the dipole S-matrix [12]. In this paper, we show that double quarkonium production in pp collisions $pp \rightarrow M_1 M_2 pX$ is a very sensitive observable for the WW gluon GTMD. This is a natural extension of the previous observation [33] (see, also, [34]) that the WW gluon TMD can be probed in single quarkonium production $pp \rightarrow MX$. The hard subprocess and the full process are respectively depicted in Fig. 1 and in Fig. 2. Two gluons are emitted from the projectile proton and scatter off the shockwave field created by the target proton. In the final state, we measure two quarkonia with momentum $\mathbf{K}_1$ and $\mathbf{K}_2$ in the forward region as well as the elastically scattered target with momentum transfer $\Delta = \mathbf{K}_1 + \mathbf{K}_2$ in the backward region. To lowest order, the shockwave consists of two gluons in the t -channel, and we can view this diagram as the square of the $gg^* \rightarrow M$ amplitude. This means that the produced quarkonia must be C -even states, such as η_c and χ_{cJ} (and their bottomonium counterparts). We study this process in the “hybrid factorization” approach (collinear gluons from one proton and noncollinear gluons from the other proton) and show that the cross section can be written as a convolution of the square of the WW gluon GTMD of the target and the double gluon PDF of the projectile.

It should be mentioned that a very similar idea came out recently in Ref. [31] where the authors proposed to measure the gluon GTMD (including its spin dependence) in doubly-diffractive double η_c production $pp \rightarrow pp\eta_c\eta_c$ for moderate values of x . Their argument is limited to the two-gluon exchange level where the difference between

¹Naively, there is another term proportional to the tensor structure $\mathbf{K}^i \Delta^j + \mathbf{K}^j \Delta^i - \mathbf{K} \cdot \Delta \delta^{ij}$. However, as pointed out in [32], this term is not independent and can be absorbed in $\mathcal{G}_{2,3}$. See, also, [5].

FIG. 1. Production of 2 quarkonia from 2 gluons.

FIG. 2. Production of a quarkonium pair in hybrid factorization with double scattering on the projectile side.

WW and dipole distributions becomes immaterial. Our process is single-diffractive and we do not consider spin effects. Instead, we focus on the forward, small- x region and carefully examine the structure of the relevant Wilson lines. Our result suggests that the GTMD discussed in [31] likely becomes the WW GTMD once higher order rescattering effects are taken into account.

III. QUARKONIUM PRODUCTION AMPLITUDE

A. Single quarkonium production

As a warm-up, we first compute the amplitude for the single quarkonium production in the hybrid factorization approach combined with the nonrelativistic QCD (NRQCD) framework. Related calculations can be found in the literature, e.g., [33,34]. The right-moving projectile

FIG. 3. Open charm production.

proton is treated as a dilute object, i.e., partons are extracted from the projectile using regular parton distribution functions, whereas the left-moving target is treated as a shockwave. The subamplitude for the open production of a massive $q\bar{q}$ pair from a gluon (see Fig. 3) can be straightforwardly written as, in $D = d + 2$ dimensions,²

$$(\mathcal{A}_\sigma)^c = -ig \int d^D z_0 \bar{u}(p_q, z_0) \gamma^\mu t^b \epsilon_\mu^{cb}(k, z_0) v(p_{\bar{q}}, z_0), \quad (7)$$

where, with similar methods as in [35,36] one can derive the effective quark, antiquark, and gluon lines as obtained for example in [17,37]:

$$\begin{aligned} \bar{u}(p_q, z_0) = & \frac{1}{2} \left(\frac{p_q^+}{2\pi} \right)^{\frac{d}{2}} \int d^d \mathbf{x}_1 e^{i p_q^+ (z_0 - \frac{(x_1 - z_0)^2}{2z_0^+} + i0) - i \mathbf{p}_q \cdot \mathbf{x}_1 + i \frac{z_0^+}{2p_q^+} (m^2 + i0)} \\ & \times \left(\frac{i}{z_0^+} \right)^{\frac{d}{2}} \bar{u}_{p_q} \gamma^+ [U_{\mathbf{x}_1} \theta(-z_0^+) + \theta(z_0^+)] \\ & \times \left(\gamma^- - \frac{\not{x}_{1\perp} - \not{z}_{0\perp}}{z_0^+} + \frac{m}{p_q^+} \right), \end{aligned} \quad (8)$$

$$\begin{aligned} v(p_{\bar{q}}, z_0) = & \frac{1}{2} \left(\frac{p_{\bar{q}}^+}{2\pi} \right)^{\frac{d}{2}} \int d^d \mathbf{x}_2 e^{i p_{\bar{q}}^+ (z_0 - \frac{(x_2 - z_0)^2}{2z_0^+} + i0) - i \mathbf{p}_{\bar{q}} \cdot \mathbf{x}_2 + i \frac{z_0^+}{2p_{\bar{q}}^+} (m^2 + i0)} \\ & \times \left(\frac{i}{z_0^+} \right)^{\frac{d}{2}} \left(\gamma^- - \frac{\not{x}_{2\perp} - \not{z}_{0\perp}}{z_0^+} - \frac{m}{p_{\bar{q}}^+} \right) \\ & \times [U_{\mathbf{x}_2}^\dagger \theta(-z_0^+) + \theta(z_0^+)] \gamma^+ v_{p_{\bar{q}}}, \end{aligned} \quad (9)$$

²Our notation is as follows: We define two lightlike vectors n_1, n_2 such that the projectile flies along n_1 and the target flies along n_2 , and such that $n_1 \cdot n_2 = 1$. Lightcone coordinates are then defined as

$$k^\mu \equiv k^+ n_1^\mu + k^- n_2^\mu + k_\perp^\mu, \quad (4)$$

$$k \cdot l \equiv k^+ l^- + k^- l^+ + k_\perp \cdot l_\perp \equiv k^+ l^- + k^- l^+ - (\mathbf{k} \cdot \mathbf{l}). \quad (5)$$

The metric tensor will be decomposed into its longitudinal and transverse parts as

$$g^{\mu\nu} \equiv n_1^\mu n_2^\nu + n_1^\nu n_2^\mu + g_\perp^{\mu\nu}. \quad (6)$$

$$\begin{aligned} \varepsilon_\mu^{ba}(k, z_0) &= \left(\frac{k^+}{2\pi}\right)^{\frac{d}{2}} \int d^d \mathbf{x}_0 e^{-ik^+(z_0 - \frac{(x_0 - z_0)^2}{2z_0^+} - i0) + ik \cdot \mathbf{x}_0} \\ &\times \left(\frac{-i}{z_0^+}\right)^{\frac{d}{2}} \left(g_{\perp\mu\sigma} + \frac{x_{0\perp\sigma} - z_{0\perp\sigma}}{z_0^+} n_{2\mu}\right) \\ &\times [U_{x_0}^{ba}\theta(z_0^+) + \delta^{ab}\theta(-z_0^+)] \varepsilon_{k\perp}^\sigma. \end{aligned} \quad (10)$$

In the above, m is the heavy quark mass and we have kept the transverse momentum of the incoming gluon $\mathbf{k}$. U is the Wilson line which arises after the eikonal interaction with the target field A^-

$$U_x \equiv \mathcal{P} e^{ig \int_{-\infty}^{\infty} dx^+ A^-(x)}, \quad (11)$$

We will use the conventions that lines with color indices as superscripts U_x^{ab} are in the adjoint representation while lines without color indices are in the fundamental representation.

Integrating over the interaction point z_0 , we get

$$(\mathcal{A}_\sigma)^c = \varepsilon_{k\perp}^\sigma \bar{u}_{p_q} \mathcal{M}_\sigma^c v_{p_{\bar{q}}}, \quad (12)$$

where

$$\begin{aligned} \mathcal{M}_\sigma^c &= -\frac{ig}{2} (2\pi) \delta(p_q^+ + p_{\bar{q}}^+ - k^+) \left(\frac{p_q^+ p_{\bar{q}}^+ k^+}{2\pi}\right)^{\frac{d}{2}} \int d^d \mathbf{x}_1 d^d \mathbf{x}_2 d^d \mathbf{x}_0 e^{-ip_q \cdot \mathbf{x}_1 - ip_{\bar{q}} \cdot \mathbf{x}_2 + ik \cdot \mathbf{x}_0} \delta(p_q^+ \mathbf{x}_1 + p_{\bar{q}}^+ \mathbf{x}_2 - k^+ \mathbf{x}_0) \\ &\times \left[\frac{1}{z_0^+} \left(\gamma_{\perp\sigma} \not{x}_{12\perp} - 2 \frac{p_{\bar{q}}^+}{k^+} x_{12\perp\sigma} \right) - m \frac{k^+}{p_q^+ p_{\bar{q}}^+} \gamma_{\perp\sigma} \right] \gamma^+ \left[(-i)^{\frac{d}{2}} \int_{-\infty}^0 dz_0^+ (-z_0^+)^{-\frac{d}{2}} (U_{x_1} t^c U_{x_2}^\dagger) e^{i \frac{k^+(m^2 - i0)}{2p_q^+ p_{\bar{q}}^+} z_0^+ - i \frac{p_{\bar{q}}^+ p_{\bar{q}}^+ x_{12\perp}^2}{2k^+ z_0^+}} \right. \\ &\left. + i^{\frac{d}{2}} \int_0^{+\infty} dz_0^+ (z_0^+)^{-\frac{d}{2}} (t^d U_{x_0}^{cd}) e^{i \frac{k^+(m^2 + i0)}{2p_q^+ p_{\bar{q}}^+} z_0^+ - i \frac{p_{\bar{q}}^+ p_{\bar{q}}^+ x_{12\perp}^2}{2k^+ z_0^+}} \right]. \end{aligned} \quad (13)$$

Using the integrals, for $Q, Z > 0$:

$$\begin{aligned} \int_{-\infty}^0 dz^+ (-z^+)^{-n} e^{i(Q-i0)z^+ - i \frac{Z+i0}{z^+}} &= 2(-i)^{1-n} \left(\sqrt{\frac{Z}{Q}}\right)^{1-n} K_{n-1}(2\sqrt{QZ}) \\ \int_0^{+\infty} dz^+ (z^+)^{-n} e^{i(Q+i0)z^+ - i \frac{Z-i0}{z^+}} &= 2i^{1-n} \left(\sqrt{\frac{Z}{Q}}\right)^{1-n} K_{n-1}(2\sqrt{QZ}), \end{aligned} \quad (14)$$

we get

$$\begin{aligned} \mathcal{M}_\sigma^c &= ig \delta(p_q^+ + p_{\bar{q}}^+ - k^+) \int d^d \mathbf{x}_1 d^d \mathbf{x}_2 d^d \mathbf{x}_0 e^{-ip_q \cdot \mathbf{x}_1 - ip_{\bar{q}} \cdot \mathbf{x}_2 + ik \cdot \mathbf{x}_0} \\ &\times (2\pi) \delta(p_q^+ \mathbf{x}_1 + p_{\bar{q}}^+ \mathbf{x}_2 - k^+ \mathbf{x}_0) \left(\frac{m(k^+)^2}{2\pi |\vec{x}_{12}|}\right)^{\frac{d}{2}} (U_{x_1} t^c U_{x_2}^\dagger - t^d U_{x_0}^{cd}) \\ &\times \left[K_{\frac{d}{2}}(m|\mathbf{x}_{12}|) \left(\gamma_{\perp\sigma} \not{x}_{12\perp} - 2 \frac{p_{\bar{q}}^+}{k^+} x_{12\perp\sigma} \right) - i |\mathbf{x}_{12}| K_{\frac{d}{2}-1}(m|\mathbf{x}_{12}|) \gamma_{\perp\sigma} \right] \gamma^+. \end{aligned} \quad (15)$$

In four dimensions and after a few simple changes of variables, this can finally be rewritten as the known result:

$$\begin{aligned} \mathcal{M}_\sigma^c &= img \delta(K^+ - k^+) \int d^2 \mathbf{b} d^2 \mathbf{r} e^{-i(K-k) \cdot \mathbf{b} + iq^+ (\frac{\mathbf{k}}{K^+} - \frac{\mathbf{q}}{q^+}) \cdot \mathbf{r}} (U_{b+\frac{\mathbf{r}}{2}} t^c U_{b-\frac{\mathbf{r}}{2}}^\dagger - t^d U_b^{cd}) \\ &\times \left[\frac{\gamma_{\perp\sigma} \not{\mathbf{r}}_{\perp} - (1 - 2 \frac{q^+}{K^+}) \mathbf{r}_{\perp\sigma}}{|\mathbf{r}|} K_1(m|\mathbf{r}|) - i K_0(m|\mathbf{r}|) \gamma_{\perp\sigma} \right] \gamma^+. \end{aligned} \quad (16)$$

Our next step is to convolute this amplitude with the transition probability to a quarkonium in the NRQCD approach where one expands the charmonium wave function as a series in powers of the relative velocity of its constituents. As we have noted already, we shall be interested in C -even quarkonia, η and $(\chi_J)_{J=0,1,2}$. The corresponding projectors for the pseudoscalar $n^{2s+1}L_0$ and vector $n^{2s+1}L_1$ states are

$$\begin{aligned}\Pi_0 &\equiv \frac{1}{\sqrt{8m^3}} \left(\frac{\mathbf{K}}{2} - \mathbf{q} - m \right) \gamma_5 \left(\frac{\mathbf{K}}{2} + \mathbf{q} + m \right), \\ \Pi_1^\rho &\equiv \frac{1}{\sqrt{8m^3}} \left(\frac{\mathbf{K}}{2} - \mathbf{q} - m \right) \gamma^\rho \left(\frac{\mathbf{K}}{2} + \mathbf{q} + m \right).\end{aligned}\quad (17)$$

Then the perturbative part of the 1S_0 wave and 3P_J wave transitions read respectively:

$$\begin{aligned}\mathcal{A}^{\sigma,c}(^1S_0) &\equiv [\text{Tr}(\Pi_0(\mathcal{M}^\sigma)^c)]_{q=0} \\ &= \sqrt{2m}g\delta(K^+ - k^+)K^+ \int d^2\mathbf{r}d^2\mathbf{b}e^{-i(\mathbf{K}-\mathbf{k})\cdot\mathbf{b}}(U_{b+\frac{\mathbf{r}}{2}}t^c U_{b-\frac{\mathbf{r}}{2}}^\dagger - U_b t^c U_b^\dagger) \epsilon^{\sigma\perp\mu\perp+-} \frac{r_{\perp\mu}}{|\mathbf{r}|} K_1(m|\mathbf{r}|),\end{aligned}\quad (18)$$

and

$$\begin{aligned}\mathcal{A}^{\sigma,c}(^3P_J) &\equiv \epsilon_{(J)}^{\rho\mu} \left[\frac{d}{dq^\rho} \text{Tr}(\Pi_{1\mu}(\mathcal{M}^\sigma)^c) \right]_{q=0} \\ &= ig\sqrt{2m}\epsilon_{(J)\rho\mu}\delta(K^+ - k^+) \int d^2\mathbf{r}d^2\mathbf{b}e^{-i(\mathbf{K}-\mathbf{k})\cdot\mathbf{b}} \left\{ r_{\perp\alpha} K_0(m|\mathbf{r}|) (K^+ g_{\perp}^{\sigma\mu} - K_{\perp}^{\sigma\mu} n_2^\mu) \right. \\ &\quad \times \left[\left(g_{\perp}^{\alpha\rho} - \frac{k_{\perp}^\alpha}{K^+} n_2^\rho \right) \left(U_{b+\frac{\mathbf{r}}{2}} t^c U_{b-\frac{\mathbf{r}}{2}}^\dagger \right) - \left(g_{\perp}^{\alpha\rho} - \frac{K_{\perp}^\alpha}{K^+} n_2^\rho \right) (U_b t^c U_b^\dagger) \right] + \frac{r_{\perp\alpha} K_1(m|\mathbf{r}|)}{m|\mathbf{r}|} \left(U_{b+\frac{\mathbf{r}}{2}} t^c U_{b-\frac{\mathbf{r}}{2}}^\dagger - U_b t^c U_b^\dagger \right) \\ &\quad \times \left[K^+ (g_{\perp}^{\rho\sigma} g_{\perp}^{\alpha\mu} - g_{\perp}^{\alpha\rho} g_{\perp}^{\sigma\mu}) + (K^\mu g_{\perp}^{\alpha\sigma} + K_{\perp}^\alpha g_{\perp}^{\sigma\mu} - K_{\perp}^\sigma g_{\perp}^{\alpha\mu}) n_2^\rho + \left(K_{\perp}^\sigma g_{\perp}^{\alpha\rho} - K_{\perp}^\alpha g_{\perp}^{\rho\sigma} - g_{\perp}^{\alpha\sigma} \frac{m^2}{K^+} n_2^\rho \right) n_2^\mu \right] \Big\}.\end{aligned}\quad (19)$$

Next we use the fact that the modified Bessel functions are peaked around 0 to expand the integrand as

$$\int d^2\mathbf{r} K_n(m|\mathbf{r}|) F(\mathbf{r}) \approx \int d^2\mathbf{r} [K_n(m|\mathbf{r}|) F(\mathbf{0}) + K_n(m|\mathbf{r}|) \partial_\perp^\mu \partial_\mu F(\mathbf{0})]. \quad (20)$$

and perform the $d^2\mathbf{r}$ integrals

$$\int d^2\mathbf{r} K_0(m|\mathbf{r}|) r_{\perp\alpha} r_{\perp}^\nu = -\frac{4\pi}{m^4} g_{\perp\alpha}^\nu \quad \int d^2\mathbf{r} \frac{K_1(m|\mathbf{r}|)}{m|\mathbf{r}|} r_{\perp\alpha} r_{\perp}^\nu = -\frac{2\pi}{m^4} g_{\perp\alpha}^\nu. \quad (21)$$

Finally, we notice that the leading NRQCD contribution for η mesons and for χ_J mesons are color singlet contributions for 1S_0 and for 3P_J waves, respectively. Introducing the NRQCD long distance matrix elements (LDME's) and the color singlet projector

$$\frac{\delta^{ij}}{N_c} \langle \mathcal{O}_\eta(^1S_0^1) \rangle^{\frac{1}{2}}, \quad \frac{\delta^{ij}}{N_c} \langle \mathcal{O}_{\chi_J}(^3P_J^1) \rangle^{\frac{1}{2}}, \quad (22)$$

(ij are color indices and the second superscript “1” denotes color singlet) we arrive at the gluon-to-meson transition amplitudes:

$$\mathcal{A}^{\sigma,c}(\eta) = \frac{2g\pi}{m^3} \sqrt{2m}\delta(K^+ - k^+) \int d^2\mathbf{b}e^{-i(\mathbf{K}-\mathbf{k})\cdot\mathbf{b}} \frac{1}{N_c} \text{Tr}[(\partial_\nu U_b^\dagger) U_b t^c] K^+ \epsilon^{\sigma\perp\nu\perp+-} \langle \mathcal{O}_\eta(^1S_0^1) \rangle^{\frac{1}{2}} \quad (23)$$

for η transitions, and

$$\mathcal{A}^{\sigma,c}(\chi_J) = \frac{2g\pi}{m^3} \sqrt{2m}\delta(K^+ - k^+) \int d^2\mathbf{b}e^{-i(\mathbf{K}-\mathbf{k})\cdot\mathbf{b}} \frac{1}{N_c} \text{Tr}[(\partial_\alpha U_b^\dagger) U_b t^c] \frac{i}{m} \epsilon_{(J)\rho\mu} \mathcal{P}^{\sigma\alpha\rho\mu} \langle \mathcal{O}_{\chi_J}(^3P_J^1) \rangle^{\frac{1}{2}}, \quad (24)$$

for χ_J transitions. We used the transversity condition $K^\rho \epsilon_{J\rho\mu} = 0$ to obtain (25). The tensor structure in (25) reads

$$\mathcal{P}^{\sigma\alpha\rho\mu} \equiv K^+ (g_{\perp}^{\sigma\mu} g_{\perp}^{\alpha\rho} + g_{\perp}^{\alpha\mu} g_{\perp}^{\rho\sigma}) - (K_{\perp}^\sigma g_{\perp}^{\alpha\rho} + K_{\perp}^\alpha g_{\perp}^{\rho\sigma}) n_2^\mu + [(K_{\perp}^\alpha - 2k_{\perp}^\alpha) g_{\perp}^{\sigma\mu} - K_{\perp}^\sigma g_{\perp}^{\alpha\mu}] n_2^\rho + \frac{2}{K^+} (k_{\perp}^\alpha K_{\perp}^\sigma - 2m^2 g_{\perp}^{\alpha\sigma}) n_2^\mu n_2^\rho. \quad (25)$$

We shall use the following compact notation which summarizes the above results

$$\mathcal{A}^{\sigma,c}(^{2S+1}L_J) \equiv \frac{2g\pi}{m^3} \sqrt{2m}\delta(K^+ - k^+) \langle \mathcal{O}_M(^{2S+1}L_J^1) \rangle^{\frac{1}{2}} \int d^2\mathbf{b}e^{-i(\mathbf{K}-\mathbf{k})\cdot\mathbf{b}} \frac{1}{N_c} \text{Tr}[(\partial_\alpha U_b^\dagger) U_b t^c] \mathcal{P}^{\sigma\alpha}(^{2S+1}L_J), \quad (26)$$

where the tensor structures read

$$\mathcal{P}^{\sigma\alpha}(^3S_0) \equiv K^+ \epsilon^{\sigma\perp\alpha\perp+-}, \quad (28)$$

$$\mathcal{P}^{\sigma\alpha}(^3P_J) \equiv \frac{i}{m} \epsilon_{(J)\rho\mu} \mathcal{P}^{\sigma\alpha\rho\mu}. \quad (29)$$

In Appendix, we compute the single-inclusive cross section of 3S_0 and 3P_J states in pp (or pA) collisions, in order to check the compatibility our results with the single-inclusive J/ψ production computed in [34].

B. Double quarkonium production

We are now ready to write down the generic diffractive amplitude for the process $gg \rightarrow M_1(^{2S_1+1}L_{1J_1})M_2(^{2S_2+1}L_{2J_2})$. We basically square the $g \rightarrow M$ amplitude (27) and project onto the color singlet state. First let us assume that the final state consists of a $c\bar{c}$ quarkonium and a $b\bar{b}$ quarkonium. In this case the formula

$$\begin{aligned} & \text{Tr}[U_{b_1} t^a (\partial_{a_1} U_{b_1}^\dagger)] \text{Tr}[U_{b_2} t^a (\partial_{a_2} U_{b_2}^\dagger)] \\ &= \frac{1}{2} \text{Tr}[(\partial_{a_1} U_{b_1}^\dagger) U_{b_1} (\partial_{a_2} U_{b_2}^\dagger) U_{b_2}] \end{aligned} \quad (30)$$

immediately gives the core structure of the WW GTMD (2). The full amplitude is

$$\begin{aligned} (\mathcal{S}^{\sigma_1\sigma_2})(M_1, M_2) &= \frac{4g^2\pi^2}{\sqrt{m_1^5 m_2^5}} \frac{\delta(K_1^+ - k_1^+) \delta(K_2^+ - k_2^+)}{N_c^2(N_c^2 - 1)} \\ &\times \int d^2b_1 d^2b_2 e^{-i(K_1 - k_1) \cdot b_1 - i(K_2 - k_2) \cdot b_2} \\ &\times \langle P_T' | \text{Tr}[(\partial_{a_1} U_{b_1}^\dagger) U_{b_1} (\partial_{a_2} U_{b_2}^\dagger) U_{b_2}] | P_T \rangle \\ &\times \langle \mathcal{O}_{M_1}(^{2S_1+1}L_{1J_1}^1) \rangle^{\frac{1}{2}} \langle \mathcal{O}_{M_2}(^{2S_2+1}L_{2J_2}^1) \rangle^{\frac{1}{2}} \\ &\times \mathcal{P}^{\sigma_1\alpha_1}(M_1) \mathcal{P}^{\sigma_2\alpha_2}(M_2). \end{aligned} \quad (31)$$

A complication arises when the pair consists of quarkonia with the same flavor. In this case, there exists an “exchange” diagram in which a quark from one gluon recombines with an antiquark from the other gluon and the remaining $q\bar{q}$ pair forms the second quarkonium. In order for this process to occur, the two quarks and antiquarks have to be all within the distance of order $1/m$. We thus cannot simply Taylor-expand the Wilson line separately in each gluon wave function as we have done above. Instead we must go back to the original expression and find the structure

$$\begin{aligned} & \text{Tr}[U_{x_1} t^c U_{x_2}^\dagger U_{y_1} t^c U_{y_2}^\dagger] \\ &= \frac{1}{2} \text{Tr}[U_{x_2}^\dagger U_{y_1}] \text{Tr}[U_{y_2}^\dagger U_{x_1}] - \frac{1}{2N_c} \text{Tr}[U_{x_1} U_{x_2}^\dagger U_{y_1} U_{y_2}^\dagger], \end{aligned} \quad (32)$$

which is not associated with the WW gluon distribution. While this may seem a problem, it is intuitively clear that such a contribution is negligible in the limit of large quark mass. The probability to find all the four quarks and antiquarks within a small area of order $1/m^2$ is

power suppressed compared to the “direct” contribution³ (see Fig. 1). However, this suppression is difficult to see in the NRQCD framework where to leading order one just multiplies the partonic cross section by the constant LDMEs. We think this is an artifact of the NRQCD approach, and the exchange diagram will be suppressed in a more complete treatment of the problem. Therefore, while (32) and similar results below are valid for the production of a different-flavor pair, strictly speaking, we think they can be also used for a same-flavor pair up to small corrections.

Before leaving this section, we should comment on the color-octet production mechanism. In NRQCD, quarkonia can be produced in a color-octet state [40]. It is well known that, in the collinear factorization framework, the color octet contribution actually dominates over the color-singlet contribution for J/ψ production, and this is also the case for χ_c production at large transverse momentum [41]. In this paper, we instead focus on low transverse momentum quarkonia production $p_T \sim m$ where the color-singlet channel may actually dominate [41], although the NRQCD factorization in this region needs further investigation to clarify this issue.⁴ On the other hand, it has been found that η_c production is always dominated by the color-singlet channel in the whole range of transverse momentum [42,43]. Moreover, in the k_T -factorization approach, χ_c production is clearly dominated by the color-singlet channel [44,45]. Indeed, in the $\chi_{J=1}$ production channel which will be our main focus, the subprocess $gg^* \rightarrow \chi_1^{\text{singlet}}$ is not forbidden by the Landau-Yang theorem because one of the gluons is off-shell in the k_T and hybrid factorizations, although it is forbidden in the collinear factorization. As we are considering the forward production of quarkonia at low to moderate p_T , the use of k_T or hybrid factorization is more appropriate. We thus concentrate on the color-singlet production mechanism in this paper, and leave the color-octet case for future work.

IV. HYBRID FACTORIZATION WITH DOUBLE SCATTERING

A. Double gluon PDF

The 2 gluons-to-2 quarkonia production amplitude (31) is to be squared and convoluted with the double gluon distribution of the projectile proton with momentum P_p . In doing so, one has to be careful about the fact that the Lorentz indices ($\sigma_1\sigma_2$) of the gluons in the amplitude and the complex-conjugate amplitude can in general be different. This forces us to consider the most general double gluon PDF [46]

³For studies of such contributions, the reader is referred to [38] for massive quarks, or [39] for an earlier study with massless quarks.

⁴From our derivation we find that the color-octet contribution is not sensitive to the WWW distribution that we are after. A cleaner approach to avoid this problem is to consider doubly diffractive events, $pp \rightarrow pM_1M_2p$ as in [31]. The cross section is then quartic in the proton Wigner distribution.

$$\mathcal{F}_{a_1 a_2}(x_1, x_2, \mathbf{k}_1, \mathbf{k}_2, q) = \frac{2}{x_1 x_2 P_P^+} \int d^2 \mathbf{k}_1 d^2 \mathbf{k}_2 \int \frac{dr^-}{2\pi} \frac{dz^-}{2\pi} \frac{d^2 \mathbf{r}}{(2\pi)^2} \frac{d^2 \mathbf{z}}{(2\pi)^2} e^{ix_1 P_P^+ r^- + ix_2 P_P^+ z^- - i\mathbf{k}_1 \cdot \mathbf{r} - i\mathbf{k}_2 \cdot \mathbf{z}} \\ \times \int dy^- d^2 \mathbf{y} e^{iq \cdot \mathbf{y}} \langle P_P | \Pi_{a_1}^{i' l'} G^{+i'} \left(-\frac{r}{2} \right) G^{+i} \left(\frac{r}{2} \right) \Pi_{a_2}^{j j'} G^{+j'} \left(y - \frac{z}{2} \right) G^{+j} \left(y + \frac{z}{2} \right) | P_P \rangle_{y^+ = r^+ = z^+ = 0}, \quad (33)$$

where $a_{1,2} = \{g, \Delta g, \delta g\}$ and $\Pi_g^{ii'} \equiv \delta^{ii'}$, $\Pi_{\Delta g}^{ii'} \equiv i\epsilon^{ii'}$, $\Pi_{\delta g}^{ii'} \equiv \tau^{ii', ll'} \equiv \frac{1}{2}(\delta^{il} \delta^{i' l'} + \delta^{i' l} \delta^{i l'} - \delta^{ii'} \delta^{ll'})$ are respectively the unpolarized, longitudinally polarized and linearly polarized projectors. The momentum $\mathbf{q}$ is conjugate to the relative transverse coordinate of the two gluons, and its dependence cannot be completely eliminated [46].

B. Polarization sum

For unpolarized quarkonium production, we square the amplitude (31) and sum over quarkonium polarizations:

$$\mathcal{P}^{\sigma\alpha}(^3S_0) \mathcal{P}^{\sigma'\alpha'}(^3S_0) = (K^+)^2 \epsilon^{\sigma\perp\alpha\perp+-} \epsilon^{\sigma'\perp\alpha'\perp+-}, \quad (34)$$

$$\mathcal{P}^{\sigma\alpha}(^3P_J) \mathcal{P}^{\sigma'\alpha'}(^3P_J) = \sum_{\text{pol}} \frac{\mathcal{E}_{(J)\rho\mu} \mathcal{E}_{(J)\rho'\mu'}}{m^2} \mathcal{P}^{\sigma\alpha\rho\mu} \mathcal{P}^{\sigma'\alpha'\rho'\mu'}, \quad (35)$$

where the indices in the complex conjugate amplitude are denoted with a prime. This can be evaluated explicitly using the following result for 3P_J polarization sums:

$$\begin{aligned} \mathcal{E}_{(0)\rho\mu} \mathcal{E}_{(0)\rho'\mu'}^* &= \frac{1}{3} \Pi_{\rho\mu} \Pi_{\rho'\mu'}, \\ \sum \mathcal{E}_{(1)\rho\mu} \mathcal{E}_{(1)\rho'\mu'}^* &= \frac{1}{2} (\Pi_{\rho\rho'} \Pi_{\mu\mu'} - \Pi_{\rho\mu'} \Pi_{\rho'\mu}), \\ \sum \mathcal{E}_{(2)\rho\mu} \mathcal{E}_{(2)\rho'\mu'}^* &= \frac{1}{2} (\Pi_{\rho\rho'} \Pi_{\mu\mu'} + \Pi_{\rho\mu'} \Pi_{\rho'\mu}) - \frac{1}{3} \Pi_{\rho\mu} \Pi_{\rho'\mu'}, \end{aligned} \quad (36)$$

where

$$\Pi_{\rho\mu} \equiv -g_{\rho\mu} + \frac{P_\mu P_\rho}{4m^2}, \quad (37)$$

and the contractions

$$\begin{aligned} \Pi_{\rho\mu} \Pi_{\rho'\mu'} \mathcal{K}^{\sigma\alpha\rho\mu} \mathcal{K}^{\sigma'\alpha'\rho'\mu'} &= 9(K^+)^2 g_\perp^{\alpha\sigma} g_\perp^{\alpha'\sigma'}, \\ \Pi_{\rho\rho'} \Pi_{\mu\mu'} \mathcal{P}^{\sigma\alpha\rho\mu} \mathcal{P}^{\sigma'\alpha'\rho'\mu'} &= (K^+)^2 \left[2g_\perp^{\alpha\alpha'} g_\perp^{\sigma\sigma'} + 2g_\perp^{\alpha\sigma'} g_\perp^{\alpha'\sigma} + g_\perp^{\alpha'\sigma'} g_\perp^{\alpha\sigma} - g_\perp^{\sigma\sigma'} \frac{(K_\perp^\alpha - k_\perp^\alpha)(K_\perp^{\alpha'} - \ell_\perp^{\alpha'})}{m^2} \right], \\ \Pi_{\rho\mu'} \Pi_{\rho'\mu} \mathcal{P}^{\sigma\alpha\rho\mu} \mathcal{P}^{\sigma'\alpha'\rho'\mu'} &= (K^+)^2 [2g_\perp^{\alpha\sigma'} g_\perp^{\alpha'\sigma} + 2g_\perp^{\alpha\alpha'} g_\perp^{\sigma\sigma'} + g_\perp^{\alpha\sigma} g_\perp^{\alpha'\sigma'}]. \end{aligned} \quad (38)$$

The result is

$$\begin{aligned} \mathcal{P}^{\sigma\alpha}(^3P_0) \mathcal{P}^{\sigma'\alpha'}(^3P_0) &= 3 \frac{(K^+)^2}{m^2} g_\perp^{\alpha\sigma} g_\perp^{\alpha'\sigma'} \\ \mathcal{P}^{\sigma\alpha}(^3P_1) \mathcal{P}^{\sigma'\alpha'}(^3P_1) &= -\frac{(K^+)^2}{2m^4} (K_\perp^\alpha - k_\perp^\alpha)(K_\perp^{\alpha'} - \ell_\perp^{\alpha'}) g_\perp^{\sigma\sigma'} \\ \mathcal{P}^{\sigma\alpha}(^3P_2) \mathcal{P}^{\sigma'\alpha'}(^3P_2) &= \frac{2(K^+)^2}{m^2} \left[g_\perp^{\alpha\alpha'} g_\perp^{\sigma\sigma'} - g_\perp^{\alpha\sigma} g_\perp^{\alpha'\sigma'} + g_\perp^{\alpha\sigma'} g_\perp^{\alpha'\sigma} - \frac{(K_\perp^\alpha - k_\perp^\alpha)(K_\perp^{\alpha'} - \ell_\perp^{\alpha'})}{4m^2} g_\perp^{\sigma\sigma'} \right]. \end{aligned} \quad (39)$$

Note that we wrote these quantities with distinct incoming gluon transverse momenta in the amplitude $k_\perp$ and in the complex conjugate amplitude $\ell_\perp$. This is required for the proper use of the double PDF (33).

C. Full cross section

We now have all the machinery to finally compute the full differential cross section. This is straightforward but tedious, as we have to consider all possible Lorentz index structures, namely, $\mathcal{G}_{1,2,3,4}$ from (3) and $a_{1,2} = \{g, \Delta g, \delta g\}$ in (33). The generic result is

$$\begin{aligned}
\frac{d\sigma(M_1, M_2)}{dY_1 dY_2 d^2\Delta d^2\mathbf{K}} &= \frac{\alpha_s^2}{16m_1^5 m_2^5 N_c^4 (N_c^2 - 1)^2} \langle \mathcal{O}_{M_1}(^{2S_1+1}L_{1J_1}^1) \rangle \langle \mathcal{O}_{M_2}(^{2S_2+1}L_{2J_2}^1) \rangle \\
&\times x_1 x_2 \int d^2\mathbf{q} [\delta^{ii'} \delta^{jj'} \mathcal{F}_{g,g}(x_1, x_2, \mathbf{q}) - i\delta^{ii'} \epsilon^{jj'} \mathcal{F}_{g,\Delta g}(x_1, x_2, \mathbf{q}) + 2\delta^{ii'} \tau^{jj',nn'} \mathcal{F}_{g,\delta g}^{nn'}(x_1, x_2, \mathbf{q}) \\
&- i\epsilon^{ii'} \delta^{jj'} \mathcal{F}_{\Delta g,g}(x_1, x_2, \mathbf{q}) - \epsilon^{ii'} \epsilon^{jj'} \mathcal{F}_{\Delta g,\Delta g}(x_1, x_2, \mathbf{q}) - 2i\epsilon^{ii'} \tau^{jj',nn'} \mathcal{F}_{\Delta g,\delta g}^{nn'}(x_1, x_2, \mathbf{q}) \\
&+ 2\delta^{jj'} \tau^{ii',mm'} \mathcal{F}_{\delta g,g}^{mm'}(x_1, x_2, \mathbf{q}) - 2i\epsilon^{jj'} \tau^{ii',mm'} \mathcal{F}_{\delta g,\Delta g}^{mm'}(x_1, x_2, \mathbf{q}) + 4\tau^{jj',nn'} \tau^{ii',mm'} \mathcal{F}_{\delta g,\delta g}^{mm',nn'}(x_1, x_2, \mathbf{q})] \\
&\times \Pi_1^{ii',kk'}(M_1) \Pi_2^{jj',\ell\ell'}(M_2) x \mathcal{G}^{k\ell'}\left(\mathbf{K} - \frac{\mathbf{q}}{2}, \Delta\right) x \mathcal{G}^{k'\ell'^*}\left(\mathbf{K} + \frac{\mathbf{q}}{2}, \Delta\right), \quad (40)
\end{aligned}$$

where $x_i \equiv \frac{K_i^+}{P^+}$, $\Delta \equiv \mathbf{K}_1 + \mathbf{K}_2$, and from now on, $\mathbf{K} \equiv \frac{\mathbf{K}_1 - \mathbf{K}_2}{2}$. Y_i the rapidity of meson M_i , and the hard subparts $\Pi_{1,2}$ are given by

$$\begin{aligned}
\Pi_1^{ii',kk'}(\eta) &= \delta^{ii'} \delta^{kk'} - \delta^{ik'} \delta^{i'k}, \\
\Pi_1^{ii',kk'}(\chi_0) &= 3 \frac{\delta^{ik} \delta^{i'k'}}{m_1^2}, \\
\Pi_1^{ii',kk'}(\chi_1) &= \frac{(\mathbf{K}^k + \frac{\Delta^k - \mathbf{q}^k}{2})(\mathbf{K}^{k'} + \frac{\Delta^{k'} + \mathbf{q}^{k'}}{2})}{2m_1^4} \delta^{ii'}, \\
\Pi_1^{ii',kk'}(\chi_2) &= \frac{2}{m_1^2} \left[\delta^{ii'} \delta^{kk'} - \delta^{ik} \delta^{i'k'} + \delta^{i'k} \delta^{ik'} + \frac{(\mathbf{K}^k + \frac{\Delta^k - \mathbf{q}^k}{2})(\mathbf{K}^{k'} + \frac{\Delta^{k'} + \mathbf{q}^{k'}}{2})}{4m_1^2} \delta^{ii'} \right], \quad (41)
\end{aligned}$$

for meson 1 and

$$\begin{aligned}
\Pi_2^{jj',\ell\ell'}(\eta) &= \delta^{jj'} \delta^{\ell\ell'} - \delta^{j\ell'} \delta^{j'\ell}, \\
\Pi_2^{jj',\ell\ell'}(\chi_0) &= 3 \frac{\delta^{j\ell} \delta^{j'\ell'}}{m_2^2}, \\
\Pi_2^{jj',\ell\ell'}(\chi_1) &= \frac{(\mathbf{K}^\ell - \frac{\Delta^\ell - \mathbf{q}^\ell}{2})(\mathbf{K}^{\ell'} - \frac{\Delta^{\ell'} + \mathbf{q}^{\ell'}}{2})}{2m_2^4} \delta^{jj'}, \\
\Pi_2^{jj',\ell\ell'}(\chi_2) &= \frac{2}{m_2^2} \left[\delta^{jj'} \delta^{\ell\ell'} - \delta^{j\ell} \delta^{j'\ell'} + \delta^{j'\ell} \delta^{j\ell'} + \frac{(\mathbf{K}^\ell - \frac{\Delta^\ell - \mathbf{q}^\ell}{2})(\mathbf{K}^{\ell'} - \frac{\Delta^{\ell'} + \mathbf{q}^{\ell'}}{2})}{4m_2^2} \delta^{jj'} \right], \quad (42)
\end{aligned}$$

for meson 2. Interestingly, the intrinsic momentum $\mathbf{q}$ of the double PDF enters the argument of the WW GTMD. Physically, $\mathbf{q}$ is conjugate to the relative distance of the two quarkonia. More precisely, it is conjugate to $\frac{\mathbf{b}_1 + \mathbf{b}'_1}{2} - \frac{\mathbf{b}_2 + \mathbf{b}'_2}{2}$ where $\mathbf{b}_{1,2}$ is as in (31) and $\mathbf{b}'_{1,2}$ are the corresponding coordinates in the complex-conjugate amplitude.

V. EXPLICIT CROSS SECTIONS

In the following, we will assume that $|\mathbf{K}| \gg |\mathbf{q}|$. The dependence on $\mathbf{q}$ is then completely absorbed in the projectile double PDF. We will thus define the integrated double PDFs

$$\mathcal{F}_{a_1, a_2}(x_1, x_2) \equiv \int d^2\mathbf{q} \mathcal{F}_{a_1, a_2}(x_1, x_2, \mathbf{q}). \quad (43)$$

At this point, we can already cancel some contributions. Indeed the symmetry and tracelessness properties of

the $(a_1 = g, a_2 = \delta g)$ double PDF allows one to write it as:

$$\mathcal{F}_{g, \delta g}^{mn}(x_1, x_2, \mathbf{q}) = \left(\frac{\mathbf{q}^m \mathbf{q}^n}{\mathbf{q}^2} - \frac{\delta^{mn}}{2} \right) \frac{\mathbf{q}^2}{M_P^2} \mathcal{H}_{g, \delta g}(x_1, x_2, \mathbf{q}^2), \quad (44)$$

where M_P is the projectile's mass. It is then easy to show that the integral given in (43) cancels for this integrated double PDF. Similarly, we can cancel the $(a_1 = \delta g, a_2 = g)$ integrated double PDF.

It will also be useful to write the $(a_1, a_2) = (\delta g, \delta g)$ integrated double PDF as

$$\begin{aligned}
\mathcal{F}_{\delta g, \delta g}^{mm', nn'}(x_1, x_2) &\equiv \frac{1}{2} (\delta^{mn} \delta^{m'n'} + \delta^{mn'} \delta^{m'n} - \delta^{mm'} \delta^{nn'}) \\
&\times \mathcal{H}_{\delta g, \delta g}(x_1, x_2). \quad (45)
\end{aligned}$$

A. $\chi_1\chi_1$ cross section

We find that the most concise formula is obtained for the double $\chi_{f=1}$ production. It is given by

$$\frac{d\sigma(\chi_{f_1 1}, \chi_{f_2 1})}{dY_1 dY_2 d^2\mathbf{\Delta} d^2\mathbf{K}} = \frac{x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2)}{64 m_1^9 m_2^9 N_c^4 (N_c^2 - 1)^2} \alpha_s^4 \langle \mathcal{O}_{\chi_{f_1 1}}(^3P_1^1) \rangle \langle \mathcal{O}_{\chi_{f_2 1}}(^3P_1^1) \rangle \left| \left(\mathbf{K}^i + \frac{\mathbf{\Delta}^i}{2} \right) \left(\mathbf{K}^j - \frac{\mathbf{\Delta}^j}{2} \right) x \mathcal{G}^{ij}(\mathbf{K}, \mathbf{\Delta}) \right|^2, \quad (46)$$

with $f_{1,2}$ the meson flavors. To check that the right-hand side has the correct dimensions, we note that $\dim \mathcal{G} = -2$, $\dim \mathcal{F} = 2$, and $\dim \langle \mathcal{O}_\chi \rangle = 5$ ($\dim \langle \mathcal{O}_\eta \rangle = 3$). Explicitly, the second line of (46) reads

$$\left| \left(\mathbf{K}^i + \frac{\mathbf{\Delta}^i}{2} \right) \left(\mathbf{K}^j - \frac{\mathbf{\Delta}^j}{2} \right) x \mathcal{G}^{ij}(\mathbf{K}, \mathbf{\Delta}) \right|^2 = K^4 \left| \left(1 - \frac{\mathbf{\Delta}^2}{4K^2} \right) \mathcal{G}_1 + \left(1 - \frac{(\mathbf{K} \cdot \mathbf{\Delta})^2}{2K^4} + \frac{\mathbf{\Delta}^2}{4K^2} \right) \frac{K^2}{2M^2} \mathcal{G}_2 - \left(\frac{\mathbf{\Delta}^2}{2M^2} - \frac{(\mathbf{K} \cdot \mathbf{\Delta})^2}{K^2 M^2} + \frac{\mathbf{\Delta}^4}{8K^2 M^2} \right) \mathcal{G}_3 - \left(\frac{\mathbf{\Delta}^2}{M^2} - \frac{(\mathbf{K} \cdot \mathbf{\Delta})^2}{K^2 M^2} \right) \mathcal{G}_4 \right|^2 \quad (47)$$

$$\approx K^4 \left(\mathcal{G}_1 + \frac{K^2}{2M^2} \mathcal{G}_2 \right)^2, \quad (48)$$

where the last line is obtained by assuming $|\mathbf{K}| \gg |\mathbf{\Delta}|$. Remarkably, the cross section is directly proportional to the WW GTMD squared, without any convolution in momentum.

B. All averaged cross sections

We will now present all the full cross sections for our (M_1, M_2) process, for $M_i \in \{\eta, \chi_0, \chi_1, \chi_2\}$. In order to get rid of the contributions with a longitudinally polarized double PDF, we will take the average with respect to the angle between $\mathbf{K}$ and $\mathbf{\Delta}$. This allows to keep only $\mathcal{F}_{g,g}(x_1, x_2)$ and $\mathcal{H}_{\delta g, \delta g}(x_1, x_2)$ as the nonperturbative distributions on the projectile side, since our goal is to focus rather on the nonperturbative effects on the target side.

1. $(\chi_1\chi_1)$

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\chi_{f_1 1}, \chi_{f_2 1})}{dY_1 dY_2 d^2\mathbf{K} d^2\mathbf{\Delta}} = \frac{\alpha_s^4 x^2 K^4}{32 m_1^9 m_2^9 N_c^4 (N_c^2 - 1)^2} x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \langle \mathcal{O}_{\chi_{f_1 1}}(^3P_1^1) \rangle \langle \mathcal{O}_{\chi_{f_2 1}}(^3P_1^1) \rangle \times \left[\left(\mathcal{G}_1 + \frac{K^2}{2M^2} \mathcal{G}_2 \right)^2 - \frac{\mathbf{\Delta}^2}{2K^2} \left(\mathcal{G}_1 + \frac{K^2}{2M^2} \mathcal{G}_2 \right) \left(\mathcal{G}_1 + 2 \frac{K^2}{M^2} \mathcal{G}_4 \right) \right]. \quad (49)$$

2. $(\chi_1\chi_0)$

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\chi_1, \chi_0)}{dY_1 dY_2 d^2\mathbf{K} d^2\mathbf{\Delta}} = \frac{3\alpha_s^4 x^2 K^2}{32 m_1^9 m_2^7 N_c^4 (N_c^2 - 1)^2} x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \langle \mathcal{O}_{\chi_1}(^3P_1^1) \rangle \langle \mathcal{O}_{\chi_0}(^3P_0^1) \rangle \times \left[\left(\mathcal{G}_1 + \frac{K^2}{2M^2} \mathcal{G}_2 \right)^2 + \frac{\mathbf{\Delta}^2}{4K^2} \left(\mathcal{G}_1^2 + \frac{K^4}{4M^4} (\mathcal{G}_2^2 - 8\mathcal{G}_2 \mathcal{G}_4 + 8\mathcal{G}_4^2) \right) \right] \quad (50)$$

3. $(\chi_1\chi_2)$

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\chi_1, \chi_2)}{dY_1 dY_2 d^2\mathbf{K} d^2\mathbf{\Delta}} = \frac{\alpha_s^4 K^2 x^2}{8 m_1^7 m_2^7 N_c^4 (N_c^2 - 1)^2} \langle \mathcal{O}_{\chi_1}(^3P_1^1) \rangle \langle \mathcal{O}_{\chi_2}(^3P_2^1) \rangle x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \left\{ \left(1 + \frac{K^2}{4m_2^2} \right) \left(\mathcal{G}_1 + \frac{K^2}{2M^2} \mathcal{G}_2 \right)^2 + \frac{\mathbf{\Delta}^2}{4K^2} \left[\left(1 + \frac{K^2}{m_2^2} \right) \mathcal{G}_1^2 + \frac{3K^4}{4m_2^2 M^2} \mathcal{G}_1 \mathcal{G}_2 + \frac{K^4}{4M^4} \left(1 + \frac{K^2}{2m_2^2} \right) \mathcal{G}_2^2 - 2 \frac{K^4}{M^4} (\mathcal{G}_2 - \mathcal{G}_4) \mathcal{G}_4 \right] \right\} \quad (51)$$

4. ($\chi_1\eta$)

$$\begin{aligned} \frac{1}{2\pi} \int_0^{2\pi} d\phi \frac{d\sigma(\chi_1, \eta)}{dY_1 dY_2 d\Delta^2 d^2\mathbf{K}} &= \frac{\alpha_s^4 x^2 K^2}{32m_1^7 m_2^5 N_c^4 (N_c^2 - 1)^2} x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \langle \mathcal{O}_{\chi_1}(^3P_1^1) \rangle \langle \mathcal{O}_\eta(^1S_0^1) \rangle \\ &\times \left[\left(\mathcal{G}_1 + \frac{K^2}{2M^2} \mathcal{G}_2 \right)^2 + \frac{\Delta^2}{4K^2} \left(\mathcal{G}_1^2 + \frac{K^4}{4M^4} (\mathcal{G}_2^2 - 8\mathcal{G}_2\mathcal{G}_4 + 8\mathcal{G}_4^2) \right) \right] \end{aligned} \quad (52)$$

5. ($\chi_0\chi_0$)

$$\begin{aligned} \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\chi_{f_1 0}, \chi_{f_2 0})}{dY_1 dY_2 d^2\mathbf{K} d\Delta^2} &= \frac{9\alpha_s^4 x^2}{16m_1^7 m_2^7 N_c^4 (N_c^2 - 1)^2} \langle \mathcal{O}_{\chi_{f_1 0}}(^3P_0^1) \rangle \langle \mathcal{O}_{\chi_{f_2 0}}(^3P_0^1) \rangle \left\{ x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \left(\mathcal{G}_1^2 + \frac{K^4}{4M^4} \mathcal{G}_2^2 + \frac{K^2 \Delta^2}{2M^4} \mathcal{G}_4^2 \right) \right. \\ &\left. + 4x_1 x_2 \mathcal{H}_{\delta g, \delta g}(x_1, x_2) \left[\mathcal{G}_1^2 - 2 \frac{\Delta^2}{M^2} \left(\mathcal{G}_1 \mathcal{G}_3 - \frac{K^2}{4M^2} \mathcal{G}_4^2 \right) \right] \right\} \end{aligned} \quad (53)$$

6. ($\chi_0\chi_2$)

$$\begin{aligned} \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\chi_0, \chi_2)}{dY_1 dY_2 d^2\mathbf{K} d\Delta^2} &= \frac{3\alpha_s^4 x^2}{16m_1^7 m_2^7 N_c^4 (N_c^2 - 1)^2} x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \langle \mathcal{O}_{\chi_0}(^3P_0^1) \rangle \langle \mathcal{O}_{\chi_2}(^3P_2^1) \rangle \\ &\times \left[4\mathcal{G}_1^2 + \frac{K^4}{M^4} \mathcal{G}_2^2 + \frac{K^2}{2m_2^2} \left(\mathcal{G}_1 + \frac{K^2}{2M^2} \mathcal{G}_2 \right)^2 \right. \\ &\left. + \frac{\Delta^2}{8M^2} \left(\frac{M^2}{m_2^2} \mathcal{G}_1^2 + 16 \frac{K^2}{M^2} \mathcal{G}_4^2 + \frac{K^4}{4m_2^2 M^2} (\mathcal{G}_2^2 + 8\mathcal{G}_2\mathcal{G}_4 + 8\mathcal{G}_4^2) \right) \right] \end{aligned} \quad (54)$$

7. ($\chi_0\eta$)

$$\begin{aligned} \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\chi_0, \eta)}{dY_1 dY_2 d^2\mathbf{K} d\Delta^2} &= \frac{3\alpha_s^4 x^2}{16m_1^7 m_2^5 N_c^4 (N_c^2 - 1)^2} \langle \mathcal{O}_{\chi_0}(^3P_0^1) \rangle \langle \mathcal{O}_\eta(^1S_0^1) \rangle \left\{ x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \left(\mathcal{G}_1^2 + \frac{K^4}{4M^4} \mathcal{G}_2^2 + \frac{K^2 \Delta^2}{2M^4} \mathcal{G}_4^2 \right) \right. \\ &\left. - 4x_1 x_2 \mathcal{H}_{\delta g, \delta g}(x_1, x_2) \left[\mathcal{G}_1^2 - 2 \frac{\Delta^2}{M^2} \left(\mathcal{G}_1 \mathcal{G}_3 - \frac{K^2}{4M^2} \mathcal{G}_4^2 \right) \right] \right\} \end{aligned} \quad (55)$$

8. ($\chi_2\chi_2$)

$$\begin{aligned} \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\chi_{f_1 2}, \chi_{f_2 2})}{dY_1 dY_2 d^2\mathbf{K} d\Delta^2} &= \frac{\alpha_s^4 x^2}{8m_1^7 m_2^7 N_c^4 (N_c^2 - 1)^2} x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \langle \mathcal{O}_{\chi_{f_1 2}}(^3P_2^1) \rangle \langle \mathcal{O}_{\chi_{f_2 2}}(^3P_2^1) \rangle \\ &\times \left\{ 8 \left(\mathcal{G}_1^2 + \frac{K^4}{4M^4} \mathcal{G}_2^2 \right) + \left(\frac{K^2}{m_1^2} + \frac{K^2}{m_2^2} + \frac{K^4}{4m_1^2 m_2^2} \right) \left(\mathcal{G}_1 + \frac{K^2}{2M^2} \mathcal{G}_2 \right)^2 \right. \\ &+ \frac{\Delta^2}{32m_1^2 m_2^2} \left[8\mathcal{G}_1^2 (K^2 + m_1^2 + m_2^2) + 6K^2 \frac{K^2}{M^2} \mathcal{G}_1 \mathcal{G}_2 + \frac{K^4}{M^4} (K^2 + 2m_1^2 + 2m_2^2) \mathcal{G}_2^2 \right] \\ &\left. + \frac{\Delta^2}{2m_1^2 m_2^2} \frac{K^2}{M^2} \left[(m_1^2 - m_2^2) \frac{K^2}{M^2} \mathcal{G}_2 \mathcal{G}_4 + \left(\frac{K^2 m_1^2}{M^2} + \frac{K^2 m_2^2}{M^2} + 8 \frac{m_1^2 m_2^2}{M^2} \right) \mathcal{G}_4^2 \right] \right\} \end{aligned} \quad (56)$$

9. ($\chi_2\eta$)

$$\begin{aligned} \int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\chi_2, \eta)}{dY_1 dY_2 d^2\mathbf{K} d\Delta^2} &= \frac{\alpha_s^4 x^2}{16m_1^7 m_2^5 N_c^4 (N_c^2 - 1)^2} \langle \mathcal{O}_{\chi_2}(^3P_2^1) \rangle \langle \mathcal{O}_\eta(^1S_0^1) \rangle \\ &\times x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \left[4 \left(\mathcal{G}_1^2 + \frac{K^4}{4M^4} \mathcal{G}_2^2 \right) + \frac{K^2}{2m_1^2} \left(\mathcal{G}_1 + \frac{K^2}{2M^2} \mathcal{G}_2 \right)^2 \right. \\ &\left. + \frac{\Delta^2}{8m_1^2} \left(\mathcal{G}_1^2 + \frac{K^4}{4M^4} (\mathcal{G}_2^2 - 8\mathcal{G}_2\mathcal{G}_4 + 8\mathcal{G}_4^2) + 16 \frac{K^2 m_1^2}{M^2 M^2} \mathcal{G}_4^2 \right) \right] \end{aligned} \quad (57)$$

10. ($\eta\eta$)

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \frac{d\sigma(\eta_{f_1}, \eta_{f_2})}{dY_1 dY_2 d^2\mathbf{K} d\Delta^2} = \frac{\alpha_s^4 x^2}{16m_1^5 m_2^5 N_c^4 (N_c^2 - 1)^2} \langle \mathcal{O}_{\eta_{f_1}}(^1S_0) \rangle \langle \mathcal{O}_{\eta_{f_2}}(^1S_0) \rangle \left\{ x_1 x_2 \mathcal{F}_{g,g}(x_1, x_2) \left(\mathcal{G}_1^2 + \frac{\mathbf{K}^4}{4M^4} \mathcal{G}_2^2 + \frac{\mathbf{K}^2 \Delta^2}{2M^4} \mathcal{G}_4^2 \right) \right. \\ \left. + 4x_1 x_2 \mathcal{H}_{\delta g, \delta g}(x_1, x_2) \left[\mathcal{G}_1^2 - 2 \frac{\Delta^2}{M^2} \left(\mathcal{G}_1 \mathcal{G}_3 - \frac{\mathbf{K}^2}{4M^2} \mathcal{G}_4^2 \right) \right] \right\}. \quad (58)$$

VI. CONCLUSIONS

In summary, in this paper we derive the diffractive double quarkonia production in pp and pA collisions to probe the WW gluon GTMDs. In particular, we applied the double parton scattering mechanism from the projectile, where the two gluons scatter off the nucleon/nucleus target diffractively to produce the final state two quarkonium states. The amplitudes are found to be sensitive to the WW gluon GTMDs.

More importantly, we found that the differential cross sections can be much simplified if we integrate out the transverse momenta of the gluons from the projectile. The explicit expressions show that the cross sections can be written as squared of the WW gluon GTMDs from the target.

Our result is a first example of direct access to the WW gluon GTMDs in hard diffractive processes. Experimentally, this process may be a challenge to measure. We hope that our derivations will stimulate further theoretical developments to explore the physics of GTMD and possibility to measure them in experiments.

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APPENDIX: GLUON TO 1S_0 AND 3P_J WAVES TRANSITION IN THE HYBRID FORMALISM

For completeness, here we write down the cross section for the single-inclusive quarkonium production $gp \rightarrow MX$ for 1S_0 and 3P_J waves. Squaring (29) and using the projectors (34)–(38), we find

$$\frac{d\sigma_g(M)}{dK^+ d^2\mathbf{K}} = \frac{\alpha_s}{2m^5 N_c (N_c^2 - 1)} \delta(K^+ - k^+) \\ \times \int \frac{d^2\mathbf{b} d^2\mathbf{b}'}{(2\pi)^2} e^{i\mathbf{K} \cdot (\mathbf{b}' - \mathbf{b})} \\ \times \left\langle \frac{1}{N_c} \text{Tr}[U_{\mathbf{b}}^\dagger (\partial_i U_{\mathbf{b}}) U_{\mathbf{b}'}^\dagger (\partial_j U_{\mathbf{b}'})] \right\rangle \\ \times \varphi^{ij}(^{2S+1}L_J) \langle \mathcal{O}_M(^{2S+1}L_J^\dagger) \rangle^{\frac{1}{2}},$$

where

$$\varphi^{ij}(^1S_0) = \delta^{ij}, \quad \varphi^{ij}(^3P_0) = \frac{3\delta^{ij}}{m^2}, \quad \varphi^{ij}(^3P_1) = \frac{K^i K^j}{m^4}, \\ \varphi^{ij}(^3P_2) = \frac{4}{m^2} \left(\delta^{ij} + \frac{K^i K^j}{4m^2} \right), \quad (A1)$$

and the brackets denote the forward matrix element $\frac{\langle P | \dots | P \rangle}{\langle P | P \rangle}$, or equivalently the CGC averaging. Taking the case where $M = J/\psi$ and taking the small-dipole limit in the collinear limit of results described in [34] shows full compatibility of the present results with previous calculations.

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