Erratum: Partially massless graviton on beyond Einstein spacetimes [Phys. Rev. D **95**, 124036 (2017)]

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II. MASSIVE AND PARTIALLY MASSLESS GRAVITONS ON CURVED BACKGROUND SPACETIMES

C. Covariant constraint counting on a general background

Equation (31) should read

$$A^{\beta\lambda} \equiv m^2 S_\rho^\beta \left[\left(\beta_0 \beta_1 + \beta_0 \beta_2 e_1 + \frac{1}{2} \beta_1^2 e_1 + \frac{1}{2} \beta_1 \beta_2 e_1^2 \right) g^{\rho\lambda} + \left(-2\beta_0 \beta_2 - \frac{1}{2} \beta_1^2 - \beta_1 \beta_2 e_1 - 2\beta_2^2 e_2 + \beta_2^2 e_1^2 \right) S^{\rho\lambda} \right. \\ \left. - (\beta_1 \beta_2 + \beta_2^2 e_1) [S^2]^{\rho\lambda} \right]. \quad (31)$$

Equations (32) and (33) are unchanged. The new form of Eq. (31) above differs from the old one only by terms proportional to the product $\beta_1 \beta_2$, and hence the discussions and results given in the original version when this product vanishes are unaffected. This holds in particular for Sec. III A. The correction of Eq. (31) implies several modifications of equations in Sec. III D. However the main result of the paper, i.e., the proof that a PM graviton can propagate on a non-Einstein spacetime still holds. The modification of Eq. (31) results in modifications of the relations between the parameters of the massive graviton model and the characteristics of the non-Einstein spacetimes where a PM graviton can exist.

III. BACKGROUND SPACETIMES FOR PARTIAL MASSLESSNESS

D. Ricci symmetric spacetimes admitting partial masslessness

1. Conditions for partial masslessness on Ricci symmetric spacetimes

Equation (67) now reads

$$\delta_\lambda^\beta \left(\beta_2 \beta_0 e_1 + \beta_0 \beta_1 + \frac{\beta_1^2}{2} e_1 + \frac{1}{2} \beta_1 \beta_2 e_1^2 \right) + S_\lambda^\beta \left(-2\beta_2 \beta_0 + \beta_2^2 e_1^2 - 2\beta_2^2 e_2 - \frac{\beta_1^2}{2} - \beta_1 \beta_2 e_1 \right) - (S^2)_\lambda^\beta (\beta_1 \beta_2 + e_1 \beta_2^2) = 0. \quad (67)$$

The cases $\beta_1 = 0$ and $\beta_2 = 0$ are still not admissible (and the discussion is unchanged with respect to the first version, as explained above). Then the other particular case which may give interesting solutions is $\beta_1 + \beta_2 e_1 = 0$. We will comment on this case below. When $\beta_1 + \beta_2 e_1$ does not vanish, the linear relation (68) between the tensor $S_{\mu\nu}$ and the Ricci tensor $R_{\mu\nu}$ reads

$$R_\nu^\rho = \frac{m^2}{\beta_1} \left(-2\beta_0 \beta_2 + \frac{3}{2} \beta_1^2 \right) S^\rho_\nu. \quad (68)$$

We then slightly modify definition (69) of the parameters v and c by

$$u \equiv \frac{\beta_0 \beta_2}{\beta_1^2}, \quad v \equiv m^2 \beta_0, \quad \text{and} \quad c \equiv \beta_1^2 (-4u + 3). \quad (69)$$

Using these new parameters Eq. (70) becomes

$$R^\rho_\nu = \frac{m^2 c}{2\beta_1} S^\rho_\nu. \quad (70)$$

The inverse relation (71) expressing S^ρ_ν in terms of R^ρ_ν becomes

$$S^\rho_\nu = \frac{2\beta_1}{m^2 c} R^\rho_\nu. \quad (71)$$

Equation (72) no longer holds. Equation (73) which gives a condition on the curvature now reads

$$\left[v^2 \left(4u^2 - 6u + \frac{9}{4} \right) + vR \left(-u + \frac{3}{4} \right) \right] g^{\beta\lambda} + \left[v \left(-4u^2 + 4u - \frac{3}{4} \right) + uR \right] R^{\beta\lambda} - u[R^2]^{\beta\lambda} = 0. \quad (73)$$

Under the conditions

$$\beta_1 \beta_2 \neq 0, \quad (74a)$$

$$\beta_1 + \beta_2 e_1 \neq 0, \quad (74b)$$

$$v \neq 0, \quad (74c)$$

$$c \neq 0, \quad (74d)$$

the system of two equations (75) to be solved for u and v is now

$$\begin{aligned} v^2 \left(4u^2 - 6u + \frac{9}{4} \right) + vR \left(-u + \frac{3}{4} \right) - ur_2 &= 0, \\ v \left(-4u^2 + 4u - \frac{3}{4} \right) + u(R - r_1) &= 0. \end{aligned} \quad (75)$$

2. Examples of Ricci symmetric spacetimes with partial masslessness

Petrov type O.—There are no partially massless solutions of type O. Indeed, in this case, the above system (75) does not yield any valid solution with $r_1 = R/3$ and $r_2 = 0$.

Petrov type D.—In this case we have $r_1 = \frac{R}{2}$ and $r_2 = -\frac{1}{16}(R^2 - E^2)$. The general system of equations reduces to

$$\begin{aligned} v^2 \left(4u^2 - 6u + \frac{9}{4} \right) + vR \left(-u + \frac{3}{4} \right) - u \frac{E^2 - R^2}{16} &= 0, \\ v \left(-4u^2 + 4u - \frac{3}{4} \right) + \frac{uR}{2} &= 0. \end{aligned} \quad (76)$$

The two values $u = \frac{1}{4}$ and $u = \frac{3}{4}$ are not valid solutions so we can solve for v , and we get

$$v = \frac{2u}{(4u-3)(4u-1)} R. \quad (77)$$

Inserting this back into Eq. (76), we obtain the following relation between E and R :

$$E^2 = \frac{(4u-3)^2}{(4u-1)^2} R^2. \quad (78)$$

For given values of R and E , v and u are determined by the above two Eqs. (78) and (77) and are given by

$$u = \frac{E - 3R}{4(E - R)}, \quad v = \frac{(E - 3R)(E - R)}{8E} \quad \text{and} \quad u = \frac{E + 3R}{4(E + R)}, \quad v = -\frac{(E + 3R)(E + R)}{8E}. \quad (79)$$

We also have the particular solutions corresponding to $\beta_1 + \beta_2 e_1 = 0$. The conditions are

$$E^2 = 2R^2 \frac{8u - 3}{2(-3 + 4u)^2}, \quad v = m^2 \beta_0 = \frac{2u}{-3 + 4u} R, \quad (80)$$

and the matrix S is given by

$$S^\mu_\nu = -\frac{\beta_1}{\beta_2} \left(u - \frac{1}{2} \right) \delta^\mu_\nu + \frac{\beta_1}{\beta_2 R} (-3 + 4u) R^\mu_\nu. \quad (81)$$

The conditions that the matrix S should fulfill, namely that it is invertible and does not have common eigenvalues with $-S$, further imply that

$$ER \neq 0, \quad E \neq \pm R, \quad \frac{E}{R} \neq \frac{2u - 1}{4u - 3}, \quad \frac{E}{2R} \neq \pm \frac{u - 1}{4u - 3}. \quad (82)$$

These conditions also translate into some forbidden values for u and v . To summarize, for Petrov-type D spacetimes, we have an infinite number of solutions. More precisely, for any given value of the set (R, E) [except for the forbidden values of Eq. (82)], we have up to two different solutions given by Eq. (79), or by Eq. (80).

B. Example of a non-Einstein PM theory

The Einstein static universe is no longer a solution.

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