Erratum: Equivalent and inequivalent canonical structures of higher order theories of gravity [Phys. Rev. D **96, 084025 (2017)]**

Ranajit Mandal and Abhik Kumar Sanyal

(Received 13 August 2018; published 13 September 2018)

In the manuscript, we concluded that Horowitz's as well as Dirac's and Ostrogradski's formalisms fail to produce a viable quantum description of the modified Einstein-Gauss-Bonnet-dilatonic (MEGBD) action. However, later we note that such conclusion stems from an error in the solution to the classical field equations. Here, we rectify the error, along with all the expressions associated with it. In the process, we find that both Horowitz's formalism (as well as Dirac's and Ostrogradski's formalisms) and modified Horowitz's formalism produce a viable, although inequivalent, quantum description of the MEGBD action. Nevertheless, the main motivation of the manuscript, viz. "canonical formulation of higher-order theories of gravity following different prescriptions in some cases lead to equivalent and in some other cases inequivalent phase-space structures", remains unaltered.

DOI: [10.1103/PhysRevD.98.069901](https://doi.org/10.1103/PhysRevD.98.069901)

Section IIIB:

The correct solutions (86) to the field equations (84) and (85) are

$$a = a_0 e^{Ht}; \quad \phi = \phi_0 e^{-Ht}; \quad \Lambda(\phi) = -\frac{\phi^2}{48H^2}; \quad V = 6\alpha H^2 + \frac{1}{2} H^2 \phi^2. \quad (1)$$

The correct expression for Eq. (87), therefore, is

$$-\frac{\hbar^2 \sqrt{z}}{36\beta x} \left(\frac{\partial^2}{\partial x^2} + \frac{n}{x} \frac{\partial}{\partial x} \right) \Psi - \frac{\hbar^2}{2xz^{\frac{3}{2}}} \frac{\partial^2 \Psi}{\partial \phi^2} - i\hbar \frac{\partial \Psi}{\partial z} + \frac{i\hbar \alpha z}{6\beta x} \frac{\partial \Psi}{\partial x} - \frac{i\hbar \phi^2 x}{288\beta H^2 z} \frac{\partial \Psi}{\partial x} + \mathcal{V} \Psi = 0,$$

$$\text{where , } \mathcal{V} = \left(\frac{\alpha^2 z^{\frac{3}{2}}}{4\beta x} + \frac{\phi^4 x^3}{9216\beta H^4 z^{\frac{5}{2}}} - \frac{\alpha \phi^2 x}{96\beta H^2 \sqrt{z}} - \frac{\phi^2 x^3}{32H^2 z^{\frac{3}{2}}} - \frac{i\hbar \alpha z}{12\beta x^2} - \frac{i\hbar \phi^2}{576\beta H^2 z} + \frac{6\alpha H^2 z^{\frac{3}{2}}}{x} + \frac{H^2 \phi^2 z^{\frac{3}{2}}}{2x} \right). \quad (2)$$

Finally, only one expression of Eq. (90a) and the last expressions of (90b) and (90c) are also changed as

$$\frac{\sqrt{z}}{36\beta x} S_{0,x}^2 + \frac{S_{0,\phi}^2}{2xz^{\frac{3}{2}}} + S_{0,z} - \frac{\alpha z}{6\beta x} S_{0,x} + \frac{\phi^2 x}{288\beta H^2 z} S_{0,x} + \mathcal{V}(x, z, \phi) = 0 \quad (3a)$$

$$-\frac{i\sqrt{z}}{36\beta x} S_{0,xx} - \frac{in\sqrt{z}}{36\beta x^2} S_{0,x} - \frac{iS_{0,\phi\phi}}{2xz^{\frac{3}{2}}} + S_{1,z} + \frac{\sqrt{z}S_{0,x}S_{1,x}}{18\beta x} + \frac{S_{0,\phi}S_{1,\phi}}{xz^{\frac{3}{2}}} - \frac{\alpha z}{6\beta x} S_{1,x} + \frac{\phi^2 x}{288\beta H^2 z} S_{1,x} = 0 \quad (3b)$$

$$-\frac{i\sqrt{z}}{36\beta x} S_{1,xx} - \frac{in\sqrt{z}}{36\beta x^2} S_{1,x} - \frac{iS_{1,\phi\phi}}{2xz^{\frac{3}{2}}} + S_{2,z} + \frac{\sqrt{z}S_{0,x}S_{2,x}}{18\beta x} + \frac{S_{0,\phi}S_{2,\phi}}{xz^{\frac{3}{2}}} - \frac{\alpha z}{6\beta x} S_{2,x} + \frac{\phi^2 x}{288\beta H^2 z} S_{2,x} = 0. \quad (3c)$$

The expressions for momenta appearing in Eqs. (92b) and (92c) of the manuscript need correction,

$$p_x = Q = \frac{3\alpha\sqrt{x}}{\sqrt{2H}} + \frac{72\beta H^{\frac{3}{2}}\sqrt{x}}{\sqrt{2}} - \frac{\sqrt{H}a_0^2\phi_0^2}{2\sqrt{2}\sqrt{x}} \quad (4a)$$

$$p_z = -3\alpha H\sqrt{z} - 72\beta H^3\sqrt{z} - \frac{Ha_0^2\phi_0^2}{4\sqrt{z}}. \quad (4b)$$

Consequently, the integrals appearing in (93a) and (93b) also need correction,

$$\int p_x dx = \frac{2\alpha}{\sqrt{2H}} x^{\frac{3}{2}} + \frac{48\beta H^{\frac{3}{2}}}{\sqrt{2}} x^{\frac{3}{2}} - \frac{\sqrt{2}Ha_0^2\phi_0^2}{2} \sqrt{x} = 4\alpha H z^{\frac{3}{2}} + 96\beta H^{\frac{3}{2}} z^{\frac{3}{2}} - a_0^2\phi_0^2 H \sqrt{z} \quad (5a)$$

$$\int p_z dz = -2\alpha H z^{\frac{3}{2}} - 48\beta H^{\frac{3}{2}} z^{\frac{3}{2}} - \frac{Ha_0^2\phi_0^2}{2} \sqrt{z}. \quad (5b)$$

Therefore, the correct expression for the Hamilton-Jacobi function (S_0) appearing in Eq. (94) of the manuscript is

$$S_0 = 2\alpha H z^{\frac{3}{2}} + 48\beta H^{\frac{3}{2}} z^{\frac{3}{2}} - \frac{Ha_0^2\phi_0^2}{2} \sqrt{z}. \quad (6)$$

The zeroth-order on-shell action appearing in (95) and (96) is found upon substituting classical solutions (1) in actions (59) or in (74), and even in the canonical action (79). The correct expression is

$$A = A_{cl} = \int \left[6\alpha H^2 a_0^3 e^{3Ht} + 144\beta H^4 a_0^3 e^{3Ht} - \frac{H^2 a_0^3 \phi_0^2 e^{Ht}}{2} \right] dt = 2\alpha H a_0^3 e^{3Ht} + 48\beta H^3 a_0^3 e^{3Ht} - \frac{H a_0^3 \phi_0^2 e^{Ht}}{2}, \quad (7)$$

which is the same as S_0 obtained in (6), i.e., $S_0 = A_{cl}$. This is a significant outcome of corrected solutions (1), because formerly we found $S_0 \neq A_{cl}$, and concluded that Horowitz's formalism (as well as Dirac's and Ostrogradski's) does not produce a viable quantum dynamics. So one can compute the semiclassical wave function as well, which up to first-order approximation reads as

$$\Psi = \psi_{01} e^{i\hbar[2\alpha H z^{\frac{3}{2}} + 48\beta H^{\frac{3}{2}} z^{\frac{3}{2}} - \frac{Ha_0^2\phi_0^2}{2} \sqrt{z}]}, \quad \text{where } \psi_{01} = \psi_0 e^{g(z)}, \quad (8)$$

where ψ_0 is a constant. Since the wave function is oscillatory about the classical inflationary solution (1), Horowitz's as well as Ostrogradski's and Dirac's formalisms lead to a viable quantum dynamics. This is the important consequence of the correction.

D. Modified Horowitz formalism

As a result of the preceding correction, the statement with which this subsection started turns out to be wrong. Instead, it should read as follows: In this subsection, we invoke a modified Horowitz's canonical formulation to find that it also produces a phase-space Hamiltonian which results in a viable quantum dynamics. Nevertheless, what we find is a different Hamiltonian altogether, which is not canonically related to the earlier ones.

Due to the corrected solutions (1), some changes also appear in the expressions of the modified Horowitz formalism. Although such changes, unlike the previous case, do not at all make any significant change in the conclusion, we still give the correct expressions.

Equations (108)–(110) appearing in the modified Horowitz's formalism also get changed as

$$\begin{aligned} \frac{i\hbar}{\sqrt{z}} \frac{\partial \Psi}{\partial z} = & -\frac{\hbar^2}{36\beta x} \left(\frac{\partial^2}{\partial x^2} + \frac{n}{x} \frac{\partial}{\partial x} \right) \Psi - \frac{\hbar^2}{2xz^2} \frac{\partial^2 \Psi}{\partial \phi^2} + i\hbar \frac{x^2}{48H^2 z^{\frac{7}{2}}} \left(\phi \frac{\partial \Psi}{\partial \phi} + \frac{\partial}{\partial \phi} (\phi \Psi) \right) \\ & + \left(\frac{3\alpha x}{2z} + \frac{\phi^2 x^5}{576H^4 z^5} + \left(6\alpha H^2 + \frac{1}{2} H^2 \phi^2 \right) \frac{z^{\frac{3}{2}}}{x} \right) \Psi, \end{aligned} \quad (9)$$

$$i\hbar \frac{\partial \Psi}{\partial \sigma} = -\frac{\hbar^2}{54\beta} \left(\frac{1}{x} \frac{\partial^2}{\partial x^2} + \frac{n}{x^2} \frac{\partial}{\partial x} \right) \Psi - \frac{\hbar^2}{3x\sigma^{\frac{4}{3}}} \frac{\partial^2 \Psi}{\partial \phi^2} + i\hbar \frac{x^2}{72H^2 \sigma^{\frac{4}{3}}} \left(2\phi \frac{\partial \Psi}{\partial \phi} + \Psi \right) + V_e \Psi = \hat{H}_e \Psi, \quad (10)$$

$$V_e = \frac{\alpha x}{\sigma^{\frac{2}{3}}} + \frac{\phi^2 x^5}{864H^4 \sigma^{\frac{10}{3}}} + \frac{2\sigma}{3x} \left(6\alpha H^2 + \frac{1}{2} H^2 \phi^2 \right). \quad (11)$$

$\mathbf{J}_\phi$ appearing in (112b) also changes,

$$\mathbf{J}_\phi = \frac{i\hbar}{3x\sigma^{\frac{4}{3}}} (\Psi\Psi_{,\phi}^* - \Psi^*\Psi_{,\phi}) - \frac{x^2\phi}{36H^2\sigma^{\frac{7}{3}}} \Psi^*\Psi. \quad (12)$$

Equations (113) and (114) take the forms

$$-\frac{\hbar^2\sqrt{z}}{36\beta x} \left(\frac{\partial^2}{\partial x^2} + \frac{n}{x} \frac{\partial}{\partial x} \right) \Psi - \frac{\hbar^2}{2xz^{\frac{3}{2}}} \frac{\partial^2 \Psi}{\partial \phi^2} - i\hbar \frac{\partial \Psi}{\partial z} + i\hbar \frac{x^2}{24H^2 z^3} \phi \frac{\partial \Psi}{\partial \phi} + \mathcal{V}\Psi = 0, \quad (13)$$

where

$$\mathcal{V} = \frac{3\alpha x}{2\sqrt{z}} + \frac{\phi^2 x^5}{576H^4 z^{\frac{9}{2}}} + \frac{i\hbar x^2}{48H^2 z^3} + \left(6\alpha H^2 + \frac{1}{2} H^2 \phi^2 \right) \frac{z^2}{x}. \quad (14)$$

Finally, one expression of Eq. (117a) and the last expressions of (117b) and (117c) are also changed as

$$\frac{\sqrt{z}}{36\beta x} S_{0,x}^2 + \frac{S_{0,\phi}^2}{2xz^{\frac{3}{2}}} + S_{0,z} - \frac{x^2}{48H^2 z^3} \phi S_{0,\phi} + \mathcal{V}(x, z, \phi) = 0 \quad (15a)$$

$$-\frac{i\sqrt{z}}{36\beta x} S_{0,xx} - \frac{in\sqrt{z}}{36\beta x^2} S_{0,x} - \frac{iS_{0,\phi\phi}}{2xz^{\frac{3}{2}}} + S_{1,z} + \frac{\sqrt{z}S_{0,x}S_{1,x}}{18\beta x} + \frac{S_{0,\phi}S_{1,\phi}}{xz^{\frac{3}{2}}} - \frac{x^2}{48H^2 z^3} \phi S_{1,\phi} = 0. \quad (15b)$$

$$\frac{\sqrt{z}S_{0,x}S_{2,x}}{18\beta x} - i\frac{\sqrt{z}S_{1,xx}}{36\beta x} - i\frac{n\sqrt{z}S_{1,x}}{36\beta x^2} + \frac{S_{0,\phi}S_{2,\phi}}{xz^{\frac{3}{2}}} - i\frac{S_{1,\phi\phi}}{xz^{\frac{3}{2}}} + S_{2,z} - \frac{x^2}{48H^2 z^3} \phi S_{2,\phi} = 0. \quad (15c)$$

The corrected expressions for momenta appearing in Eqs. (119c) and (119d) are

$$p_z = -6\alpha H\sqrt{z} - 72\beta H^3\sqrt{z} - \frac{Ha_0^2\phi_0^2}{2\sqrt{z}} \quad (16a)$$

$$p_\phi = -\frac{2Ha_0^3\phi_0^3}{3\phi^2}. \quad (16b)$$

The correct form of the integrals appearing in expressions (120) are

$$\int p_x dx = 24\sqrt{2}\beta H^{\frac{3}{2}} x^{\frac{3}{2}} = 96\beta H^{\frac{3}{2}} z^{\frac{3}{2}}; \quad (17a)$$

$$\int p_z dz = -4\alpha H z^{\frac{3}{2}} - 48\beta H^3 z^{\frac{3}{2}} - Ha_0^2\phi_0^2\sqrt{z}; \quad (17b)$$

$$\int p_\phi d\phi = \frac{2Ha_0^3\phi_0^3}{3\phi} = \frac{2}{3}Ha_0^2\phi_0^2\sqrt{z}. \quad (17c)$$

Therefore, the corrected expression for S_0 appearing in Eq. (121) is

$$S_0 = -4\alpha H z^{\frac{3}{2}} + 48\beta H^3 z^{\frac{3}{2}} - \frac{H}{3} a_0^2 \phi_0^2 \sqrt{z}. \quad (18)$$

Since the modified Horowitz formalism initiates from action (97), or equivalently (99), of the manuscript, the corrected expressions for the on-shell actions (122) and (123) are obtained using classical field equations (1) in actions (97) as well as in (99), instead of (59),

$$A = A_{cl} = \int \left[-12\alpha H^2 a_0^3 e^{3Ht} + 144\beta H^4 a_0^3 e^{3Ht} - \frac{H^2}{3} a_0^3 \phi_0^2 e^{Ht} \right] dt = -4\alpha H a_0^3 e^{3Ht} + 48\beta H^3 a_0^3 e^{3Ht} - \frac{H}{3} a_0^3 \phi_0^2 e^{Ht}, \quad (19)$$

which is the same as the Hamilton-Jacobi function S_0 , obtained in (18). Therefore, the corrected form of the wave function (124) takes the form

$$\psi = \psi_0 e^{\frac{i}{\hbar}[-4\alpha H z^{\frac{3}{2}} + 48\beta H^3 z^{\frac{3}{2}} - \frac{H}{3} a_0^2 \phi_0^2 \sqrt{z}]}, \quad (20)$$

and, finally, the wave function (128) up to first-order approximation is expressed as

$$\psi = \psi_{01} e^{\frac{i}{\hbar}[-4\alpha H z^{\frac{3}{2}} + 48\beta H^3 z^{\frac{3}{2}} - \frac{H}{3} a_0^2 \phi_0^2 \sqrt{z}]}, \quad \text{where } , \psi_{01} = \Psi_0 e^{f(z)}. \quad (21)$$