Probing the seesaw scale with gravitational waves

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The $U(1)_{B-L}$ gauge symmetry is a promising extension of the standard model of particle physics, which is supposed to be broken at some high energy scale. Associated with the $U(1)_{B-L}$ gauge symmetry breaking, right-handed neutrinos acquire their Majorana masses and then tiny light neutrino masses are generated through the seesaw mechanism. In this paper, we demonstrate that the first-order phase transition of the $U(1)_{B-L}$ gauge symmetry breaking can generate a large amplitude of stochastic gravitational wave (GW) radiation for some parameter space of the model, which is detectable in future experiments. Therefore, the detection of GWs is an interesting strategy to probe the seesaw scale which can be much higher than the energy scale of collider experiments.

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I. INTRODUCTION

The nonvanishing neutrino masses have been established through various neutrino oscillation phenomena. The most attractive idea to explain the tiny neutrino masses is the so-called seesaw mechanism with heavy Majorana right-handed (RH) neutrinos [1]. Then, the origin of neutrino masses is ultimately reduced to questions on the origin of RH neutrino masses. It is natural to suppose that masses of RH neutrinos are also generated associated with developing the vacuum expectation value (VEV) of a Higgs field which breaks a certain (gauge) symmetry at a high energy scale.

As a promising and minimal extension of the standard model (SM), we may consider models based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ [2] where the $U(1)_{B-L}$ (baryon number minus lepton number) gauge symmetry is supposed to be broken at a high energy scale. In this class of models with a natural/conventional $U(1)_{B-L}$ charge assignment, the gauge and gravitational anomaly cancellations require us to introduce three RH neutrinos whose Majorana masses are generated by the spontaneous breakdown of the $U(1)_{B-L}$ gauge symmetry. In the case that the $U(1)_{B-L}$ symmetry breaking takes place at an energy

scale higher than the TeV scale, it is very difficult for any collider experiments to address the mechanism of the symmetry breaking and the RH neutrino mass generation.

Detection of gravitational waves brings information about the evolution of the very early Universe. Cosmological GWs could originate from, for instance, quantum fluctuations during inflationary expansion [3] and phase transitions [4,5]. If a first-order phase transition occurs in the early Universe, the dynamics of bubble collision [6–10] and subsequent turbulence of the plasma [11–15] and sonic waves generate GWs [16–18]. These might be within a sensitivity of future space interferometer experiments such as eLISA [19]; the Big Bang Observer (BBO) [20] and DECi-hertz Interferometer Observatory (DECIGO) [21]; or even ground-based detectors such as Advanced LIGO (aLIGO) [22], KAGRA [23] and VIRGO [24].

The spectrum of stochastic GWs produced by the first-order phase transition in the early Universe, in particular, by the SM Higgs doublet field, has been investigated in the literature. Here, the phase transition occurs at the weak scale. See, for instance, Ref. [25] for a recent review.

In this paper, we focus on GWs from the first-order phase transition associated with the spontaneous $U(1)_{B-L}$ gauge symmetry breaking at a scale higher than the TeV scale. GWs generated by a $U(1)_{B-L}$ extended model with the classical conformal invariance [26,27], where its phase transition takes place around the weak scale, have been studied in Ref. [28]. GWs from a second-order $B-L$ phase transition during reheating have been studied in Ref. [29]. In this paper, we consider a slightly extended Higgs sector from the minimal model and introduce an additional $U(1)_{B-L}$ charged

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Higgs field with its charge $+1$. This is one of the key ingredients in this paper. GWs generated by a phase transition in this extended scalar potential, but at TeV scale, have been studied in Ref. [30]. As we will show below, the new Higgs field plays a crucial role in causing the first-order phase transition of the $U(1)_{B-L}$ symmetry breaking and the amplitude of resultant GWs generated by the phase transition can be much larger than the one we naively expect.

II. GW GENERATION BY A COSMOLOGICAL FIRST-ORDER PHASE TRANSITION

In this section, we briefly summarize the properties of GWs produced by a first-order phase transition in the early Universe. There are three main GW production processes and mechanisms: bubble collisions, turbulence [11] and sound waves after bubble collisions [16]. The GW spectrum generated by a first-order phase transition is mainly characterized by two quantities: the ratio of the latent heat energy to the radiation energy density, which is expressed by a parameter α and the transition speed β defined below. In this section, we introduce those parameters and the fitting formula of the GW spectrum.

A. Scalar potential parameters related to the GW spectrum

We consider the system composed of radiation and a scalar field ϕ at temperature T . The energy density of radiation is given by

$$\rho_{\text{rad}} = \frac{\pi^2 g_*}{30} T^4, \quad (1)$$

with g_* being the number of relativistic degrees of freedom in the thermal plasma. At the moment of a first-order phase transition, the potential energy of the scalar field includes the latent energy density given by

$$\epsilon = \left(V - T \frac{\partial V}{\partial T} \right) \Big|_{\{\phi_{\text{high}}, T_*\}} - \left(V - T \frac{\partial V}{\partial T} \right) \Big|_{\{\phi_{\text{low}}, T_*\}}, \quad (2)$$

where $\phi_{\text{high(low)}}$ denotes the field value of ϕ at the high (low) vacuum. Here and hereafter, quantities with the subscript $\star$ stand for those at the time when the phase transition takes place [31]. Then, a parameter α is defined as

$$\alpha \equiv \frac{\epsilon}{\rho_{\text{rad}}}. \quad (3)$$

The bubble nucleation rate per unit volume at a finite temperature is given by

$$\Gamma(T) = \Gamma_0 e^{-S(T)} \simeq \Gamma_0 e^{-S_E^3(T)/T}, \quad (4)$$

where Γ_0 is a coefficient of the order of the transition energy scale, S is the action in the four-dimensional

Minkowski space, and S_E^3 is the three-dimensional Euclidean action [9]. The inverse of the transition timescale can be defined as

$$\beta \equiv - \frac{dS}{dt} \Big|_{t_*}. \quad (5)$$

Its dimensionless parameter $\beta/H_\star$ can be expressed as

$$\frac{\beta}{H_\star} \simeq T \frac{dS}{dT} \Big|_{T_*} = T \frac{d(S_E^3/T)}{dT} \Big|_{T_*}. \quad (6)$$

B. GW spectrum

1. Bubble collisions

Under the envelope approximation¹ and for $\beta/H_\star \gg 1$ [10], the peak frequency and the peak amplitude of GWs generated by bubble collisions are given by

$$f_{\text{peak}} \simeq 17 \left(\frac{f_\star}{\beta} \right) \left(\frac{\beta}{H_\star} \right) \left(\frac{T_\star}{10^8 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}, \quad (7)$$

$$h^2 \Omega_{\text{GW}}(f_{\text{peak}}) \simeq 1.7 \times 10^{-5} \kappa^2 \Delta \left(\frac{\beta}{H_\star} \right)^{-2} \left(\frac{\alpha}{1+\alpha} \right)^2 \times \left(\frac{g_*}{100} \right)^{-1/3}, \quad (8)$$

with the following fitting functions:

$$\Delta = \frac{0.11 v_b^3}{0.42 + v_b^2}, \quad (9)$$

$$\frac{f_\star}{\beta} = \frac{0.62}{1.8 - 0.1 v_b + v_b^2}, \quad (10)$$

where v_b denotes the bubble wall velocity. The efficiency factor (κ) is given by [11]

$$\kappa = \frac{1}{1 + A\alpha} \left(A\alpha + \frac{4}{27} \sqrt{\frac{3\alpha}{2}} \right), \quad (11)$$

with $A = 0.715$. The full GW spectrum is expressed as [31]

$$\Omega_{\text{GW}}(f) = \Omega_{\text{GW}}(f_{\text{peak}}) \frac{(a+b)f_{\text{peak}}^b f^a}{b f_{\text{peak}}^{a+b} + a f^{a+b}}, \quad (12)$$

with numerical factors $a \in [2.66, 2.82]$ and $b \in [0.90, 1.19]$. We set the values of $(a, b, v_b) = (2.7, 1.0, 0.6)$ in our analysis.

¹For a recent development beyond the envelope approximation, see Ref. [32].

2. Sound waves

The peak frequency and the peak amplitude of GWs generated by sound waves are given by [16,17]

$$f_{\text{peak}} \simeq 19 \frac{1}{v_b} \left(\frac{\beta}{H_\star} \right) \left(\frac{T_\star}{10^8 \text{ GeV}} \right) \left(\frac{g_\star}{100} \right)^{1/6} \text{ Hz}, \quad (13)$$

$$h^2 \Omega_{\text{GW}}(f_{\text{peak}}) \simeq 2.7 \times 10^{-6} \kappa_v^2 v_b \left(\frac{\beta}{H_\star} \right)^{-1} \left(\frac{\alpha}{1+\alpha} \right)^2 \times \left(\frac{g_\star}{100} \right)^{-1/3}. \quad (14)$$

The efficiency factor (κ_v) is given by [33]

$$\kappa_v \simeq \begin{cases} v_b^{6/5} \frac{6.9\alpha}{1.36-0.037\sqrt{\alpha}+\alpha} & \text{for } v_b \ll c_s \\ \frac{\alpha}{0.73+0.083\sqrt{\alpha}+\alpha} & \text{for } v_b \simeq 1 \end{cases}, \quad (15)$$

with c_s being the sonic speed. The spectrum shape is expressed as [34]

$$\left(\frac{f}{f_{\text{peak}}} \right)^3 \left(\frac{7}{4 + 3 \left(\frac{f}{f_{\text{peak}}} \right)^2} \right)^{7/2}. \quad (16)$$

3. Turbulence

The peak frequency and amplitude of GWs generated by turbulence are given by [11]

$$f_{\text{peak}} \simeq 27 \frac{1}{v_b} \left(\frac{\beta}{H_\star} \right) \left(\frac{T_\star}{10^8 \text{ GeV}} \right) \left(\frac{g_\star}{100} \right)^{1/6} \text{ Hz}, \quad (17)$$

$$h^2 \Omega_{\text{GW}}(f_{\text{peak}}) \simeq 3.4 \times 10^{-4} v_b \left(\frac{\beta}{H_\star} \right)^{-1} \left(\frac{\kappa_{\text{turb}} \alpha}{1+\alpha} \right)^{3/2} \times \left(\frac{g_\star}{100} \right)^{-1/3}. \quad (18)$$

In our analysis, we conservatively set the efficiency factor for turbulence to be $\kappa_{\text{turb}} \simeq 0.05 \kappa_v$ as in Ref. [34]. The spectrum shape is given by [15,34,35]

$$\frac{\left(\frac{f}{f_{\text{peak}}} \right)^3}{\left(1 + \frac{f}{f_{\text{peak}}} \right)^{11/3} \left(1 + \frac{8\pi f}{h_\star} \right)}, \quad (19)$$

with

$$h_\star = 17 \left(\frac{T_\star}{10^8 \text{ GeV}} \right) \left(\frac{g_\star}{100} \right)^{1/6} \text{ Hz}. \quad (20)$$

III. GWS GENERATED BY SEESAW PHASE TRANSITION

A. $B-L$ seesaw model

Our model is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, where three RH neutrinos

TABLE I. The particle content of our $U(1)_{B-L}$ model. In addition to the SM particle content ($i = 1, 2, 3$), three RH neutrinos [N_R^i ($i = 1, 2, 3$)] and two $U(1)_{B-L}$ Higgs fields (Φ_1 and Φ_2) are introduced.

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L^i	3	2	1/6	1/3
u_R^i	3	1	2/3	1/3
d_R^i	3	1	-1/3	1/3
ℓ_L^i	1	2	-1/2	-1
e_R^i	1	1	-1	-1
H	1	2	-1/2	0
N_R^i	1	1	0	-1
Φ_1	1	1	0	+1
Φ_2	1	1	0	+2

(N_R^i with i running 1,2,3) and two SM singlet $B-L$ Higgs fields (Φ_1 and Φ_2) are introduced. Under these gauge groups, three generations of RH neutrinos have to be introduced for the anomaly cancellation. The particle content is listed in Table I. The Yukawa sector of the SM is extended to have

$$\mathcal{L}_{\text{Yukawa}} \supset - \sum_{i=1}^3 \sum_{j=1}^3 Y_D^{ij} \bar{\ell}_L^i H N_R^j - \frac{1}{2} \sum_{k=1}^3 Y_{N^k} \Phi_2 \bar{N}_R^k N_R^k + \text{H.c.}, \quad (21)$$

where the first term is the neutrino Dirac Yukawa coupling, and the second is the Majorana Yukawa couplings. Once the $U(1)_{B-L}$ Higgs field Φ_2 develops a nonzero VEV, the $U(1)_{B-L}$ gauge symmetry is broken and the Majorana mass terms of the RH neutrinos are generated. Then, the seesaw mechanism is automatically implemented in the model after the electroweak symmetry breaking.

We consider the following scalar potential:

$$V(\Phi_1, \Phi_2) = \frac{1}{2} \lambda_1 (\Phi_1 \Phi_1^\dagger)^2 + \frac{1}{2} \lambda_2 (\Phi_2 \Phi_2^\dagger)^2 + \lambda_3 \Phi_1 \Phi_1^\dagger (\Phi_2 \Phi_2^\dagger) + M_{\Phi_1}^2 \Phi_1 \Phi_1^\dagger - M_{\Phi_2}^2 \Phi_2 \Phi_2^\dagger - A(\Phi_1 \Phi_1 \Phi_2^\dagger + \Phi_1^\dagger \Phi_1^\dagger \Phi_2). \quad (22)$$

Here, we omit the SM Higgs field (H) part and its interaction terms for not only simplicity but also little importance in the following discussion, since we are interested in the case that the VEVs of $B-L$ Higgs fields are much larger than that of the SM Higgs field.² All parameters in the potential (22) are taken to be real and

²For the case of a phase transition of the SM Higgs field interacting with new Higgs fields, see, for example, Ref. [36].

positive. At the $U(1)_{B-L}$ symmetry breaking vacuum, the $B-L$ Higgs fields are expanded around those VEVs v_1 and v_2 , as

$$\Phi_1 = \frac{v_1 + \phi_1 + i\chi_1}{\sqrt{2}}, \quad (23)$$

$$\Phi_2 = \frac{v_2 + \phi_2 + i\chi_2}{\sqrt{2}}. \quad (24)$$

Here, ϕ_1 and ϕ_2 correspond to two real degrees of freedom as CP -even scalars, one linear combination of χ_1 and χ_2 is the Nambu-Goldstone mode eaten by the $U(1)_{B-L}$ gauge boson (Z' boson) and the other is left as a physical CP -odd scalar. Mass terms of particles are expressed as

$$\begin{aligned} \mathcal{L}_{\text{mass}} = & -\frac{1}{2}(\phi_2 \phi_1) \begin{pmatrix} \frac{1}{2}\lambda_3 v_1^2 + \frac{3}{2}\lambda_2 v_2^2 - M_{\Phi_2}^2 & v_1(-\sqrt{2}A + \lambda_3 v_2) \\ v_1(-\sqrt{2}A + \lambda_3 v_2) & \frac{3\lambda_1 v_1^2}{2} + \frac{\lambda_3 v_2^2}{2} + M_{\Phi_1}^2 - \sqrt{2}A v_2 \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_1 \end{pmatrix} \\ & -\frac{1}{2}(\chi_2 \chi_1) \begin{pmatrix} \frac{1}{2}\lambda_3 v_1^2 + \frac{1}{2}\lambda_2 v_2^2 - M_{\Phi_2}^2 & -\sqrt{2}A v_1 \\ -\sqrt{2}A v_1 & \frac{1}{2}\lambda_1 v_1^2 + M_{\Phi_1}^2 + \frac{1}{2}\lambda_3 v_2^2 + \sqrt{2}A v_2 \end{pmatrix} \begin{pmatrix} \chi_2 \\ \chi_1 \end{pmatrix} \\ & -\frac{1}{2}g_{B-L}^2(4v_2^2 + v_1^2)Z'^\mu Z'_\mu - \frac{1}{2}\sum_i \bar{N}_i^c \frac{Y_{N^i} v_2}{\sqrt{2}} N_i. \end{aligned} \quad (25)$$

With the $U(1)_{B-L}$ symmetry breaking, the RH neutrinos N_R^i and the Z' boson acquire their masses, respectively, as

$$m_{N_R^i} = \frac{Y_{N^i}}{\sqrt{2}} v_2, \quad (26)$$

$$m_{Z'}^2 = g_{B-L}^2(4v_2^2 + v_1^2), \quad (27)$$

where g_{B-L} is the $U(1)_{B-L}$ gauge coupling. The mass matrix of CP -even Higgs bosons (ϕ_1 and ϕ_2) and the mass of the physical CP -odd scalar P can be, respectively, simplified as

$$\begin{pmatrix} \lambda_2 v_2^2 + \frac{A v_1^2}{\sqrt{2} v_2} & v_1(\lambda_3 v_2 - \sqrt{2}A) \\ v_1(\lambda_3 v_2 - \sqrt{2}A) & \lambda_1 v_1^2 \end{pmatrix}, \quad (28)$$

and

$$m_P^2 = \frac{A}{\sqrt{2} v_2}(v_1^2 + 4v_2^2), \quad (29)$$

by eliminating $M_{\Phi_2}^2$ and $M_{\Phi_1}^2$ under the stationary conditions,

$$\frac{\lambda_2 v_2^3}{2} + \frac{1}{2}\lambda_3 v_1^2 v_2 - M_{\Phi_2}^2 v_2 - \frac{A v_1^2}{\sqrt{2}} = 0, \quad (30)$$

$$\frac{\lambda_1 v_1^3}{2} + \frac{1}{2}\lambda_3 v_1 v_2^2 + M_{\Phi_1}^2 v_1 - \sqrt{2}A v_1 v_2 = 0. \quad (31)$$

FIG. 1. The three-dimensional plot of the one-loop corrected scalar potential of two $B-L$ Higgs fields Φ_1 and Φ_2 at zero temperature, which induces a first-order phase transition in the early Universe.

Let us here note the LEP constraint $m_{Z'}/g_{B-L} = \sqrt{4v_2^2 + v_1^2} \gtrsim 6$ TeV [37,38] and the constraint from the LHC Run-2 (see, for example, Refs. [39–42])

$$m_{Z'} \gtrsim 3.9 \text{ TeV}, \quad (32)$$

for $g_{B-L} \simeq 0.7$.

With a suitable choice of parameters, we find that the phase transition of the $B-L$ gauge symmetry breaking by the Higgs fields Φ_1 and Φ_2 becomes of the first order in the early Universe. In the following analysis, we set $\lambda_1 = 0.1$,

TABLE II. Input and output parameters for several benchmark points are listed. All dimensionful quantities are shown in units of TeV.

Point	g_{B-L}	v_1	v_2	A	α	β/H_*	T_*
A	0.7	4	4	1.1	0.086	109	1.272
B	0.7	100	100	29	0.69	104	16.18
C	0.71	10^4	10^4	2900	0.89	52.24	1515
D	0.72	10^5	10^5	29000	0.77	57.9	15719

TABLE III. The mass spectrum of the Z' boson and new Higgs bosons for point A is shown in units of TeV.

Point	$m_{Z'}$	m_P	m_{H_1}	m_{H_2}
A	6.26	2.49	0.65	2.10

FIG. 2. The predicted GW spectra for the benchmark points with $v_b = 0.6$ are shown. Green, blue, red and purple curves from left to right correspond to points A, B, C and D, respectively. The future experimental sensitivity curves of LISA [44], DECIGO and BBO [45], aLIGO [46] and Cosmic Explore (CE) [47] are also shown as black curves.

$\lambda_2 = 0.1$, and $\lambda_3 = 0.3$ and all corrections through neutrino Yukawa couplings Y_{Ni} have been neglected, assuming $Y_{Ni} \ll g_{B-L}$, for simplicity. We show in Fig. 1 the shape of the one-loop scalar potential (22).

Implementing our model into the public code CosmoTransitions [43], we have evaluated the parameters α, β and T_* at a renormalization scale $Q^2 = (v_2^2 + v_1^2)/2$. We list our results for four benchmark points in Table II. In Table III, we list the new particles' mass spectrum for point A, which can be tested by the future LHC experiment. Except for point A, one can easily see the benchmark points are far beyond the reach of collider experiments.

In Fig. 2, we show predicted GW spectra for our benchmark points along with expected sensitivities of future interferometer experiments. Here, the resultant spectra have been calculated with a bubble wall speed of $v_b = 0.6$. We have confirmed that the results are not so significantly changed for other v_b values of $\mathcal{O}(0.1)$. Green, blue, red and purple curves from left to right correspond to points A, B, C and D, respectively. Black solid curves denote the expected sensitivities of each indicated experiment, according to Ref. [44] for LISA, Ref. [45] for DECIGO and BBO, Ref. [46] for aLIGO and Ref. [47] for Cosmic

Explore (CE). Curves are drawn by gwplotter [48]. The sensitivities of DECIGO and BBO reach the results of points A and B. Point C is an example which is not marginally able to be detected by DECIGO/BBO but its peak is within the reach of CE.

IV. SUMMARY

The origin of heavy Majorana RH neutrino masses is one of the essential pieces to understand the origin of neutrino masses through the seesaw mechanism. Gauged $B-L$ symmetry and its breakdown are a natural framework to introduce the RH neutrinos into the SM and to generate their Majorana masses. The seesaw scale is in general far beyond the reach of future collider experiments. In this paper, we have investigated a possibility to probe the seesaw scale through the observation of stochastic GW radiation. We have shown in the context of a simple $U(1)_{B-L}$ extended SM that the first-order phase transition of the $B-L$ Higgs potential can generate an amplitude of GWs large enough to be detected in future experiments. Such a detection is informative to estimate the seesaw scale. Grojean and Servant have shown that GWs generated by phase transitions at $T_* \sim 10^7$ GeV are in reach of future experiments [49]. (For recent studies, see e.g., Refs. [50,51]). We have demonstrated that the detection of GWs is indeed possible in our model context.

At last, we should note a delicate and critical caveat about the issue of gauge dependence of the effective Higgs potential. See, for example, Ref. [52] for recent discussions. So far, we have no clear resolution to this issue. According to Ref. [52], the resultant GW spectrum has one order of magnitude uncertainties under a specific gauge choice. Thus, even for the worst case, our benchmark points A and B can still be within the reach of future experiments. Once a better prescription has been developed, we will reevaluate the amplitude of GWs.

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