Looking for a $ud\bar{s}\,\bar{b}$ bound state in the chiral quark model

Xiaoyun Chen[1,*] and Jialun Ping[2,†]

[1]*Department of Basic Courses, Jinling Institute of Technology, Nanjing 211169, People's Republic of China*
[2]*Department of Physics, Nanjing Normal University, Nanjing 210023, People's Republic of China*

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Inspired by the report of D0 Collaboration on the state $X(5568)$ with four different flavors, a similar state, $ud\bar{s}\,\bar{b}$, is investigated in the present work. The advantage of looking for this state over the state $X(5568)$ with quark contents, $bu\bar{d}\,\bar{s}$ or $bd\bar{u}\,\bar{s}$, is that the BK threshold is 270 MeV higher than that of $B_s\pi$, and it allows a large mass region for $ud\bar{s}\,\bar{b}$, which cannot decay to $B_s\pi$, to be stable. The chiral quark model and Gaussian expansion method are employed to do the calculations of four-quark states, $ud\bar{s}\,\bar{b}$, with quantum numbers IJ^P ($I=0,1;J=0,1,2;P=+$). Two structures, diquark-antidiquark and meson-meson, with all possible color configurations are considered. The results indicate that energies of the tetraquark with diquark-antiquark configuration are all higher than the threshold of BK, but the resonances are still possible because of their structures. For the state of $IJ^P=00^+$ in the meson-meson structure, the energies are just below the corresponding thresholds, where the color channel coupling plays an important role. The bound state is possible. The distances between two objects (quark/antiquark) show that the state is a molecular one.

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I. INTRODUCTION

Since the first exotic resonance $X(3872)$ was observed by the Bell Collaboration in 2003 [1], many other exotic states, so-called XYZ states, have emerged from the reports of Belle, *BABAR*, BESIII, LHCb, CDF, D0, and other collaborations. In the traditional quark models, the meson consists of a quark and antiquark, and the baryon is made up of three quarks. How to explain these exotic states is a big challenge for quark models. The study of the exotic states is helpful for improving quark models and deepening our understanding of nonperturbative quantum chromodynamics (QCD).

Although these XYZ states are difficult to explain as ordinary hadrons, their quantum numbers can be constructed by quark-antiquark combinations. To find an unambiguous tetraquark state, the particle with four different flavors is expected. Not so long ago, the D0 Collaboration announced a new resonance named $X(5568)$ with mass $M=5567.8\pm 2.9^{+0.9}_{-1.9}$ MeV and narrow width $\Gamma=21.9\pm 6.4^{+5.0}_{-2.5}$ MeV. [2]. Recently, the D0 Collaboration reported further evidence about this state in the weak decay of B with a significance of 6.7σ [3], which is consistent with their previous measurement [2]. Subsequently, searches for $X(5568)$ in decays to $B_s^0\pi^\pm$, $B_s^0\to J/\psi\phi$ were performed by the LHCb [4], CMS [5], and ATLAS [6] Collaborations in pp collisions and by the CDF Collaboration [7] at the Tevatron, and all of these experiments revealed no signal. Clearly, more measurements are needed.

On the theoretical side, the early work based on QCD sum rules supported the existence of the state $X(5568)$ [8–12]. Some quark model calculations also claimed to have possible explanations for the results of the D0 Collaboration [13–15]. However, a detailed examination of the various interpretations of the state $X(5568)$ showed that the threshold, cusp, molecular, and tetraquark models are all unfavored [16,17]. Based on the general properties of QCD, F. K. Guo *et al.* argued that QCD does not support the existence of the state $X(5568)$ [18]. Our previous quark model calculation of diquark-antidiquark states and molecules also obtained negative results [19].

In this work, another particle with four different flavors, $ud\bar{s}\,\bar{b}$, which is different from $X(5568)$ consisting of $us\bar{d}\,\bar{b}$, is considered. For simplicity, we denote the particle $ud\bar{s}\,\bar{b}$ as T_{bs}. The reasons for looking for this particle are as follows. First, the breaking up state of T_{bs} is BK, in which the threshold is higher than the $B_s\pi$ threshold of $X(5568)$, and it leads to a large mass region for T_{bs} to be stable. Second, for diquark-antidiquark configuration, the ud quark pair is more stable than the us, owing to the d quark having lower mass than the s quark. In other words, if

*xychen@jit.edu.cn
†jlping@njnu.edu.cn

$X(5568)$ does exist, the T_{bs} must be a stable state. If $X(5568)$ is proved to be nonexistent, there's still probability for T_{bs} to be stable. F. S. Yu also suggested that $bs\bar{u}\,\bar{d}$ is a promising detectable tetraquark state, and the most favorable decay mode to observe it will be $J/\psi K^- K^- \pi^+$ experimentally [20].

In order to look for the particle of T_{bs} theoretically, we calculate the masses of the states with quantum numbers IJ^P ($I=0,1; J=0,1,2; P=+$), including two different structures, diquark-antidiquark and meson-meson in the chiral quark model, and all possible color configurations are investigated by using the Gaussian expansion method (GEM) [21]. In the calculation, two ways of using σ meson exchange are adopted. One is that the σ meson exchange only occurs between the u quark and/or d quark. Another is that the effective σ is exchanged between u, d, and s quarks. If a bound state is obtained, the average distances between quarks or antiquarks, which can be used to clarify the structures of the states, a compact tetraquark or a molecular state, are calculated.

The chiral quark model, the wave functions of T_{bs}, and the method for solving the four quark states are detailed in Sec. II; Sec. III is devoted to a discussion of the results. Section IV is a summary.

II. CHIRAL QUARK MODEL AND WAVE FUNCTIONS OF T_{bs}

A. The chiral quark model

The chiral quark model has been successful in describing both the hadron spectra and hadron-hadron interactions. The details of the model can be found in Refs. [22,23]. Here, only the Hamiltonian of the chiral quark model is given as follows:

$$H = \sum_{i=1}^{4} m_i + \frac{p_{12}^2}{2\mu_{12}} + \frac{p_{34}^2}{2\mu_{34}} + \frac{p_{1234}^2}{2\mu_{1234}} + \sum_{i<j=1}^{4} V_{ij}, \tag{1}$$

$$V_{ij} = V_{ij}^C + V_{ij}^G + \sum_{\chi=\pi,K,\eta} V_{ij}^{\chi} + V_{ij}^{\sigma}. \tag{2}$$

The potential energy is constituted of pieces describing quark confinement (C), one-gluon-exchange (G), one Goldstone boson exchange ($\chi = \pi$, K, η), and a σ exchange. The form for the low-lying four-quark states is [22]

$$V_{ij}^C = (-a_c r_{ij}^2 - \Delta)\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c, \tag{3a}$$

$$V_{ij}^G = \frac{\alpha_s}{4}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \left[\frac{1}{r_{ij}} - \frac{2\pi}{3m_i m_j}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \delta(\boldsymbol{r}_{ij})\right], \tag{3b}$$

$$\delta(\boldsymbol{r}_{ij}) = \frac{e^{-r_{ij}/r_0(\mu_{ij})}}{4\pi r_{ij} r_0^2(\mu_{ij})}, \tag{3c}$$

$$V_{ij}^{\pi} = \frac{g_{ch}^2}{4\pi}\frac{m_\pi^2}{12 m_i m_j}\frac{\Lambda_\pi^2}{\Lambda_\pi^2 - m_\pi^2} m_\pi v_{ij}^{\pi} \sum_{a=1}^{3} \lambda_i^a \lambda_j^a, \tag{3d}$$

$$V_{ij}^{K} = \frac{g_{ch}^2}{4\pi}\frac{m_K^2}{12 m_i m_j}\frac{\Lambda_K^2}{\Lambda_K^2 - m_K^2} m_K v_{ij}^{K} \sum_{a=4}^{7} \lambda_i^a \lambda_j^a, \tag{3e}$$

$$V_{ij}^{\eta} = \frac{g_{ch}^2}{4\pi}\frac{m_\eta^2}{12 m_i m_j}\frac{\Lambda_\eta^2}{\Lambda_\eta^2 - m_\eta^2} m_\eta v_{ij}^{\eta} \times [\lambda_i^8 \lambda_j^8 \cos\theta_P - \lambda_i^0 \lambda_j^0 \sin\theta_P], \tag{3f}$$

$$v_{ij}^{\chi} = \left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij})\right]\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \tag{3g}$$

$$V_{ij}^{\sigma} = -\frac{g_{ch}^2}{4\pi}\frac{\Lambda_\sigma^2}{\Lambda_\sigma^2 - m_\sigma^2} m_\sigma \times \left[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij})\right], \tag{3h}$$

where $Y(x) = e^{-x}/x$; m_i is the constituent mass of ith quarks/antiquarks, and μ_{ij} is the reduced mass of the two interacting particles and

$$\mu_{1234} = \frac{(m_1 + m_2)(m_3 + m_4)}{m_1 + m_2 + m_3 + m_4}, \tag{4}$$

where $\mathbf{p}_{ij} = (\mathbf{p}_i - \mathbf{p}_j)/2$, $\mathbf{p}_{1234} = (\mathbf{p}_{12} - \mathbf{p}_{34})/2$; $r_0(\mu_{ij}) = s_0/\mu_{ij}$; $\boldsymbol{\sigma}$ are the $SU(2)$ Pauli matrices; $\boldsymbol{\lambda}$, $\boldsymbol{\lambda}^c$ are $SU(3)$ flavor and color Gell-Mann matrices, respectively; $g_{ch}^2/4\pi$ is the chiral coupling constant, determined from the π-nucleon coupling; and α_s is an effective scale-dependent running coupling [23],

$$\alpha_s(\mu_{ij}) = \frac{\alpha_0}{\ln[(\mu_{ij}^2 + \mu_0^2)/\Lambda_0^2]}. \tag{5}$$

Although the chiral quark model describes properties of hadrons and hadron-hadron interactions well, in any model, the predictions or the results can only be trusted to a certain level of precision because the model parameters themselves have some associated level of uncertainty. Thus, it is essential to give the errors of the model parameters and the calculated results for drawing appropriate conclusions.

In this paper, the model parameters of quark masses and the Goldstone boson exchanges are taken directly from our previous work [22], which is shown in Table I, and the other six parameters of quark confinement and one-gluon-exchange a_c, Δ, α_0, Λ_0, μ_0, s_0 are taken as free parameters and fixed with uncertainties by fitting the mass spectra of the ground states of mesons from light to heavy using the MINUIT program. These are presented in Tables II and III, respectively. From Table II, we can see that the newly obtained six parameters are similar to those in Ref. [22].

TABLE I. Model parameters, determined by fitting the meson spectrum in Ref. [22].

Quark masses (MeV)	$m_u = m_d$	313
	m_s	536
	m_c	1728
	m_b	5112
Goldstone bosons (fm^{-1})	m_π	0.70
	m_σ	3.42
	m_η	2.77
	m_K	2.51
	$\Lambda_\pi = \Lambda_\sigma$	4.2
	$\Lambda_\eta = \Lambda_K$	5.2
	$g_{ch}^2/(4\pi)$	0.54
	$\theta_p(°)$	−15

TABLE II. Adjustable parameters of the quark confinement and one-gluon-exchange potential in the chiral quark model (units: Δ, μ_0, s_0, Mev; a_c, MeV fm^{-2}; Λ_0, fm^{-1}; α_0, dimensionless).

Parameters	x_i	Δx_i	Parameters	x_i	Δx_i
a_c	100.0	1.946	Λ_0	0.026	0.001
Δ	−80.0	1.646	μ_0	30.002	17.81
α_0	3.627	0.031	s_0	25.638	0.295

B. The wave functions of T_{bs}

We will introduce the wave functions for the two structures, diquark-antidiquark and meson-meson, respectively. For each degree of freedom, first we construct the wave functions for two-body clusters, then coupling the wave functions of two clusters to the wave functions of four-quark states.

1. Diquark-antidiquark structure

For spin, the wave functions for two-body clusters are

$$\chi_{11} = \alpha\alpha, \qquad \chi_{10} = \frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha), \qquad \chi_{1-1} = \beta\beta,$$
$$\chi_{00} = \frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha), \tag{6}$$

then the wave functions for four-quark states are obtained,

$$\chi_0^{\sigma 1} = \chi_{00}\chi_{00}, \tag{7a}$$

$$\chi_0^{\sigma 2} = \sqrt{\frac{1}{3}}(\chi_{11}\chi_{1-1} - \chi_{10}\chi_{10} + \chi_{1-1}\chi_{11}), \tag{7b}$$

$$\chi_1^{\sigma 3} = \chi_{00}\chi_{11}, \tag{7c}$$

$$\chi_1^{\sigma 4} = \chi_{11}\chi_{00}, \tag{7d}$$

$$\chi_1^{\sigma 5} = \frac{1}{\sqrt{2}}(\chi_{11}\chi_{10} - \chi_{10}\chi_{11}), \tag{7e}$$

$$\chi_2^{\sigma 6} = \chi_{11}\chi_{11}, \tag{7f}$$

where the subscript of χ represents the total spin of T_{bs}, and it takes the values $S = 0,\ 1,\ 2$.

For flavor, the wave functions for four-quark states are

$$\chi_{d0}^{f1} = \frac{1}{\sqrt{2}}(ud - du)\bar{s}\,\bar{b}, \tag{8a}$$

$$\chi_{d1}^{f2} = \frac{1}{\sqrt{2}}(ud + du)\bar{s}\,\bar{b}. \tag{8b}$$

Analogously, the subscript of χ_d represents the isospin of T_{bs}, $I = 0,\ 1$.

For color, there are two color configurations for the quark pair, [11] and [2], respectively.

$$\chi_1^{[11]} = \frac{1}{\sqrt{2}}(rg - gr), \tag{9a}$$

$$\chi_2^{[11]} = \frac{1}{\sqrt{2}}(rb - br), \tag{9b}$$

$$\chi_3^{[11]} = \frac{1}{\sqrt{2}}(gb - bg), \tag{9c}$$

$$\chi_1^{[2]} = rr, \qquad \chi_2^{[2]} = gg, \qquad \chi_3^{[2]} = bb, \tag{10a}$$

TABLE III. Ground state light and heavy meson spectra in fit (unit: MeV).

States	$E + \Delta E$	PDG	States	$E + \Delta E$	PDG
π	139.6 ± 20.6	139.6	D^*	1986.4 ± 12.1	2006.9
ρ	777.1 ± 15.0	775.3	D_s	1967.0 ± 8.6	1968.3
ω	708.6 ± 14.1	782.7	D_s^*	2085.9 ± 10.5	2112.1
K	502.5 ± 13.9	493.7	η_c	3003.1 ± 6.2	2983.6
K^*	918.0 ± 13.8	891.7	J/ψ	3103.2 ± 8.5	3096.9
B	5290.3 ± 10.9	5279.0	ϕ	1020.7 ± 12.2	1019.5
B^*	5326.4 ± 11.4	5325.2	η'	724.3 ± 10.5	957.8
B_s^0	5378.2 ± 9.1	5366.8	η_b	9323.3 ± 0.7	9399.0
B_s^*	5417.1 ± 9.8	5415.4	Υ	9454.9 ± 2.8	9460.3
D	1875.1 ± 10.6	1869.6			

$$\chi_4^{[2]} = \frac{1}{\sqrt{2}}(rg + gr), \tag{10b}$$

$$\chi_5^{[2]} = \frac{1}{\sqrt{2}}(rb + br), \tag{10c}$$

$$\chi_6^{[2]} = \frac{1}{\sqrt{2}}(gb + bg). \tag{10d}$$

For the antiquark pair, the two color configurations are [211] and [22], respectively,

$$\chi_1^{[211]} = \frac{1}{\sqrt{2}}(\bar{g}\,\bar{b} - \bar{b}\,\bar{g}),$$
$$\chi_2^{[211]} = \frac{1}{\sqrt{2}}(\bar{r}\,\bar{b} - \bar{b}\,\bar{r}),$$
$$\chi_3^{[211]} = \frac{1}{\sqrt{2}}(\bar{r}\,\bar{g} - \bar{g}\,\bar{r}). \tag{11}$$

$$\chi_1^{[22]} = \bar{r}\,\bar{r}, \qquad \chi_2^{[22]} = \bar{g}\,\bar{g}, \qquad \chi_3^{[22]} = \bar{b}\,\bar{b},$$
$$\chi_4^{[22]} = -\frac{1}{\sqrt{2}}(\bar{g}\,\bar{b} + \bar{b}\,\bar{g}),$$
$$\chi_5^{[22]} = \frac{1}{\sqrt{2}}(\bar{r}\,\bar{b} + \bar{b}\,\bar{r}),$$
$$\chi_6^{[22]} = -\frac{1}{\sqrt{2}}(\bar{r}\,\bar{g} + \bar{g}\,\bar{r}). \tag{12}$$

The color wave functions of T_{bs} in the diquark-antidiquark structure should be color singlet [222], and it can be obtained by using the $SU(3)$ Clebsh-Gordan coefficients,

$$\chi_{d1}^c = \frac{\sqrt{3}}{6}(rg\bar{r}\,\bar{g} - rg\bar{g}\,\bar{r} + gr\bar{g}\,\bar{r} - gr\bar{r}\,\bar{g} + rb\bar{r}\,\bar{b} - rb\bar{b}\,\bar{r} + br\bar{b}\,\bar{r} - br\bar{r}\,\bar{b} + gb\bar{g}\,\bar{b} - gb\bar{b}\,\bar{g} + bg\bar{b}\,\bar{g} - bg\bar{g}\,\bar{b}). \tag{13a}$$

$$\chi_{d2}^c = \frac{\sqrt{6}}{12}(2rr\bar{r}\,\bar{r} + 2gg\bar{g}\,\bar{g} + 2bb\bar{b}\,\bar{b} + rg\bar{r}\,\bar{g} + rg\bar{g}\,\bar{r} + gr\bar{g}\,\bar{r} + gr\bar{r}\,\bar{g} + rb\bar{r}\,\bar{b} + rb\bar{b}\,\bar{r} + br\bar{b}\,\bar{r} + br\bar{r}\,\bar{b} + gb\bar{g}\,\bar{b} + gb\bar{b}\,\bar{g} + bg\bar{b}\,\bar{g} + bg\bar{g}\,\bar{b}), \tag{13b}$$

where χ_{d1}^c and χ_{d2}^c represent the color antitriplet-triplet ($\bar{3} \times 3$) and sextet-antisextet ($6 \times \bar{6}$), respectively.

2. Meson-meson structure

For spin, the wave functions are the same as those of the diquark-antidiquark structure, Eq. (7).

The flavor wave functions of T_{bs} are as follows,

$$\chi_{m0}^{f1} = \frac{1}{\sqrt{2}}(\bar{s}u\bar{b}d - \bar{s}d\bar{b}u), \tag{14a}$$

$$\chi_{m1}^{f2} = \frac{1}{\sqrt{2}}(\bar{s}u\bar{b}d + \bar{s}d\bar{b}u), \tag{14b}$$

where the subscript of χ_m represents the isospin of T_{bs}, $I = 0$, 1.

For color wave functions, the possible color configurations of a cluster consisting of a quark and an antiquark are [111] and [21],

$$\chi_1^{[111]} = \frac{1}{\sqrt{3}}(\bar{r}r + \bar{g}g + \bar{b}b). \tag{15}$$

$$\chi_1^{[21]} = \bar{b}r, \qquad \chi_2^{[21]} = \bar{b}g, \qquad \chi_3^{[21]} = -\bar{g}r,$$
$$\chi_4^{[21]} = \frac{1}{\sqrt{2}}(\bar{r}r - \bar{g}g), \qquad \chi_5^{[21]} = \frac{1}{\sqrt{6}}(2\bar{b}b - \bar{r}r - \bar{g}g),$$
$$\chi_6^{[21]} = \bar{r}g, \qquad \chi_7^{[21]} = -\bar{g}b, \qquad \chi_8^{[21]} = \bar{r}b. \tag{16}$$

At last, the color singlet wave functions of T_{bs} in the meson-meson structure are

$$\chi_{m1}^c = \frac{1}{3}(\bar{r}r + \bar{g}g + \bar{b}b)(\bar{r}r + \bar{g}g + \bar{b}b), \tag{17a}$$

$$\chi_{m2}^c = \frac{\sqrt{2}}{12}(3\bar{b}r\bar{r}b + 3\bar{g}r\bar{r}g + 3\bar{b}g\bar{g}b + 3\bar{g}b\bar{b}g + 3\bar{r}g\bar{g}r + 3\bar{r}b\bar{b}r + 2\bar{r}r\bar{r}r + 2\bar{g}g\bar{g}g + 2\bar{b}b\bar{b}b - \bar{r}r\bar{g}g - \bar{g}g\bar{r}r - \bar{b}b\bar{g}g - \bar{b}b\bar{r}r - \bar{g}g\bar{b}b - \bar{r}r\bar{b}b), \tag{17b}$$

where χ_{m1}^c and χ_{m2}^c represent the color singlet-singlet (1×1) and color octet-octet (8×8), respectively.

As for the spatial wave functions, the total orbital wave functions can be constructed by coupling the orbital wave function for each relative motion of the system,

$$\Psi_L^{M_L} = [[\Psi_{l_1}(\mathbf{r}_{12})\Psi_{l_2}(\mathbf{r}_{34})]_{l_{12}}\Psi_{L_r}(\mathbf{r}_{1234})]_L^{M_L}, \tag{18}$$

where L is the total orbital angular momentum of T_{bs}, $\Psi_{L_r}(\mathbf{r}_{1234})$ is the wave function of the relative motion between two subclusters with orbital angular momentum L_r, and the Jacobi coordinates are defined as

$$\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2,$$
$$\mathbf{r}_{34} = \mathbf{r}_3 - \mathbf{r}_4,$$
$$\mathbf{r}_{1234} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} - \frac{m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_3 + m_4}. \tag{19}$$

For diquark-antidiquark structure, the quarks are numbered as 1,2, and the antiquarks are numbered as 3,4; for meson-meson structure, one cluster with antiquark and quark is marked as 1,2, the other cluster with antiquark and quark is marked as 3,4. In GEM, the spatial wave function is expanded by Gaussians [21],

$$\Psi_l^m(\mathbf{r}) = \sum_{n=1}^{n_{\max}} c_n \psi_{nlm}^G(\mathbf{r}), \tag{20a}$$

$$\psi_{nlm}^G(\mathbf{r}) = N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}), \tag{20b}$$

where N_{nl} are normalization constants,

$$N_{nl} = \left[\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}(2l+1)}\right]^{\frac{1}{2}}, \tag{21}$$

and c_n are the variational parameters, which are determined dynamically. The Gaussian size parameters are chosen according to the following geometric progression:

$$\nu_n = \frac{1}{r_n^2}, \qquad r_n = r_1 a^{n-1}, \qquad a = \left(\frac{r_{n_{\max}}}{r_1}\right)^{\frac{1}{n_{\max}-1}}. \tag{22}$$

This procedure enables optimization of the ranges using just a small number of Gaussians. Finally, the complete channel wave function for the four-quark system for diquark-antidiquark structure is written as

$$\Psi_{IJ,i,j,k}^{M_I M_J} = [\Psi_L \chi_S^{\sigma i}]_J^{M_J} \chi_{dI}^{fj} \chi_{dk}^c,$$
$$(i = 1 \sim 6, j = 1,2, k = 1,2; S = 0,1,2; I = 0,1). \tag{23}$$

For the meson-meson structure, the complete wave function is written as

$$\Psi_{IJ,i,j,k}^{M_I M_J} = \mathcal{A}[\Psi_L \chi_S^{\sigma i}]_J^{M_J} \chi_{mI}^{fj} \chi_{mk}^c,$$
$$(i = 1 \sim 6, j = 1,2, k = 1,2; S = 0,1,2; I = 0,1). \tag{24}$$

Here, $\mathcal{A}$ is the antisymmetrization operator: if all quarks (antiquarks) are taken as identical particles, then

$$\mathcal{A} = \frac{1}{2}(1 - P_{13} - P_{24} + P_{13}P_{24}). \tag{25}$$

In the present work, for the T_{bs} system, only two quarks are identical particles, so the antisymmetrization operator used is

$$\mathcal{A} = \frac{1}{\sqrt{2}}(1 - P_{13}). \tag{26}$$

The eigenenergy of the T_{bs} system is obtained by solving a Schrödinger equation,

$$H\Psi_{IJ}^{M_I M_J} = E^{IJ}\Psi_{IJ}^{M_I M_J}, \tag{27}$$

where $\Psi_{IJ}^{M_I M_J}$ is the wave function of the T_{bs}, which is the linear combination of the above channel wave functions—Eq. (23) in the diquark-antidiquark structure or Eq. (24) in the meson-meson structure, respectively.

The calculation of Hamiltonian matrix elements is complicated if any one of the relative orbital angular momenta is nonzero. In this case, it is useful to employ the method of infinitesimally shifted Gaussians [21], where the spherical harmonics are absorbed into the Gaussians,

$$\begin{aligned}\psi_{nlm}^G(\mathbf{r}) &= N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}) \\ &= N_{nl} \lim_{\epsilon \to 0} \frac{1}{\epsilon^l} \sum_k^{k_{\max}} C_{lm,k} \mathrm{e}^{-\nu_n (\mathbf{r} - \epsilon \mathbf{D}_{lm,k})^2},\end{aligned} \tag{28}$$

and where, plainly, the quantities $C_{lm,k}$, $D_{lm,k}$ are fixed by the particular spherical harmonic under consideration.

III. NUMERICAL RESULTS AND DISCUSSIONS

In the present work, we try to look for the particle with quantum numbers $IJ^P (I = 0,1; J = 0,1,2; P = +)$ consisting of four different flavors $ud\bar{s}\,\bar{b}$, denoted as T_{bs}, in the chiral quark model. All the orbital angular momenta are set to zero because we are interested in the low-lying states. Two structures of T_{bs}, diquark-antidiquark and meson-meson, are investigated. In each structure, all possible states are considered. For diquark-antidiquark structure, two color configurations, color antitriplet-triplet $(\bar{3} \times 3)$ and sexet-antisexet $(6 \times \bar{6})$, are examined. And for meson-meson structure, color singlet-singlet (1×1) and octet-octet (8×8), are taken into account.

In $SU(2)$ flavor symmetry, the σ meson exchange only occurs between u quark and d quark. In this situation, the eigenvalues with their errors of T_{bs} for diquark-antidiquark and meson-meson structure, are given in Tables IV and V, respectively.

In Table IV, the second column gives the index of the antisymmetry wave functions of T_{bs}. E_s is the single channel eigenenergy for the different channels; E_{cc} represents the eigenenergy with the effect of channel-coupling of different spin-color configurations. From the Table, we can see that the channels with different spin-color configurations have similar energies and the coupling of them is rather strong. However, the energies are all higher than the threshold of BK, 5773 MeV, which indicates that there are no bound states with the diquark-antidiquark structure in our model calculation.

Because of the color structure, colorful subclusters cannot fall apart; there may be a resonance even with the higher energy of the state. To check the possibility, we perform an adiabatic calculation of energy for the 00^+ state. In this case, the number of Gaussians used for the relative motion between the diquark and antidiquark subclusters is set to 1, that is to do the calculation with fixed separation

TABLE IV. The eigenenergies of T_{bs} in $SU(2)$ flavor symmetry for diquark-antidiquark structure (unit: MeV). E_{th1} represents the theoretical threshold and E_{th2} denotes the experimental threshold.

IJ^P	Channel	E_s	E_{cc}	E_{th1}	E_{th2}
00^+	$\chi_0^{\sigma1}\chi_{d0}^{f1}\chi_{d1}^{c}$	6042.3 ± 26.9	6024.3 ± 26.5	5792.8	5773.3
	$\chi_0^{\sigma2}\chi_{d0}^{f1}\chi_{d2}^{c}$	6424.9 ± 26.2		6244.4	6216.9
01+	$\chi_1^{\sigma3}\chi_{d0}^{f1}\chi_{d1}^{c}$	6056.2 ± 27.1	6046.5 ± 26.9	5828.9	5818.9
	$\chi_1^{\sigma4}\chi_{d0}^{f1}\chi_{d2}^{c}$	6528.7 ± 27.8		6208.3	6171.3
	$\chi_1^{\sigma5}\chi_{d0}^{f1}\chi_{d2}^{c}$	6477.3 ± 27.1		6244.4	6216.9
02+	$\chi_2^{\sigma6}\chi_{d0}^{f1}\chi_{d2}^{c}$	6568.8 ± 28.4	6568.8 ± 28.4	6244.4	6216.9
10+	$\chi_0^{\sigma1}\chi_{d1}^{f2}\chi_{d2}^{c}$	6561.3 ± 28.3	6443.6 ± 26.8	5792.8	5773.3
	$\chi_0^{\sigma2}\chi_{d1}^{f2}\chi_{d1}^{c}$	6476.4 ± 27.3		6244.4	6216.9
11+	$\chi_1^{\sigma3}\chi_{d1}^{f2}\chi_{d2}^{c}$	6557.5 ± 28.3	6455.5 ± 27.1	5828.9	5818.9
	$\chi_1^{\sigma4}\chi_{d1}^{f2}\chi_{d1}^{c}$	6504.7 ± 27.8		6208.3	6171.3
	$\chi_1^{\sigma5}\chi_{d1}^{f2}\chi_{d1}^{c}$	6497.9 ± 27.7		6244.4	6216.9
12+	$\chi_2^{\sigma6}\chi_{d1}^{f2}\chi_{d1}^{c}$	6538.0 ± 28.3	6538.0 ± 28.3	6244.4	6216.9

between the two subclusters. The results are shown in Fig. 1. we can see that when the two subclusters approach closely or fall apart, the energy is increasing, and the minimum of the energy occurs around the separation 0.6 fm. The results indicate that the two subclusters cannot fall apart or get too close. So that the state turned to be two mesons B and K is hindered because of the separation. 00^+ state may be a resonance state in our present calculation.

TABLE V. The eigenenergies of T_{bs} in $SU(2)$ flavor symmetry for meson-meson structure (unit: MeV).

IJ^P	Channel	E_s	E_{cc1}	E_{cc2}	E_{cc3}	E_{th1}
00^+	$\chi_0^{\sigma1}\chi_{m0}^{f1}\chi_{m1}^{c}$	5792.9 ± 24.8	5792.6 ± 24.8	5792.5 ± 24.8	5792.5 ± 24.8	5792.8
	$\chi_0^{\sigma1}\chi_{m0}^{f1}\chi_{m2}^{c}$	6467.8 ± 27.1				
	$\chi_0^{\sigma2}\chi_{m0}^{f1}\chi_{m1}^{c}$	6244.5 ± 25.2	6238.2 ± 24.9			6244.4
	$\chi_0^{\sigma2}\chi_{m0}^{f1}\chi_{m2}^{c}$	6283.6 ± 24.8				
01^+	$\chi_1^{\sigma3}\chi_{m0}^{f1}\chi_{m1}^{c}$	5829.6 ± 25.4	5829.2 ± 25.4	5829.2 ± 25.4	5829.2 ± 25.4	5828.9
	$\chi_1^{\sigma3}\chi_{m0}^{f1}\chi_{m2}^{c}$	6488.7 ± 26.9				
	$\chi_1^{\sigma4}\chi_{m0}^{f1}\chi_{m1}^{c}$	6209.5 ± 24.6	6209.2 ± 24.6			6208.3
	$\chi_1^{\sigma4}\chi_{m0}^{f1}\chi_{m2}^{c}$	6486.9 ± 26.9				
	$\chi_1^{\sigma5}\chi_{m0}^{f1}\chi_{m1}^{c}$	6245.9 ± 25.1	6245.1 ± 25.2			6244.4
	$\chi_1^{\sigma5}\chi_{m0}^{f1}\chi_{m2}^{c}$	6389.1 ± 25.8				
02^+	$\chi_2^{\sigma6}\chi_{m0}^{f1}\chi_{m1}^{c}$	6245.1 ± 25.2	6245.1 ± 25.2	6245.1 ± 25.2	6245.1 ± 25.2	6244.4
	$\chi_2^{\sigma6}\chi_{m0}^{f1}\chi_{m2}^{c}$	6607.5 ± 28.5				
10^+	$\chi_0^{\sigma1}\chi_{m1}^{f2}\chi_{m1}^{c}$	5794.9 ± 24.8	5794.9 ± 24.8	5794.9 ± 24.8	5794.9 ± 24.8	5792.8
	$\chi_0^{\sigma1}\chi_{m1}^{f2}\chi_{m2}^{c}$	6538.6 ± 27.8				
	$\chi_0^{\sigma2}\chi_{m1}^{f2}\chi_{m1}^{c}$	6246.1 ± 25.2	6246.1 ± 25.2			6244.4
	$\chi_0^{\sigma2}\chi_{m1}^{f2}\chi_{m2}^{c}$	6493.6 ± 26.8				
11+	$\chi_1^{\sigma3}\chi_{m1}^{f2}\chi_{m1}^{c}$	5831.0 ± 25.4	5831.0 ± 25.4	5831.0 ± 25.4	5831.0 ± 25.4	5828.9
	$\chi_1^{\sigma3}\chi_{m1}^{f2}\chi_{m2}^{c}$	6530.9 ± 27.6				
	$\chi_1^{\sigma4}\chi_{m1}^{f2}\chi_{m1}^{c}$	6210.4 ± 24.6	6210.3 ± 24.6			6208.27
	$\chi_1^{\sigma4}\chi_{m1}^{f2}\chi_{m2}^{c}$	6528.2 ± 27.6				
	$\chi_1^{\sigma5}\chi_{m1}^{f2}\chi_{m1}^{c}$	6246.3 ± 25.2	6246.3 ± 25.2			6244.4
	$\chi_1^{\sigma5}\chi_{m1}^{f2}\chi_{m2}^{c}$	6504.4 ± 27.1				
12+	$\chi_2^{\sigma6}\chi_{m1}^{f2}\chi_{m1}^{c}$	6246.6 ± 25.2	6246.6 ± 25.2	6246.6 ± 25.2	6246.6 ± 25.2	6244.4
	$\chi_2^{\sigma6}\chi_{m1}^{f2}\chi_{m2}^{c}$	6539.6 ± 27.9				

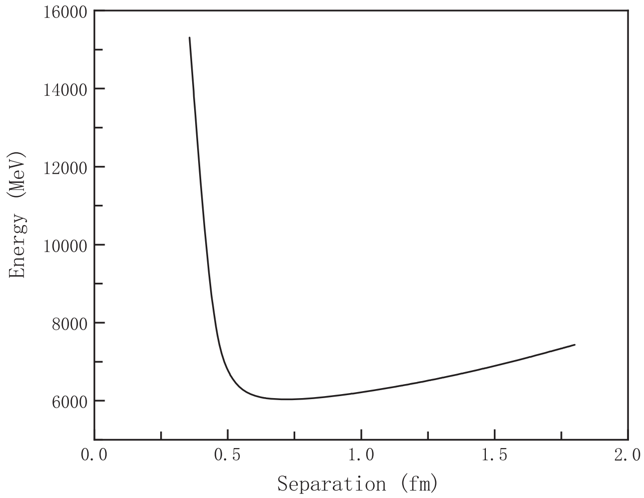

FIG. 1. The eigenenergy of the 00^+ state as a function of the distance between the diquark and antidiquark.

In Ref. [24], the hypothesis that the diquarks and antidiquarks in tetraquarks are separated by a potential barrier is proposed to explain the properties of exotic resonances such as X and Z. Also, Brodsky *et al.* present a new dynamical picture which can explain the nature of the exotic XYZ states based on the separation between diquark and antidiquark subclusters [25]. Our calculations support both work. Experimentally, for the state with isospin $= 1$, the $uu\bar{s}\,\bar{b}$ with the electric charge $+2$ may be a much easier detectable state.

With regard to meson-meson structure, the results are shown in Table V. In our calculations, the color singlet-singlet configurations always have the lower energies than those of color octet-octet ones. E_{cc1} is the eigenenergy from the channel coupling of the two color configurations, which is close to that of single channel (color singlet-singlet) result, E_s. This indicates that the effect of the hidden color is very small. E_{cc2} gives the eigenenergy from the channel coupling of all the color singlet-singlet ones, and the results

TABLE VII. The rms distances between quarks and antiquarks for the state 00^+ in meson-meson structure in $SU(2)$ flavor symmetry (unit: fm).

Channel	$u\bar{s}$	$d\bar{b}$	ud	$\bar{s}\,\bar{b}$	$u\bar{b}$	$d\bar{s}$
$0\times 0\rightarrow 0$	0.5	0.6	7.0	7.0	7.0	7.0
$1\times 1\rightarrow 0$	0.8	0.6	1.7	1.5	1.5	1.6
Coupling	0.5	0.6	6.4	6.3	6.3	6.3

show that the coupling is also very small. E_{cc3} represents the eigen-energy from the channel coupling of all channels with the same quantum numbers. Naturally, the coupling tends to be small. The obtained central energies E_{cc3} are all higher and approach to the theoretical thresholds in all case except the state, 00^+. For $IJ^P = 00^+$ state, the eigenenergies from the single channel calculation are higher than their theoretical thresholds. With the help of channel coupling to the color octet-octet configuration, the central energies of the states are lower than their corresponding thresholds. For the first channel with spin $0\times 0\rightarrow 0$, the calculated central energy is 5792.6 MeV, which is a little smaller than the theoretical threshold 5792.8 MeV, and the binding energy is -0.2 MeV. For the second channel with spin $1\times 1\rightarrow 0$, the obtained central energy is 6238.2 MeV, lower than the theoretical threshold 6244.4 MeV, and the binding energy is -6.2 MeV. All channels coupling obtain the lowest state with binding energy -0.3 MeV and push $1\times 1\rightarrow 0$ state above its corresponding threshold. To identify which terms in the Hamiltonian making the state be bound, the contributions from each term of Hamiltonian for the four-quark state and the sum of two mesons are given in Table VI for both two 00^+ states. From the Table, we can see that the color confinement (V_{ij}^C), one-gluon-exchange (V_{ij}^G) and σ-meson exchange (V_{ij}^σ) contribute the binding of the states. For spin $1\times 1\rightarrow 0$ state, π-meson exchange makes a considerable binding due to the compact structure of the state (see Table VII). For spin $0\times 0\rightarrow 0$

TABLE VI. The contributions of each term of the Hamiltonian for the 00^+ state in the meson-meson structure in $SU(2)$ flavor symmetry (unit: MeV). $\Delta_i (i = 1, 2)$ is the difference between the contributions in four-quark state and the sum of the contributions of two mesons.

	$0\times 0\rightarrow 0$			$1\times 1\rightarrow 0$		
	T_{bs}	BK	Δ_1	T_{bs}	B^*K^*	Δ_2
Rest mass	6274.0	6274.0	0	6274.0	6274.0	0
Kinetic	1518.5	1512.5	6.0	899.0	726.7	172.3
V_{ij}^G	−1440.2	−1436.6	−3.6	−539.9	−449.9	−90.0
V_{ij}^C	−489.9	−489.6	−0.3	−326.7	−309.6	−17.1
V_{ij}^η	−67.6	−67.5	−0.1	7.4	3.2	4.2
V_{ij}^π	0	0	0	−62.0	0	−62.0
V_{ij}^K	0	0	0	0	0	0
V_{ij}^σ	−2.2	0	−2.2	−13.6	0	−13.6
Eigenenergy	5792.6	5792.8	−0.2	6238.2	6244.4	−6.2

TABLE VIII. The eigenenergies of T_{bs} in $SU(3)$ flavor symmetry for diquark-antidiquark structure (unit: MeV).

IJ^P	Channel	E_s	E_{cc}	E_{th1}	E_{th2}
00^+	$\chi_0^{\sigma 1}\chi_{d0}^{f1}\chi_{d1}^{c}$	5986.7 ± 26.6	5966.3 ± 26.1	5792.8	5773.3
	$\chi_0^{\sigma 2}\chi_{d0}^{f1}\chi_{d2}^{c}$	6373.6 ± 25.9		6244.4	6216.9
01+	$\chi_1^{\sigma 3}\chi_{d0}^{f1}\chi_{d1}^{c}$	6001.4 ± 26.8	5990.5 ± 26.5	5828.9	5818.9
	$\chi_1^{\sigma 4}\chi_{d0}^{f1}\chi_{d2}^{c}$	6482.9 ± 27.4		6208.3	6171.3
	$\chi_1^{\sigma 5}\chi_{d0}^{f1}\chi_{d2}^{c}$	6428.8 ± 26.7		6244.4	6216.9
02+	$\chi_2^{\sigma 6}\chi_{d0}^{f1}\chi_{d2}^{c}$	6524.7 ± 28.0	6524.7 ± 28.0	6244.4	6216.9
10+	$\chi_0^{\sigma 1}\chi_{d1}^{f2}\chi_{d2}^{c}$	6517.3 ± 28.0	6393.4 ± 26.4	5792.8	5773.3
	$\chi_0^{\sigma 2}\chi_{d1}^{f2}\chi_{d1}^{c}$	6428.2 ± 26.9		6244.4	6216.9
11+	$\chi_1^{\sigma 3}\chi_{d1}^{f2}\chi_{d2}^{c}$	6513.3 ± 27.9	6406.2 ± 26.6	5828.9	5818.9
	$\chi_1^{\sigma 4}\chi_{d1}^{f2}\chi_{d1}^{c}$	6458.5 ± 27.3		6208.3	6171.3
	$\chi_1^{\sigma 5}\chi_{d1}^{f2}\chi_{d1}^{c}$	6451.1 ± 27.2		6244.4	6216.9
12+	$\chi_2^{\sigma 6}\chi_{d1}^{f2}\chi_{d1}^{c}$	6493.5 ± 27.9	6493.5 ± 27.9	6244.4	6216.9

state, π-meson exchange makes no contribution because of the large separation between two mesons. F. Close *et al.* also found that the pion exchange between hadrons can lead to deeply bound quasimolecular states [26,27].

Furthermore, the root-mean-square (rms) distances between quarks and antiquarks in meson-meson structure for 00^+ state are calculated and shown in Table VII. For the $0 \times 0 \to 0$ channel, the distances between the two meson

TABLE IX. The eigenenergies of T_{bs} in $SU(3)$ flavor symmetry for meson-meson structure (unit: MeV). E_b represents the binding energy of states.

IJ^P	Channel	E_s	E_{cc1}	E_{cc2}	E_{cc3}	E_{th1}	E_b
00^+	$\chi_0^{\sigma 1}\chi_{m0}^{f1}\chi_{m1}^{c}$	5725.3 ± 25.0	5722.7 ± 24.9	5722.7 ± 24.9	5722.3 ± 24.9	5792.8	−70.1
	$\chi_0^{\sigma 1}\chi_{m0}^{f1}\chi_{m2}^{c}$	6417.2 ± 26.6					
	$\chi_0^{\sigma 2}\chi_{m0}^{f1}\chi_{m1}^{c}$	6213.7 ± 24.9	6187.3 ± 24.3			6244.4	−57.1
	$\chi_0^{\sigma 2}\chi_{m0}^{f1}\chi_{m2}^{c}$	6222.8 ± 24.4					
01^+	$\chi_1^{\sigma 3}\chi_{m0}^{f1}\chi_{m1}^{c}$	5762.3 ± 25.5	5760.8 ± 25.5	5762.1 ± 25.5	5760.4 ± 25.5	5828.9	−68.1
	$\chi_1^{\sigma 3}\chi_{m0}^{f1}\chi_{m2}^{c}$	6437.4 ± 26.5					
	$\chi_1^{\sigma 4}\chi_{m0}^{f1}\chi_{m1}^{c}$	6178.6 ± 24.3	6176.8 ± 24.3			6208.3	−31.5
	$\chi_1^{\sigma 4}\chi_{m0}^{f1}\chi_{m2}^{c}$	6435.5 ± 26.5					
	$\chi_1^{\sigma 5}\chi_{m0}^{f1}\chi_{m1}^{c}$	6215.3 ± 24.9	6209.8 ± 24.8			6244.4	−34.6
	$\chi_1^{\sigma 5}\chi_{m0}^{f1}\chi_{m2}^{c}$	6333.0 ± 25.4					
02^+	$\chi_2^{\sigma 6}\chi_{m0}^{f1}\chi_{m1}^{c}$	6213.9 ± 24.9	6213.8 ± 24.9	6213.8 ± 24.9	6213.8 ± 24.9	6244.4	−30.6
	$\chi_2^{\sigma 6}\chi_{m0}^{f1}\chi_{m2}^{c}$	6561.1 ± 28.0					
10^+	$\chi_0^{\sigma 1}\chi_{m1}^{f2}\chi_{m1}^{c}$	5731.1 ± 24.9	5731.1 ± 24.9	5731.1 ± 24.9	5731.1 ± 24.9	5792.8	−61.7
	$\chi_0^{\sigma 1}\chi_{m1}^{f2}\chi_{m2}^{c}$	6491.6 ± 27.3					
	$\chi_0^{\sigma 2}\chi_{m1}^{f2}\chi_{m1}^{c}$	6216.3 ± 24.9	6216.3 ± 24.9			6244.4	−28.1
	$\chi_0^{\sigma 2}\chi_{m1}^{f2}\chi_{m2}^{c}$	6442.6 ± 26.3					
11+	$\chi_1^{\sigma 3}\chi_{m1}^{f2}\chi_{m1}^{c}$	5767.2 ± 25.5	5767.2 ± 25.5	5767.2 ± 25.5	5767.2 ± 25.5	5828.9	−61.7
	$\chi_1^{\sigma 3}\chi_{m1}^{f2}\chi_{m2}^{c}$	6483.6 ± 27.2					
	$\chi_1^{\sigma 4}\chi_{m1}^{f2}\chi_{m1}^{c}$	6180.7 ± 24.3	6180.7 ± 24.3			6208.3	−27.6
	$\chi_1^{\sigma 4}\chi_{m1}^{f2}\chi_{m2}^{c}$	6480.7 ± 27.1					
	$\chi_1^{\sigma 5}\chi_{m1}^{f2}\chi_{m1}^{c}$	6216.6 ± 24.9	6216.6 ± 24.9			6244.4	−27.8
	$\chi_1^{\sigma 5}\chi_{m1}^{f2}\chi_{m2}^{c}$	6454.9 ± 26.6					
12+	$\chi_2^{\sigma 6}\chi_{m1}^{f2}\chi_{m1}^{c}$	6216.9 ± 25.0	6216.9 ± 25.0	6216.9 ± 25.0	6216.9 ± 25.0	6244.4	−27.5
	$\chi_2^{\sigma 6}\chi_{m1}^{f2}\chi_{m2}^{c}$	6493.2 ± 27.4					

TABLE X. The contributions of each term of the Hamiltonian for the 00^+ state in the meson-meson structure in $SU(3)$ flavor symmetry (unit: MeV).

	$0\times 0\to 0$			$1\times 1\to 0$		
	T_{bs}	BK	Δ_1	T_{bs}	B^*K^*	Δ_2
Rest mass	6274.0	6274.0	0	6274.0	6274.0	0
Kinetic	1629.6	1512.5	117.1	1050.7	726.7	324.0
V_{ij}^G	−1521.9	−1436.6	−85.3	−620.8	−449.9	−170.9
V_{ij}^C	−503.3	−489.6	−13.7	−353.2	−309.6	−43.6
V_{ij}^η	−72.8	−67.5	−5.3	10.1	3.2	6.9
V_{ij}^π	0	0	0	−97.7	0	−97.7
V_{ij}^K	0	0	0	0	0	0
V_{ij}^σ	−82.9	0	−82.9	−75.8	0	−75.8
Eigenenergy	5722.7	5792.8	−70.1	6187.3	6244.4	−57.1

clusters are much larger than those of $u-\bar{s}$ or $d-\bar{b}$ within one cluster and it may tend to be a molecular state; for the $1\times 1\to 0$ channel, the distances between the two meson clusters are about twice of that between the quark and antiquark in one cluster which indicates that it may be a little compact molecular state in our present calculation. When the coupling of two channels is considered, the dominant component of the lowest state is $0\times 0\to 0$ color singlet-singlet state, and the distances between the two meson clusters are a little smaller but still far larger than that between the quark and antiquark in one cluster, so the 00^+ state may be a molecular state in the present work when considering the uncertainties of the model.

The Salamanca version of the chiral quark model can describe the meson spectrum well, where the σ meson exchange is considered between u, d and s quark. So it is interesting to calculate the T_{bs} in $SU(3)$ flavor symmetry, and the results for both diquark-antidiquark structure and meson-meson structure are demonstrated in Tables VIII and IX, respectively. From the Tables, we found that the central energies are much lower than those in the $SU(2)$ flavor symmetry no matter in diquark-antidiquark structure or meson-meson structure. In the diquark-antidiquark structure, the central energies are still all higher than the threshold of BK. In the meson-meson structure, the energies are all below the corresponding thresholds. For comparison, for the 00^+ state, the binding energies are −70.1 MeV for the spin $0\times 0\to 0$ state and −57.1 MeV for the spin $1\times 1\to 0$ state, which are much deeper than those (−0.2 MeV and −6.2 MeV) in the $SU(2)$ symmetry. From the contributions of each term of the Hamiltonian for the 00^+ state in the $SU(3)$ symmetry (see Table X), we can see that the σ meson exchange leads to the deeper binding energies for two channels. Furthermore, the rms distances between four particles for the 00^+ states in the $SU(3)$ flavor symmetry are demonstrated in Table XI. From the Table, we found that the distances between the two meson clusters are much closer than those in the $SU(2)$ flavor symmetry due to the σ meson exchange. For the $0\times 0\to 0$ channel, it still may be a molecular one and for the $1\times 1\to 0$ channel, it may be a compact tetraquark state. The effect of the channel coupling is still tiny.

TABLE XI. The root-mean-square (rms) radiuses of quarks and antiquarks of the state 00^+ in meson-meson structure in $SU(3)$ flavor symmetry (unit: fm).

Channel	$u\bar{s}$	$d\bar{b}$	ud	$\bar{s}\bar{b}$	$u\bar{b}$	$d\bar{s}$
$0\times 0\to 0$	0.5	0.6	1.9	1.8	1.9	1.9
$1\times 1\to 0$	0.8	0.6	1.1	0.8	0.8	0.9
Coupling	0.5	0.6	1.8	1.7	1.8	1.8

IV. SUMMARY

One benefit of the $ud\bar{s}\bar{b}$'s higher threshold than $B_s\pi$ is that it has a larger mass region than $X(5568)$ to be stable, and it may be a promising detectable tetraquark state. In this paper, we try to calculate the state $ud\bar{s}\bar{b}$ (T_{bs}) with quantum numbers $IJ^P(I=0,1;J=0,1,2;P=+)$ by using GEM. The constituent chiral quark model with flavor symmetries $SU(2)$ and $SU(3)$, which describes the light and heavy meson spectra well, is employed in the calculation. Meanwhile, we give the uncertainty of the model parameters and the calculated results, which are important for us to draw the conclusions. Two structures—diquark-antidiquark and meson-meson—are investigated. We found that the energies of T_{bs} with diquark-antidiquark structure are all higher than the threshold of BK, leaving no space for the bound state, but for the lowest energy 00^+ state, it may be a resonance state in the $SU(2)$ flavor symmetry in our calculation. Besides, in the $SU(2)$ flavor symmetry with the meson-meson structure, the mass of the 00^+ state is just a little below the threshold of BK with a small binding energy, −0.3 MeV, which may be a molecular state in the present work. As for the $SU(3)$ flavor symmetry, the results for the diquark-antidiquak structure are unaltered, and the energies with the meson-meson structure are much lower owing to the medium-range attraction supplied by the σ meson exchange between the u, d, and s quarks. From the experimental side, it is not expected that there exist so many states, so using the σ-meson exchange between the u, d, and s quarks is not proper in the $SU(3)$ flavor symmetry. A better way is to employ the scalar nonet in the $SU(3)$ flavor symmetry instead of one σ meson exchange in the $SU(2)$ flavor symmetry. Last, it is worth mentioning that $uu\bar{s}\,\bar{b}$ with electric charge $+2$ may be an easier state to identify.

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