Resonance $Y(4660)$ as a vector tetraquark and its strong decay channels

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The spectroscopic parameters and partial widths of the strong decay channels of the vector meson $Y(4660)$ are calculated by treating it as a bound state of a diquark and antidiquark. The mass and coupling of the $J^{PC} = 1^{--}$ tetraquark $Y(4660)$ are evaluated in the context of the two-point sum rule method by taking into account the quark, gluon, and mixed condensates up to dimension 10. The widths of the $Y(4660)$ resonance's strong S -wave decays to $J/\psi f_0(980)$ and $\psi(2S)f_0(980)$ as well as to $J/\psi f_0(500)$ and $\psi(2S)f_0(500)$ final states are computed. To this end, strong couplings in the relevant vertices are extracted from the QCD sum rule on the light cone supplemented by the technical methods of the soft approximation. The obtained result for the mass of the resonance $m_Y = 4677^{+71}_{-63}$ MeV, and prediction for its total width $\Gamma_Y = (64.8 \pm 10.8)$ MeV is in nice agreements with the experimental information.

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I. INTRODUCTION

The last 15 years were very fruitful for hadron physics due to valuable information on properties of the hadrons collected by numerous experimental collaborations and owing to new theoretical ideas and predictions that extended the boundaries of our knowledge about the quark-gluon structure of elementary particles. An observation of the resonances that may be interpreted as four- and five-quark states is one of most interesting discoveries to be mentioned among these achievements. Strictly speaking, the existence of the multiquark states does not contradict the fundamental principles of QCD and was foreseen in the first years of QCD [1], but only results of the Belle Collaboration about the narrow resonance $X(3872)$ placed the physics of multiquark hadrons on a firm basis of experimental data [2]. Now experimentally detected and theoretically investigated, four-quark resonances form a family of particles known as XYZ states [3,4].

The resonance $Y(4660)$, which is the subject of the present study, was observed for the first time by the Belle Collaboration in the process $e^+e^- \rightarrow \gamma_{\text{ISR}}\psi(2S)\pi^+\pi^-$ via initial-state radiation (ISR) as one of two resonant structures in the $\psi(2S)\pi^+\pi^-$ invariant mass distribution [5,6].

The second state discovered in this experiment received the label $Y(4360)$. The analysis carried out in Refs. [5,6] showed that these structures cannot be interpreted as known charmonium states. The measured the mass and total width of the resonance $Y(4660)$ are [6]

$$\begin{aligned} m_Y &= 4652 \pm 10 \pm 8 \text{ MeV}, \\ \Gamma_Y &= 68 \pm 11 \pm 1 \text{ MeV}. \end{aligned} \quad (1)$$

The state $Y(4630)$, which is usually identified with the $Y(4660)$, was detected in the process $e^+e^- \rightarrow \Lambda_c^+\Lambda_c^-$ as a peak in the $\Lambda_c^+\Lambda_c^-$ invariant mass distribution [7]. Making an assumption on a resonance nature of this peak, its mass and width were found equal to $m_Y = 4634^{+8}_{-7}(\text{stat.})^{+5}_{-8}(\text{sys.})$ MeV and $\Gamma_Y = 92^{+40}_{-24}(\text{stat.})^{+10}_{-21}(\text{sys.})$ MeV, respectively. Independent confirmation of the $Y(4660)$ state came from the *BABAR* Collaboration [8], which studied the same process $e^+e^- \rightarrow \gamma_{\text{ISR}}\psi(2S)\pi^+\pi^-$ and fixed two resonant structures in the $\pi^+\pi^-\psi(2S)$ invariant mass distribution. Resonant structures mass and width confirm that they can be identified with $Y(4660)$ and $Y(4360)$. Besides two resonances under discussion, there are also states $Y(4260)$ and $Y(4390)$, which together constitute the family of at least four Y hidden-charmed particles with $J^{PC} = 1^{--}$.

The numerous theoretical articles claiming to interpret the $Y(4660)$ and $Y(4360)$ embrace the variety of models and schemes available in high-energy physics. Thus, attempts were made to consider the new resonance

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$Y(4660)$ as an excited state of conventional charmonium: in Refs. [9,10], it was interpreted as the excited 5^3S_1 and 6^3S_1 charmonia, respectively. To explain the experimental information on the resonance $Y(4660)$, it was examined as a compound of the scalar $f_0(980)$ and vector $\psi(2S)$ mesons [11–13] or as a baryonium state [14,15]. The hadrocharmonium model for these resonances was suggested in Ref. [16].

The most popular models for the states $Y(4360)$ and $Y(4660)$, however, are the diquark-antidiquark models, which suggest that these resonances are tightly bound states of a diquark and an antidiquark with required quantum numbers. Within this picture, the resonance $Y(4360)$ was analyzed in Ref. [17] as an excited $1P$ tetraquark built of an axial-vector diquark and antidiquark, whereas $Y(4660)$ [and also $Y(4630)$] was found to be the $2P$ state of a scalar diquark-antidiquark. Calculations there were carried out in the context of the relativistic diquark picture. The resonance $Y(4360)$ was interpreted as a radial excitation of the tetraquark $Y(4008)$ in Ref. [18]. A similar idea but in the framework of the QCD sum rule method was realized in Ref. [19]: the $Y(4660)$ was considered as the P -wave $[cs][\bar{c}\bar{s}]$ state and modeled by a $C\gamma_5 \otimes D_\mu\gamma_5 C$ -type interpolating current, where C is the charge conjugation matrix. The tetraquark $[cs][\bar{c}\bar{s}]$ with interpolating current $C\gamma_5 \otimes \gamma_5\gamma_\mu C$ was used in Ref. [20] to treat $Y(4660)$, and the mass of this state was evaluated by employing the QCD sum rule approach in nice agreement with experimental data. There are many other interesting models of the vector resonances, details of which can be found in the reviews (see Refs. [3,4]).

In general, the vector tetraquarks with different P and C parities can be built using the five independent diquark fields with spin 0 and 1 and different P parities [21]. This implies the existence of numerous diquark-antidiquark structures and, as a result, different interpolating currents with the same quantum numbers $J^{PC} = 1^{--}$. Within the framework of the two-point sum rule method, these currents, excluding ones with derivatives, were used in Ref. [21] for calculating masses of the vector tetraquarks with $J^{PC} = 1^{--}, 1^{--}, 1^{++}, 1^{+-}$ and quark contents $[cs][\bar{c}\bar{s}]$ and $[cq][\bar{c}\bar{q}]$. For the mass of the $1^{--} [cq][\bar{c}\bar{q}]$ state, all of the explored currents led to the result $m \sim 4.6\text{--}4.7$ GeV, which implies a possible tetraquark interpretation of $Y(4660)$. But this fact does not exclude interpretation of $Y(4660)$ as the state $1^{--} [cs][\bar{c}\bar{s}]$, because the $C\gamma^\nu \otimes \sigma_{\mu\nu} C - C\sigma_{\mu\nu} \otimes \gamma^\nu C$ -type current gives for the mass of such a state $m = 4.64 \pm 0.09$ GeV, comparable with the mass of the $Y(4660)$ resonance. The sum rule approach was also employed in Refs. [22–24] to investigate the resonance $Y(4660)$ by considering it a tetraquark with $[cq][\bar{c}\bar{q}]$ or $[cs][\bar{c}\bar{s}]$ quark content and using the interpolating currents of $C\gamma_\mu \otimes \gamma_\nu C - C\gamma_\nu \otimes \gamma_\mu C$ and $C \otimes \gamma_\mu C$ types.

In the present work, we treat the $Y(4660)$ resonance as the vector tetraquark with $[cs][\bar{c}\bar{s}]$ content and compute its

total width. To this end, we first recalculate the mass and coupling of $Y(4660)$, which enter as the important input parameters into its partial decay widths. We utilize the two-point QCD sum rule approach, which is one of the powerful nonperturbative methods for investigating the features of the hadrons [25,26]. It is suitable for studying not only conventional hadrons but also multiquark systems. In our computations, we take into account vacuum condensates up to dimension 10, which lead to reliable predictions for quantities of interest.

The next problem addressed in the present article is investigation of the $Y(4660)$ state's strong decays. Some of the possible decay channels of the vector tetraquarks were written down in Ref. [21]. Our aim is to evaluate the width of the main S -wave decays $Y \rightarrow J/\psi f_0(980)$, $Y \rightarrow \psi(2S)f_0(980)$, $Y \rightarrow J/\psi f_0(500)$, and $Y \rightarrow \psi(2S)f_0(500)$ of the resonance $Y(4660)$ and estimate its full width that can be confronted with existing data. To this end, we employ the QCD sum rule on the light cone (LCSR) in conjunction with a technique of the soft approximation [27,28]. For investigation of the tetraquarks, this approach was adapted in Ref. [29] and used successfully to investigate their numerous strong decays.

This article is structured in the following manner. In Sec. II, we calculate the mass m_Y and coupling f_Y of the vector $Y(4660)$ resonance using the two-point sum rule method and include in the analysis the quark, gluon, and mixed condensates up to dimension 10. The obtained results for these parameters are applied in Sec. III to evaluate strong couplings and widths of the $Y(4660)$ state's partial S -wave decays. In Sec. IV, we present our conclusions. The Appendix contains technical details of calculations.

II. MASS AND COUPLING OF THE VECTOR TETRAQUARK $Y(4660)$

In this section, we revisit the sum rule calculation of the mass and coupling of the resonance $Y(4660)$ to extract their values. In the context of the QCD sum rule method this problem was originally addressed in Refs. [20–24], in which $Y(4660)$ was considered as the state with $[cq][\bar{c}\bar{q}]$ or $[cs][\bar{c}\bar{s}]$ content. In these papers the relevant interpolating current was constructed using different assumptions on quantum numbers of the constituent diquark and antidiquark.

Here, we treat $Y(4660)$ as the $[cs][\bar{c}\bar{s}]$ tetraquark composed of the scalar diquark and vector antidiquark with the $C\gamma_5 \otimes \gamma_5\gamma_\mu C$ -type interpolating current. The same assumption about the quark content and structure of the $Y(4660)$ resonance was made in Refs. [20,21], in which its mass was found by employing various interpolating currents and quark, gluon, and mixed vacuum condensates up to dimension 8. In our calculations, we take into account condensates up to dimension 10 and include in the analysis the gluon condensate $\langle g_s^3 G^3 \rangle$ neglected in these papers and improve accuracy of the obtained results. We do not restrict

ourselves by calculation of the mass of the resonance $Y(4660)$, as was done in the aforementioned works, and also extract the current coupling of the tetraquark $Y(4660)$, which is necessary for investigating its decay channels.

After these preliminary comments, let us turn to our problem and start from the analysis of the correlation function

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle. \quad (2)$$

Here, $J_\mu(x)$ is the interpolating current of the resonance $Y(4660)$ chosen in the form

$$J_\mu(x) = \epsilon \tilde{\epsilon} [s_b^T(x) C \gamma_5 c_c(x) \bar{s}_d(x) \gamma_5 \gamma_\mu C \bar{c}_e^T(x) + s_b^T(x) C \gamma_\mu \gamma_5 c_c(x) \bar{s}_d(x) \gamma_5 C \bar{c}_e^T(x)], \quad (3)$$

where $\epsilon \tilde{\epsilon} = \epsilon_{abc} \epsilon_{ade}$ and a, b, c, d , and e are color indices.

In general, $\Pi_{\mu\nu}(p)$ has the Lorentz decomposition

$$\Pi_{\mu\nu}(p) = \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) \Pi_V(p^2) - \frac{p_\mu p_\nu}{p^2} \Pi_S(p^2), \quad (4)$$

where the invariant functions $\Pi_V(p^2)$ and $\Pi_S(p^2)$ are contributions of the vector and scalar states, respectively. Because we are interested only in the analysis of $\Pi_V(p^2)$, it is convenient to choose such a structure in Eq. (4), which accumulates effects due to only the vector particles. It is seen that such a Lorentz structure is $g_{\mu\nu}$; in fact, the terms proportional to $p_\mu p_\nu$ are formed owing to both the vector and scalar particles.

Deriving the sum rules for the mass m_Y and coupling f_Y proceeds through two main stages. In the first step, we express the correlation function in terms of the physical parameters of the tetraquark $Y(4660)$, which give rise to the function $\Pi_{\mu\nu}^{\text{Phys}}(p)$. In the next phase, we employ the explicit expression of the interpolating current $J_\mu(x)$, and calculate $\Pi_{\mu\nu}(p)$ contracting relevant quark fields and replacing the obtained propagators with their

nonperturbative expressions. As a result of these manipulations, we get $\Pi_{\mu\nu}^{\text{OPE}}(p)$, which depends on the various quark, gluon, and mixed vacuum condensates. By invoking assumptions about the quark-hadron duality, we can equate the functions $\Pi_{\mu\nu}^{\text{Phys}}(p)$ and $\Pi_{\mu\nu}^{\text{OPE}}(p)$ to each other, fix invariant amplitudes corresponding to the chosen Lorentz structure, and after well-known operations extract required sum rules.

Let us begin from the phenomenological side of the sum rules, i.e., from function $\Pi_{\mu\nu}^{\text{Phys}}(p)$. We assume that the tetraquark $Y(4660)$ with the chosen quark content and diquark-antidiquark structure is the ground-state particle in its class. Then, by introducing into Eq. (2) the full set of corresponding states, performing the integration over x and isolating contribution to $\Pi_{\mu\nu}^{\text{Phys}}(p)$ of the ground state, we obtain [for brevity, in formulas, we use $Y \equiv Y(4660)$]

$$\Pi_{\mu\nu}^{\text{Phys}}(p) = \frac{\langle 0 | J_\mu | Y(p) \rangle \langle Y(p) | J_\nu^\dagger | 0 \rangle}{m_Y^2 - p^2} + \dots, \quad (5)$$

where m_Y is the mass of $Y(4660)$ and dots show the contribution of the higher resonances and continuum. We simplify this formula by introducing the matrix element

$$\langle 0 | J_\mu | Y(p) \rangle = m_Y f_Y \epsilon_\mu \quad (6)$$

with f_Y and ϵ_μ being the coupling and polarization vector of the resonance $Y(4660)$, respectively. After some simple calculations, we get

$$\Pi_{\mu\nu}^{\text{Phys}}(p) = \frac{m_Y^2 f_Y^2}{m_Y^2 - p^2} \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \dots \quad (7)$$

It is evident that $\Pi_V^{\text{Phys}}(p^2) = m_Y^2 f_Y^2 / (m_Y^2 - p^2)$ is the invariant amplitude that can be used later to derive sum rules.

To find $\Pi_{\mu\nu}^{\text{OPE}}(p)$, we follow the recipes that have just been outlined above and express it in terms of the quark propagators

$$\begin{aligned} \Pi_{\mu\nu}^{\text{OPE}}(p) = i \int d^4x e^{ipx} \epsilon \tilde{\epsilon} \epsilon' \tilde{\epsilon}' \{ & \text{Tr}[\gamma_5 \tilde{S}_s^{bb'}(x) \gamma_5 S_c^{cc'}(x)] \text{Tr}[\gamma_5 \gamma_\mu \tilde{S}_c^{e'e}(-x) \gamma_\nu \gamma_5 S_s^{d'd}(-x)] + \text{Tr}[\gamma_5 \gamma_\mu \tilde{S}_c^{e'e}(-x) \gamma_5 \\ & \times S_s^{d'd}(-x)] \text{Tr}[\gamma_5 \gamma_\nu \tilde{S}_s^{bb'}(x) \gamma_5 S_c^{cc'}(x)] + \text{Tr}[\gamma_5 \tilde{S}_c^{e'e}(-x) \gamma_\nu \gamma_5 S_s^{d'd}(-x)] \text{Tr}[\gamma_5 \tilde{S}_s^{bb'}(x) \gamma_\mu \gamma_5 S_c^{cc'}(x)] \\ & + \text{Tr}[\gamma_5 \gamma_\nu \tilde{S}_s^{bb'}(x) \gamma_\mu \gamma_5 S_c^{cc'}(x)] \text{Tr}[\gamma_5 \tilde{S}_c^{e'e}(-x) \gamma_5 S_s^{d'd}(-x)] \}, \end{aligned} \quad (8)$$

where

$$\tilde{S}_{c(s)}(x) = C S_{c(s)}^T(x) C$$

and $S_{c(s)}(x)$ is the heavy c -quark (the light s -quark) propagator.

The expressions of the quark propagators are well known, and therefore we do not provide them here explicitly (see, e.g., Appendix in Ref. [30]). We calculate $\Pi_{\mu\nu}^{\text{OPE}}(p)$ by taking into account various vacuum condensates up to dimension 10 and write the QCD counterpart of the phenomenological function $\Pi_V^{\text{OPE}}(p^2)$ in terms of the corresponding spectral density $\rho(s)$,

$$\Pi_V^{\text{OPE}}(p^2) = \int_{\mathcal{M}^2}^{\infty} \frac{\rho(s)}{s - p^2} ds, \quad (9)$$

where $\mathcal{M}^2 = 4(m_c + m_s)^2$. Now, to extract the required sum rules, we equate these invariant amplitudes to each other, apply the Borel transformation to both sides of the obtained expression to suppress contributions arising from the higher resonances and continuum, and perform the continuum subtraction by utilizing the assumption about the quark-hadron quality. The second equality can be derived by acting to the first expression by the operator $d/d(-1/M^2)$: these two equalities can be used to extract the sum rules for m_Y and f_Y

$$m_Y^2 = \frac{\int_{\mathcal{M}^2}^{s_0} ds \rho(s) s e^{-s/M^2}}{\int_{\mathcal{M}^2}^{s_0} ds \rho(s) e^{-s/M^2}} \quad (10)$$

and

$$f_Y^2 = \frac{1}{m_Y^2} \int_{\mathcal{M}^2}^{s_0} ds \rho(s) e^{(m_Y^2 - s)/M^2}. \quad (11)$$

In the sum rules given by Eqs. (10) and (11), M^2 is the Borel parameter that has been introduced when applying the corresponding transformation, and s_0 is the continuum threshold parameter that separates the ground-state contribution from other effects.

Apart from the auxiliary parameters M^2 and s_0 , the sum rules depend also on the numerous vacuum condensates. In numerical computations, we use their values fixed at the normalization scale $\mu_0^2 = 1 \text{ GeV}^2$: for the quark and mixed condensates, $\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3$, $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\langle \bar{q}q_s \sigma G q \rangle = m_0^2 \langle \bar{q}q \rangle$, and $\langle \bar{s}g_s \sigma G s \rangle = m_0^2 \langle \bar{s}s \rangle$, and for the gluon condensates, $\langle \alpha_s G^2/\pi \rangle = (0.012 \pm 0.004) \text{ GeV}^4$ and $\langle g_s^3 G^3 \rangle = (0.57 \pm 0.29) \text{ GeV}^6$. For the masses of the quarks, we employ $m_s = (128 \pm 10) \text{ MeV}$ and $m_c = (1.27 \pm 0.03) \text{ GeV}$ borrowed from Ref. [31].

The vacuum condensates have fixed numerical values, whereas the Borel and continuum threshold parameters can be varied within some regions, which have to satisfy the standard restrictions of the sum rules computations. Thus, the window for $M^2 \in [M_{\text{max}}^2, M_{\text{min}}^2]$ is fixed from the constraints imposed on the pole contribution (PC)

$$\text{PC} = \frac{\Pi_V(M_{\text{max}}^2, s_0)}{\Pi_V(M_{\text{max}}^2, \infty)} \geq 0.15, \quad (12)$$

which determines M_{max}^2 , and on the ratio $R(M_{\text{min}}^2)$,

$$R(M_{\text{min}}^2) = \frac{\Pi_V^{\text{DimN}}(M_{\text{min}}^2, s_0)}{\Pi_V(M_{\text{min}}^2, s_0)} < 0.05, \quad (13)$$

necessary to find M_{min}^2 . In the expressions above, $\Pi_V(M^2, s_0)$ is the Borel transformed and subtracted

expression of the invariant function $\Pi_V^{\text{OPE}}(p^2)$, and M_{max}^2 and M_{min}^2 are the maximal and minimal allowed values of the Borel parameter. In Eq. (13), $\Pi_V^{\text{DimN}}(M_{\text{min}}^2, \infty)$ is the contribution to the correlation function of the last Nth term (or a sum of the last few terms) in the operator product expansion (OPE). The ratio $R(M_{\text{min}}^2)$ quantifies the convergence of the OPE and will be used for the numerical analysis. The last restriction on the lower limit M_{min}^2 is the prevalence of the perturbative contribution over the non-perturbative one.

It is clear that m_Y and f_Y should not depend on the auxiliary parameters M^2 and s_0 . But in real calculations, these quantities are nevertheless sensitive to the choice of both M^2 and s_0 . Therefore, the parameters M^2 and s_0 should also be determined in such a way as to minimize the dependence of m_Y and f_Y on them.

The analysis carried out by taking into account all of the aforementioned constraints allows us to determine

$$M^2 \in [4.9, 6.8] \text{ GeV}^2, \quad s_0 \in [23.2, 25.2] \text{ GeV}^2, \quad (14)$$

as the optimal regions for M^2 and s_0 . In fact, at M_{min}^2 , the convergence of the operator product expansion is fulfilled with high accuracy, and $R(4.8 \text{ GeV}^2) = 0.017$, which is estimated by employing the sum of the last three terms, i.e., $\text{DimN} \equiv \text{Dim8} + \text{Dim9} + \text{Dim10}$. Moreover, at M_{min}^2 , the perturbative contribution amounts to more than 74% of the full result, considerably overshooting the nonperturbative effects. The pole contribution is $\text{PC} = 0.16$, which is typical for sum rules involving multi-quark aggregations. It is worth noting that PC at M_{min}^2 reaches its maximal value and becomes equal to 0.78.

In Figs. 1 and 2, we plot the predictions for m_Y and f_Y , which visually demonstrate their dependence on the used values of M^2 and s_0 . It is seen that the dependence of the mass and coupling on the Borel parameter is very weak; the predictions for m_Y and f_Y demonstrate a high stability against changes of M^2 inside of the optimized working interval. But m_Y and f_Y are sensitive to the choice of the continuum threshold parameter s_0 . Namely, this dependence generates a main part of uncertainties in the present sum rules, which, nevertheless, remain within standard limits accepted for such a kind of computations. From these studies, we extract the mass and coupling of the resonance $Y(4660)$ as

$$m_Y = 4677_{-63}^{+71} \text{ MeV}, \quad f_Y = (0.99 \pm 0.16) \times 10^{-2} \text{ GeV}^4. \quad (15)$$

Our result for m_Y is in reasonable agreement with experimental data [6]. It is also instructive to compare m_Y with results of other theoretical studies. As we have mentioned above, in the context of the sum rule method, the

 FIG. 1. The dependence of the $Y(4660)$ resonance's mass on the Borel (left) and continuum threshold (right) parameters.

 FIG. 2. The same as in Fig. 1 but for the coupling f_Y .

mass of the resonance $Y(4660)$ was evaluated in different papers. Thus, in Ref. [19], the mass of $Y(4660)$ was found equal to $m_Y = (4.69 \pm 0.36)$ GeV, where the authors examined it as P -wave excitation of the scalar tetraquark $[cs][\bar{c}\bar{s}]$.

The resonance $Y(4660)$ was treated as the vector $[cs][\bar{c}\bar{s}]$ tetraquark in Ref. [20], the mass of which was found equal to

$$m_Y = (4.65 \pm 0.10) \text{ GeV}. \quad (16)$$

These predictions are compatible with experimental data, and by taking into account the theoretical errors, also with our result.

The vector tetraquarks with positive and negative C parities were explored in Ref. [21], and their masses were extracted from two-point sum rules by taking into account vacuum condensates up to dimension 8. The resonance $Y(4660)$ was identified in Ref. [21] as the tetraquarks with $J^{PC} = 1^{--}$ and $[cq][\bar{c}\bar{q}]$ or $[cs][\bar{c}\bar{s}]$ contents. In the case of the $[cs][\bar{c}\bar{s}]$ state built of the scalar diquark and vector

antidiquark, the authors used two interpolating currents denoted in Ref. [21] as $J_{1\mu}$ and $J_{3\mu}$, the first of which overshoots the mass of the $Y(4660)$ resonance

$$m_{J_1} = (4.92 \pm 0.10) \text{ GeV}, \quad (17)$$

whereas the second one underestimates it, leading to the result

$$m_{J_3} = (4.52 \pm 0.10) \text{ GeV}. \quad (18)$$

These predictions contradict to the experimental data and also do not coincide with our present result not the result of Ref. [20] obtained using the current Eq. (3).

The $Y(4660)$ was assigned in Ref. [24] to be the $C \otimes \gamma_\mu C$ -type vector tetraquark with the mass $m_Y = (4.66 \pm 0.09)$ GeV and the pole residue $\lambda_Y = (6.74 \pm 0.88) \times 10^{-2} \text{ GeV}^5$, which for the coupling f_Y leads to $f_Y = (1.45 \pm 0.19) \times 10^{-2} \text{ GeV}^4$. The discrepancy between this prediction and our result (15) for f_Y can be explained by the different assumptions on the internal structure of the

vector resonance $Y(4660)$. Indeed, in the present work, we consider it a state composed of a scalar diquark and vector antidiquark, whereas in Ref. [24], it was treated as a bound state of a pseudoscalar diquark and axial-vector antidiquark.

As is seen, within the sum rule method the $Y(4660)$ resonance can be interpreted as the vector tetraquark $[cs][\bar{c}\bar{s}]$, but with the different internal structures and interpolating currents. Therefore, one has to deepen the analysis and consider decays of the state $Y(4660)$ to make a choice between existing models. In the next section we are going to concentrate on the strong decay modes of $Y(4660)$, in which our results for m_Y and f_Y will be used as the input parameters.

III. STRONG DECAYS OF THE RESONANCE $Y(4660)$

The strong decays of the tetraquark $Y(4660)$ can be fixed using the kinematical restriction, which is evident from Eq. (15). Because we are interested in S -wave decays of $Y(4660)$, the spin in these processes should be conserved. Another constraint on possible partial decay modes of the $Y(4660)$ tetraquark is imposed by P parities of the final particles. Performed analysis allows us to see that partial decays to $J/\psi f_0(980)$, $\psi(2S)f_0(980)$ and $J/\psi f_0(500)$, $\psi(2S)f_0(500)$ are among important decay modes of $Y(4660)$.

The $Y(4660)$ resonance's decays contain in the final state the scalar mesons $f_0(980)$ and $f_0(500)$, which we are going to treat as diquark-antidiquark states. The interpretation of the mesons belonging to the light scalar nonet as four-quark systems is not new and starts from the analyses of Refs. [1,32]. In the model suggested recently in Ref. [33], the isoscalar mesons $f_0(980)$ and $f_0(500)$ are considered as mixtures of the basic tetraquark states $\mathbf{L} = [ud][\bar{u}\bar{d}]$ and $\mathbf{H} = ([su][\bar{s}\bar{u}] + [ds][\bar{d}\bar{s}])/\sqrt{2}$. Calculations performed using this new model led to reasonable predictions for the mass and full width of the mesons $f_0(980)$ and $f_0(500)$ [34,35]; these will be used in the present work, as well. It is worth noting that this mixing phenomenon allows one to study the decays of the $Y(4660)$ resonance to the $f_0(980)$ and $f_0(500)$ mesons within the same framework, because both of them interact with $Y(4660)$ through their $\mathbf{H}$ components.

We concentrate on the decays $J/\psi f_0(980)$ and $\psi(2S)f_0(980)$ and calculate the strong couplings $g_{YJf_0(980)}$ and $g_{Y\psi f_0(980)}$ corresponding to the vertices

$YJ/\psi f_0(980)$ and $Y\psi(2S)f_0(980)$, respectively. For these purposes, we employ the LCSR method and consider the correlation function

$$\Pi_{\mu\nu}(p, q) = i \int d^4x e^{ipx} \langle f_0(q) | T \{ J_\mu^\psi(x) J_\nu^\dagger(0) \} | 0 \rangle, \quad (19)$$

where $J_\nu(x)$ and $J_\mu^\psi(x)$ are the interpolating currents to $Y(4660)$ and J/ψ , respectively. The current $J_\nu(x)$ has been defined in Eq. (3), whereas $J_\mu^\psi(x)$ is given by the expression

$$J_\mu^\psi(x) = \bar{c}_i(x) i \gamma_\mu c_i(x). \quad (20)$$

In the vertices p, q , and $p' = p + q$ are the momenta of J/ψ or $\psi(2S)$, $f_0(980)$, and $Y(4660)$, respectively.

To derive the sum rules for $g_{YJf_0(980)}$ and $g_{Y\psi f_0(980)}$, we first calculate $\Pi_{\mu\nu}(p, q)$ in terms of the physical parameters of involved particles. It is not difficult to get

$$\begin{aligned} \Pi_{\mu\nu}^{\text{Phys}}(p, q) &= \frac{\langle 0 | J_\mu^\psi | J/\psi(p) \rangle}{p^2 - m_J^2} \langle J/\psi(p) f_0(q) | Y(p') \rangle \\ &\times \frac{\langle Y(p') | J_\nu^\dagger | 0 \rangle}{p'^2 - m_Y^2} + \frac{\langle 0 | J_\mu^\psi | \psi(2S)(p) \rangle}{p^2 - m_\psi^2} \\ &\times \langle \psi(2S)(p) f_0(q) | Y(p') \rangle \frac{\langle Y(p') | J_\nu^\dagger | 0 \rangle}{p'^2 - m_Y^2} \dots, \end{aligned} \quad (21)$$

where m_J and m_ψ are the masses of the mesons J/ψ and $\psi(2S)$, respectively. The dots in Eq. (21) denote a contribution of the higher resonances and continuum states. As is seen, $\Pi_{\mu\nu}^{\text{Phys}}(p, q)$ contains two terms and corresponds to the “ground-state + first radially excited state + continuum” scheme.

Further simplification of $\Pi_{\mu\nu}^{\text{Phys}}(p, q)$ can be achieved by employing the matrix element (6) and new ones from Eq. (22),

$$\begin{aligned} \langle 0 | J_\mu^\psi | J/\psi(p) \rangle &= f_J m_J \varepsilon_\mu, \\ \langle 0 | J_\mu^\psi | \psi(2S)(p) \rangle &= f_\psi m_\psi \varepsilon_\mu, \end{aligned} \quad (22)$$

as well as by introducing two elements that describe the vertices:

$$\begin{aligned} \langle J/\psi(p) f_0(q) | Y(p') \rangle &= g_{YJf_0(980)} [(p \cdot p')(\varepsilon^* \cdot \varepsilon') - (p \cdot \varepsilon')(p' \cdot \varepsilon^*)], \\ \langle \psi(2S)(p) f_0(q) | Y(p') \rangle &= g_{Y\psi f_0(980)} [(p \cdot p')(\varepsilon^* \cdot \varepsilon') - (p \cdot \varepsilon')(p' \cdot \varepsilon^*)]. \end{aligned} \quad (23)$$

In the expressions above, f_J (f_ψ) is the J/ψ [$\psi(2S)$] meson's decay constant, and ε_μ and ε'_ν are the polarization vectors of the J/ψ [$\psi(2S)$] meson and the resonance $Y(4660)$, respectively.

Then, the correlation function takes the following form:

$$\begin{aligned} \Pi_{\mu\nu}^{\text{Phys}}(p, q) = & \frac{g_Y J f_0(980) f_J m_J f_Y m_Y}{(p'^2 - m_Y^2)(p^2 - m_J^2)} \left(-p'_\mu p_\nu \right. \\ & \left. + \frac{m_Y^2 + m_J^2}{2} g_{\mu\nu} \right) + \frac{g_Y \Psi f_0(980) f_\psi m_\psi f_Y m_Y}{(p'^2 - m_Y^2)(p^2 - m_\psi^2)} \\ & \times \left(-p'_\mu p_\nu + \frac{m_Y^2 + m_\psi^2}{2} g_{\mu\nu} \right) + \dots \quad (24) \end{aligned}$$

We extract the sum rules for the strong couplings using the invariant functions corresponding to the structure $\sim g_{\mu\nu}$. The correlation function $\Pi_{\mu\nu}(p, q)$ contains inside of the $\mathcal{T}$ operation a tetraquark and conventional meson currents; therefore, this situation does not differ considerably from the analysis of the tetraquark-meson-meson vertices elaborated in Ref. [29]. These vertices can be investigated using the $q \rightarrow 0$ limit of the full LCSR method, which is known as the “soft-meson approximation” [28,36]. This approximation was applied numerously to study decays of the tetraquarks, e.g., in Refs. [37–39].

In the general case the invariant function $\Pi^{\text{Phys}}(p^2, p'^2)$ depends on two variables, but in the soft approximation when $p = p'$ it reduces to $\Pi^{\text{Phys}}(p^2)$. In this approach, we replace $1/[(p'^2 - m_Y^2)(p^2 - m_J^2)]$ by the double pole factor $1/[(p^2 - m_1^2)^2]$, where $m_1^2 = (m_Y^2 + m_J^2)/2$. The same is true also for the second term in Eq. (24) with the clear replacement $m_1^2 \rightarrow m_2^2 = (m_Y^2 + m_\psi^2)/2$. Then, the Borel transformation of the $\Pi^{\text{Phys}}(p^2)$ reads

$$\begin{aligned} \mathcal{B}\Pi^{\text{Phys}}(p^2) = & g_Y J f_0(980) f_J m_J f_Y m_Y m_1^2 \frac{e^{-m_1^2/M^2}}{M^2} \\ & + g_Y \Psi f_0(980) f_\psi m_\psi f_Y m_Y m_2^2 \frac{e^{-m_2^2/M^2}}{M^2} \dots \quad (25) \end{aligned}$$

In the next step, one has to find the expression of the correlation function in terms of the quark propagators. After some calculations, we get

$$\begin{aligned} \Pi_{\mu\nu}^{\text{OPE}}(p, q) = & \int d^4x e^{ipx} \epsilon \tilde{e} [\gamma_5 \tilde{S}_c^{ic}(x) \gamma_\mu \\ & \times \tilde{S}_c^{ei}(-x) \gamma_\nu \gamma_5 - \gamma_\nu \gamma_5 \tilde{S}_c^{ic}(x) \gamma_\mu \tilde{S}_c^{ei}(-x) \gamma_5]_{\alpha\beta} \\ & \times \langle f_0(q) | \bar{s}_\alpha(0) s_\beta^d(0) | 0 \rangle, \quad (26) \end{aligned}$$

where α and β are the spinor indices.

The matrix element $\langle f_0(q) | \bar{s}_\alpha^b(0) s_\beta^d(0) | 0 \rangle$ has to be rewritten in a form suitable for further analysis. To this end, we apply the expansion

$$\bar{s}_\alpha^b s_\beta^d \rightarrow \frac{1}{12} \delta_{bd} \Gamma_{\beta\alpha}^j (\bar{s} \Gamma^j s), \quad (27)$$

where $\Gamma^j = \mathbf{1}, \gamma_5, \gamma_\lambda, i\gamma_5 \gamma_\lambda, \sigma_{\lambda\rho}/\sqrt{2}$ form the full set of Dirac matrices, and express $\Pi_{\mu\nu}^{\text{OPE}}(p, q)$ in terms of the local matrix elements of the scalar meson $f_0(980)$. Calculations prove that the matrix elements with $\Gamma^j = \gamma_5$ and $i\gamma_5 \gamma_\lambda$, i.e., ones with an odd number of γ_5 matrices are identically equal to zero. The matrix elements in Eq. (27) with γ_λ and $\sigma_{\lambda\rho}/\sqrt{2}$ should be proportional to q_λ and $q_\lambda q_\rho$ because only the momentum of $f_0(980)$ has the required Lorentz index. But in the soft approximation, $q = 0$, and therefore these elements do not contribute to $\Pi_{\mu\nu}^{\text{OPE}}(p, q)$. In the matrix element with $\sigma_{\lambda\rho}/\sqrt{2}$ components, $\sim g_{\lambda\rho}$ may lead to some effects, but in the present work, we neglect them. We also ignore matrix elements $\sim G$ with insertions of the gluon field strength tensor, contributions of which in the soft approximation, as a rule, vanish. Hence, the only matrix element that we take into account is

$$\langle f_0(980)(q) | \bar{s}(0) s(0) | 0 \rangle = \lambda_{f'}, \quad (28)$$

which forms the correlation function $\Pi_{\mu\nu}^{\text{OPE}}(p, q = 0)$. The $\lambda_{f'}$ and the similar matrix element $\langle f_0(500)(q) | \bar{s}(0) s(0) | 0 \rangle = \lambda_f$ can be computed using the two-point sum rule method, the details of which are presented in the Appendix.

After standard calculations for the Borel transformed correlation function $\Pi^{\text{OPE}}(M^2)$, we find

$$\begin{aligned} \Pi^{\text{OPE}}(M^2) = & \frac{\lambda_{f'}}{24\pi^2} \int_{4m_c^2}^{\infty} \frac{ds}{s} \sqrt{s(s - 4m_c^2)} \\ & \times (s + 8m_c^2) + \lambda_{f'} \int_0^1 dz e^{-m_c^2/M^2 Z} F(z, M^2), \quad (29) \end{aligned}$$

where the first term is the perturbative contribution, whereas the nonperturbative effects are encoded by the second term. The function $F(z, M^2)$ in Eq. (29) has the form

$$\begin{aligned} F(z, M^2) = & -\frac{\langle \alpha_s G^2/\pi \rangle m_c^2}{72M^4} \frac{1}{Z} [m_c^2(1 - 2Z) - M^2 Z(3 - 7Z)] + \frac{\langle g_s^3 G^3 \rangle}{45 \cdot 2^9 \pi^2 M^8 Z^5} \\ & \times \{m_c^6(1 - 2z)^2(9 - 11Z) + 2m_c^4 M^4 Z^2[-42 + Z(122 + 9Z)] - 2M^6 Z^3 \\ & \times [6 - Z(22 - 9Z)] + m_c^4 M^2 Z(-11 + 119Z - 190Z^2)\} + \frac{\langle \alpha_s G^2/\pi \rangle^2 m_c^2 \pi^2}{648M^{10} Z^3} [m_c^4 - m_c^2 M^2 \\ & \times (1 + 4Z) + 2M^4 Z(2 - Z)], \quad (30) \end{aligned}$$

where $Z = z(1 - z)$.

The perturbative term in Eq. (29) is calculated as an imaginary part of the relevant component in $\Pi_{\mu\nu}^{\text{OPE}}(p, q = 0)$, and afterward, the Borel transformations are carried out. The Borel transformation of the non-perturbative contribution is computed directly from $\Pi_{\mu\nu}^{\text{OPE}}(p, q = 0)$ and contains vacuum condensates up to dimension 8. By equating $\mathcal{B}\Pi^{\text{Phys}}(p^2)$ to $\Pi^{\text{OPE}}(M^2)$ and performing the continuum subtraction, we find an expression that depends on two unknown variables $g_{YJf_0(980)}$ and $g_{Y\Psi f_0(980)}$. Let us note that continuum subtraction in the perturbative part is done by $\infty \rightarrow s_0$ replacement. Because all terms in Eq. (30) are proportional to inverse powers of the Borel parameter M^2 , in accordance with accepted methodology (see, Ref. [28]) the nonperturbative contribution should be left in an unsubtracted form preserving its original version. The second equation necessary for our purposes can be derived by applying the operator $d/d(-1/M^2)$ to both sides of this expression. These two equalities allow us to find sum rules for both $g_{YJf_0(980)}$ and $g_{Y\Psi f_0(980)}$, the explicit formulas of which are too cumbersome to present here.

The width of the decay process, e.g., $Y \rightarrow \psi(2S)f_0(980)$, can be found by means of the formula

$$\Gamma(Y \rightarrow \psi(2S)f_0(980)) = \frac{g_{Y\Psi f_0(980)}^2 m_\psi^2}{24\pi} \Lambda \left(3 + \frac{2\Lambda^2}{m_\psi^2} \right), \quad (31)$$

where $\Lambda = \Lambda(m_Y, m_\psi, m_{f_0})$ and

$$\Lambda(a, b, c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2(a^2b^2 + a^2c^2 + b^2c^2)}}{2a}.$$

The numerical computations of the strong couplings are performed using the values of the different vacuum condensates (see Sec. II) as well as spectroscopic parameters of the mesons J/ψ and $\psi(2S)$ (in units of MeV): $m_J = 3096.900 \pm 0.006$ and $f_J = 411 \pm 7$ and $m_\psi = 3686.097 \pm 0.005$ and $f_\psi = 279 \pm 8$. The parameters of the resonance $Y(4660)$ have been found in the present work, and for the mass of the $f_0(980)$ meson, we use its experimentally measured value $m_{f_0} = 990 \pm 20$ MeV. The parameters M^2 and s_0 are varied inside of the regions: $M^2 = (4.9\text{--}6.8)$ GeV² and $s_0 = (23.2\text{--}25.2)$ GeV². The obtained results for the strong couplings read

$$\begin{aligned} |g_{YJf_0(980)}| &= (0.22 \pm 0.07) \text{ GeV}^{-1}, \\ g_{Y\Psi f_0(980)} &= (1.22 \pm 0.33) \text{ GeV}^{-1}. \end{aligned} \quad (32)$$

Then, widths of the corresponding partial decay channels become equal to (in units of MeV)

$$\begin{aligned} \Gamma(Y \rightarrow J/\psi f_0(980)) &= 18.8 \pm 5.4, \\ \Gamma(Y \rightarrow \psi(2S)f_0(980)) &= 30.2 \pm 8.5. \end{aligned} \quad (33)$$

Analysis of the remaining two decays does not differ from previous ones and leads to predictions

$$\begin{aligned} g_{YJf_0(500)} &= (0.07 \pm 0.02) \text{ GeV}^{-1}, \\ |g_{Y\Psi f_0(500)}| &= (0.18 \pm 0.05) \text{ GeV}^{-1}, \end{aligned} \quad (34)$$

and (in MeV)

$$\begin{aligned} \Gamma(Y \rightarrow J/\psi f_0(500)) &= 2.7 \pm 0.7, \\ \Gamma(Y \rightarrow \psi(2S)f_0(500)) &= 13.1 \pm 3.7. \end{aligned} \quad (35)$$

The total width of the $Y(4660)$ resonance estimated using these four strong decay channels,

$$\Gamma_Y = (64.8 \pm 10.8) \text{ MeV}, \quad (36)$$

is in nice agreement with the experimental value $68 \pm 11 \pm 1$ MeV. For the total width of the $Y(4660)$ resonance, the Particle Data Group provides the world average $\Gamma_Y = 72 \pm 11$ MeV [31]. This is higher than the result of Ref. [6]; nevertheless, within uncertainties of theoretical calculations and errors of experimental measurements the prediction obtained here is compatible with the world average, as well. One has also to take into account that the diquark-antidiquark model for the $Y(4660)$ implies the existence of the S -wave decay channels $Y(4660) \rightarrow D_s^\pm D_{s1}^\mp(2460)$ and $Y(4660) \rightarrow D_s^{*\pm} D_{s0}^\mp(2317)$ that also contribute to Γ_Y and may improve this agreement.

IV. CONCLUSIONS

In the present work, we have calculated the full width of the resonance $Y(4660)$ by interpreting it as the diquark-antidiquark state with quantum numbers $J^{PC} = 1^{--}$. Its partial decay widths depend, as important input parameters, on the mass m_Y and coupling f_Y . The mass of the $Y(4660)$ as a scalar diquark-vector antidiquark $[cs][\bar{c}\bar{s}]$ state was originally calculated in Refs. [20,21]. But in these articles the coupling of the resonance $Y(4660)$ was not evaluated. Therefore, we have computed the spectroscopic parameters of the $Y(4660)$ state by employing the QCD two-point sum rules and taking into account quark, gluon, and mixed condensates up to dimension 10. This has allowed us to improve the accuracy of the aforementioned computations as well as to find the coupling of the resonance $Y(4660)$. Our result for $m_Y = 4677_{-63}^{+71}$ MeV within theoretical ambiguities agrees with experimental data and the prediction made in Ref. [20] but is not compatible with predictions of Ref. [21]. The coupling f_Y in the framework of the sum rule method was evaluated in Ref. [24], in which the another suggestion about the structure of the resonance

$Y(4660)$, namely, a pseudoscalar diquark-axial-vector anti-diquark picture, was employed. The coupling f_Y found there is larger than our result, which can be attributed to different structures used in Ref. [24] and in the present study.

In other words, calculation of a tetraquark's mass does not provide information enough to interpret it unambiguously as a bound state of a diquark and an antidiquark with fixed quantum numbers. Additional important information can be extracted from analysis of its decay channels. In the present article, we have computed the full width of the resonance $Y(4660)$ by taking into account its S -wave strong decays $Y \rightarrow J/\psi f_0(500)$, $Y \rightarrow \psi(2S)f_0(500)$, $Y \rightarrow J/\psi f_0(980)$, and $Y \rightarrow \psi(2S)f_0(980)$ and found reasonable agreement with the measurements. However, the process $Y(4660) \rightarrow \psi(2S)\pi^+\pi^-$ is the only decay mode of the state $Y(4660)$ observed experimentally. It is known that the dominant decay channels of the $f_0(500)$ and $f_0(980)$ mesons are processes $f_0 \rightarrow \pi^+\pi^-$ and $f_0 \rightarrow \pi^0\pi^0$. Therefore, the chains $Y(4660) \rightarrow \psi(2S)f_0(980) \rightarrow \psi(2S)\pi^+\pi^-$ and $Y(4660) \rightarrow \psi(2S)f_0(500) \rightarrow \psi(2S)\pi^+\pi^-$ explain a dominance of the observed final state in the decay of the resonance $Y(4660)$. In the tetraquark model, as we have seen, the width of the channel $Y(4660) \rightarrow J/\psi f_0(980)$ is sizeable. Additionally, the final states $\psi(2S)\pi^0\pi^0$ and $J/\psi\pi^0\pi^0$ should also be detected. But neither $J/\psi\pi^+\pi^-$ nor $\pi^0\pi^0$ were observed in the $Y(4660)$ decays. It is worth noting that most of the aforementioned final particles were discovered in decays of the vector resonance $Y(4260)$: its partial decays to $J/\psi\pi^+\pi^-$ and $J/\psi\pi^0\pi^0$ as well as to $J/\psi K^+K^-$ were seen experimentally. Therefore, more accurate measurements may reveal these modes in decays of the resonance $Y(4660)$, as well.

A situation with decays to D_s mesons is more difficult because in the tetraquark model there are not evident reasons for these channels of the $Y(4660)$ state to be highly suppressed or even forbidden. Decays to a pair of D mesons were not seen in the case of the resonance $Y(4260)$, either. It is quite possible that partial widths of decays to D_s mesons are numerically small. But this is only an assumption that must be confirmed by explicit calculations. Further experimental investigations of the $Y(4660)$ resonance, more precious measurements of relevant decays channels can enlighten existing problems with its nature.

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APPENDIX: THE LOCAL MATRIX ELEMENTS

In this Appendix, we calculate the couplings λ_f and $\lambda_{f'}$ [hereafter, $f = f_0(500)$ and $f' = f_0(980)$] defined as the matrix elements of the current $J_{\bar{s}s}(x) = \bar{s}(x)s(x)$ sandwiched between the exotic meson and vacuum states

$$\langle f(q)|\bar{s}s|0\rangle = \lambda_f, \quad \langle f'(q)|\bar{s}s|0\rangle = \lambda_{f'}. \quad (\text{A1})$$

To this end, we explore the two-point correlation function (see, e.g., Ref. [40])

$$\Pi^{f(f')}(q) = i \int d^4x e^{iqx} \langle 0 | T \{ J^{f(f')}(x) J_{\bar{s}s}^\dagger(0) \} | 0 \rangle, \quad (\text{A2})$$

where $J^{f(f')}(x)$ is the interpolating current for the scalar tetraquark f or f' . In the two-angles mixing scheme, these currents are given by the expression [34]

$$\begin{pmatrix} J^f(x) \\ J^{f'}(x) \end{pmatrix} = U(\varphi_H, \varphi_L) \begin{pmatrix} J^H(x) \\ J^L(x) \end{pmatrix}, \quad (\text{A3})$$

where $U(\varphi_H, \varphi_L)$ is the mixing matrix

$$U(\varphi_H, \varphi_L) = \begin{pmatrix} \cos \varphi_H & -\sin \varphi_L \\ \sin \varphi_H & \cos \varphi_L \end{pmatrix}, \quad (\text{A4})$$

which is responsible also for the couplings' mixing.

The currents $J^L(x)$ and $J^H(x)$ correspond to the basic states $\mathbf{L} = [ud][\bar{u}\bar{d}]$ and $\mathbf{H} = ([su][\bar{s}\bar{u}] + [ds][\bar{d}\bar{s}])/\sqrt{2}$ and have the forms

$$J^H(x) = \frac{\epsilon\tilde{\epsilon}}{\sqrt{2}} \{ [u_a^T(x) C \gamma_5 s_b(x)] [\bar{u}_c(x) \gamma_5 C \bar{s}_e^T(x)] + [d_a^T(x) C \gamma_5 s_b(x)] [\bar{d}_c(x) \gamma_5 C \bar{s}_e^T(x)] \}, \quad (\text{A5})$$

and

$$J^L(x) = \epsilon\tilde{\epsilon} [u_a^T(x) C \gamma_5 d_b(x)] [\bar{u}_c(x) \gamma_5 C \bar{d}_e^T(x)], \quad (\text{A6})$$

where $\epsilon\tilde{\epsilon} = \epsilon^{dab}\epsilon^{dce}$.

For an example, let us write down all expressions for the f meson. To find the phenomenological side of the sum rule, we use the “ground-state + continuum” scheme and get

$$\Pi^{f\text{Phys}}(q) = \frac{\langle 0 | J^f(x) | f(q) \rangle \langle f(q) | J_{\bar{s}s}^\dagger(0) | 0 \rangle}{m_f^2 - q^2} + \dots, \quad (\text{A7})$$

where the dots traditionally stand for the higher resonances and continuum. We continue using explicit expressions of the matrix elements $\langle 0 | J^f(x) | f(q) \rangle$ and $\langle f(q) | J_{\bar{s}s}^\dagger(0) | 0 \rangle$. The former element has just been introduced by Eq. (A1) and after some manipulations can be recast to the final form

$$\langle 0 | J^f | f(q) \rangle = m_f (F_H \cos^2 \varphi_H + F_L \sin^2 \varphi_L). \quad (\text{A8})$$

During this process, we have used the current J^f as it is given in Eq. (A3) and also the matrix elements

$$\langle 0 | J^i | f(p) \rangle = F_f^i m_f, \quad i = H, L. \quad (\text{A9})$$

We also benefited from the suggestion made in Ref. [34] that the couplings F_f^i follow a pattern of state mixing, which in the two-angles mixing scheme implies

$$\begin{pmatrix} F_f^H & F_f^L \\ F_{f'}^H & F_{f'}^L \end{pmatrix} = U(\varphi_H, \varphi_L) \begin{pmatrix} F_H & 0 \\ 0 & F_L \end{pmatrix}, \quad (\text{A10})$$

where F_H and F_L may be formally interpreted as couplings of the “particles” $|\mathbf{H}\rangle$ and $|\mathbf{L}\rangle$.

Then, we get

$$\Pi^{f\text{Phys}}(q) = \frac{\lambda_f m_f (F_H \cos^2 \varphi_H + F_L \sin^2 \varphi_L)}{m_f^2 - q^2} + \dots \quad (\text{A11})$$

The following task is a computation of $\Pi^{\text{OPE}}(q)$, which leads to

$$\Pi^{f\text{OPE}}(q) = \cos \varphi_H \Pi_0^{\text{OPE}}(q), \quad (\text{A12})$$

where

$$\begin{aligned} \Pi_0^{\text{OPE}}(q) &= i^2 \int d^4 x e^{iqx} \frac{\epsilon_{dab} \epsilon_{dae}}{6\sqrt{2}} \langle \bar{q} q \rangle \\ &\times \text{Tr}[\gamma_5 \tilde{S}_s^{ie}(-x) \tilde{S}_s^{bi}(x) \gamma_5]. \end{aligned} \quad (\text{A13})$$

The matrix element λ_f can be evaluated from the sum rule

$$\lambda_f = \frac{\Pi_0^{\text{OPE}}(M^2, s_0) \cos \varphi_H}{m_f (F_H \cos^2 \varphi_H + F_L \sin^2 \varphi_L)}, \quad (\text{A14})$$

where $\Pi_0^{\text{OPE}}(M^2, s_0)$ is the Borel transform of the correlation function $\Pi_0^{\text{OPE}}(q)$. The matrix element of the f' meson can be computed by means of the same expression with trivial replacements $m_f \rightarrow m_{f'}$, $\lambda_f \rightarrow \lambda_{f'}$, $\cos \varphi_H \rightarrow \sin \varphi_H$ and $\sin \varphi_L \rightarrow \cos \varphi_L$.

In numerical computations, we have utilized the parameters of the $f - f'$ system from Ref. [34], i.e., for the mixing angles, we have used $\varphi_H = -28^\circ.87 \pm 0^\circ.42$ and $\varphi_L = -27^\circ.66 \pm 0^\circ.31$, whereas for the couplings, $F_H = (1.35 \pm 0.34) \times 10^{-3} \text{ GeV}^4$ and $F_L = (0.68 \pm 0.17) \times 10^{-3} \text{ GeV}^4$ have been employed. The masses of the scalar particles $m_f = (518 \pm 74) \text{ MeV}$ and $m_{f'} = (996 \pm 130) \text{ MeV}$ have been borrowed from Ref. [34], as well. In calculations of λ_f , the Borel and continuum threshold parameters have been chosen as $M^2 = (0.75 - 1.0) \text{ GeV}^2$ and $s_0 = (0.8 - 1.1) \text{ GeV}^2$, whereas in the case of $\lambda_{f'}$ we have used $M^2 = (1.1 - 1.3) \text{ GeV}^2$ and $s_0 = (1.4 - 1.6) \text{ GeV}^2$. As a result, we have found

$$\begin{aligned} \lambda_f &= (0.015 \pm 0.004) \text{ GeV}^2, \\ |\lambda_{f'}| &= (0.052 \pm 0.013) \text{ GeV}^2, \end{aligned} \quad (\text{A15})$$

which have been used in Sec. III.

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