Weak decays of stable doubly heavy tetraquark states

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In light of the recent discovery of the Ξ_{cc}^{++} by the LHCb Collaboration, we study the stable doubly heavy tetraquarks. These states are compact exotic hadrons which can be approximated as the diquark-antidiquark correlations. In the flavor SU(3) symmetry, they form a SU(3) triplet or antisextet. The spectra of the stable doubly heavy tetraquark states are predicted by the Sakharov-Zeldovich formula. We find that the $T^-_{bb\bar{u}\bar{d}}(\mathbf{3})$ is about 73 MeV below the BB^* threshold at SET I. We then study the semileptonic and nonleptonic weak decays of the stable doubly heavy tetraquark states. The doubly heavy tetraquark decay amplitudes are parametrized in terms of flavor SU(3)-irreducible parts. Ratios between decay widths of different channels are also derived. At the end, we collect the Cabibbo allowed two-body and three-body decay channels, which are most promising to search for the stable doubly heavy tetraquark states at the LHCb and Belle II experiments.

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I. INTRODUCTION

Up to now most hadrons found by experiments can be well established as quark-antiquark pair and triquarks configurations [1]. Based on the principle of color confining, the multiquark color singlet states such as $qq\bar{q}\bar{q}$ (tetraquarks) and $qqqq\bar{q}$ (pentaquarks) can also exist. On the experimental aspect, many multiquark candidates have been observed even though their physical figures are still not established. The most aged of these exotic resonances is the neutral $X(3872)$ discovered in $B^\pm \to K^\pm X(X \to J/\psi\pi^+\pi^-)$ by Belle in 2003 [2]. Four years later, the Belle Collaboration observed a charged hidden charm tetraquark candidate, i.e., $Z^+(4430)$ [3]. In 2013, the BESIII Collaboration discovered $Z_c^+(3900)$ through the channel $Y(4260) \to \pi^-\pi^+J/\psi$ [4], which directly hadronic decays into $J/\psi\pi^+$, and then implies that it shall be a meson with quark contents $c\bar{c}u\bar{d}$. In 2015, the LHCb Collaboration discovered two exotic baryons $P_c(4380)$ and $P_c(4450)$ hadronic decaying into $J/\psi p$, which are candidates for pentaquark states and shall be a baryon with quark contents $c\bar{c}uud$ [5].

The LHCb Collaboration have recently observed the doubly charmed baryon Ξ_{cc}^{++} in the $\Lambda_c^+K^-\pi^+\pi^+$ invariant mass spectrum, whose mass is measured to be $3621.40 \pm 0.72(\mathrm{stat}) \pm 0.27(\mathrm{syst}) \pm 0.14(\Lambda_c^+)$ MeV [6]. This discovery has attracted wide attention from both the theoretical and experimental sides in high energy physics. From the diquark-based model, the doubly heavy quarks can provide a static color source as the attractive diquark in the color $\bar{\mathbf{3}}$ representation. The attractive heavy diquark and the light quark in the color $\mathbf{3}$ representation then form a color singlet hadron. Thus, it is natural to conceive the doubly heavy tetraquark states with attractive heavy diquark and attractive light diquark. From the basic principles of QCD, the long-distance interactions among light quarks and gluons has a characteristic scale of the order of 300 MeV. When the two heavy quarks attract each other and their separation is smaller than the separation to the light quark, the two heavy quarks interact with a perturbative one-gluon-exchange Coulomb-like potential. When the two heavy quarks have a large separation, the four quarks will form two weakly interacting mesons. It is an important issue to be discussed about whether or not the stable doubly heavy tetraquark states exist. When they steadily exist, it is another important issue on how to detect them.

On the theoretical aspect, the mass spectra of the doubly heavy tetraquark states have been studied in much of the literature [7–21]. Most of them supported the existence of the doubly heavy tetraquark states; however, they predicted differently the spectra of the

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doubly heavy tetraquark states in these works. The structures of the doubly heavy tetraquark states were also different in their descriptions. Unlike the $Qq\bar{Q}\bar{q}$ system which can be classified into four kinds of four quark structures [22], the structures of the $QQ\bar{q}\bar{q}$ system are relatively simple. Take the $bb\bar{q}\bar{q}$, for example. The four quark structures may be classified into two groups: one is treated as a bound state made of a loosely bound BB meson pair or two far separated and essentially weak interacting B mesons; the other one is treated as a bound state made of a heavy diquark with color antitriplet and a light antidiquark with color triplet.

Theoretical descriptions of doubly heavy tetraquark states decays are few in current studies. Whether or not the QCD factorization is valid in the doubly heavy tetraquark states decays is an open question. An alternative and model-independent approach is to employ the flavor SU(3) symmetry, which has been successfully applied into the B meson and the heavy baryon decays [23–41]. In this paper, we will investigate the amplitudes and decay widths of doubly heavy tetraquark states under the flavor SU(3) symmetry.

The paper will be presented as follows. In Sec. II, we classify the doubly heavy tetraquark states into a SU(3) triplet or antisextet according to the decomposition of $\bar{3}\otimes\bar{3}=3\oplus\bar{6}$. Other related baryons and mesons are also listed in SU(3) flavor symmetry. In Sec. III, we give the spectra of the doubly heavy tetraquark states. Their stability properties are essential for the discovery and will be discussed. In Secs. IV and V, we mainly study the semi-leptonic and nonleptonic weak decays of the stable doubly heavy tetraquarks. The decay amplitudes are explored with the SU(3) flavor symmetry. The ratios between the decay widths of different decay channels are predicted. We summarize and conclude in the end.

II. PARTICLE MULTIPLETS

Following the flavor SU(3) group, the doubly heavy tetraquark states and their decay products can be grouped into the particle multiplets.

In principle, doubly heavy tetraquark states with the $QQ\bar{q}\bar{q}$ are similar to the $\bar{Q}\bar{Q}qq$. We are focusing on the $QQ\bar{q}\bar{q}$ states for simplification. The doubly heavy tetraquark ($QQ\bar{q}\bar{q}$) can form a SU(3) triplet or antisextet by the decomposition of $\bar{3}\otimes\bar{3}=3\oplus\bar{6}$. The triplet has the expression

$$T_{cc3}=\begin{pmatrix}0 & T^{+}_{cc\bar{u}\bar{d}} & T^{+}_{cc\bar{u}\bar{s}}\\ -T^{+}_{cc\bar{u}\bar{d}} & 0 & T^{++}_{cc\bar{d}\bar{s}}\\ -T^{+}_{cc\bar{u}\bar{s}} & -T^{++}_{cc\bar{d}\bar{s}} & 0\end{pmatrix},\quad T_{bc3}=\begin{pmatrix}0 & T^{+}_{bc\bar{u}\bar{d}} & T^{+}_{bc\bar{u}\bar{s}}\\ -T^{+}_{bc\bar{u}\bar{d}} & 0 & T^{++}_{bc\bar{d}\bar{s}}\\ -T^{+}_{bc\bar{u}\bar{s}} & -T^{++}_{bc\bar{d}\bar{s}} & 0\end{pmatrix},\quad T_{bb3}=\begin{pmatrix}0 & T^{+}_{bb\bar{u}\bar{d}} & T^{+}_{bb\bar{u}\bar{s}}\\ -T^{+}_{bb\bar{u}\bar{d}} & 0 & T^{++}_{bb\bar{d}\bar{s}}\\ -T^{+}_{bb\bar{u}\bar{s}} & -T^{++}_{bb\bar{d}\bar{s}} & 0\end{pmatrix}. \tag{1}$$

Since the doubly heavy tetraquark in an antisextet can usually strongly decay into the triplets and are not stable, we will not consider them here.

When we study the weak decays of the doubly heavy tetraquarks under the flavor SU(3) symmetry, we should classify the products. The charmed bottom baryons can form a SU(3) triplet with $F_{bc}=(\Xi^{+}_{bc}(bcu),\Xi^{0}_{bc}(bcd),\Omega^{0}_{bc}(bcs))$. The charmed antibaryons are classified into a triplet and an antisextet

$$F_{\bar{\mathbf{c}}\mathbf{3}}=\begin{pmatrix}0 & \Lambda^{-}_{\bar{c}} & \Xi^{-}_{\bar{c}}\\ -\Lambda^{-}_{\bar{c}} & 0 & \bar{\Xi}^{0}_{\bar{c}}\\ -\Xi^{-}_{\bar{c}} & -\bar{\Xi}^{0}_{\bar{c}} & 0\end{pmatrix},\qquad F_{\bar{\mathbf{c}}\bar{\mathbf{6}}}=\begin{pmatrix}\Sigma^{--}_{\bar{c}} & \frac{1}{\sqrt{2}}\Sigma^{-}_{\bar{c}} & \frac{1}{\sqrt{2}}\Xi'^{-}_{\bar{c}}\\ \frac{1}{\sqrt{2}}\Sigma^{-}_{\bar{c}} & \bar{\Sigma}^{0}_{\bar{c}} & \frac{1}{\sqrt{2}}\bar{\Xi}'^{0}_{\bar{c}}\\ \frac{1}{\sqrt{2}}\Xi'^{-}_{\bar{c}} & \frac{1}{\sqrt{2}}\bar{\Xi}'^{0}_{\bar{c}} & \bar{\Omega}^{0}_{\bar{c}}\end{pmatrix}. \tag{2}$$

Similarly, the singly charmed baryons $F_{\mathbf{c}\bar{\mathbf{3}}}$ and $F_{\mathbf{c}\mathbf{6}}$ can be described in the same way, whose explicit expressions can be found in Refs. [35,38].

The antibaryons with light quarks can be classified into an octet and an antidecuplet. We write the octet as

$$F_{8}=\begin{pmatrix}\frac{1}{\sqrt{2}}\bar{\Sigma}^{0}+\frac{1}{\sqrt{6}}\bar{\Lambda}^{0} & \bar{\Sigma}^{+} & \bar{\Xi}^{+}\\ \bar{\Sigma}^{-} & -\frac{1}{\sqrt{2}}\bar{\Sigma}^{0}+\frac{1}{\sqrt{6}}\bar{\Lambda}^{0} & \bar{\Xi}^{0}\\ \bar{p} & \bar{n} & -\sqrt{\frac{2}{3}}\bar{\Lambda}^{0}\end{pmatrix}, \tag{3}$$

while the antidecuplet can be written as [34,35]

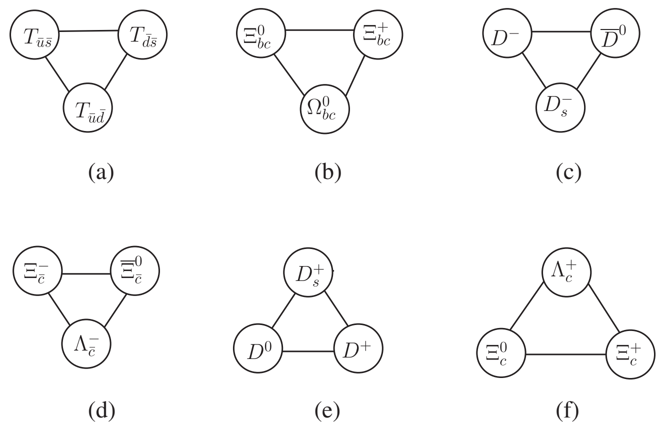

FIG. 1. Flavor SU(3) weight diagrams for the triplet and antitriplet hadrons. Panel (a) corresponds with doubly heavy tetraquark triplet; panel (b) corresponds with the charmed bottom baryons triplet; panel (c) and (e) stand for the anticharm mesons triplet and the charmed mesons antitriplet; panel (d) and (f) represent the anticharm antibaryons triplet and the charmed baryons antitriplet respectively.

$$
\begin{aligned}
&(F_{\overline{10}})^{111} = \overline{\Delta}^{--},\\
&(F_{\overline{10}})^{112} = (F_{\overline{10}})^{121} = (F_{\overline{10}})^{211} = \frac{1}{\sqrt{3}}\overline{\Delta}^{-},\\
&(F_{\overline{10}})^{222} = \overline{\Delta}^{+},\\
&(F_{\overline{10}})^{122} = (F_{\overline{10}})^{212} = (F_{\overline{10}})^{221} = \frac{1}{\sqrt{3}}\overline{\Delta}^{0},\\
&(F_{\overline{10}})^{113} = (F_{\overline{10}})^{131} = (F_{\overline{10}})^{311} = \frac{1}{\sqrt{3}}\overline{\Sigma}'^{-},\\
&(F_{\overline{10}})^{223} = (F_{\overline{10}})^{232} = (F_{\overline{10}})^{322} = \frac{1}{\sqrt{3}}\overline{\Sigma}'^{+},\\
&(F_{\overline{10}})^{123} = (F_{\overline{10}})^{132} = (F_{\overline{10}})^{213} = (F_{\overline{10}})^{231} = (F_{\overline{10}})^{312}\\
&\qquad = (F_{\overline{10}})^{321} = \frac{1}{\sqrt{6}}\overline{\Sigma}'^{0},\\
&(F_{\overline{10}})^{133} = (F_{\overline{10}})^{313} = (F_{\overline{10}})^{331} = \frac{1}{\sqrt{3}}\overline{\Xi}'^{0},\\
&(F_{\overline{10}})^{233} = (F_{\overline{10}})^{323} = (F_{\overline{10}})^{332} = \frac{1}{\sqrt{3}}\overline{\Xi}'^{+},\\
&(F_{\overline{10}})^{333} = \overline{\Omega}^{+}.
\end{aligned} \tag{4}
$$

The light pseudoscalar mesons belong to an octet and a singlet. The explicit expression of the octet can be found in Refs. [35,38]. The flavor singlet η_1 will not be considered here for simplicity. For the heavy flavor mesons, we can write them as $B_i = (B^-, \overline{B}^0, \overline{B}^0_s)$ and $B^i = (B^+, B^0, B^0_s)$. Similarly, the charmed mesons can be written as $D_i = (D^0, D^+, D^+_s)$ and $D^i = (\overline{D}^0, D^-, D^-_s)$. To a summary of the particle multiplets, we plot the flavor SU(3) weight diagrams for hadrons with different representations in Figs. 1 and 2.

III. SPECTRA OF THE DOUBLY HEAVY TETRAQUARKS

The wave function of a tetraquark consists of four parts: space-coordinate, flavor, color, and spin subspaces,

$$
\begin{aligned}
\Psi(Q, Q', \bar{q}, \bar{q}') = {}& R(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) \otimes \chi_f(f_1, f_2, f_3, f_4)\\
& \otimes \chi_\lambda(\lambda_1, \lambda_2, \lambda_3, \lambda_4) \otimes \chi_s(s_1, s_2, s_3, s_4),
\end{aligned} \tag{5}
$$

where $R(\mathbf{x}_i)$, $\chi_f(f_i)$, $\chi_\lambda(\lambda_i)$, and $\chi_s(s_i)$ denote the radial, flavor, color, and spin wave functions, respectively. The sublabels 1, 2, 3, 4 in the above equation denote Q, Q', $\bar{q}$, $\bar{q}'$, respectively.

For the two quark system, there are eight distinct diquark multiplets in flavor $\otimes$ color $\otimes$ spin space. According to the Pauli exclusion principle, the diquark-antidiquark configuration $[QQ'][\bar{q}\,\bar{q}']$ of doubly heavy tetraquark states only has four possible topologies[1]

$$|\mathbf{1}_f(S), \mathbf{3}_f(A)\rangle \otimes |\bar{\mathbf{3}}_c(A), \mathbf{3}_c(A)\rangle \otimes |1_s(S), 0_s(A)\rangle, \tag{6a}$$

$$|\mathbf{1}_f(S), \mathbf{3}_f(A)\rangle \otimes |\mathbf{6}_c(S), \bar{\mathbf{6}}_c(S)\rangle \otimes |0_s(A), 1_s(S)\rangle, \tag{6b}$$

$$|\mathbf{1}_f(S), \bar{\mathbf{6}}_f(S)\rangle \otimes |\bar{\mathbf{3}}_c(A), \mathbf{3}_c(A)\rangle \otimes |1_s(S), 1_s(S)\rangle, \tag{6c}$$

$$|\mathbf{1}_f(S), \bar{\mathbf{6}}_f(S)\rangle \otimes |\mathbf{6}_c(S), \bar{\mathbf{6}}_c(S)\rangle \otimes |0_s(A), 0_s(A)\rangle, \tag{6d}$$

where the sublabels f, c, s denote the flavor, color, spin spaces, respectively. S and A denote the symmetric and antisymmetric properties. Each half bracket denotes the diquark configuration. For example, $|\mathbf{1}_f(S), \mathbf{3}_f(A)\rangle$ denotes that the diquark $[QQ']$ is the singlet in flavor space and thus is symmetric (S), while the diquark $[\bar{q}\,\bar{q}']$ is the triplet in flavor space and thus is antisymmetric (A). Consider that the color sextet diquarks have larger color electrostatic energy and thus are not a well-favored configuration, and the odd parity diquark operators will vanish in the single mode configuration. The diquarks $|\mathbf{3}_c(A)\rangle \otimes |0_s(A)\rangle$ and $|\bar{\mathbf{3}}_c(A)\rangle \otimes |1_s(S)\rangle$ in Eqs. (6a) and (6c) are the "scalar" and "axial-vector" diquarks, respectively, indicated as the "good" and "bad" diquarks by Jaffe [42]. According to Jaffe's diquark, the good and bad diquarks have mass splitting. Approximately, for the up and down diquark, the mass difference between the good and bad diquarks is around 210 MeV, and for the up and strange diquark, the mass difference becomes around 150 MeV [42]. While for

[1]One should note that the two quarks and two antiquarks can be encoded by three different ways of the color basis. One of them is the color basis of $\{|\bar{\mathbf{3}}_{12}, \mathbf{3}_{34}\rangle, |\mathbf{6}_{12}, \bar{\mathbf{6}}_{34}\rangle\}$, and the other two are $\{|\mathbf{1}_{13}, \mathbf{1}_{24}\rangle, |\mathbf{8}_{13}, \mathbf{8}_{24}\rangle\}$ and $\{|\mathbf{1}_{14}, \mathbf{1}_{23}\rangle, |\mathbf{8}_{14}, \mathbf{8}_{23}\rangle\}$, respectively. Through decomposition, the singlet-singlet and octet-octet basis can be described by the triplet-antitriplet and antisextet-sextet base.

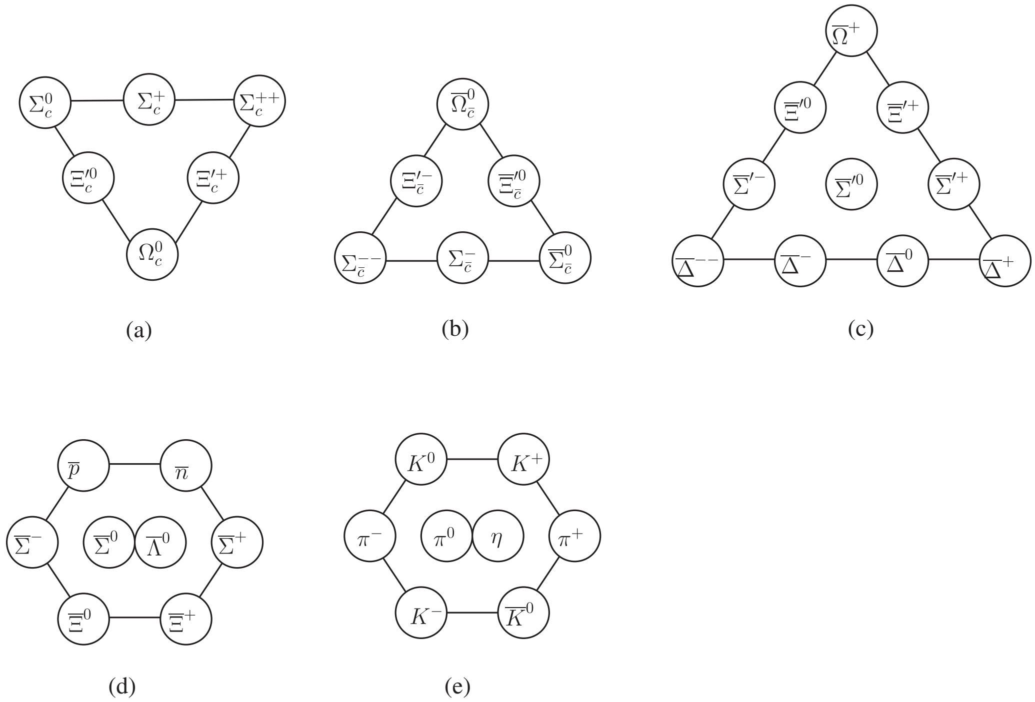

FIG. 2. Flavor SU(3) weight diagrams for the sextet, antisextet, antidecuplet, antioctet, and octet hadrons. Panel (a) corresponds with the charmed baryons sextet; panel (b) corresponds with the anticharm baryons antisextet; panel (c) and (d) represent the light antibaryons antidecuplet and the light antibaryon antioctet respectively; panel (e) stands for the light mesons octet.

the charm and light diquark, the mass difference is around 60 MeV [43]. One can assume the diquark mass difference for the doubly heavy quark is around 50 MeV. Other configurations of the diquark are "worse" diquarks. For simplification, we will not consider these worse diquarks in the prediction of the spectra.

The constituent quark models have a robust power to predict hadron spectra, especially for the S-wave states. In these quark models, the hadrons are bound states composed of the constituent quarks. In the Sakharov-Zeldovich formula, the interaction Hamiltonian through the color-spin interaction is given by [42,44]

$$\mathcal{H}_{\text{color spin}} = \sum_{i<j}\left(-\frac{3}{8}\right)\frac{C^{ij}}{m_i m_j}\vec{\lambda}_i \cdot \vec{\lambda}_j \vec{s}_i \cdot \vec{s}_j, \tag{7}$$

where the overall strength can be given as $C^{ij} = v^{ij}\langle\delta(r_{ij})\rangle$ with the coupling v^{ij} and the strength of the radial wave function at zero separation $\langle\delta(r_{ij})\rangle$ which is dependent on the hadron constituent quark flavors. The $\vec{\lambda}_i$ is the Gell-Mann matrix for the color SU(3) group, and $\vec{s}_i = \vec{\sigma}_i/2$ is the quark spin operator with the Pauli matrix $\vec{\sigma}_i$. The effects of the color-Coulomb potential and the color confining potential are included in the fitted quark mass. We give the matrix elements of color and spin operators in Table I.

The parameters in Eq. (7) can be fitted by the hadron spectra [45], which have been given in Table II. The overall factor $C^{ij}/(m_i m_j)$ can be extracted from the hadron mass differences. From Table II, one gets the following results: $C^{qq}/m_u^2 = 193$ MeV, $C^{sq}/(m_u m_s) = 118$ MeV, $C^{cq}/(m_u m_c) = 23$ MeV, and $C^{bq}/(m_u m_b) = 2.3$ MeV for the diquark configuration; and $C^{q\bar{q}}/m_u^2 = 318$ MeV, $C^{s\bar{q}}/(m_u m_s) = 199$ MeV, $C^{c\bar{q}}/(m_u m_c) = 69$ MeV, $C^{b\bar{q}}/(m_u m_b) = 23$ MeV, $C^{s\bar{s}}/m_s^2 = 118$ MeV, $C^{c\bar{s}}/(m_s m_c) = 72$ MeV, $C^{b\bar{s}}/(m_s m_b) = 24$ MeV, $C^{c\bar{c}}/m_c^2 = 57$ MeV, and $C^{b\bar{b}}/m_b^2 = 31$ MeV for the quark-antiquark configuration. These fittings are consistent with the previous literature [43,46–48]. The effective quark masses are fitted as follows: $m_{u,d} = 305$ MeV, $m_s = 490$ MeV, $m_c = 1670$ MeV, and $m_b = 5008$ MeV for SET I (fitted from the mesons [43,46]); and $m_{u,d} = 330$ MeV, $m_s = 500$ MeV, $m_c = 1550$ MeV, and $m_b = 4880$ MeV for SET III (widely used in quark potential model [49]); and $m_{u,d} = 362$ MeV, $m_s = 546$ MeV, $m_c = 1721$ MeV, and $m_b = 5050$ MeV for SET II (fitted from the baryons

TABLE I. Matrix elements $\vec{\lambda}_i \cdot \vec{\lambda}_j \vec{s}_i \cdot \vec{s}_j$ for two quarks in color $\bar{\mathbf{3}}$, $\mathbf{6}$, $\mathbf{1}$, $\mathbf{8}$ configurations.

	$\bar{\mathbf{3}}$	$\mathbf{6}$	$\mathbf{1}$	$\mathbf{8}$
$\langle\vec{\lambda}_i \cdot \vec{\lambda}_j\rangle$	$-\frac{8}{3}$	$\frac{4}{3}$	$-\frac{16}{3}$	$\frac{2}{3}$
$\langle\vec{\lambda}_i \cdot \vec{\lambda}_j \vec{s}_i \cdot \vec{s}_j\rangle$ $(s=0)$	2	-1	4	$-\frac{1}{2}$
$\langle\vec{\lambda}_i \cdot \vec{\lambda}_j \vec{s}_i \cdot \vec{s}_j\rangle$ $(s=1)$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{4}{3}$	$\frac{1}{6}$

TABLE II. Fitting the overall factors $C^{ij}/(m_i m_j)$ from the baryon and meson spectra. The fitted results for these factors become the following: $C^{qq}/m_u^2 = 193$ MeV, $C^{sq}/(m_u m_s) = 118$ MeV, $C^{cq}/(m_u m_c) = 23$ MeV, and $C^{bq}/(m_u m_b) = 2.3$ MeV for the diquark configuration; $C^{q\bar{q}}/m_u^2 = 318$ MeV, $C^{s\bar{q}}/(m_u m_s) = 199$ MeV, $C^{c\bar{q}}/(m_u m_c) = 69$ MeV, $C^{b\bar{q}}/(m_u m_b) = 23$ MeV, $C^{s\bar{s}}/m_s^2 = 118$ MeV, $C^{c\bar{s}}/(m_s m_c) = 72$ MeV, $C^{b\bar{s}}/(m_s m_b) = 24$ MeV, $C^{c\bar{c}}/m_c^2 = 57$ MeV, and $C^{b\bar{b}}/m_b^2 = 31$ MeV for the quark-antiquark configuration. These fitted results are consistent with the previous literature [43,46–48].

Mass	$M_\Delta - M_n$	$M_\Sigma - M_\Lambda$	$M_{\Sigma_c} - M_{\Lambda_c}$	$M_{\Sigma_b} - M_{\Lambda_b}$
Form.	$3C^{qq}/(2m_u^2)$	$C^{qq}/m_u^2 - C^{sq}/(m_s m_u)$	$C^{qq}/m_u^2 - C^{cq}/(m_c m_u)$	$C^{qq}/m_u^2 - C^{bq}/(m_b m_u)$
Exp. [1]	290 MeV	75 MeV	170 MeV	191 MeV
Mass	$M_\rho - M_\pi$	$M_{K^*} - M_K$	$M_{D^*} - M_D$	$M_{B^*} - M_B$
Form.	$2C^{q\bar{q}}/m_u^2$	$2C^{s\bar{q}}/(m_s m_u)$	$2C^{c\bar{q}}/(m_c m_u)$	$2C^{b\bar{q}}/(m_b m_u)$
Exp. [1]	635 MeV	397 MeV	137 MeV	46 MeV
Mass	$M_\omega - M_\eta$	$M_{D_s^*} - M_{D_s}$	$M_{B_s^*} - M_{B_s}$	
Form.	$2C^{s\bar{s}}/m_s^2$	$2C^{c\bar{s}}/(m_c m_s)$	$2C^{b\bar{s}}/(m_b m_s)$	
Exp. [1]	235 MeV	144 MeV	48 MeV	
Mass	$M_{J/\psi} - M_{\eta_c}$	$M_\Upsilon - M_{\eta_b}$		
Form.	$2C^{c\bar{c}}/m_c^2$	$2C^{b\bar{b}}/m_b^2$		
Exp. [1]	113 MeV	61 MeV		

[43,46]). For the tetraquark, it is approximate to use the quark mass parameter values extracted from the mesons.

The spectra of the triplet doubly charmed tetraquark for SET I are determined as

$$m(T^+_{cc\bar{u}\bar{d}}(\mathbf{3})) = 3.86\ \text{GeV}, \qquad J^P = 1^+, \tag{8}$$

$$m(T^+_{cc\bar{u}\bar{s}}(\mathbf{3})) = m(T^{++}_{cc\bar{d}\bar{s}}(\bar{\mathbf{3}})) = 4.10\ \text{GeV}, \qquad J^P = 1^+. \tag{9}$$

Above, $T^+_{cc\bar{u}\bar{d}}(\mathbf{3})$ lies on the spectrum at about 16 MeV below the DD^* threshold and at about 120 MeV above the DD threshold. However, the $T^+_{cc\bar{u}\bar{d}}(\mathbf{3})$ is an axial-vector meson and cannot directly hadronic decay to DD. Thus, $T^+_{cc\bar{u}\bar{d}}(\mathbf{3})$ with spin-parity 1^+ may be a stable tetraquark, but the situation is not optimistic if we consider the large uncertainty from the good and bad diquark's mass splitting. The binding energy 16 MeV may be polished if the good and bad diquark's mass splitting increases 16 MeV. $T^+_{cc\bar{u}\bar{s}}(\mathbf{3})$ and $T^{++}_{cc\bar{d}\bar{s}}(\mathbf{3})$ lie on the spectrum at about 124 MeV above the D_sD^* threshold and at about 118 MeV above the D_s^*D threshold. These two states can hadronic decay and thus, are not stable.

The spectra of the triplet charm-beauty tetraquark for SET I are determined as

$$m(T^0_{bc\bar{u}\bar{d}}(\mathbf{3})) = 7.20\ \text{GeV}, \qquad J^P = 1^+, \tag{10}$$

$$m(T^0_{bc\bar{u}\bar{s}}(\mathbf{3})) = m(T^+_{bc\bar{d}\bar{s}}(\bar{\mathbf{3}})) = 7.43\ \text{GeV}, \qquad J^P = 1^+. \tag{11}$$

In this kind, $T^0_{bc\bar{u}\bar{d}}(\mathbf{3})$ lies on the spectrum at about 86 MeV below the BD^* threshold but at about 5.8 MeV above the B^*D threshold. Thus, $T^0_{bc\bar{u}\bar{d}}(\mathbf{3})$ can hadronically decay to B^*D and has a large decay width. $T^0_{bc\bar{u}\bar{s}}(\mathbf{3})$ and $T^+_{bc\bar{d}\bar{s}}(\mathbf{3})$ lie on the spectrum at about 56 MeV above the B_sD^* threshold and at about 137 MeV above the B^*D_s threshold. These two states are also not stable.

The spectra of the triplet doubly bottomed tetraquark for SET I are determined as

$$m(T^-_{bb\bar{u}\bar{d}}(\mathbf{3})) = 10.53\ \text{GeV}, \qquad J^P = 1^+, \tag{12}$$

$$m(T^-_{bb\bar{u}\bar{s}}(\mathbf{3})) = m(T^0_{bb\bar{d}\bar{s}}(\bar{\mathbf{3}})) = 10.77\ \text{GeV}, \qquad J^P = 1^+. \tag{13}$$

For the bottom sector, $T^-_{bb\bar{u}\bar{d}}(\mathbf{3})$ lies on the spectrum at about 73 MeV below the BB^* threshold. Thus, $T^-_{bb\bar{u}\bar{d}}(\mathbf{3})$ with spin-parity 1^+ is a stable tetraquark, which shall be tested in experiment. $T^-_{bb\bar{u}\bar{s}}(\mathbf{3})$ and $T^0_{bb\bar{d}\bar{s}}(\mathbf{3})$ lie on the spectrum at about 78 MeV above the B_sB^* threshold and at about 75 MeV above the B_s^*B threshold, which are not stable.

Considering the uncertainties from the quark masses and the heavy diquark mass splitting, we adopt another input for these parameters. When we adopt the quark masses with SET II and increase the heavy diquark mass splitting by 100 MeV, we can easily get the tetraquark mass differences and then predict the doubly heavy quark tetraquark spectra. The mass of $T^+_{cc\bar{u}\bar{d}}(\mathbf{3})$ will been reduced by 90 MeV, while $T^+_{cc\bar{u}\bar{s}}(\mathbf{3})$ and $T^{++}_{cc\bar{d}\bar{s}}(\mathbf{3})$ will be reduced by 105 MeV. The conclusions of the stability discussions of them are unchanged. For the triplet charm-beauty tetraquark with SET II, the mass of $T^0_{bc\bar{u}\bar{d}}(\mathbf{3})$ will be reduced by 98 MeV, while $T^0_{bc\bar{u}\bar{s}}(\mathbf{3})$ and $T^+_{bc\bar{d}\bar{s}}(\mathbf{3})$ will be reduced by 113 MeV. Then $T^0_{bc\bar{u}\bar{d}}(\mathbf{3})$ is about 179 MeV below the BD^* threshold and about 87 MeV below the B^*D threshold, which indicates that $T^0_{bc\bar{u}\bar{d}}(\mathbf{3})$ may be a stable state. For the triplet doubly bottomed tetraquark with SET II, the mass of $T^-_{bb\bar{u}\bar{d}}(\mathbf{3})$ will be reduced by 86 MeV, while $T^-_{bb\bar{u}\bar{s}}(\mathbf{3})$ and $T^0_{bb\bar{d}\bar{s}}(\mathbf{3})$ will be reduced by 101 MeV. Thus, $T^-_{bb\bar{u}\bar{d}}(\mathbf{3})$ becomes more stable. $T^-_{bb\bar{u}\bar{s}}(\mathbf{3})$ and $T^0_{bb\bar{d}\bar{s}}(\mathbf{3})$ are about 43 MeV below the B_sB^* threshold and about 46 MeV below the B^*_sB threshold, which become stable. If one uses the quark mass values of SET III and considers the uncertainty of the heavy bad diquark mass, only one state $T^-_{bb\bar{u}\bar{d}}(\mathbf{3})$ is near its hadron threshold. Other states stay away from their hadron threshold.

In Ref. [8], Karliner and Rosner predicted a stable doubly bottomed tetraquark $T(bb\bar{u}\bar{d})$ with $J^P = 1^+$ at 10.389 ± 0.012 GeV. In Ref. [9], Eichten and Quigg predicted two stable doubly bottomed tetraquarks $T(bb\bar{u}\bar{d})$ with $J^P = 1^+$ at 10.482 GeV and $T(bb\bar{q}\bar{s})$ with $J^P = 1^+$ at 10.643 GeV. Our work also supports the possibility of a stable doubly bottomed tetraquark with $J^P = 1^+$ at (10.45–10.53) GeV. On the other hand, Karliner and Rosner predicted a doubly charmed tetraquark $T(cc\bar{u}\bar{d})$ with $J^P = 1^+$ at 3.882 ± 0.012 GeV, which is 7 MeV above the DD^* threshold in Ref. [8]. Our work gives the $T^+_{cc\bar{u}\bar{d}}(\mathbf{3})$ state also near the DD^* threshold but above that by 16 MeV. But one should note that the theoretical uncertainties for the doubly charmed tetraquark become larger than that of the doubly bottomed tetraquark.

To hunt for these possible doubly heavy tetraquarks in the flavor triplet, we will study their weak decay properties. Their semileptonic and nonleptonic decay amplitudes can be parametrized in terms of SU(3)-irreducible amplitudes. For completeness, we will investigate the weak two-body, three-body, and four-body decays of the doubly heavy tetraquarks.

IV. SEMILEPTONIC DECAYS

A. Semileptonic $T_{bb\bar{q}\bar{q}}$ decays

1. Decays into mesons and $\ell\bar{\nu}_\ell$

First we study the decays into mesons and $\ell\bar{\nu}_\ell$, where the electroweak Hamiltonian is

$$\mathcal{H}^{\mathrm{e.w.}}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}[V_{q'b}\bar{q}'\gamma^\mu(1-\gamma_5)b\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell] + \mathrm{H.c.}, \quad (14)$$

with $q' = u, c$. The corresponding Feynman diagrams are given in Fig. 3. The transition of $b \to c\ell^-\bar{\nu}_\ell$ belongs to a SU(3) singlet, while the $b \to u\ell^-\bar{\nu}_\ell$ transition belongs to a SU(3) triplet and they can be described as H_3 which has the matrix elements $(H_3)^1 = V_{ub}$ and $(H_3)^{2(3)} = 0$ with the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub}.

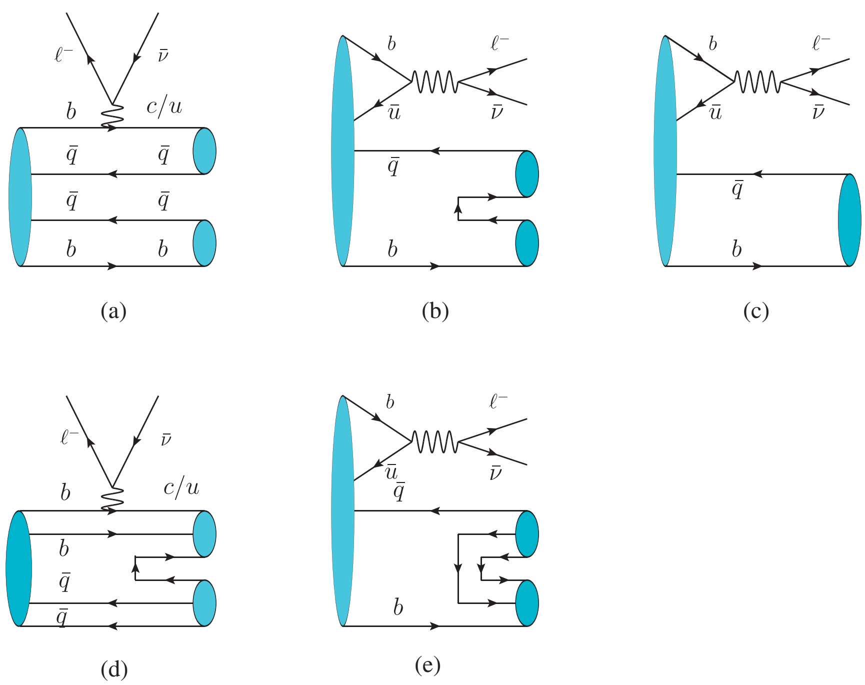

FIG. 3. Feynman diagrams for semileptonic decays of doubly bottomed tetraquark. Panels (a),(b) correspond to the decays into a pair of mesons. In panel (c), there is only one meson in the final states. Panels (d),(e) denote the decays into baryonic states. In panels (b),(c), (e), the two $b\bar{u}$ quarks in the initial state can annihilate, but such contributions are usually power suppressed.

TABLE III. Decay amplitudes for doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ semileptonic decays into mesons.

Channel	Amplitude	Channel	Amplitude
$T^-_{bb\bar{u}\bar{s}} \to B^- D^+_s \ell^- \bar{\nu}_\ell$	$a_3 V_{cb}$	$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s D^0 \ell^- \bar{\nu}_\ell$	$-a_3 V_{cb}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 D^+_s \ell^- \bar{\nu}_\ell$	$a_3 V_{cb}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s D^+ \ell^- \bar{\nu}_\ell$	$-a_3 V_{cb}$
$T^-_{bb\bar{u}\bar{d}} \to B^- D^+ \ell^- \bar{\nu}_\ell$	$a_3 V_{cb}$	$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 D^0 \ell^- \bar{\nu}_\ell$	$-a_3 V_{cb}$

Channel	Amplitude	Channel	Amplitude
$T^-_{bb\bar{u}\bar{s}} \to B^- K^+ \ell^- \bar{\nu}_\ell$	$(a_4 + a_5) V_{ub}$	$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0 K^0 \ell^- \bar{\nu}_\ell$	$a_4 V_{ub}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s \pi^0 \ell^- \bar{\nu}_\ell$	$-\frac{a_5}{\sqrt{2}} V_{ub}$	$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s \eta \ell^- \bar{\nu}_\ell$	$-\frac{(2a_4+a_5)}{\sqrt{6}} V_{ub}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 K^+ \ell^- \bar{\nu}_\ell$	$a_5 V_{ub}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s \pi^+ \ell^- \bar{\nu}_\ell$	$-a_5 V_{ub}$
$T^-_{bb\bar{u}\bar{d}} \to B^- \pi^+ \ell^- \bar{\nu}_\ell$	$(a_4 + a_5) V_{ub}$	$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 \pi^0 \ell^- \bar{\nu}_\ell$	$-\frac{(a_4+a_5)}{\sqrt{2}} V_{ub}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 \eta \ell^- \bar{\nu}_\ell$	$\frac{(a_4-a_5)}{\sqrt{6}} V_{ub}$	$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0_s \overline{K}^0 \ell^- \bar{\nu}_\ell$	$a_4 V_{ub}$

Except for the two three-body decay channels shown in Fig. 3(c), i.e.,

$$T^-_{bb\bar{u}\bar{d}} \to \bar{B}^0 \ell^- \bar{\nu}_\ell, \qquad T^-_{bb\bar{u}\bar{s}} \to \bar{B}^0_s \ell^- \bar{\nu}_\ell,$$

most modes involve four particles in the final states. The semileptonic T_{bb3} decays can be described through the hadron-level Hamiltonian which is written as

$$\mathcal{H} = a_3 (T_{bb3})_{[ij]} \bar{B}^i \bar{D}^j \bar{\ell} \nu_\ell + a_4 (T_{bb3})_{[ij]} (H_3)^i \bar{B}^k M^j_k \bar{\ell} \nu_\ell + a_5 (T_{bb3})_{[ij]} (H_3)^k \bar{B}^i M^j_k \bar{\ell} \nu_\ell. \tag{15}$$

Here the a_i's are the nonperturbative model-independent parameters. The a_3 and a_5 will be present in Fig. 3(a), and a_4 is related to the annihilation diagrams in Fig. 3(b). Through the Hamiltonian, we can obtain the decay amplitudes for different decay modes, which are given in Table III.

To satisfy the SU(3) flavor symmetry, one can ignore the effects of phase space when analyzing their decay widths. From Table III, all six decay channels into a B and a D meson have the same decay widths. Besides, we can get the relations for the decays into a B meson and a light meson:

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{B}^0_s \pi^0 l^- \bar{\nu}) = \frac{1}{2} \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{B}^0 K^+ l^- \bar{\nu}) = \frac{1}{2} \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{B}^0_s \pi^+ l^- \bar{\nu}),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to B^- \pi^+ l^- \bar{\nu}) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to B^- K^+ l^- \bar{\nu}) = 2\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{B}^0 \pi^0 l^- \bar{\nu}),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{B}^0 K^0 l^- \bar{\nu}) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{B}^0_s \bar{K}^0 l^- \bar{\nu}).$$

2. Decays into a bottom baryon, a light antibaryon, and $\ell \bar{\nu}_\ell$

As shown in the last two panels in Fig. 3, the T_{bb3} can transit into a bottom baryon and a light antibaryon. Since the decuplet is antisymmetric for light quarks in flavor space, the two spectator light quarks will not go into the decuplet. We write the Hamiltonian as

$$\begin{aligned}\mathcal{H} = {} & b_1 (T_{bb3})_{[ij]} \epsilon^{xjk} (F_8)^l_x (H_3)^i (\overline{F}_{b\bar{3}})_{[kl]} \bar{\ell} \nu_\ell + b_2 (T_{bb3})_{[ij]} \epsilon^{xkl} (F_8)^j_x (H_3)^i (\overline{F}_{b\bar{3}})_{[kl]} \bar{\ell} \nu_\ell \\ & + b_3 (T_{bb3})_{[ij]} \epsilon^{xij} (F_8)^l_x (H_3)^k (\overline{F}_{b\bar{3}})_{[kl]} \bar{\ell} \nu_\ell + b_4 (T_{bb3})_{[ij]} \epsilon^{xil} (F_8)^j_x (H_3)^k (\overline{F}_{b\bar{3}})_{[kl]} \bar{\ell} \nu_\ell \\ & + b_5 (T_{bb3})_{[ij]} \epsilon^{xjk} (F_8)^l_x (H_3)^i (\overline{F}_{b6})_{\{kl\}} \bar{\ell} \nu_\ell + b_6 (T_{bb3})_{[ij]} \epsilon^{xij} (F_8)^l_x (H_3)^k (\overline{F}_{b6})_{\{kl\}} \bar{\ell} \nu_\ell \\ & + b_7 (T_{bb3})_{[ij]} \epsilon^{xil} (F_8)^j_x (H_3)^k (\overline{F}_{b6})_{\{kl\}} \bar{\ell} \nu_\ell + b_8 (T_{bb3})_{[ij]} (F_{\overline{10}})^{\{jkl\}} (H_3)^i (\overline{F}_{b6})_{\{kl\}} \bar{\ell} \nu_\ell. \end{aligned} \tag{16}$$

For convenience, we label the decay channels with different final states: class I for an octet baryon plus a heavy triplet baryon, class II for an octet baryon plus a heavy sextet baryon, and class III for a decuplet light baryon plus a heavy sextet baryon. The last type of decays can occur only through the annihilation of $b\bar{u}$ shown as in Fig. 3(c). The explicit amplitudes can be found in Table IV.

TABLE IV. Amplitudes for doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a light antibaryon octet plus a bottom baryon antitriplet (class I) or sextet (class II), and a light antibaryon decuplet plus a bottom baryon sextet (class III).

Class I	Amplitude	Class III	Amplitude
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Lambda}^0 \Lambda^0_b \ell^- \bar{\nu}_\ell$	$\frac{(2b_1-4b_2-2b_3-b_4)V_{ub}}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^- \Sigma^+_b \ell^- \bar{\nu}_\ell$	$\frac{b_8 V_{ub}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^0 \Lambda^0_b \ell^- \bar{\nu}_\ell$	$\frac{(2b_3+b_4)V_{ub}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^0 \Sigma^0_b \ell^- \bar{\nu}_\ell$	$\frac{b_8 V_{ub}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^+ \Xi^-_b \ell^- \bar{\nu}_\ell$	$-(b_1-2b_2)V_{ub}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^+ \Sigma^-_b \ell^- \bar{\nu}_\ell$	$\frac{b_8 V_{ub}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0 \Xi^0_b \ell^- \bar{\nu}_\ell$	$(b_1-2b_2-2b_3-b_4)V_{ub}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0 \Xi'^0_b \ell^- \bar{\nu}_\ell$	$\sqrt{\frac{2}{3}} b_8 V_{ub}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+ \Lambda^0_b \ell^- \bar{\nu}_\ell$	$(2b_3+b_4)V_{ub}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^+ \Xi'^-_b \ell^- \bar{\nu}_\ell$	$\sqrt{\frac{2}{3}} b_8 V_{ub}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+ \Xi^0_b \ell^- \bar{\nu}_\ell$	$(2b_3+b_4)V_{ub}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Omega}^+ \Omega^-_b \ell^- \bar{\nu}_\ell$	$b_8 V_{ub}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Lambda}^0 \Xi^0_b \ell^- \bar{\nu}_\ell$	$\frac{(b_1-2(b_2+2b_3+b_4))V_{ub}}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^- \Sigma^+_b \ell^- \bar{\nu}_\ell$	$\frac{b_8 V_{ub}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0 \Xi^0_b \ell^- \bar{\nu}_\ell$	$-\frac{(b_1-2b_2)V_{ub}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^0 \Sigma^0_b \ell^- \bar{\nu}_\ell$	$\sqrt{\frac{2}{3}} b_8 V_{ub}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^+ \Xi^-_b \ell^- \bar{\nu}_\ell$	$-(b_1-2b_2)V_{ub}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^+ \Sigma^-_b \ell^- \bar{\nu}_\ell$	$b_8 V_{ub}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{n} \Lambda^0_b \ell^- \bar{\nu}_\ell$	$-(b_1-2b_2-2b_3-b_4)V_{ub}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^0 \Xi'^0_b \ell^- \bar{\nu}_\ell$	$\frac{b_8 V_{ub}}{\sqrt{3}}$
		$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^+ \Xi'^-_b \ell^- \bar{\nu}_\ell$	$\sqrt{\frac{2}{3}} b_8 V_{ub}$
		$T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}'^+ \Omega^-_b \ell^- \bar{\nu}_\ell$	$\frac{b_8 V_{ub}}{\sqrt{3}}$

Class II	Amplitude	Class II	Amplitude
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Lambda}^0 \Sigma^0_b \ell^- \bar{\nu}_\ell$	$-\frac{(2b_6+b_7)V_{ub}}{2\sqrt{3}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+ \Sigma^0_b \ell^- \bar{\nu}_\ell$	$\frac{(2b_6+b_7)V_{ub}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^- \Sigma^+_b \ell^- \bar{\nu}_\ell$	$(b_5-2b_6-b_7)V_{ub}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+ \Xi'^0_b \ell^- \bar{\nu}_\ell$	$\frac{(2b_6+b_7)V_{ub}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^0 \Sigma^0_b \ell^- \bar{\nu}_\ell$	$\frac{1}{6}(-6b_5+6b_6+3b_7)V_{ub}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Lambda}^0 \Xi'^0_b \ell^- \bar{\nu}_\ell$	$\frac{(3b_5-2(2b_6+b_7))V_{ub}}{2\sqrt{3}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^+ \Sigma^-_b \ell^- \bar{\nu}_\ell$	$-b_5 V_{ub}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0 \Xi'^0_b \ell^- \bar{\nu}_\ell$	$\frac{b_5 V_{ub}}{2}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^+ \Xi'^-_b \ell^- \bar{\nu}_\ell$	$-\frac{b_5 V_{ub}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^+ \Xi'^-_b \ell^- \bar{\nu}_\ell$	$\frac{b_5 V_{ub}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0 \Xi'^0_b \ell^- \bar{\nu}_\ell$	$\frac{(b_5-2b_6-b_7)V_{ub}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{p} \Sigma^+_b \ell^- \bar{\nu}_\ell$	$-(b_5-2b_6-b_7)V_{ub}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0 \Sigma^+_b \ell^- \bar{\nu}_\ell$	$\frac{(2b_6+b_7)V_{ub}}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{n} \Sigma^0_b \ell^- \bar{\nu}_\ell$	$-\frac{(b_5-2b_6-b_7)V_{ub}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0 \Sigma^+_b \ell^- \bar{\nu}_\ell$	$\frac{(2b_6+b_7)V_{ub}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}^+ \Omega^-_b \ell^- \bar{\nu}_\ell$	$b_5 V_{ub}$

From them, the relations for class I decays can be found as follows:

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^0 \Lambda^0_b l^- \bar{\nu}) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+ \Xi^0_b l^- \bar{\nu}) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+ \Lambda^0_b l^- \bar{\nu}),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0 \Xi^0_b l^- \bar{\nu}) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^+ \Xi^-_b l^- \bar{\nu}) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^+ \Xi^-_b l^- \bar{\nu}),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{n} \Lambda^0_b l^- \bar{\nu}) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0 \Xi^0_b l^- \bar{\nu}).$$

The relations for class II decays become

TABLE V. Amplitudes for doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a light antibaryon octet and a charmed and bottomed baryon.

Channel	Amplitude	Channel	Amplitude
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Lambda}^0 \Xi^0_{bc} \ell^- \bar{\nu}_\ell$	$-\frac{2b_9+b_{10}}{\sqrt{6}} V_{cb}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^- \Xi^+_{bc} \ell^- \bar{\nu}_\ell$	$(-2b_9 - b_{10}) V_{cb}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^0 \Xi^0_{bc} \ell^- \bar{\nu}_\ell$	$\frac{2b_9+b_{10}}{\sqrt{2}} V_{cb}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0 \Omega^0_{bc} \ell^- \bar{\nu}_\ell$	$(-2b_9 - b_{10}) V_{cb}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0 \Xi^+_{bc} \ell^- \bar{\nu}_\ell$	$\frac{2b_9+b_{10}}{\sqrt{6}} V_{cb}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0 \Xi^+_{bc} \ell^- \bar{\nu}_\ell$	$\frac{2b_9+b_{10}}{\sqrt{2}} V_{cb}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+ \Xi^0_{bc} \ell^- \bar{\nu}_\ell$	$(2b_9 + b_{10}) V_{cb}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+ \Omega^0_{bc} \ell^- \bar{\nu}_\ell$	$(2b_9 + b_{10}) V_{cb}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Lambda}^0 \Omega^0_{bc} \ell^- \bar{\nu}_\ell$	$-\sqrt{\frac{2}{3}}(2b_9 + b_{10}) V_{cb}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{p} \Xi^+_{bc} \ell^- \bar{\nu}_\ell$	$(2b_9 + b_{10}) V_{cb}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{n} \Xi^0_{bc} \ell^- \bar{\nu}_\ell$	$(2b_9 + b_{10}) V_{cb}$		

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Lambda}^0 \Sigma^0_b l^- \bar{\nu}) = \frac{1}{6}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+ \Xi'^0_b l^- \bar{\nu}) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0 \Sigma^+_b l^- \bar{\nu}) = \frac{1}{6}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+ \Sigma^0_b l^- \bar{\nu}) = \frac{1}{6}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0 \Sigma^+_b l^- \bar{\nu}),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^- \Sigma^+_b l^- \bar{\nu}) = 2\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0 \Xi'^0_b l^- \bar{\nu}) = 2\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{n} \Sigma^0_b l^- \bar{\nu}) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{p} \Sigma^+_b l^- \bar{\nu}),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^+ \Sigma^-_b l^- \bar{\nu}) = 2\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^+ \Xi'^-_b l^- \bar{\nu}) = 4\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0 \Xi'^0_b l^- \bar{\nu}) = 2\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^+ \Xi'^-_b l^- \bar{\nu}) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}^+ \Omega^-_b l^- \bar{\nu}).$$

The relations for class III decay widths are

$$\begin{aligned}
\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^- \Sigma^+_b l^- \bar{\nu}) &= \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^0 \Sigma^0_b l^- \bar{\nu}) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^+ \Sigma^-_b l^- \bar{\nu}) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0 \Xi'^0_b l^- \bar{\nu}) \\
&= \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^+ \Xi'^-_b l^- \bar{\nu}) = \frac{1}{3}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Omega}^+ \Omega^-_b l^- \bar{\nu}) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^0 \Sigma^0_b l^- \bar{\nu}) = \frac{1}{3}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^+ \Sigma^-_b l^- \bar{\nu}) \\
&= \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^0 \Xi'^0_b l^- \bar{\nu}) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^+ \Xi'^-_b l^- \bar{\nu}) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}'^+ \Omega^-_b l^- \bar{\nu}) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^- \Sigma^+_b l^- \bar{\nu}).
\end{aligned}$$

3. Decays into a charmed and bottomed baryon plus a light antibaryon and $\ell\bar{\nu}_\ell$

T_{bb3} can also decay into a light octet or antidecuplet antibaryon and a charmed and bottomed baryon for the $b \to c$ transition. After removing the forbidden constructions, the Hamiltonian becomes

$$\mathcal{H}_{\mathrm{eff}} = b_9 (T_{bb3})_{[ij]} \epsilon^{xij} (F_8)^k_x (\bar{F}_{bc})_k \bar{\ell} \nu_\ell + b_{10} (T_{bb3})_{[ij]} \epsilon^{xik} (F_8)^j_x (\bar{F}_{bc})_k \bar{\ell} \nu_\ell. \tag{17}$$

The amplitudes are derived and given in Table V. From them, the relations of the related decay widths are

$$\begin{aligned}
\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Lambda}^0 \Xi^0_{bc} l^- \bar{\nu}) &= \frac{1}{6}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0 \Omega^0_{bc} l^- \bar{\nu}) = \frac{1}{6}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+ \Omega^0_{bc} l^- \bar{\nu}) = \frac{1}{4}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Lambda}^0 \Omega^0_{bc} l^- \bar{\nu}) \\
&= \frac{1}{6}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^- \Xi^+_{bc} l^- \bar{\nu}) = \frac{1}{3}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^0 \Xi^0_{bc} l^- \bar{\nu}) = \frac{1}{6}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+ \Xi^0_{bc} l^- \bar{\nu}) = \frac{1}{6}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{n} \Xi^0_{bc} l^- \bar{\nu}) \\
&= \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0 \Xi^+_{bc} l^- \bar{\nu}) = \frac{1}{3}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0 \Xi^+_{bc} l^- \bar{\nu}) = \frac{1}{6}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{p} \Xi^+_{bc} l^- \bar{\nu}).
\end{aligned}$$

B. $T_{cc\bar{q}\bar{q}}$ decays

The effective Hamiltonian from the charmed semileptonic decays into a light quark is

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} [V^*_{cq} \bar{q} \gamma^\mu (1 - \gamma_5) c \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell] + \text{H.c.}, \tag{18}$$

where $q = d, s$. A SU(3) triplet denoted as H_3 with the elements $(H_3)^1 = 0$, $(H_3)^2 = V^*_{cd}$, and $(H_3)^3 = V^*_{cs}$ is introduced for the heavy-to-light quark operators. We plotted the corresponding Feynman diagrams in Fig. 4. The triplet state T_{cc3} can decay to a charmed meson plus $\ell^+\nu$:

$$T^+_{cc\bar{u}\bar{s}}/T^+_{cc\bar{u}\bar{d}} \to D^0 l^+ \nu, \qquad T^{++}_{cc\bar{d}\bar{s}} \to D^+ l^+ \nu, \qquad T^{++}_{cc\bar{d}\bar{s}} \to D^+_s l^+ \nu, \tag{19}$$

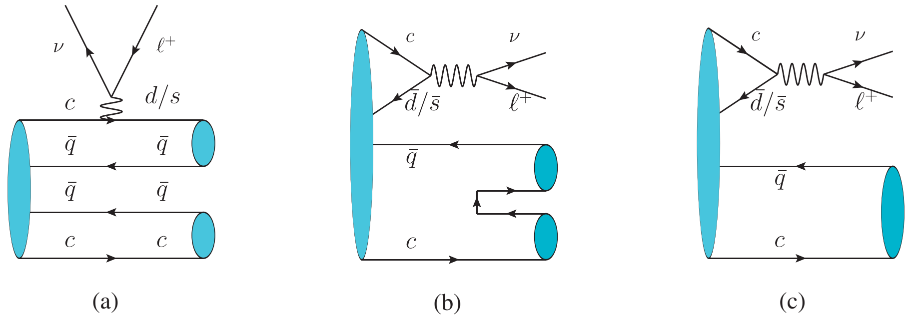

FIG. 4. Feynman diagrams for semileptonic decays of doubly charmed tetraquarks. Panels (a),(b) correspond to the decays into a pair of mesons, and in panel (c), there is only one meson in the final state. In panels (b),(c), the two $c\bar{d}/\bar{s}$ quarks in the initial state can annihilate, and usually such contributions are power suppressed.

and their Feynman diagram is given in Fig. 4(c). Thus we obtained

$$\Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^0 l^+\nu) = \Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+ l^+\nu), \qquad \Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+_s l^+\nu) = \Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^0 l^+\nu).$$

The effective Hamiltonian for decays into a charmed meson and a light meson is written as

$$\mathcal{H} = a_1 (T_{cc3})_{[ij]} (H_3)^i (\bar{D})^k M^j_k \bar{\nu}_\ell \ell + a_2 (T_{cc3})_{[ij]} (H_3)^k (\bar{D})^i M^j_k \bar{\nu}_\ell \ell. \tag{20}$$

The related Feynman diagrams are plotted in Figs. 4(a) and 4(b), and the related results for the decay width relations are given in Table VI. Thus, we have

$$\begin{aligned}
&\Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^0\pi^0 l^+\nu) = \Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+\pi^0 l^+\nu) = \frac{1}{2}\Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^+\pi^- l^+\nu) = \frac{1}{2}\Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^0\pi^+ l^+\nu),\\
&\Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^0 K^+ l^+\nu) = \Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+_s K^- l^+\nu), \qquad \Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+_s \bar{K}^0 l^+\nu) = \Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^+_s K^- l^+\nu),\\
&\Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^0 \bar{K}^0 l^+\nu) = \Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+ K^- l^+\nu), \qquad \Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^0 \eta l^+\nu) = \Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+ \eta l^+\nu),\\
&\Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^+_s \pi^- l^+\nu) = \Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^0 K^0 l^+\nu) = 2\Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+_s \pi^0 l^+\nu),\\
&\Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^0\pi^0 l^+\nu) = \frac{1}{2}\Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+ K^0 l^+\nu) = \frac{1}{2}\Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+\pi^- l^+\nu).
\end{aligned}$$

TABLE VI. Amplitudes for doubly charmed tetraquark $T_{cc\bar{q}\bar{q}}$ decays into a charmed meson and a light meson.

Channel	Amplitude	Channel	Amplitude
$T^{+}_{cc\bar{u}\bar{s}} \to D^0\pi^0 l^+\nu$	$-\frac{a_1 V^*_{cs}}{\sqrt{2}}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^0 K^0 l^+\nu$	$a_2 V^*_{cd}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^0\eta l^+\nu$	$-\frac{(a_1+2a_2)V^*_{cs}}{\sqrt{6}}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+\pi^- l^+\nu$	$-a_1 V^*_{cs}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\pi^- l^+\nu$	$-a_2 V^*_{cd}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s K^- l^+\nu$	$-(a_1+a_2)V^*_{cs}$
$T^{++}_{cc\bar{d}\bar{s}} \to D^0\pi^+ l^+\nu$	$-a_1 V^*_{cs}$	$T^{++}_{cc\bar{d}\bar{s}} \to D^0 K^+ l^+\nu$	$a_1 V^*_{cd}$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+\pi^0 l^+\nu$	$\frac{a_1 V^*_{cs}}{\sqrt{2}}$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+ K^0 l^+\nu$	$(a_1+a_2)V^*_{cd}$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+\eta l^+\nu$	$-\frac{(a_1+2a_2)V^*_{cs}}{\sqrt{6}}$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\pi^0 l^+\nu$	$\frac{a_2 V^*_{cd}}{\sqrt{2}}$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\overline{K}^0 l^+\nu$	$-(a_1+a_2)V^*_{cs}$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\eta l^+\nu$	$-\frac{(2a_1+a_2)V^*_{cd}}{\sqrt{6}}$
$T^{+}_{cc\bar{u}\bar{d}} \to D^0\pi^0 l^+\nu$	$-\frac{(a_1+a_2)V^*_{cd}}{\sqrt{2}}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^0\overline{K}^0 l^+\nu$	$a_2 V^*_{cs}$
$T^{+}_{cc\bar{u}\bar{d}} \to D^0\eta l^+\nu$	$\frac{(a_2-a_1)V^*_{cd}}{\sqrt{6}}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+\pi^- l^+\nu$	$-(a_1+a_2)V^*_{cd}$
$T^{+}_{cc\bar{u}\bar{d}} \to D^+K^- l^+\nu$	$-a_2 V^*_{cs}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s K^- l^+\nu$	$-a_1 V^*_{cd}$

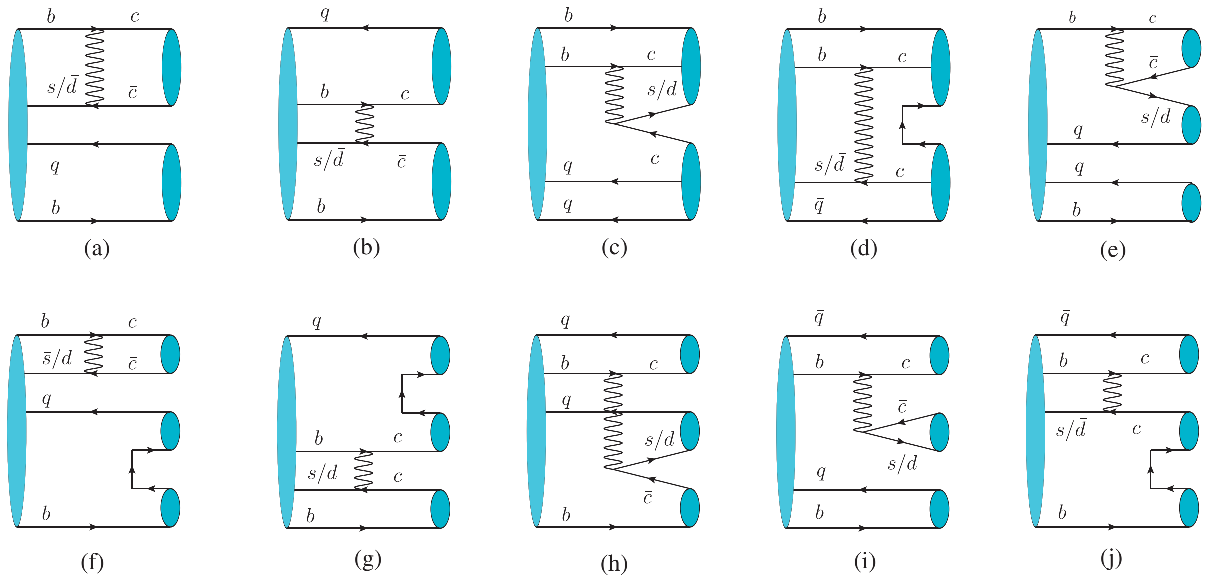

FIG. 5. Feynman diagrams for nonleptonic decays of doubly heavy tetraquarks. Panels (a),(b) correspond to the two mesons W-exchange process; panels (c),(d) correspond to the baryon and antibaryon processes; and panels (e),(f),(g),(h),(i),(j) correspond to three mesons processes [Panels (e),(f) match with J/ψ plus the B meson and the light meson; panels (g),(h) match with the $B_{\bar{c}}$ plus D and light meson; and panels (i),(j) match with the B plus $\overline{D}$ and D meson].

C. Semileptonic $T_{bc\bar{q}\bar{q}}$ decays

Both the bottom and charm quarks can decay in the semileptonic $T_{bc\bar{q}\bar{q}}$ decays. For the bottom decay in $T_{bc\bar{q}\bar{q}}$, one can easily get the decay amplitude from those for $T_{bb\bar{q}\bar{q}}$ decays with $T_{bb\bar{q}\bar{q}} \to T_{bc\bar{q}\bar{q}}$, $B \to D$. For the charm decays in $T_{bc\bar{q}\bar{q}}$, one can easily get them from those for $T_{cc\bar{q}\bar{q}}$ decays with the replacement of $T_{cc\bar{q}\bar{q}} \to T_{bc\bar{q}\bar{q}}$, $D \to B$. Thus, we do not need to give the tedious results here.

V. NONLEPTONIC $T_{bb\bar{q}\bar{q}}$ DECAYS

Next, we will study the nonleptonic decay amplitudes. For the bottom quark decay, there are four types:

$$b \to c\bar{c}d/s, \quad b \to c\bar{u}d/s, \quad b \to u\bar{c}d/s, \quad b \to q_1\bar{q}_2q_3, \tag{21}$$

where q_i with $i = 1, 2, 3$ denote the light flavors. We will discuss these decay modes one by one in the following.

A. $b \to c\bar{c}d/s$ transition

1. W-exchange topology

The transition $b \to c$ or $\bar{d}/\bar{s} \to \bar{c}$ can be signed as a W-exchange topology, and we plotted the corresponding Feynman diagrams in Fig. 5. The effective Hamiltonian by this kind of transition is

$$\mathcal{H} = f_1(T_{bb3})_{[ij]}(\overline{B})^j(H_3)^iJ/\psi + f_2(T_{bb3})_{[ij]}(\overline{D})^j(H_3)^i\overline{B}_c, \tag{22}$$

where $(H_3)_2 = V^*_{cd}$ and $(H_3)_3 = V^*_{cs}$. We gave the decay amplitudes in Table VII, from which the relations of the decay widths are

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to B^-J/\psi) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0J/\psi),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to B^-J/\psi) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_sJ/\psi),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to D^0B^-_c) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+B^-_c),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to D^0B^-_c) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+_sB^-_c).$$

2. Decays into an anticharmed antibaryon and a charmed bottom baryon

The transition $b \to c\bar{c}d/s$ can lead to the process of an antibaryon plus a baryon, where the anticharmed antibaryons form a triplet or antisextet and the charmed bottom baryon forms a SU(3) triplet. The effective Hamiltonian is described as

TABLE VII. Amplitudes for the W-exchange $T_{bb\bar{q}\bar{q}}$ decays induced by the $b \to c\bar{c}d/s$ transition. Note that these amplitudes have an additional identical CKM factor V_{cb}.

Channel	Amplitude $(/V_{cb})$	Channel	Amplitude $(/V_{cb})$
$T^-_{bb\bar{u}\bar{s}} \to B^-J/\psi$	$-f_1V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0J/\psi$	$-f_1V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_sJ/\psi$	$f_1V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to B^-J/\psi$	$-f_1V^*_{cd}$
$T^-_{bb\bar{u}\bar{s}} \to D^0B^-_c$	$-f_2V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to D^+B^-_c$	$-f_2V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to D^+_sB^-_c$	$f_2V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to D^0B^-_c$	$-f_2V^*_{cd}$

TABLE VIII. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into an anticharmed antibaryon triplet (class I) or antisextet (class II) and a charmed bottom baryon.

Class I	Amplitude $(/V_{cb})$	Class II	Amplitude $(/V_{cb})$
$T^-_{bb\bar{u}\bar{s}} \to \Xi^0_{bc}\Lambda^-_{\bar{c}}$	$-a_1 V^*_{cs}$	$T^-_{bb\bar{u}\bar{s}} \to \Xi^+_{bc}\Sigma^{--}_{\bar{c}}$	$-a_3 V^*_{cs}$
$T^-_{bb\bar{u}\bar{s}} \to \Xi^0_{bc}\Xi^-_{\bar{c}}$	$2a_2 V^*_{cd}$	$T^-_{bb\bar{u}\bar{s}} \to \Xi^0_{bc}\Sigma^-_{\bar{c}}$	$-\frac{a_3 V^*_{cs}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to \Omega^0_{bc}\Xi^-_{\bar{c}}$	$-(a_1-2a_2)V^*_{cs}$	$T^-_{bb\bar{u}\bar{s}} \to \Omega^0_{bc}\Xi'^-_{\bar{c}}$	$-\frac{a_3 V^*_{cs}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \Xi^+_{bc}\Lambda^-_{\bar{c}}$	$a_1 V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to \Xi^+_{bc}\Sigma^-_{\bar{c}}$	$-\frac{a_3 V^*_{cs}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \Xi^+_{bc}\Xi^-_{\bar{c}}$	$-a_1 V^*_{cd}$	$T^0_{bb\bar{d}\bar{s}} \to \Xi^+_{bc}\Xi'^-_{\bar{c}}$	$\frac{a_3 V^*_{cd}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \Xi^0_{bc}\bar{\Xi}^0_{\bar{c}}$	$-(a_1-2a_2)V^*_{cd}$	$T^0_{bb\bar{d}\bar{s}} \to \Xi^0_{bc}\bar{\Sigma}^0_{\bar{c}}$	$-a_3 V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to \Omega^0_{bc}\bar{\Xi}^0_{\bar{c}}$	$-(a_1-2a_2)V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to \Xi^0_{bc}\bar{\Xi}'^0_{\bar{c}}$	$\frac{a_3 V^*_{cd}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to \Xi^0_{bc}\Lambda^-_{\bar{c}}$	$-(a_1-2a_2)V^*_{cd}$	$T^0_{bb\bar{d}\bar{s}} \to \Omega^0_{bc}\bar{\Xi}'^0_{\bar{c}}$	$-\frac{a_3 V^*_{cs}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to \Omega^0_{bc}\Lambda^-_{\bar{c}}$	$2a_2 V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to \Omega^0_{bc}\bar{\Omega}^0_{\bar{c}}$	$a_3 V^*_{cd}$
$T^-_{bb\bar{u}\bar{d}} \to \Omega^0_{bc}\Xi^-_{\bar{c}}$	$-a_1 V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to \Xi^+_{bc}\Sigma^{--}_{\bar{c}}$	$-a_3 V^*_{cd}$
		$T^-_{bb\bar{u}\bar{d}} \to \Xi^0_{bc}\Sigma^-_{\bar{c}}$	$-\frac{a_3 V^*_{cd}}{\sqrt{2}}$
		$T^-_{bb\bar{u}\bar{d}} \to \Omega^0_{bc}\Xi'^-_{\bar{c}}$	$-\frac{a_3 V^*_{cd}}{\sqrt{2}}$

$$\begin{aligned}\mathcal{H} = {}& a_1(T_{bb3})_{[ij]}(F_{\bar{c}3})^{[jk]}(H_3)^i(\bar{F}_{bc})_k \\ &+ a_2(T_{bb3})_{[ij]}(F_{\bar{c}3})^{[ij]}(H_3)^k(\bar{F}_{bc})_k \\ &+ a_3(T_{bb3})_{[ij]}(F_{\bar{c}6})^{\{jk\}}(H_3)^i(\bar{F}_{bc})_k. \end{aligned} \tag{23}$$

The related decay amplitudes are given in Table VIII, in which class I represents a triplet antibaryon plus the charmed bottom baryon in the final states, and class II denotes the antisextet antibaryon plus the charmed bottom baryon.

For class I, we have the relations:

$$\begin{aligned}&\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Omega^0_{bc}\Xi^-_{\bar{c}}) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \Omega^0_{bc}\bar{\Xi}^0_{\bar{c}}),\\ &\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Xi^0_{bc}\Lambda^-_{\bar{c}}) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \Xi^0_{bc}\bar{\Xi}^0_{\bar{c}}),\\ &\Gamma(T^0_{bb\bar{d}\bar{s}} \to \Xi^+_{bc}\Lambda^-_{\bar{c}}) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \Xi^0_{bc}\Lambda^-_{\bar{c}}),\\ &\Gamma(T^0_{bb\bar{d}\bar{s}} \to \Xi^+_{bc}\Xi^-_{\bar{c}}) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \Omega^0_{bc}\Xi^-_{\bar{c}}).\end{aligned}$$

For class II, we have the following:

$$\begin{aligned}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Xi^+_{bc}\Sigma^{--}_{\bar{c}}) &= 2\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Xi^0_{bc}\Sigma^-_{\bar{c}}) = 2\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Omega^0_{bc}\Xi'^-_{\bar{c}}) = 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \Xi^+_{bc}\Sigma^-_{\bar{c}})\\ &= \Gamma(T^0_{bb\bar{d}\bar{s}} \to \Xi^0_{bc}\bar{\Sigma}^0_{\bar{c}}) = 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \Omega^0_{bc}\bar{\Xi}'^0_{\bar{c}}),\\ \Gamma(T^0_{bb\bar{d}\bar{s}} \to \Xi^+_{bc}\Xi'^-_{\bar{c}}) &= \Gamma(T^0_{bb\bar{d}\bar{s}} \to \Xi^0_{bc}\bar{\Xi}'^0_{\bar{c}}) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \Omega^0_{bc}\bar{\Omega}^0_{\bar{c}}) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Xi^+_{bc}\Sigma^{--}_{\bar{c}})\\ &= \Gamma(T^-_{bb\bar{u}\bar{d}} \to \Xi^0_{bc}\Sigma^-_{\bar{c}}) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \Omega^0_{bc}\Xi'^-_{\bar{c}}).\end{aligned}$$

3. Decays into three mesons

The transition $b \to c\bar{c}d/s$ leads to three-body decays where the effective Hamiltonian becomes

$$\begin{aligned}\mathcal{H} = {}& a_1(T_{bb3})_{[ij]}(H_3)^k M^j_k(\bar{B})^i J/\psi + a_2(T_{bb3})_{[ij]}(H_3)^i M^j_k(\bar{B})^k J/\psi + a_3(T_{bb3})_{[ij]}(H_3)^k M^j_k(\bar{D})^i \bar{B}_c\\ &+ a_4(T_{bb3})_{[ij]}(H_3)^i M^j_k(\bar{D})^k \bar{B}_c + a_5(T_{bb3})_{[ij]}(H_3)^j D_k(\bar{D})^i \bar{B}^k + a_6(T_{bb3})_{[ij]}(H_3)^k D_k(\bar{D})^i \bar{B}^j\\ &+ a_7(T_{bb3})_{[ij]}(H_3)^i D_k(\bar{D})^k \bar{B}^j.\end{aligned} \tag{24}$$

The T_{bb3} decay amplitudes into J/ψ plus a bottom meson and a light meson are given in Table IX, from which we have

$$\begin{aligned}&\Gamma(T^-_{bb\bar{u}\bar{s}} \to B^-\pi^0 J/\psi) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{B}^0\pi^- J/\psi) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{B}^0\pi^0 J/\psi) = \frac{1}{8}\Gamma(T^0_{bb\bar{d}\bar{s}} \to B^-\pi^+ J/\psi),\\ &\Gamma(T^0_{bb\bar{d}\bar{s}} \to B^-K^+ J/\psi) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{B}^0_s K^- J/\psi), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{B}^0_s \bar{K}^0 J/\psi) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{B}^0_s K^- J/\psi),\\ &\Gamma(T^-_{bb\bar{u}\bar{d}} \to B^-\bar{K}^0 J/\psi) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{B}^0 K^- J/\psi), \qquad \Gamma(T^-_{bb\bar{u}\bar{s}} \to B^-\eta J/\psi) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{B}^0\eta J/\psi),\\ &\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{B}^0_s\pi^- J/\psi) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to B^-K^0 J/\psi) = 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{B}^0_s\pi^0 J/\psi),\\ &\Gamma(T^-_{bb\bar{u}\bar{d}} \to B^-\pi^0 J/\psi) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{B}^0 K^0 J/\psi) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{B}^0\pi^- J/\psi).\end{aligned}$$

Decay amplitudes for T_{bb3} decays into B_c meson plus a charmed meson and a light meson are given in Table X. The decay width relations are

TABLE IX. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a J/ψ, a bottom meson and a light meson.

Channel	Amplitude $(/V_{cb})$	Channel	Amplitude $(/V_{cb})$
$T^-_{bb\bar{u}\bar{s}} \to B^-\pi^0 J/\psi$	$-\frac{a_2 V^*_{cs}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{s}} \to B^- K^0 J/\psi$	$a_1 V^*_{cd}$
$T^-_{bb\bar{u}\bar{s}} \to B^-\eta J/\psi$	$-\frac{(2a_1+a_2)V^*_{cs}}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0\pi^- J/\psi$	$-a_2 V^*_{cs}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s\pi^- J/\psi$	$-a_1 V^*_{cd}$	$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s K^- J/\psi$	$-(a_1+a_2)V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to B^-\pi^+ J/\psi$	$-a_2 V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to B^- K^+ J/\psi$	$a_2 V^*_{cd}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0\pi^0 J/\psi$	$\frac{a_2 V^*_{cs}}{\sqrt{2}}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 K^0 J/\psi$	$(a_1+a_2)V^*_{cd}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0\eta J/\psi$	$-\frac{(2a_1+a_2)V^*_{cs}}{\sqrt{6}}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\pi^0 J/\psi$	$\frac{a_1 V^*_{cd}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\overline{K}^0 J/\psi$	$-(a_1+a_2)V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\eta J/\psi$	$-\frac{(a_1+2a_2)V^*_{cd}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\pi^0 J/\psi$	$-\frac{(a_1+a_2)V^*_{cd}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{d}} \to B^-\overline{K}^0 J/\psi$	$a_1 V^*_{cs}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\eta J/\psi$	$\frac{(a_1-a_2)V^*_{cd}}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0\pi^- J/\psi$	$-(a_1+a_2)V^*_{cd}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 K^- J/\psi$	$-a_1 V^*_{cs}$	$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0_s K^- J/\psi$	$-a_2 V^*_{cd}$

TABLE X. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a charmed B meson, a charmed meson and a light meson.

Channel	Amplitude $(/V_{cb})$	Channel	Amplitude $(/V_{cb})$
$T^-_{bb\bar{u}\bar{s}} \to D^0\pi^0 B^-_c$	$-\frac{a_4 V^*_{cs}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{s}} \to D^0 K^0 B^-_c$	$a_3 V^*_{cd}$
$T^-_{bb\bar{u}\bar{s}} \to D^0\eta B^-_c$	$-\frac{(2a_3+a_4)V^*_{cs}}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{s}} \to D^+\pi^- B^-_c$	$-a_4 V^*_{cs}$
$T^-_{bb\bar{u}\bar{s}} \to D^+_s\pi^- B^-_c$	$-a_3 V^*_{cd}$	$T^-_{bb\bar{u}\bar{s}} \to D^+_s K^- B^-_c$	$-(a_3+a_4)V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to D^0\pi^+ B^-_c$	$-a_4 V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to D^0 K^+ B^-_c$	$a_4 V^*_{cd}$
$T^0_{bb\bar{d}\bar{s}} \to D^+\pi^0 B^-_c$	$\frac{a_4 V^*_{cs}}{\sqrt{2}}$	$T^0_{bb\bar{d}\bar{s}} \to D^+ K^0 B^-_c$	$(a_3+a_4)V^*_{cd}$
$T^0_{bb\bar{d}\bar{s}} \to D^+\eta B^-_c$	$-\frac{(2a_3+a_4)V^*_{cs}}{\sqrt{6}}$	$T^0_{bb\bar{d}\bar{s}} \to D^+_s\pi^0 B^-_c$	$\frac{a_3 V^*_{cd}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to D^+_s\overline{K}^0 B^-_c$	$-(a_3+a_4)V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to D^+_s\eta B^-_c$	$-\frac{(a_3+2a_4)V^*_{cd}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to D^0\pi^0 B^-_c$	$-\frac{(a_3+a_4)V^*_{cd}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{d}} \to D^0\overline{K}^0 B^-_c$	$a_3 V^*_{cs}$
$T^-_{bb\bar{u}\bar{d}} \to D^0\eta B^-_c$	$\frac{(a_3-a_4)V^*_{cd}}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{d}} \to D^+\pi^- B^-_c$	$-(a_3+a_4)V^*_{cd}$
$T^-_{bb\bar{u}\bar{d}} \to D^+ K^- B^-_c$	$-a_3 V^*_{cs}$	$T^-_{bb\bar{u}\bar{d}} \to D^+_s K^- B^-_c$	$-a_4 V^*_{cd}$

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to D^0\pi^0 B^-_c) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{s}} \to D^+\pi^- B^-_c) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+\pi^0 B^-_c) = \frac{1}{8}\Gamma(T^0_{bb\bar{d}\bar{s}} \to D^0\pi^+ B^-_c),$$
$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to D^0 K^+ B^-_c) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to D^+_s K^- B^-_c), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+_s \bar{K}^0 B^-_c) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to D^+_s K^- B^-_c),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to D^0 \bar{K}^0 B^-_c) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to D^+ K^- B^-_c), \qquad \Gamma(T^-_{bb\bar{u}\bar{s}} \to D^0\eta B^-_c) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+\eta B^-_c),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to D^+_s\pi^- B^-_c) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to D^0 K^0 B^-_c) = 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+_s\pi^0 B^-_c),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to D^0\pi^0 B^-_c) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+ K^0 B^-_c) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{d}} \to D^+\pi^- B^-_c).$$

The T_{bb3} decay amplitudes into a bottom meson plus a charmed meson and an anticharmed meson are given in Table XI. And the relations of decay widths become

TABLE XI. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a B meson, a charmed meson, and an anticharmed meson.

Channel	Amplitude ($/V_{cb}$)	Channel	Amplitude ($/V_{cb}$)
$T^-_{bb\bar{u}\bar{s}} \to D^0\overline{D}^0B^-$	$(a_5-a_7)V^*_{cs}$	$T^-_{bb\bar{u}\bar{s}} \to D^0D^-\overline{B}^0$	$a_5V^*_{cs}$
$T^-_{bb\bar{u}\bar{s}} \to D^0D^-\overline{B}^0_s$	$a_6V^*_{cd}$	$T^-_{bb\bar{u}\bar{s}} \to D^0D^-_s\overline{B}^0_s$	$(a_5+a_6)V^*_{cs}$
$T^-_{bb\bar{u}\bar{s}} \to D^+D^-B^-$	$-a_7V^*_{cs}$	$T^-_{bb\bar{u}\bar{s}} \to D^+_sD^-B^-$	$-a_6V^*_{cd}$
$T^-_{bb\bar{u}\bar{s}} \to D^+_sD^-_sB^-$	$-(a_6+a_7)V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to D^0\overline{D}^0\overline{B}^0$	$-a_7V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to D^0\overline{D}^0\overline{B}^0_s$	$a_7V^*_{cd}$	$T^0_{bb\bar{d}\bar{s}} \to D^+\overline{D}^0B^-$	$a_5V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to D^+D^-\overline{B}^0$	$(a_5-a_7)V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to D^+D^-\overline{B}^0_s$	$(a_6+a_7)V^*_{cd}$
$T^0_{bb\bar{d}\bar{s}} \to D^+D^-_s\overline{B}^0_s$	$(a_5+a_6)V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to D^+_s\overline{D}^0B^-$	$-a_5V^*_{cd}$
$T^0_{bb\bar{d}\bar{s}} \to D^+_sD^-\overline{B}^0$	$-(a_5+a_6)V^*_{cd}$	$T^0_{bb\bar{d}\bar{s}} \to D^+_sD^-_s\overline{B}^0$	$-(a_6+a_7)V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to D^+_sD^-_s\overline{B}^0_s$	$(a_7-a_5)V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to D^0\overline{D}^0B^-$	$(a_5-a_7)V^*_{cd}$
$T^-_{bb\bar{u}\bar{d}} \to D^0D^-\overline{B}^0$	$(a_5+a_6)V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to D^0D^-_s\overline{B}^0$	$a_6V^*_{cs}$
$T^-_{bb\bar{u}\bar{d}} \to D^0D^-_s\overline{B}^0_s$	$a_5V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to D^+D^-B^-$	$-(a_6+a_7)V^*_{cd}$
$T^-_{bb\bar{u}\bar{d}} \to D^+D^-_sB^-$	$-a_6V^*_{cs}$	$T^-_{bb\bar{u}\bar{d}} \to D^+_sD^-_sB^-$	$-a_7V^*_{cd}$

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to D^0\bar{D}^0B^-) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+D^-\bar{B}^0),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to D^+D^-B^-) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^0\bar{D}^0\bar{B}^0),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to D^+_sD^-B^-) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to D^0D^-\bar{B}^0_s),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to D^+_sD^-_sB^-) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+_sD^-_s\bar{B}^0),$$
$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+\bar{D}^0B^-) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to D^0D^-\bar{B}^0),$$
$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+_s\bar{D}^0B^-) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to D^0D^-_s\bar{B}^0_s),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to D^0\bar{D}^0B^-) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+_sD^-_s\bar{B}^0_s),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to D^+D^-B^-) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+D^-\bar{B}^0_s),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to D^+D^-_sB^-) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to D^0D^-_s\bar{B}^0),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to D^+_sD^-_sB^-) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^0\bar{D}^0\bar{B}^0_s),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to D^0D^-_s\bar{B}^0_s) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+D^-_s\bar{B}^0_s),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to D^0D^-\bar{B}^0) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to D^+_sD^-\bar{B}^0).$$

B. $b \to c\bar{u}d/s$ transition

1. Decays into a bottom meson and a charmed meson by the W-exchange process

For the bottom quark decays to a charm quark, the effective Hamiltonian is given by

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb}V^*_{uq}[C_1O_1^{\bar{c}u} + C_2O_2^{\bar{c}u}] + \text{H.c.} \quad (25)$$

The corresponding Feynman diagrams are showed in Fig. 6. From the above Hamiltonian, the light quarks reduce an octet where the nonzero component is $(H_{\mathbf{8}})^2_1 = V^*_{ud}$ for the $b \to c\bar{u}d$ or $b\bar{d} \to c\bar{u}$ transition, and $(H_{\mathbf{8}})^3_1 = V^*_{us}$ for the $b \to c\bar{u}s$ or $b\bar{s} \to c\bar{u}$ transition. We then obtain the hadron-level effective Hamiltonian

$$\mathcal{H} = f_3(T_{bb3})_{[ij]}(\bar{B})^j(H_8)^i_k(\bar{D})^k + f_4(T_{bb3})_{[ij]}(\bar{B})^k(H_8)^i_k(\bar{D})^j. \quad (26)$$

Decay amplitudes are collected in Table XII, from which the relations of decay widths become

$$\frac{\Gamma(T^0_{bb\bar{d}\bar{s}} \to B^-D^+_s)}{\Gamma(T^0_{bb\bar{d}\bar{s}} \to B^-D^+)} = \frac{\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{B}^0_sD^0)}{\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{B}^0D^0)} = \frac{\Gamma(T^-_{bb\bar{u}\bar{d}} \to B^-D^0)}{\Gamma(T^-_{bb\bar{u}\bar{s}} \to B^-D^0)} = \frac{|V^*_{ud}|^2}{|V^*_{us}|^2}.$$

2. Decays into a light antibaryon and a charmed bottom baryon

There are two kinds of multiplets for the final states, which lead to the Hamiltonian

$$\mathcal{H} = a_4(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^l_x(H_8)^i_k(\overline{F}_{bc})_l + a_5(T_{bb3})_{[ij]}\epsilon^{xjl}(F_8)^k_x(H_8)^i_k(\overline{F}_{bc})_l + a_6(T_{bb3})_{[ij]}\epsilon^{xkl}(F_8)^j_x(H_8)^i_k(\overline{F}_{bc})_l$$
$$+ a_7(T_{bb3})_{[ij]}\epsilon^{xij}(F_8)^k_x(H_8)^l_k(\overline{F}_{bc})_l + a_8(T_{bb3})_{[ij]}(F_{\overline{10}})^{\{jkl\}}(H_8)^i_k(\overline{F}_{bc})_l + \bar{a}_7(T_{bb3})_{[ij]}\epsilon^{xik}(F_8)^j_x(H_8)^l_k(\overline{F}_{bc})_l. \quad (27)$$

TABLE XII. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a bottom meson and a charmed meson.

Channel	Amplitude $(/V_{cb})$	Channel	Amplitude $(/V_{cb})$
$T^-_{bb\bar{u}\bar{s}} \to B^- D^0$	$-(f_3+f_4)V^*_{us}$	$T^0_{bb\bar{d}\bar{s}} \to B^- D^+$	$-f_4 V^*_{us}$
$T^0_{bb\bar{d}\bar{s}} \to B^- D^+_s$	$f_4 V^*_{ud}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 D^0$	$-f_3 V^*_{us}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s D^0$	$f_3 V^*_{ud}$	$T^-_{bb\bar{u}\bar{d}} \to B^- D^0$	$-(f_3+f_4)V^*_{ud}$

Decay amplitudes are presented in Table XIII, where different final states are labeled as class I or II. Note that the factor $2a_7+\bar{a}_7$ always appears in the results; thus, we remove $\bar{a}_7$ in the final results. For class I, we have the relations:

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Sigma^-\Omega^0_{bc}) = 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0\Omega^0_{bc}).$$

For class II, we have the relations:

$$\begin{aligned}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^{--}\Xi^+_{bc}) &= 3\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^-\Xi^0_{bc}) = 3\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^-\Omega^0_{bc}) = 3\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^0\Xi^0_{bc})\\ &= 6\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0\Omega^0_{bc}) = 3\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^-\Xi^+_{bc}).\\ \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^-\Xi^+_{bc}) &= 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0\Xi^0_{bc}) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^0\Omega^0_{bc}) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^-\Xi^0_{bc})\\ &= \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^-\Omega^0_{bc}) = \frac{1}{3}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^{--}\Xi^+_{bc}).\end{aligned}$$

3. Decays into a bottom meson, a charmed meson and a light meson

The effective Hamiltonian for decays into a bottom meson, a charmed meson, and a light meson is

$$\begin{aligned}\mathcal{H} = {}& a_5(T_{bb3})_{[ij]}(\overline{B})^i(\overline{D})^l M^k_l (H_{\mathbf{8}})^j_k + a_6(T_{bb3})_{[ij]}(\overline{B})^i(\overline{D})^j M^k_l (H_{\mathbf{8}})^l_k + a_7(T_{bb3})_{[ij]}(\overline{B})^i(\overline{D})^k M^j_l (H_{\mathbf{8}})^l_k\\ &+ a_8(T_{bb3})_{[ij]}(\overline{B})^l(\overline{D})^j M^k_l (H_{\mathbf{8}})^i_k + a_9(T_{bb3})_{[ij]}(\overline{B})^l(\overline{D})^k M^j_k (H_{\mathbf{8}})^i_l + a_{10}(T_{bb3})_{[ij]}(\overline{B})^k(\overline{D})^l M^j_k (H_{\mathbf{8}})^i_l\\ &+ a_{11}(T_{bb3})_{[ij]}(\overline{B})^l(\overline{D})^i M^j_k (H_{\mathbf{8}})^k_l. \end{aligned} \tag{28}$$

Decay amplitudes are collected in Table XIV, where no relation for the decay widths is found.

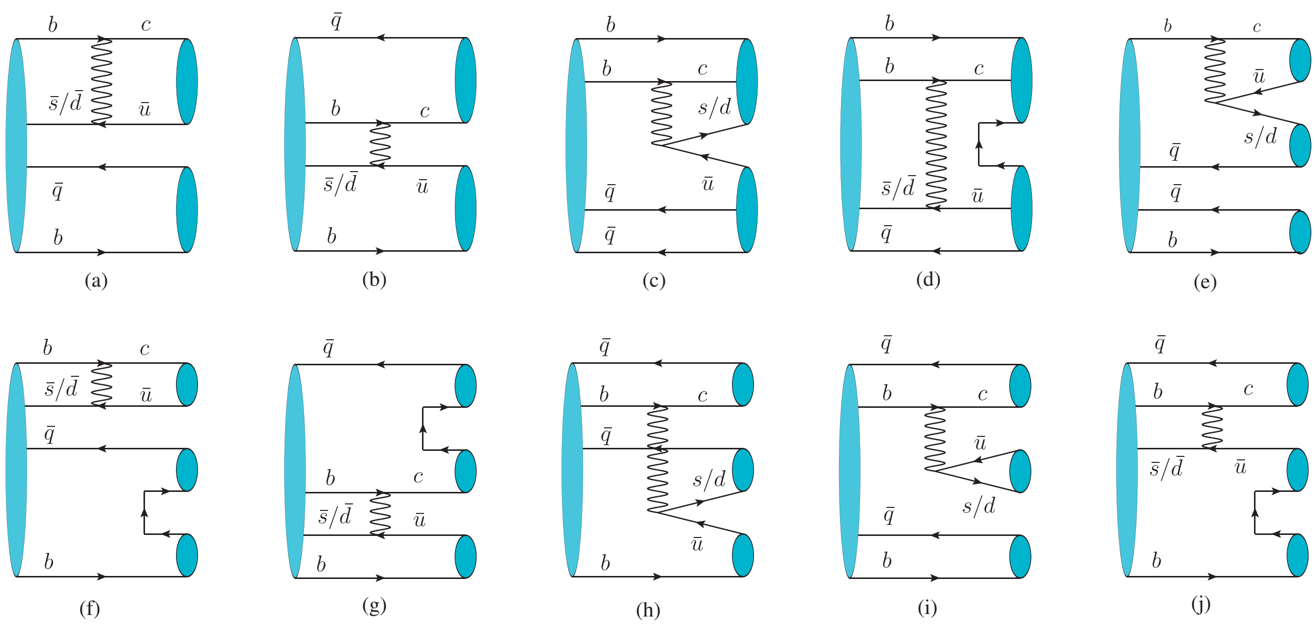

FIG. 6. Feynman diagrams for nonleptonic decays of doubly heavy tetraquarks. Panels (a),(b) correspond with the two mesons W-exchange process; panels (c),(d) correspond with the baryon and antibaryon process; and panels (e),(f),(g),(h),(i),(j) correspond with the B plus D and light meson processes.

TABLE XIII. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a light antibaryon octet (class I) or antidecuplet (class II) and a charmed bottom baryon.

Class I	Amplitude ($/V_{cb}$)	Class II	Amplitude ($/V_{cb}$)
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^- \Xi^0_{bc}$	$-2a_7 V^*_{ud}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^{--} \Xi^+_{bc}$	$-a_9 V^*_{us}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^- \Omega^0_{bc}$	$(a_5 + a_6 - 2a_7) V^*_{us}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^- \Xi^0_{bc}$	$-\frac{a_9 V^*_{us}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{p} \Xi^0_{bc}$	$-(a_5 + a_6) V^*_{us}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^- \Omega^0_{bc}$	$-\frac{a_9 V^*_{us}}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0 \Xi^0_{bc}$	$\frac{(a_4 - a_5 - 2a_6 + 2a_7) V^*_{ud}}{\sqrt{6}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^- \Xi^+_{bc}$	$-\frac{a_9 V^*_{us}}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0 \Omega^0_{bc}$	$\frac{(-2a_4 - a_5 + a_6 + 2a_7) V^*_{us}}{\sqrt{6}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^0 \Xi^0_{bc}$	$-\frac{a_9 V^*_{us}}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^- \Xi^+_{bc}$	$(a_4 + a_5) V^*_{ud}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^- \Xi^+_{bc}$	$\frac{a_9 V^*_{ud}}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0 \Xi^0_{bc}$	$-\frac{(a_4 + a_5 - 2a_7) V^*_{ud}}{\sqrt{2}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0 \Xi^0_{bc}$	$\frac{a_9 V^*_{ud}}{\sqrt{6}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0 \Omega^0_{bc}$	$-\frac{(a_5 + a_6 - 2a_7) V^*_{us}}{\sqrt{2}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0 \Omega^0_{bc}$	$-\frac{a_9 V^*_{us}}{\sqrt{6}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{p} \Xi^+_{bc}$	$(a_4 + a_5) V^*_{us}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^0 \Omega^0_{bc}$	$\frac{a_9 V^*_{ud}}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{n} \Xi^0_{bc}$	$(a_4 - a_6) V^*_{us}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^{--} \Xi^+_{bc}$	$-a_9 V^*_{ud}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^0 \Omega^0_{bc}$	$(a_4 - a_6) V^*_{ud}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^- \Xi^0_{bc}$	$-\frac{a_9 V^*_{ud}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^- \Omega^0_{bc}$	$(a_5 + a_6) V^*_{ud}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^- \Omega^0_{bc}$	$-\frac{a_9 V^*_{ud}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{p} \Xi^0_{bc}$	$-(a_5 + a_6 - 2a_7) V^*_{ud}$		
$T^-_{bb\bar{u}\bar{d}} \to \bar{p} \Omega^0_{bc}$	$2a_7 V^*_{us}$		

C. $b \to u\bar{c}d/s$ transition

1. Decays into two mesons by the W-exchange process

We write the effective Hamiltonian for the anticharm quark production as

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ub} V^*_{cq} [C_1 O_1^{\bar{u}c} + C_2 O_2^{\bar{u}c}] + \text{H.c.} \tag{29}$$

According to the flavor SU(3) group, the $H_{\bar{3}}''$ is antisymmetric while the H_6 is symmetric. The nonzero components are $(H_{\bar{3}}'')^{13} = -(H_{\bar{3}}'')^{31} = V^*_{cs}$ and $(H_{\bar{6}})^{13} = (H_{\bar{6}})^{31} = V^*_{cs}$, for the $b \to u\bar{c}s$ transition. When interchanged with $2 \leftrightarrow 3$ and $s \leftrightarrow d$, we get the nonzero components for the transition $b \to u\bar{c}d$. The corresponding Feynman diagrams are showed in Fig. 7.

We get the effective Hamiltonian

$$\begin{aligned}\mathcal{H} = {} & f_5 (T_{bb3})_{[ij]} (\overline{B})^k (H_3'')^{[ij]} (D)_k + f_6 (T_{bb3})_{[ij]} (\overline{B})^j (H_3'')^{[ik]} (D)_k + f_7 (T_{bb3})_{[ij]} (\overline{B})^j (H_6'')^{\{ik\}} (D)_k \\ & + f_8 (T_{bb3})_{[ij]} (H_3'')^{[ik]} M^j_k \overline{B}_c + f_9 (T_{bb3})_{[ij]} (H_6'')^{\{ik\}} M^j_k \overline{B}_c. \end{aligned} \tag{30}$$

For a bottom meson and an anticharmed meson produced, the amplitudes are given in Table XV and we have

$$\begin{aligned}\frac{\Gamma(T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0 D^-)}{\Gamma(T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0_s D^-_s)} &= \frac{\Gamma(T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 \overline{D}^0)}{\Gamma(T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s \overline{D}^0)} = \frac{\Gamma(T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 D^-_s)}{\Gamma(T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s D^-)} \\ &= \frac{\Gamma(T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s D^-_s)}{\Gamma(T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 D^-)} = \frac{\Gamma(T^-_{bb\bar{u}\bar{s}} \to B^- \overline{D}^0)}{\Gamma(T^-_{bb\bar{u}\bar{d}} \to B^- \overline{D}^0)} = \frac{|V^*_{cs}|^2}{|V^*_{cd}|^2}.\end{aligned}$$

TABLE XIV. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a bottom meson, a charmed meson, and a light meson.

Channel	Amplitude $(/V_{cb})$	Channel	Amplitude $(/V_{cb})$
$T^-_{bb\bar{u}\bar{s}} \to B^- D^0 \pi^0$	$\frac{(a_5-a_8-a_9-a_{10})V^*_{us}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{s}} \to B^- D^0 K^0$	$(a_7+a_{11})V^*_{ud}$
$T^-_{bb\bar{u}\bar{s}} \to B^- D^0 \eta$	$\frac{(a_5-2a_7-a_8-a_9-a_{10}-2a_{11})V^*_{us}}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{s}} \to B^- D^+ \pi^-$	$(a_5-a_9)V^*_{us}$
$T^-_{bb\bar{u}\bar{s}} \to B^- D^+_s \pi^-$	$(a_6-a_{11})V^*_{ud}$	$T^-_{bb\bar{u}\bar{s}} \to B^- D^+_s K^-$	$(a_5+a_6-a_9-a_{11})V^*_{us}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0 D^0 \pi^-$	$-(a_8+a_{10})V^*_{us}$	$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s D^0 \pi^-$	$-(a_6+a_7)V^*_{ud}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s D^0 K^-$	$-(a_6+a_7+a_8+a_{10})V^*_{us}$	$T^0_{bb\bar{d}\bar{s}} \to B^- D^0 \pi^+$	$-(a_9+a_{10})V^*_{us}$
$T^0_{bb\bar{d}\bar{s}} \to B^- D^0 K^+$	$(a_9+a_{10})V^*_{ud}$	$T^0_{bb\bar{d}\bar{s}} \to B^- D^+ \pi^0$	$\frac{(a_9-a_8)V^*_{us}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to B^- D^+ K^0$	$(a_9+a_{11})V^*_{ud}$	$T^0_{bb\bar{d}\bar{s}} \to B^- D^+ \eta$	$-\frac{(a_8+a_9+2a_{11})V^*_{us}}{\sqrt{6}}$
$T^0_{bb\bar{d}\bar{s}} \to B^- D^+_s \pi^0$	$\frac{(a_8+a_{11})V^*_{ud}}{\sqrt{2}}$	$T^0_{bb\bar{d}\bar{s}} \to B^- D^+_s \overline{K}^0$	$-(a_9+a_{11})V^*_{us}$
$T^0_{bb\bar{d}\bar{s}} \to B^- D^+_s \eta$	$\frac{(a_8-2a_9-a_{11})V^*_{ud}}{\sqrt{6}}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 D^0 \pi^0$	$\frac{(a_5+a_{10})V^*_{us}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 D^0 K^0$	$(a_7+a_{10})V^*_{ud}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 D^0 \eta$	$\frac{(a_5-2a_7-a_{10})V^*_{us}}{\sqrt{6}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 D^+ \pi^-$	$(a_5-a_8)V^*_{us}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 D^+_s \pi^-$	$(a_6+a_8)V^*_{ud}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 D^+_s K^-$	$(a_5+a_6)V^*_{us}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s D^0 \pi^0$	$\frac{(a_7-a_5)V^*_{ud}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s D^0 \overline{K}^0$	$-(a_7+a_{10})V^*_{us}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s D^0 \eta$	$-\frac{(a_5+a_7+2a_{10})V^*_{ud}}{\sqrt{6}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s D^+ \pi^-$	$-(a_5+a_6)V^*_{ud}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s D^+ K^-$	$-(a_6+a_8)V^*_{us}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s D^+_s K^-$	$(a_8-a_5)V^*_{ud}$	$T^-_{bb\bar{u}\bar{d}} \to B^- D^0 \pi^0$	$\frac{(a_5-a_7-a_8-a_9-a_{10}-a_{11})V^*_{ud}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to B^- D^0 \overline{K}^0$	$(a_7+a_{11})V^*_{us}$	$T^-_{bb\bar{u}\bar{d}} \to B^- D^0 \eta$	$\frac{(a_5+a_7-a_8-a_9-a_{10}+a_{11})V^*_{ud}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to B^- D^+ \pi^-$	$(a_5+a_6-a_9-a_{11})V^*_{ud}$	$T^-_{bb\bar{u}\bar{d}} \to B^- D^+ K^-$	$(a_6-a_{11})V^*_{us}$
$T^-_{bb\bar{u}\bar{d}} \to B^- D^+_s K^-$	$(a_5-a_9)V^*_{ud}$	$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 D^0 \pi^-$	$-(a_6+a_7+a_8+a_{10})V^*_{ud}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 D^0 K^-$	$-(a_6+a_7)V^*_{us}$	$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0_s D^0 K^-$	$-(a_8+a_{10})V^*_{ud}$

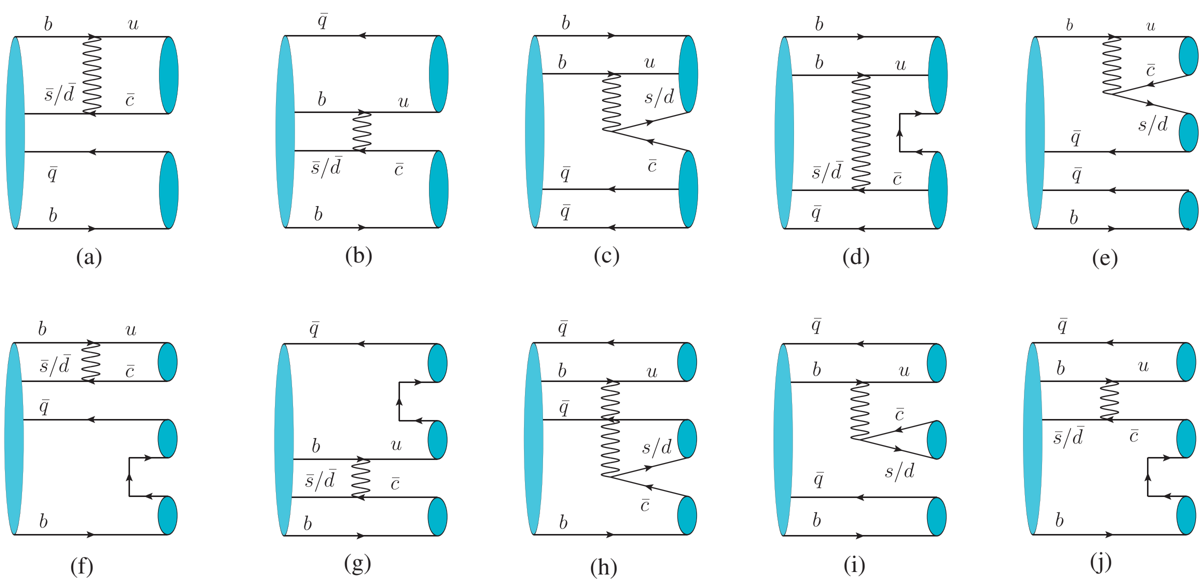

FIG. 7. Feynman diagrams for nonleptonic decays of doubly bottomed tetraquarks. Panels (a),(b) correspond with B plus $\overline{D}$ or B_c plus light meson W-exchange processes, respectively. Panels (c),(d) correspond with the baryon and antibaryon process and panels (e),(f),(i),(j) match with B plus $\overline{D}$ and light mesons. Panels (g),(h) match with B_c plus two light mesons.

TABLE XV. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a bottom meson and an anticharmed meson. Note that these amplitudes have an additional identical CKM factor V_{ub}.

Channel	Amplitude $(/V_{ub})$	Channel	Amplitude $(/V_{ub})$
$T^-_{bb\bar{u}\bar{s}} \to B^-\overline{D}^0$	$(2f_5 - f_6 - f_7)V^*_{cs}$	$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0 D^-$	$2f_5 V^*_{cs}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s D^-$	$(f_7 - f_6)V^*_{cd}$	$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s D^-_s$	$(2f_5 - f_6 + f_7)V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0\overline{D}^0$	$-(f_6 + f_7)V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\overline{D}^0$	$(f_6 + f_7)V^*_{cd}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\overline{D}^0$	$(2f_5 - f_6 - f_7)V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 D^-$	$(2f_5 - f_6 + f_7)V^*_{cd}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 D^-_s$	$(f_7 - f_6)V^*_{cs}$	$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0_s D^-_s$	$2f_5 V^*_{cd}$

TABLE XVI. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into an anticharmed B meson and a light meson.

Channel	Amplitude $(/V_{ub})$	Channel	Amplitude $(/V_{ub})$
$T^-_{bb\bar{u}\bar{s}} \to B^-_c\pi^0$	$\frac{(f_8-f_9)V^*_{cs}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{s}} \to B^-_c K^0$	$(f_8 + f_9)V^*_{cd}$
$T^-_{bb\bar{u}\bar{s}} \to B^-_c\eta$	$-\frac{(f_8+3f_9)V^*_{cs}}{\sqrt{6}}$	$T^0_{bb\bar{d}\bar{s}} \to B^-_c\pi^+$	$(f_8 - f_9)V^*_{cs}$
$T^0_{bb\bar{d}\bar{s}} \to B^-_c K^+$	$(f_9 - f_8)V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to B^-_c\pi^0$	$-\sqrt{2}f_9 V^*_{cd}$
$T^-_{bb\bar{u}\bar{d}} \to B^-_c\overline{K}^0$	$(f_8 + f_9)V^*_{cs}$	$T^-_{bb\bar{u}\bar{d}} \to B^-_c\eta$	$\sqrt{\frac{2}{3}}f_8 V^*_{cd}$

T_{bb3} decay amplitudes into a charmed bottom meson and a light meson for different channels are given in Table XVI and we have

$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to B^-_c\pi^+) = 2\Gamma(T^-_{bb\bar{u}\bar{s}} \to B^-_c\pi^0).$$

2. Decays into an anticharmed antibaryon and a bottom baryon

The effective Hamiltonian for decays into an anticharmed antibaryon and a bottom baryon is

$$\begin{aligned}\mathcal{H}_{\rm eff} = {}& b_1(T_{bb3})_{[ij]}(F_{\bar{c}3})^{[kl]}(H''_3)^{[ij]}(\overline{F}_{b\bar{3}})_{[kl]} + b_2(T_{bb3})_{[ij]}(F_{\bar{c}3})^{[jl]}(H''_3)^{[ik]}(\overline{F}_{b\bar{3}})_{[kl]} \\ &+ b_3(T_{bb3})_{[ij]}(F_{\bar{c}3})^{[ij]}(H''_3)^{[kl]}(\overline{F}_{b\bar{3}})_{[kl]} + b_4(T_{bb3})_{[ij]}(F_{\bar{c}3})^{[jl]}(H''_6)^{\{ik\}}(\overline{F}_{b\bar{3}})_{[kl]} \\ &+ b_5(T_{bb3})_{[ij]}(F_{\bar{c}3})^{[jl]}(H''_3)^{[ik]}(\overline{F}_{b6})_{\{kl\}} + b_6(T_{bb3})_{[ij]}(F_{\bar{c}3})^{[jl]}(H''_6)^{\{ik\}}(\overline{F}_{b6})_{\{kl\}} \\ &+ b_7(T_{bb3})_{[ij]}(F_{\bar{c}3})^{[ij]}(H''_6)^{\{kl\}}(\overline{F}_{b6})_{\{kl\}} + b_8(T_{bb3})_{[ij]}(F_{\bar{c}\bar{6}})^{\{jl\}}(H''_3)^{[ik]}(\overline{F}_{b\bar{3}})_{[kl]} \\ &+ b_9(T_{bb3})_{[ij]}(F_{\bar{c}\bar{6}})^{\{jl\}}(H''_6)^{\{ik\}}(\overline{F}_{b\bar{3}})_{[kl]} + b_{10}(T_{bb3})_{[ij]}(F_{\bar{c}\bar{6}})^{\{kl\}}(H''_3)^{[ij]}(\overline{F}_{b6})_{\{kl\}} \\ &+ b_{11}(T_{bb3})_{[ij]}(F_{\bar{c}\bar{6}})^{\{jl\}}(H''_3)^{[ik]}(\overline{F}_{b6})_{\{kl\}}. \end{aligned} \tag{31}$$

Different decay channel amplitudes are given in Table XVII, where class I corresponds with the triplet antibaryon plus antitriplet baryon, class II corresponds with the triplet anti-baryon plus sextet baryon, class III corresponds with the antisextet antibaryon plus antitriplet baryon, and class IV corresponds with the antisextet antibaryon plus sextet baryon.

For class I, we obtain the relations of decay widths:

$$\begin{aligned}\frac{\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Lambda^-_{\bar{c}}\Lambda^0_b)}{\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Xi^-_{\bar{c}}\Xi^0_b)} &= \frac{\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Xi^-_{\bar{c}}\Xi^0_b)}{\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Lambda^-_{\bar{c}}\Lambda^0_b)} = \frac{\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Lambda^-_{\bar{c}}\Xi^0_b)}{\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Xi^-_{\bar{c}}\Lambda^0_b)} \\ &= \frac{\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0_{\bar{c}}\Xi^-_b)}{\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}^0_{\bar{c}}\Xi^-_b)} = \frac{\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^0_{\bar{c}}\Xi^0_b)}{\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^0_{\bar{c}}\Lambda^0_b)} = \frac{|V^*_{cs}|^2}{|V^*_{cd}|^2}.\end{aligned}$$

TABLE XVII. Doubly bottomed tetraquark $T_{bb\bar q\bar q}$ decays into an anticharmed antibaryon triplet and a bottom baryon antitriplet (class I) or sextet (class II), an anticharmed antibaryon antisextet and a bottom baryon antitriplet (class III) or sextet (class IV).

Class I	Amplitude $(/V_{ub})$	Class IV	Amplitude $(/V_{ub})$
$T^-_{bb\bar u\bar s}\to\Lambda^-_{\bar c}\Lambda^0_b$	$(4b_1+b_2-b_4)V^*_{cs}$	$T^0_{bb\bar d\bar s}\to\bar\Omega^0_{\bar c}\Xi'^0_b$	$-\frac{b_{11}V^*_{cd}}{\sqrt2}$
$T^-_{bb\bar u\bar s}\to\Xi^-_{\bar c}\Lambda^0_b$	$(b_2+4b_3+b_4)V^*_{cd}$	$T^-_{bb\bar u\bar d}\to\Sigma^{--}_{\bar c}\Sigma^+_b$	$(2b_{10}+b_{11})V^*_{cd}$
$T^-_{bb\bar u\bar s}\to\Xi^-_{\bar c}\Xi^0_b$	$2(2b_1+b_2+2b_3)V^*_{cs}$	$T^-_{bb\bar u\bar d}\to\Sigma^-_{\bar c}\Sigma^0_b$	$(2b_{10}+b_{11})V^*_{cd}$
$T^-_{bb\bar u\bar s}\to\bar\Xi^0_{\bar c}\Xi^-_b$	$(4b_1+b_2+b_4)V^*_{cs}$	$T^-_{bb\bar u\bar d}\to\Sigma^-_{\bar c}\Xi'^0_b$	$\frac12 b_{11}V^*_{cs}$
$T^0_{bb\bar d\bar s}\to\bar\Xi^0_{\bar c}\Lambda^0_b$	$(b_2+4b_3-b_4)V^*_{cd}$	$T^-_{bb\bar u\bar d}\to\bar\Sigma^0_{\bar c}\Sigma^-_b$	$(2b_{10}+b_{11})V^*_{cd}$
$T^0_{bb\bar d\bar s}\to\bar\Xi^0_{\bar c}\Xi^0_b$	$(b_2+4b_3-b_4)V^*_{cs}$	$T^-_{bb\bar u\bar d}\to\bar\Sigma^0_{\bar c}\Xi'^-_b$	$\frac{b_{11}V^*_{cs}}{\sqrt2}$
$T^-_{bb\bar u\bar d}\to\Lambda^-_{\bar c}\Lambda^0_b$	$2(2b_1+b_2+2b_3)V^*_{cd}$	$T^-_{bb\bar u\bar d}\to\Xi'^-_{\bar c}\Xi'^0_b$	$\frac12(4b_{10}+b_{11})V^*_{cd}$
$T^-_{bb\bar u\bar d}\to\Lambda^-_{\bar c}\Xi^0_b$	$(b_2+4b_3+b_4)V^*_{cs}$	$T^-_{bb\bar u\bar d}\to\bar\Xi'^0_{\bar c}\Xi'^-_b$	$\frac12(4b_{10}+b_{11})V^*_{cd}$
$T^-_{bb\bar u\bar d}\to\Xi^-_{\bar c}\Xi^0_b$	$(4b_1+b_2-b_4)V^*_{cd}$	$T^-_{bb\bar u\bar d}\to\bar\Xi'^0_{\bar c}\Omega^-_b$	$\frac{b_{11}V^*_{cs}}{\sqrt2}$
$T^-_{bb\bar u\bar d}\to\bar\Xi^0_{\bar c}\Xi^-_b$	$(4b_1+b_2+b_4)V^*_{cd}$	$T^-_{bb\bar u\bar d}\to\bar\Omega^0_{\bar c}\Omega^-_b$	$2b_{10}V^*_{cd}$
Class III	Amplitude $(/V_{ub})$	$T^0_{bb\bar d\bar s}\to\bar\Xi'^0_{\bar c}\Xi'^0_b$	$\frac12 b_{11}V^*_{cs}$
$T^-_{bb\bar u\bar s}\to\Sigma^-_{\bar c}\Lambda^0_b$	$\frac{(b_8-b_9)V^*_{cs}}{\sqrt2}$	$T^-_{bb\bar u\bar s}\to\Sigma^{--}_{\bar c}\Sigma^+_b$	$(2b_{10}+b_{11})V^*_{cs}$
$T^-_{bb\bar u\bar s}\to\Xi'^-_{\bar c}\Lambda^0_b$	$-\frac{(b_8+b_9)V^*_{cd}}{\sqrt2}$	$T^-_{bb\bar u\bar s}\to\Sigma^-_{\bar c}\Sigma^0_b$	$\frac12(4b_{10}+b_{11})V^*_{cs}$
$T^-_{bb\bar u\bar s}\to\Xi'^-_{\bar c}\Xi^0_b$	$-\sqrt2 b_9V^*_{cs}$	$T^-_{bb\bar u\bar s}\to\bar\Sigma^0_{\bar c}\Sigma^-_b$	$2b_{10}V^*_{cs}$
$T^-_{bb\bar u\bar s}\to\bar\Xi'^0_{\bar c}\Xi^-_b$	$-\frac{(b_8+b_9)V^*_{cs}}{\sqrt2}$	$T^-_{bb\bar u\bar s}\to\Xi'^-_{\bar c}\Sigma^0_b$	$\frac12 b_{11}V^*_{cd}$
$T^-_{bb\bar u\bar s}\to\bar\Omega^0_{\bar c}\Xi^-_b$	$(b_8+b_9)V^*_{cd}$	$T^-_{bb\bar u\bar s}\to\Xi'^-_{\bar c}\Xi'^0_b$	$(2b_{10}+b_{11})V^*_{cs}$
$T^0_{bb\bar d\bar s}\to\bar\Sigma^0_{\bar c}\Lambda^0_b$	$(b_8-b_9)V^*_{cs}$	$T^-_{bb\bar u\bar s}\to\bar\Xi'^0_{\bar c}\Sigma^-_b$	$\frac{b_{11}V^*_{cd}}{\sqrt2}$
$T^0_{bb\bar d\bar s}\to\bar\Xi'^0_{\bar c}\Lambda^0_b$	$\frac{(b_9-b_8)V^*_{cd}}{\sqrt2}$	$T^-_{bb\bar u\bar s}\to\bar\Xi'^0_{\bar c}\Xi'^-_b$	$\frac12(4b_{10}+b_{11})V^*_{cs}$
$T^0_{bb\bar d\bar s}\to\bar\Xi'^0_{\bar c}\Xi^0_b$	$\frac{(b_8-b_9)V^*_{cs}}{\sqrt2}$	$T^-_{bb\bar u\bar s}\to\bar\Omega^0_{\bar c}\Xi'^-_b$	$\frac{b_{11}V^*_{cd}}{\sqrt2}$
$T^0_{bb\bar d\bar s}\to\bar\Omega^0_{\bar c}\Xi^0_b$	$(b_9-b_8)V^*_{cd}$	$T^-_{bb\bar u\bar s}\to\bar\Omega^0_{\bar c}\Omega^-_b$	$(2b_{10}+b_{11})V^*_{cs}$
$T^-_{bb\bar u\bar d}\to\Sigma^-_{\bar c}\Lambda^0_b$	$-\sqrt2 b_9V^*_{cd}$	$T^0_{bb\bar d\bar s}\to\Sigma^-_{\bar c}\Sigma^+_b$	$\frac{b_{11}V^*_{cs}}{\sqrt2}$
$T^-_{bb\bar u\bar d}\to\Sigma^-_{\bar c}\Xi^0_b$	$-\frac{(b_8+b_9)V^*_{cs}}{\sqrt2}$	$T^0_{bb\bar d\bar s}\to\bar\Sigma^0_{\bar c}\Sigma^0_b$	$\frac{b_{11}V^*_{cs}}{\sqrt2}$
$T^-_{bb\bar u\bar d}\to\bar\Sigma^0_{\bar c}\Xi^-_b$	$-(b_8+b_9)V^*_{cs}$	$T^0_{bb\bar d\bar s}\to\Xi'^-_{\bar c}\Sigma^+_b$	$-\frac{b_{11}V^*_{cd}}{\sqrt2}$
$T^-_{bb\bar u\bar d}\to\Xi'^-_{\bar c}\Xi^0_b$	$\frac{(b_8-b_9)V^*_{cd}}{\sqrt2}$	$T^0_{bb\bar d\bar s}\to\bar\Xi'^0_{\bar c}\Sigma^0_b$	$-\frac12 b_{11}V^*_{cd}$
$T^-_{bb\bar u\bar d}\to\bar\Xi'^0_{\bar c}\Xi^-_b$	$\frac{(b_8+b_9)V^*_{cd}}{\sqrt2}$		
Class II	Amplitude $(/V_{ub})$	Class II	Amplitude $(/V_{ub})$
$T^-_{bb\bar u\bar s}\to\Lambda^-_{\bar c}\Sigma^0_b$	$\frac{(b_5-b_6)V^*_{cs}}{\sqrt2}$	$T^-_{bb\bar u\bar s}\to\Xi^-_{\bar c}\Sigma^0_b$	$-\frac{(b_5+b_6-4b_7)V^*_{cd}}{\sqrt2}$
$T^-_{bb\bar u\bar s}\to\Xi^-_{\bar c}\Xi'^0_b$	$-\sqrt2(b_6-2b_7)V^*_{cs}$	$T^-_{bb\bar u\bar s}\to\bar\Xi^0_{\bar c}\Sigma^-_b$	$-(b_5+b_6)V^*_{cd}$
$T^-_{bb\bar u\bar s}\to\bar\Xi^0_{\bar c}\Xi'^-_b$	$-\frac{(b_5+b_6)V^*_{cs}}{\sqrt2}$	$T^0_{bb\bar d\bar s}\to\Lambda^-_{\bar c}\Sigma^+_b$	$(b_6-b_5)V^*_{cs}$
$T^0_{bb\bar d\bar s}\to\Xi^-_{\bar c}\Sigma^+_b$	$(b_5-b_6)V^*_{cd}$	$T^0_{bb\bar d\bar s}\to\bar\Xi^0_{\bar c}\Sigma^0_b$	$\frac{(b_5-b_6+4b_7)V^*_{cd}}{\sqrt2}$
$T^0_{bb\bar d\bar s}\to\bar\Xi^0_{\bar c}\Xi'^0_b$	$\frac{(b_5-b_6+4b_7)V^*_{cs}}{\sqrt2}$	$T^-_{bb\bar u\bar d}\to\Lambda^-_{\bar c}\Sigma^0_b$	$-\sqrt2(b_6-2b_7)V^*_{cd}$
$T^-_{bb\bar u\bar d}\to\Lambda^-_{\bar c}\Xi'^0_b$	$-\frac{(b_5+b_6-4b_7)V^*_{cs}}{\sqrt2}$	$T^-_{bb\bar u\bar d}\to\Xi^-_{\bar c}\Xi'^0_b$	$\frac{(b_5-b_6)V^*_{cd}}{\sqrt2}$
$T^-_{bb\bar u\bar d}\to\bar\Xi^0_{\bar c}\Xi'^-_b$	$\frac{(b_5+b_6)V^*_{cd}}{\sqrt2}$	$T^-_{bb\bar u\bar d}\to\bar\Xi^0_{\bar c}\Omega^-_b$	$(b_5+b_6)V^*_{cs}$

For class II, we obtain the relations of decay widths:

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0_{\bar{c}}\Sigma^-_b) = 2\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}^0_{\bar{c}}\Xi'^-_b), \qquad \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0_{\bar{c}}\Xi'^-_b) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}^0_{\bar{c}}\Omega^-_b),$$
$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to \Lambda^-_{\bar{c}}\Sigma^+_b) = 2\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Lambda^-_{\bar{c}}\Sigma^0_b), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to \Xi^-_{\bar{c}}\Sigma^+_b) = 2\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Xi^-_{\bar{c}}\Xi'^0_b).$$

For class III, we obtain the relations of decay widths:

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Sigma^-_{\bar{c}}\Lambda^0_b) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^0_{\bar{c}}\Xi^0_b) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0_{\bar{c}}\Lambda^0_b).$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Xi'^-_{\bar{c}}\Lambda^0_b) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}'^0_{\bar{c}}\Xi^-_b) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Omega}^0_{\bar{c}}\Xi^-_b),$$
$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^0_{\bar{c}}\Lambda^0_b) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \Xi'^-_{\bar{c}}\Xi^0_b) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Omega}^0_{\bar{c}}\Xi^0_b),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Sigma^-_{\bar{c}}\Xi^0_b) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0_{\bar{c}}\Xi^-_b) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0_{\bar{c}}\Xi^-_b).$$

For class IV, the results are

$$\begin{aligned}
\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Xi'^-_{\bar{c}}\Sigma^0_b) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^0_{\bar{c}}\Sigma^0_b) &= \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0_{\bar{c}}\Sigma^-_b) = \frac{1}{2}\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Omega}^0_{\bar{c}}\Xi'^-_b) \\
&= \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Omega}^0_{\bar{c}}\Xi'^0_b) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \Xi'^-_{\bar{c}}\Sigma^+_b), \\
\Gamma(T^0_{bb\bar{d}\bar{s}} \to \Sigma^-_{\bar{c}}\Sigma^+_b) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0_{\bar{c}}\Sigma^0_b) &= 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^0_{\bar{c}}\Xi'^0_b) = 2\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Sigma^-_{\bar{c}}\Xi'^0_b) \\
&= \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0_{\bar{c}}\Xi'^-_b) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}'^0_{\bar{c}}\Omega^-_b), \\
\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Sigma^{--}_{\bar{c}}\Sigma^+_b) &= \Gamma(T^-_{bb\bar{u}\bar{d}} \to \Sigma^-_{\bar{c}}\Sigma^0_b) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0_{\bar{c}}\Sigma^-_b), \\
\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Sigma^{--}_{\bar{c}}\Sigma^+_b) &= \Gamma(T^-_{bb\bar{u}\bar{s}} \to \Xi'^-_{\bar{c}}\Xi'^0_b) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Omega}^0_{\bar{c}}\Omega^-_b), \\
\Gamma(T^-_{bb\bar{u}\bar{d}} \to \Xi'^-_{\bar{c}}\Xi'^0_b) &= \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}'^0_{\bar{c}}\Xi'^-_b), \\
\Gamma(T^-_{bb\bar{u}\bar{s}} \to \Sigma^-_{\bar{c}}\Sigma^0_b) &= \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0_{\bar{c}}\Xi'^-_b).
\end{aligned}$$

3. Decays into three mesons

The effective Hamiltonian is written as

$$\begin{aligned}
\mathcal{H} = {}& b_1(T_{bb3})_{[ij]}(\overline{B})^i D_l M^l_k (H_{\bar{3}}'')^{[jk]} + b_2(T_{bb3})_{[ij]}(\overline{B})^i D_l M^j_k (H_{\bar{3}}'')^{[kl]} \\
&+ b_3(T_{bb3})_{[ij]}(\overline{B})^k D_l M^l_k (H_{\bar{3}}'')^{[ij]} + b_4(T_{bb3})_{[ij]}(\overline{B})^k D_l M^j_k (H_{\bar{3}}'')^{[il]} \\
&+ b_5(T_{bb3})_{[ij]}(\overline{B})^l D_l M^j_k (H_{\bar{3}}'')^{[ik]} + b_6(T_{bb3})_{[ij]}(\overline{B})^i D_l M^l_k (H_6'')^{\{jk\}} \\
&+ b_7(T_{bb3})_{[ij]}(\overline{B})^i D_l M^j_k (H_6'')^{\{kl\}} + b_8(T_{bb3})_{[ij]}(\overline{B})^k D_l M^j_k (H_6'')^{\{il\}} \\
&+ b_9(T_{bb3})_{[ij]}(\overline{B})^l D_l M^j_k (H_6'')^{\{ik\}} + b_{10}(T_{bb3})_{[ij]} M^l_k M^k_l (H_{\bar{3}}'')^{[ij]} \overline{B}_c \\
&+ b_{11}(T_{bb3})_{[ij]} M^j_k M^k_l (H_{\bar{3}}'')^{[il]} \overline{B}_c + b_{12}(T_{bb3})_{[ij]} M^i_k M^j_l (H_{\bar{3}}'')^{[kl]} \overline{B}_c \\
&+ b_{13}(T_{bb3})_{[ij]} M^j_k M^k_l (H_6'')^{\{il\}} \overline{B}_c.
\end{aligned} \tag{32}$$

The decay amplitudes into a bottom meson plus an anticharmed meson and a light meson are given in Table XVIII. Deriving the formulas, we get

$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s D^-_s \pi^+) = 2\Gamma(T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s D^-_s \pi^0), \qquad \Gamma(T^-_{bb\bar{u}\bar{d}} \to B^- D^-_s \pi^+) = 2\Gamma(T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0 D^-_s \pi^0).$$

TABLE XVIII. Doubly bottomed tetraquark $T_{bb\bar q\bar q}$ decays into a bottom meson, an anticharmed meson, and a light meson.

Channel	Amplitude ($/V_{ub}$)	Channel	Amplitude ($/V_{ub}$)
$T^-_{bb\bar u\bar s}\to B^-\overline{D}^0\pi^0$	$\frac{(-b_1+2b_3+b_4+b_5+b_6-b_8-b_9)V^*_{cs}}{\sqrt{2}}$	$T^-_{bb\bar u\bar s}\to B^-\overline{D}^0K^0$	$(-b_2+b_5+b_7+b_9)V^*_{cd}$
$T^-_{bb\bar u\bar s}\to B^-\overline{D}^0\eta$	$-\frac{(b_1-2b_2-2b_3-b_4+b_5-b_6+2b_7+b_8+3b_9)V^*_{cs}}{\sqrt{6}}$	$T^-_{bb\bar u\bar s}\to B^-D^-\pi^+$	$(-b_1+2b_3+b_6)V^*_{cs}$
$T^-_{bb\bar u\bar s}\to B^-D^-K^+$	$(b_2+b_4+b_7+b_8)V^*_{cd}$	$T^-_{bb\bar u\bar s}\to B^-D_s^-K^+$	$(-b_1+b_2+2b_3+b_4+b_6+b_7+b_8)V^*_{cs}$
$T^-_{bb\bar u\bar s}\to \overline{B}^0\overline{D}^0\pi^-$	$(2b_3+b_4-b_8)V^*_{cs}$	$T^-_{bb\bar u\bar s}\to \overline{B}^0D^-\pi^0$	$\frac{(-2b_3+b_5-b_9)V^*_{cs}}{\sqrt{2}}$
$T^-_{bb\bar u\bar s}\to \overline{B}^0D^-K^0$	$(b_4+b_5+b_8+b_9)V^*_{cd}$	$T^-_{bb\bar u\bar s}\to \overline{B}^0D^-\eta$	$\frac{(2b_3-b_5-3b_9)V^*_{cs}}{\sqrt{6}}$
$T^-_{bb\bar u\bar s}\to \overline{B}^0D_s^-K^0$	$(2b_3+b_4+b_8)V^*_{cs}$	$T^-_{bb\bar u\bar s}\to \overline{B}^0_s\overline{D}^0\pi^-$	$-(b_1-b_2+b_6+b_7)V^*_{cd}$
$T^-_{bb\bar u\bar s}\to \overline{B}^0_s\overline{D}^0K^-$	$-(b_1-b_2-2b_3-b_4+b_6+b_7+b_8)V^*_{cs}$	$T^-_{bb\bar u\bar s}\to \overline{B}^0_sD^-\pi^0$	$\frac{(b_1-b_2+b_6-b_7)V^*_{cd}}{\sqrt{2}}$
$T^-_{bb\bar u\bar s}\to \overline{B}^0_sD^-\overline{K}^0$	$-(b_1-2b_3+b_6)V^*_{cs}$	$T^-_{bb\bar u\bar s}\to \overline{B}^0_sD^-\eta$	$-\frac{(b_1+b_2+2b_4+b_6+b_7+2b_8)V^*_{cd}}{\sqrt{6}}$
$T^-_{bb\bar u\bar s}\to \overline{B}^0_sD_s^-\pi^0$	$-\frac{(b_2-b_5+b_7+b_9)V^*_{cs}}{\sqrt{2}}$	$T^-_{bb\bar u\bar s}\to \overline{B}^0_sD_s^-K^0$	$(-b_1+b_5-b_6+b_9)V^*_{cd}$
$T^-_{bb\bar u\bar s}\to \overline{B}^0_sD_s^-\eta$	$\frac{(2b_1-b_2-4b_3-2b_4-b_5+2b_6-b_7-2b_8-3b_9)V^*_{cs}}{\sqrt{6}}$	$T^0_{bb\bar d\bar s}\to B^-\overline{D}^0\pi^+$	$(b_4+b_5-b_8-b_9)V^*_{cs}$
$T^0_{bb\bar d\bar s}\to B^-\overline{D}^0K^+$	$(-b_4-b_5+b_8+b_9)V^*_{cd}$	$T^0_{bb\bar d\bar s}\to \overline{B}^0\overline{D}^0\pi^0$	$\frac{(-b_1-b_4+b_6+b_8)V^*_{cs}}{\sqrt{2}}$
$T^0_{bb\bar d\bar s}\to \overline{B}^0\overline{D}^0K^0$	$(-b_2-b_4+b_7+b_8)V^*_{cd}$	$T^0_{bb\bar d\bar s}\to \overline{B}^0\overline{D}^0\eta$	$\frac{(-b_1+2b_2+b_4+b_6-2b_7-b_8)V^*_{cs}}{\sqrt{6}}$
$T^0_{bb\bar d\bar s}\to \overline{B}^0D^-\pi^+$	$(-b_1+b_5+b_6-b_9)V^*_{cs}$	$T^0_{bb\bar d\bar s}\to \overline{B}^0D^-K^+$	$(b_2-b_5+b_7+b_9)V^*_{cd}$
$T^0_{bb\bar d\bar s}\to \overline{B}^0D_s^-K^+$	$(-b_1+b_2+b_6+b_7)V^*_{cs}$	$T^0_{bb\bar d\bar s}\to \overline{B}^0_s\overline{D}^0\pi^0$	$\frac{(b_1-b_2-b_6+b_7)V^*_{cd}}{\sqrt{2}}$
$T^0_{bb\bar d\bar s}\to \overline{B}^0_s\overline{D}^0\overline{K}^0$	$(b_2+b_4-b_7-b_8)V^*_{cs}$	$T^0_{bb\bar d\bar s}\to \overline{B}^0_s\overline{D}^0\eta$	$\frac{(b_1+b_2+2b_4-b_6-b_7-2b_8)V^*_{cd}}{\sqrt{6}}$
$T^0_{bb\bar d\bar s}\to \overline{B}^0_sD^-\pi^+$	$(b_1-b_2-b_6-b_7)V^*_{cd}$	$T^0_{bb\bar d\bar s}\to \overline{B}^0_sD_s^-\pi^+$	$-(b_2-b_5+b_7+b_9)V^*_{cs}$
$T^0_{bb\bar d\bar s}\to \overline{B}^0_sD_s^-K^+$	$(b_1-b_5-b_6+b_9)V^*_{cd}$	$T^-_{bb\bar u\bar d}\to B^-\overline{D}^0\pi^0$	$\frac{(-b_1+b_2+2b_3+b_4+b_6-b_7-b_8-2b_9)V^*_{cd}}{\sqrt{2}}$
$T^-_{bb\bar u\bar d}\to B^-\overline{D}^0\overline{K}^0$	$(-b_2+b_5+b_7+b_9)V^*_{cs}$	$T^-_{bb\bar u\bar d}\to B^-\overline{D}^0\eta$	$\frac{(-b_1-b_2+2b_3+b_4+2b_5+b_6+b_7-b_8)V^*_{cd}}{\sqrt{6}}$
$T^-_{bb\bar u\bar d}\to B^-D^-\pi^+$	$(-b_1+b_2+2b_3+b_4+b_6+b_7+b_8)V^*_{cd}$	$T^-_{bb\bar u\bar d}\to B^-D_s^-\pi^+$	$(b_2+b_4+b_7+b_8)V^*_{cs}$
$T^-_{bb\bar u\bar d}\to B^-D_s^-K^+$	$(-b_1+2b_3+b_6)V^*_{cd}$	$T^-_{bb\bar u\bar d}\to \overline{B}^0\overline{D}^0\pi^-$	$-(b_1-b_2-2b_3-b_4+b_6+b_7+b_8)V^*_{cd}$
$T^-_{bb\bar u\bar d}\to \overline{B}^0\overline{D}^0K^-$	$-(b_1-b_2+b_6+b_7)V^*_{cs}$	$T^-_{bb\bar u\bar d}\to \overline{B}^0D^-\pi^0$	$\frac{(b_1-b_2-2b_3-b_4+b_6-b_7-b_8-2b_9)V^*_{cd}}{\sqrt{2}}$
$T^-_{bb\bar u\bar d}\to \overline{B}^0D^-\overline{K}^0$	$(-b_1+b_5-b_6+b_9)V^*_{cs}$	$T^-_{bb\bar u\bar d}\to \overline{B}^0D^-\eta$	$-\frac{(b_1+b_2-2b_3-b_4-2b_5+b_6+b_7-b_8)V^*_{cd}}{\sqrt{6}}$
$T^-_{bb\bar u\bar d}\to \overline{B}^0D_s^-\pi^0$	$-\frac{(b_2+b_4+b_7+b_8)V^*_{cs}}{\sqrt{2}}$	$T^-_{bb\bar u\bar d}\to \overline{B}^0D_s^-K^0$	$-(b_1-2b_3+b_6)V^*_{cd}$
$T^-_{bb\bar u\bar d}\to \overline{B}^0D_s^-\eta$	$\frac{(2b_1-b_2+b_4+2b_6-b_7+b_8)V^*_{cs}}{\sqrt{6}}$	$T^-_{bb\bar u\bar d}\to \overline{B}^0_s\overline{D}^0K^-$	$(2b_3+b_4-b_8)V^*_{cd}$
$T^-_{bb\bar u\bar d}\to \overline{B}^0_sD^-\overline{K}^0$	$(2b_3+b_4+b_8)V^*_{cd}$	$T^-_{bb\bar u\bar d}\to \overline{B}^0_sD_s^-\pi^0$	$-\sqrt{2}b_9V^*_{cd}$
$T^-_{bb\bar u\bar d}\to \overline{B}^0_sD_s^-\overline{K}^0$	$(b_4+b_5+b_8+b_9)V^*_{cs}$	$T^-_{bb\bar u\bar d}\to \overline{B}^0_sD_s^-\eta$	$\sqrt{\frac{2}{3}}(b_5-2b_3)V^*_{cd}$

The related decay amplitudes of an anticharmed bottom meson plus two light mesons are given in Table XIX. Thus, we obtain the relations as follows:

$$\Gamma(T^-_{bb\bar u\bar s}\to B_c^-\pi^0\pi^0)=\frac{1}{2}\Gamma(T^-_{bb\bar u\bar s}\to B_c^-\pi^+\pi^-),\qquad \Gamma(T^-_{bb\bar u\bar d}\to B_c^-\pi^0\pi^0)=\frac{1}{2}\Gamma(T^-_{bb\bar u\bar d}\to B_c^-\pi^+\pi^-),$$

$$\Gamma(T^-_{bb\bar u\bar s}\to B_c^-\pi^0K^0)=3\Gamma(T^-_{bb\bar u\bar s}\to B_c^-K^0\eta)=\frac{1}{2}\Gamma(T^-_{bb\bar u\bar s}\to B_c^-\pi^-K^+),$$

$$\Gamma(T^0_{bb\bar d\bar s}\to B_c^-\pi^+K^0)=6\Gamma(T^0_{bb\bar d\bar s}\to B_c^-K^+\eta)=2\Gamma(T^0_{bb\bar d\bar s}\to B_c^-\pi^0K^+),$$

$$\Gamma(T^0_{bb\bar d\bar s}\to B_c^-K^+\overline{K}^0)=3\Gamma(T^-_{bb\bar u\bar s}\to B_c^-\pi^0\eta)=\frac{3}{2}\Gamma(T^0_{bb\bar d\bar s}\to B_c^-\pi^+\eta),$$

$$\Gamma(T^-_{bb\bar u\bar d}\to B_c^-\pi^+K^-)=6\Gamma(T^-_{bb\bar u\bar d}\to B_c^-\overline{K}^0\eta)=2\Gamma(T^-_{bb\bar u\bar d}\to B_c^-\pi^0\overline{K}^0),$$

$$\Gamma(T^-_{bb\bar u\bar d}\to B_c^-\pi^0\overline{K}^0)=3\Gamma(T^-_{bb\bar u\bar d}\to B_c^-\eta\overline{K}^0).$$

TABLE XIX. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into an anticharmed B meson and two light mesons.

Channel	Amplitude $(/V_{ub})$	Channel	Amplitude $(/V_{ub})$
$T^-_{bb\bar{u}\bar{s}} \to B_c^- \pi^+ \pi^-$	$(4b_{10}+b_{11}-b_{13})V^*_{cs}$	$T^-_{bb\bar{u}\bar{s}} \to B_c^- \pi^0 \pi^0$	$(4b_{10}+b_{11}-b_{13})V^*_{cs}$
$T^-_{bb\bar{u}\bar{s}} \to B_c^- \pi^0 K^0$	$-\frac{(b_{11}-2b_{12}+b_{13})V^*_{cd}}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{s}} \to B_c^- \pi^0 \eta$	$\frac{(b_{11}-2b_{12}-b_{13})V^*_{cs}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{s}} \to B_c^- \pi^- K^+$	$(b_{11}-2b_{12}+b_{13})V^*_{cd}$	$T^-_{bb\bar{u}\bar{s}} \to B_c^- K^+ K^-$	$2(2b_{10}+b_{11}-b_{12})V^*_{cs}$
$T^-_{bb\bar{u}\bar{s}} \to B_c^- K^0 \overline{K}^0$	$(4b_{10}+b_{11}+b_{13})V^*_{cs}$	$T^-_{bb\bar{u}\bar{s}} \to B_c^- K^0 \eta$	$-\frac{(b_{11}-2b_{12}+b_{13})V^*_{cd}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{s}} \to B_c^- \eta\eta$	$\frac{1}{3}(12b_{10}+5b_{11}-4b_{12}+3b_{13})V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to B_c^- \pi^+ K^0$	$(-b_{11}+2b_{12}+b_{13})V^*_{cd}$
$T^0_{bb\bar{d}\bar{s}} \to B_c^- \pi^+ \eta$	$\sqrt{\frac{2}{3}}(b_{11}-2b_{12}-b_{13})V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to B_c^- \pi^0 K^+$	$\frac{(-b_{11}+2b_{12}+b_{13})V^*_{cd}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to B_c^- K^+ \overline{K}^0$	$(b_{11}-2b_{12}-b_{13})V^*_{cs}$	$T^0_{bb\bar{d}\bar{s}} \to B_c^- K^+ \eta$	$\frac{(b_{11}-2b_{12}-b_{13})V^*_{cd}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to B_c^- \pi^+ \pi^-$	$2(2b_{10}+b_{11}-b_{12})V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to B_c^- \pi^+ K^-$	$(b_{11}-2b_{12}+b_{13})V^*_{cs}$
$T^-_{bb\bar{u}\bar{d}} \to B_c^- \pi^0 \pi^0$	$2(2b_{10}+b_{11}-b_{12})V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to B_c^- \pi^0 \overline{K}^0$	$-\frac{(b_{11}-2b_{12}+b_{13})V^*_{cs}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to B_c^- \pi^0 \eta$	$-\frac{2b_{13}V^*_{cd}}{\sqrt{3}}$	$T^-_{bb\bar{u}\bar{d}} \to B_c^- K^+ K^-$	$(4b_{10}+b_{11}-b_{13})V^*_{cd}$
$T^-_{bb\bar{u}\bar{d}} \to B_c^- K^0 \overline{K}^0$	$(4b_{10}+b_{11}+b_{13})V^*_{cd}$	$T^-_{bb\bar{u}\bar{d}} \to B_c^- \overline{K}^0 \eta$	$-\frac{(b_{11}-2b_{12}+b_{13})V^*_{cs}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to B_c^- \eta\eta$	$\frac{2}{3}(6b_{10}+b_{11}+b_{12})V^*_{cd}$		

D. Charmless $b \to q_1\bar{q}_2 q_3$ transition

1. Decays into a bottom meson and a light meson by the W-exchange process

The bottom to light quark transition leads to the effective Hamiltonian:

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}\left\{V_{ub}V^*_{uq}[C_1 O_1^{\bar{u}u} + C_2 O_2^{\bar{u}u}] - V_{tb}V^*_{tq}\left[\sum_{i=3}^{10} C_i O_i\right]\right\} + \mathrm{H.c.}, \tag{33}$$

where O_i is the weak four-fermion effective operator. The tree operators are described as a vector $H_{\mathbf{3}}$, a tensor $H_{\bar{\mathbf{6}}}$, and a tensor $H_{\mathbf{15}}$. The penguin operators are described as another vector $H_{\mathbf{3}}$. The nonzero components of these operators are

$$\begin{aligned}
&(H_3)^2 = 1,\\
&(H_{\bar{6}})^{12}_1 = -(H_{\bar{6}})^{21}_1 = (H_{\bar{6}})^{23}_3 = -(H_{\bar{6}})^{32}_3 = 1,\\
&2(H_{15})^{12}_1 = 2(H_{15})^{21}_1 = -3(H_{15})^{22}_2 = -6(H_{15})^{23}_3\\
&\qquad = -6(H_{15})^{32}_3 = 6,
\end{aligned} \tag{34}$$

for the nonstrange decays. After doing the exchange of $2 \leftrightarrow 3$, we will get the formulas for the $\Delta S = 1 (b \to s)$ decays. We get the effective hadron-level Hamiltonian for decays into the bottom antitriplet

TABLE XX. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a bottom meson and a light meson induced by the charmless $b \to d$ transition.

Channel	Amplitude
$T^-_{bb\bar{u}\bar{s}} \to B^- K^0$	$f_{11}+f_{13}+f_{14}+f_{15}+3f_{16}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}{}^0_s \pi^-$	$-f_{11}+f_{13}+f_{14}+3f_{15}+f_{16}$
$T^0_{bb\bar{d}\bar{s}} \to B^- K^+$	$f_{10}+2f_{12}-f_{14}+3f_{16}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}{}^0 K^0$	$f_{10}+f_{11}+2f_{12}+f_{13}+f_{15}-2f_{16}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}{}^0_s \pi^0$	$\frac{f_{11}-f_{13}-f_{14}+5f_{15}-f_{16}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}{}^0_s \eta$	$-\frac{2f_{10}+f_{11}+4f_{12}+3f_{13}+f_{14}-3f_{15}-3f_{16}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to B^- \pi^0$	$-\frac{f_{10}+f_{11}-2f_{12}-f_{13}+5f_{15}+6f_{16}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to B^- \eta$	$\frac{-f_{10}+f_{11}+2f_{12}+3f_{13}+2f_{14}-3f_{15}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}{}^0 \pi^-$	$-f_{10}-f_{11}+2f_{12}+f_{13}+3f_{15}+2f_{16}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}{}^0_s K^-$	$-f_{10}+2f_{12}-f_{14}+f_{16}$

TABLE XXI. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a bottom meson and a light meson induced by the charmless $b \to s$ transition.

Channel	Amplitude
$T^-_{bb\bar{u}\bar{s}} \to B^- \pi^0$	$\frac{-f_{10}+2f_{12}+2f_{13}+f_{14}-4f_{15}-3f_{16}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to B^- \eta$	$-\frac{f_{10}+2f_{11}-2f_{12}+f_{14}+6f_{15}+9f_{16}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}{}^0 \pi^-$	$-f_{10}+2f_{12}-f_{14}+f_{16}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}{}^0_s K^-$	$-f_{10}-f_{11}+2f_{12}+f_{13}+3f_{15}+2f_{16}$
$T^0_{bb\bar{d}\bar{s}} \to B^- \pi^+$	$-f_{10}-2f_{12}+f_{14}-3f_{16}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}{}^0 \pi^0$	$\frac{f_{10}+2f_{12}+2f_{13}+f_{14}-4f_{15}-f_{16}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}{}^0 \eta$	$-\frac{f_{10}+2f_{11}+2f_{12}-f_{14}+6f_{15}-3f_{16}}{\sqrt{6}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}{}^0_s \overline{K}^0$	$-f_{10}-f_{11}-2f_{12}-f_{13}-f_{15}+2f_{16}$
$T^-_{bb\bar{u}\bar{d}} \to B^- \overline{K}^0$	$f_{11}+f_{13}+f_{14}+f_{15}+3f_{16}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}{}^0 K^-$	$-f_{11}+f_{13}+f_{14}+3f_{15}+f_{16}$

$$
\begin{aligned}
\mathcal{H}_{\rm eff} = {} & f_{10}(T_{bb3})_{[ij]}(\overline{B})^k(H_3)^i M^j_k + f_{11}(T_{bb3})_{[ij]}(\overline{B})^i(H_3)^k M^j_k \\
& + f_{12}(T_{bb3})_{[ij]}(\overline{B})^l(H_{\bar 6})^{[ij]}_k M^k_l + f_{13}(T_{bb3})_{[ij]}(\overline{B})^j(H_{\bar 6})^{[il]}_k M^k_l \\
& + f_{14}(T_{bb3})_{[ij]}(\overline{B})^k(H_{\bar 6})^{[il]}_k M^j_l + f_{15}(T_{bb3})_{[ij]}(\overline{B})^j(H_{15})^{\{il\}}_k M^k_l \\
& + f_{16}(T_{bb3})_{[ij]}(\overline{B})^k(H_{15})^{\{il\}}_k M^j_l.
\end{aligned} \tag{35}
$$

The amplitudes are given in Table XX for the $\Delta S = 0 (b \to d)$ decays and Table XXI for the $\Delta S = 1 (b \to s)$ decays.

2. Decays into a light antibaryon and a bottom baryon

There are four kinds of different final states which are light octet or antidecuplet antibaryon plus antitriplet or sextet baryon, respectively. Thus, the Hamiltonian becomes

$$
\begin{aligned}
\mathcal{H}_{\rm eff} = {} & c_1(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^l_x(H_3)^i(\overline{F}_{b\bar 3})_{[kl]} + \bar c_1(T_{bb3})_{[ij]}\epsilon^{xkl}(F_8)^j_x(H_3)^i(\overline{F}_{b\bar 3})_{[kl]} \\
& + c_2(T_{bb3})_{[ij]}\epsilon^{xij}(F_8)^l_x(H_3)^k(\overline{F}_{b\bar 3})_{[kl]} + \bar c_2(T_{bb3})_{[ij]}\epsilon^{xik}(F_8)^j_x(H_3)^l(\overline{F}_{b\bar 3})_{[kl]} \\
& + c_3(T_{bb3})_{[ij]}\epsilon^{xkl}(F_8)^m_x(H_{\bar 6})^{[ij]}_k(\overline{F}_{b\bar 3})_{[lm]} + \bar c_3(T_{bb3})_{[ij]}\epsilon^{xlm}(F_8)^k_x(H_{\bar 6})^{[ij]}_k(\overline{F}_{b\bar 3})_{[lm]} \\
& + \bar c_{3'}(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^m_x(H_{\bar 6})^{[il]}_k(\overline{F}_{b\bar 3})_{[lm]} + c_4(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^m_x(H_{\bar 6})^{[il]}_m(\overline{F}_{b\bar 3})_{[kl]} \\
& + c_5(T_{bb3})_{[ij]}\epsilon^{xlm}(F_8)^j_x(H_{\bar 6})^{[ik]}_l(\overline{F}_{b\bar 3})_{[km]} + \bar c_5(T_{bb3})_{[ij]}\epsilon^{xij}(F_8)^m_x(H_{\bar 6})^{[kl]}_m(\overline{F}_{b\bar 3})_{[kl]} \\
& + \bar c_{5'}(T_{bb3})_{[ij]}\epsilon^{xim}(F_8)^j_x(H_{\bar 6})^{[kl]}_m(\overline{F}_{b\bar 3})_{[kl]} + c_6(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^m_x(H_{15})^{\{il\}}_k(\overline{F}_{b\bar 3})_{[lm]} \\
& + c_7(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^m_x(H_{15})^{\{il\}}_m(\overline{F}_{b\bar 3})_{[kl]} + c_8(T_{bb3})_{[ij]}\epsilon^{xlm}(F_8)^j_x(H_{15})^{\{ik\}}_l(\overline{F}_{b\bar 3})_{[km]} \\
& + d_1(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^l_x(H_3)^i(\overline{F}_{b6})_{\{kl\}} + \bar d_1(T_{bb3})_{[ij]}\epsilon^{xkl}(F_8)^m_x(H_{\bar 6})^{[ij]}_k(\overline{F}_{b6})_{\{lm\}} \\
& + \bar d_{1'}(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^m_x(H_{\bar 6})^{[il]}_k(\overline{F}_{b6})_{\{lm\}} + d_2(T_{bb3})_{[ij]}\epsilon^{xij}(F_8)^l_x(H_3)^k(\overline{F}_{b6})_{\{kl\}} \\
& + \bar d_2(T_{bb3})_{[ij]}\epsilon^{xik}(F_8)^j_x(H_3)^l(\overline{F}_{b6})_{\{kl\}} + d_3(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^m_x(H_{\bar 6})^{[il]}_m(\overline{F}_{b6})_{\{kl\}} \\
& + d_4(T_{bb3})_{[ij]}\epsilon^{xlm}(F_8)^j_x(H_{\bar 6})^{[ik]}_l(\overline{F}_{b6})_{\{km\}} + d_5(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^m_x(H_{15})^{\{il\}}_k(\overline{F}_{b6})_{\{lm\}} \\
& + d_6(T_{bb3})_{[ij]}\epsilon^{xjk}(F_8)^m_x(H_{15})^{\{il\}}_m(\overline{F}_{b6})_{\{kl\}} + d_7(T_{bb3})_{[ij]}\epsilon^{xlm}(F_8)^j_x(H_{15})^{\{ik\}}_l(\overline{F}_{b6})_{\{km\}} \\
& + d_8(T_{bb3})_{[ij]}\epsilon^{xij}(F_8)^m_x(H_{15})^{\{kl\}}_m(\overline{F}_{b6})_{\{kl\}} + \bar d_8(T_{bb3})_{[ij]}\epsilon^{xim}(F_8)^j_x(H_{15})^{\{kl\}}_m(\overline{F}_{b6})_{\{kl\}} \\
& + a_1(T_{bb3})_{[ij]}(F_{\overline{10}})^{\{jkl\}}(H_{\bar 6})^{[im]}_k(\overline{F}_{b\bar 3})_{[lm]} + a_2(T_{bb3})_{[ij]}(F_{\overline{10}})^{\{jkl\}}(H_{15})^{\{im\}}_k(\overline{F}_{b\bar 3})_{[lm]} \\
& + b_1(T_{bb3})_{[ij]}(F_{\overline{10}})^{\{jkl\}}(H_3)^i_k(\overline{F}_{b6})_{\{kl\}} + b_2(T_{bb3})_{[ij]}(F_{\overline{10}})^{\{klm\}}(H_{\bar 6})^{[ij]}_k(\overline{F}_{b6})_{\{lm\}} \\
& + b_3(T_{bb3})_{[ij]}(F_{\overline{10}})^{\{jkl\}}(H_{\bar 6})^{[im]}_k(\overline{F}_{b6})_{\{lm\}} + b_4(T_{bb3})_{[ij]}(F_{\overline{10}})^{\{jkl\}}(H_{15})^{\{im\}}_k(\overline{F}_{b6})_{\{lm\}}.
\end{aligned} \tag{36}
$$

Decay amplitudes are given in Table XXII for the transition $b \to d$, and Table XXIII for the transition $b \to s$. We remove the similar contributions in the amplitudes, such as $c_1 - 2\bar c_1, 2c_2 - \bar c_2, c_3 - 2\bar c_3 + \bar c_{3'}, c_5 + 2\bar c_5 + \bar c_{5'}, 2d_2 + \bar d_2, d_1 - 2\bar d_1 + \bar d_{1'}$, and $2d_8 + \bar d_8$. There is no relation of decay widths for class I. The relations of decay widths for class II become

$$
\Gamma(T^0_{bb\bar d\bar s} \to \bar\Sigma^+\Sigma^-_b) = 2\Gamma(T^0_{bb\bar d\bar s} \to \bar\Xi^+\Xi'^-_b).
$$

The relations of decay widths for class III become

$$
\begin{aligned}
&\Gamma(T^-_{bb\bar u\bar s} \to \bar\Sigma'^-\Lambda^0_b) = \Gamma(T^-_{bb\bar u\bar s} \to \bar\Xi'^0\Xi^-_b), && \Gamma(T^-_{bb\bar u\bar s} \to \bar\Sigma'^-\Lambda^0_b) = 2\Gamma(T^-_{bb\bar u\bar d} \to \bar\Sigma'^0\Xi^-_b), \\
&\Gamma(T^0_{bb\bar d\bar s} \to \bar\Sigma'^0\Lambda^0_b) = \frac{1}{2}\Gamma(T^0_{bb\bar d\bar s} \to \bar\Xi'^0\Xi^0_b), && \Gamma(T^0_{bb\bar d\bar s} \to \bar\Sigma'^0\Lambda^0_b) = \frac{1}{2}\Gamma(T^-_{bb\bar u\bar d} \to \bar\Sigma'^-\Xi^0_b), \\
&\Gamma(T^-_{bb\bar u\bar d} \to \bar\Sigma'^-\Xi^0_b) = \Gamma(T^0_{bb\bar d\bar s} \to \bar\Xi'^0\Xi^0_b), && \Gamma(T^-_{bb\bar u\bar d} \to \bar\Sigma'^0\Xi^-_b) = \frac{1}{2}\Gamma(T^-_{bb\bar u\bar s} \to \bar\Xi'^0\Xi^-_b).
\end{aligned}
$$

The relations of decay widths for class IV become

TABLE XXII. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a light antibaryon octet and a bottom baryon antitriplet (class I) or sextet (class II), a light antibaryon antidecuplet and a bottom baryon antitriplet (class III) or sextet (class IV) induced by the charmless $b \to d$ transition.

Class I	Amplitude		
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^- \Lambda^0_b$	$2c_2 + c_4 - c_5 + 2c_6 + 3c_7 - c_8$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0 \Xi^-_b$	$-2c_2 + c_4 - c_5 - 2c_6 + c_7 - 3c_8$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0 \Lambda^0_b$	$\frac{2c_1-2c_2+4c_3+c_4+c_5+3c_7-3c_8}{\sqrt{6}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0 \Lambda^0_b$	$\frac{-2c_2-c_4+c_5+6c_6+5c_7+c_8}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+ \Xi^-_b$	$-c_1 + 2c_2 - 2c_3 - 2c_4 + c_5 - 3c_6 - 3c_8$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^0 \Xi^0_b$	$c_1 + 2c_3 + c_4 - 3c_6 - c_7 - 2c_8$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Lambda}^0 \Xi^-_b$	$\frac{c_1-4c_2-2c_3+c_4-2c_5-3c_6-3c_7}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^- \Xi^0_b$	$-c_1 + 2c_3 + c_4 - c_6 - 3c_7 + 2c_8$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0 \Xi^-_b$	$\frac{c_1-2c_3-c_4+c_6-5c_7+6c_8}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{p} \Lambda^0_b$	$c_1 - 2c_2 - 2c_3 - 2c_4 + c_5 - c_6 - c_8$

Class II	Amplitude		
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Lambda}^0 \Sigma^-_b$	$-\frac{2d_2+3d_3+3d_4-2d_5+5d_6+7d_7-4d_8}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^- \Sigma^0_b$	$\frac{-2d_2+d_3+d_4+2d_5+3d_6+d_7-12d_8}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^0 \Sigma^-_b$	$\frac{2d_2-d_3-d_4-2d_5-3d_6-d_7-4d_8}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0 \Xi'^-_b$	$-\frac{2d_2+d_3+d_4-2d_5+d_6+3d_7-4d_8}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0 \Sigma^0_b$	$\frac{2d_2+3(d_3+d_4+2d_5-d_6-3d_7+4d_8)}{2\sqrt{3}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^- \Sigma^+_b$	$d_1 - d_3 + 3d_5 + 3d_6$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0 \Sigma^0_b$	$\frac{1}{2}(-2d_1 + 2d_2 + d_3 - d_4 - d_6 - d_7 + 12d_8)$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+ \Sigma^-_b$	$-d_1 + 2d_2 - d_4 + 3d_5 + 2d_6 - d_7 - 4d_8$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+ \Xi'^-_b$	$-\frac{d_1-2d_2+d_4-3d_5-2d_6+d_7+4d_8}{\sqrt{2}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^0 \Xi'^0_b$	$\frac{d_1+d_3+2d_4+3d_5-d_6-4d_7}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Lambda}^0 \Xi'^-_b$	$\frac{3d_1-4d_2+3d_3+d_5-d_6-2d_7+8d_8}{2\sqrt{3}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^- \Xi'^0_b$	$\frac{d_1-d_3-2d_4-d_5+3d_6+4d_7}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0 \Xi'^-_b$	$\frac{1}{2}(-d_1 + d_3 + 2d_4 + d_5 + 5d_6 + 4d_7)$	$T^-_{bb\bar{u}\bar{d}} \to \bar{p} \Sigma^0_b$	$\frac{-d_1+2d_2+d_4-d_5-6d_6-5d_7+12d_8}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{n} \Sigma^-_b$	$-d_1 + 2d_2 + d_4 - d_5 + 2d_6 + 3d_7 - 4d_8$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}^0 \Omega^-_b$	$d_1 + d_3 - d_5 - d_6$

Class III	Amplitude		
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^- \Lambda^0_b$	$\frac{2(a_1+2a_2)}{\sqrt{3}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0 \Xi^-_b$	$-\frac{2(a_1+2a_2)}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0 \Lambda^0_b$	$\sqrt{\frac{2}{3}}(a_1 - 2a_2)$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^0 \Xi^0_b$	$\frac{2(a_1-2a_2)}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^- \Lambda^0_b$	$\frac{8a_2}{\sqrt{3}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^- \Xi^0_b$	$-\frac{2(a_1-2a_2)}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^0 \Xi^-_b$	$-\sqrt{\frac{2}{3}}(a_1 + 2a_2)$		

Class IV	Amplitude		
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^- \Sigma^0_b$	$\sqrt{\frac{2}{3}}(b_3 + 2b_4)$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^0 \Sigma^-_b$	$\sqrt{\frac{2}{3}}(b_3 + 2b_4)$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0 \Xi'^-_b$	$\sqrt{\frac{2}{3}}(b_3 + 2b_4)$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^- \Sigma^+_b$	$\frac{b_1+2b_2-b_3+3b_4}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0 \Sigma^0_b$	$\frac{b_1+2b_2+b_4}{\sqrt{3}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^+ \Sigma^-_b$	$\frac{b_1+2b_2+b_3-b_4}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^0 \Xi'^0_b$	$\sqrt{\frac{2}{3}}(b_1 + 2b_2 + b_4)$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^+ \Xi'^-_b$	$\sqrt{\frac{2}{3}}(b_1 + 2b_2 + b_3 - b_4)$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Omega}^+ \Omega^-_b$	$b_1 + 2b_2 + b_3 - b_4$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^{--} \Sigma^+_b$	$-b_1 + 2b_2 + b_3 - 3b_4$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^- \Sigma^0_b$	$\sqrt{\frac{2}{3}}(-b_1 + 2b_2 + b_3 + b_4)$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^0 \Sigma^-_b$	$\frac{-b_1+2b_2+b_3+5b_4}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^- \Xi'^0_b$	$-\sqrt{\frac{2}{3}}(b_1 - 2b_2 + b_4)$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^0 \Xi'^-_b$	$\frac{-b_1+2b_2+3b_4}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Xi}'^0 \Omega^-_b$	$-\frac{b_1-2b_2+b_3-b_4}{\sqrt{3}}$		

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^- \Sigma^0_b) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^0 \Sigma^-_b), \qquad \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^- \Sigma^0_b) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0 \Xi'^-_b),$$

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^0 \Sigma^-_b) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0 \Xi'^-_b), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0 \Sigma^0_b) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^0 \Xi'^0_b),$$

$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^+ \Sigma^-_b) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^+ \Xi'^-_b), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^+ \Sigma^-_b) = \frac{1}{3}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Omega}^+ \Omega^-_b),$$

$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^+ \Xi'^-_b) = \frac{2}{3}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Omega}^+ \Omega^-_b).$$

TABLE XXIII. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a light antibaryon octet and a bottom baryon antitriplet (class I) or sextet (class II), a light antibaryon antidecuplet and a bottom baryon antitriplet (class III) or sextet (class IV) induced by the charmless $b \to s$ transition.

Class I	Amplitude		
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Lambda}^0\Xi^-_b$	$\frac{c_1+2c_2-2c_3-2c_4+c_5+3c_6-6c_7+9c_8}{\sqrt{6}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^-\Xi^0_b$	$-c_1+2c_2+2c_3+2c_4-c_5+c_6+c_8$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^0\Xi^-_b$	$\frac{c_1-2c_2-2c_3-c_5-c_6-4c_7+3c_8}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{p}\Lambda^0_b$	$c_1-2c_3-c_4+c_6+3c_7-2c_8$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0\Xi^0_b$	$\frac{-c_1-2c_2-2c_3-2c_4+c_5+9c_6+6c_7+3c_8}{\sqrt{6}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0\Xi^0_b$	$\frac{c_1-2c_2+2c_3+c_5+3c_6+4c_7-c_8}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+\Xi^-_b$	$c_1-2c_2+2c_3+2c_4-c_5+3c_6+3c_8$	$T^0_{bb\bar{d}\bar{s}} \to \bar{n}\Lambda^0_b$	$c_1+2c_3+c_4-3c_6-c_7-2c_8$
$T^-_{bb\bar{u}\bar{d}} \to \bar{p}\Xi^0_b$	$-2c_2-c_4+c_5-2c_6-3c_7+c_8$	$T^-_{bb\bar{u}\bar{d}} \to \bar{n}\Xi^-_b$	$-2c_2+c_4-c_5-2c_6+c_7-3c_8$

Class II	Amplitude		
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Lambda}^0\Xi'^-_b$	$\frac{3d_1-2d_2-3d_4-d_5-8d_6-7d_7+4d_8}{2\sqrt{3}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^-\Xi'^0_b$	$\frac{d_1-2d_2-d_4+d_5+6d_6+5d_7-12d_8}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}^0\Xi'^-_b$	$\frac{-d_1+2d_2-2d_3-d_4-d_5-2d_6-d_7-4d_8}{2}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{p}\Sigma^0_b$	$\frac{-d_1+d_3+2d_4+d_5-3d_6-4d_7}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{n}\Sigma^-_b$	$-d_1-d_3+d_5+d_6$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}^0\Omega^-_b$	$d_1-2d_2-d_4+d_5-2d_6-3d_7+4d_8$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Lambda}^0\Xi'^0_b$	$\frac{-3d_1+2d_2-3d_4-3d_5+3d_7+12d_8}{2\sqrt{3}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0\Xi'^0_b$	$\frac{-d_1+2d_2+2d_3+d_4+3d_5-2d_6-5d_7+12d_8}{2}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+\Xi'^-_b$	$-\frac{d_1-2d_2+d_4-3d_5-2d_6+d_7+4d_8}{\sqrt{2}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{p}\Sigma^+_b$	$d_1-d_3+3d_5+3d_6$
$T^0_{bb\bar{d}\bar{s}} \to \bar{n}\Sigma^0_b$	$\frac{d_1+d_3+2d_4+3d_5-d_6-4d_7}{\sqrt{2}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+\Omega^-_b$	$-d_1+2d_2-d_4+3d_5+2d_6-d_7-4d_8$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Lambda}^0\Omega^-_b$	$\sqrt{\frac{2}{3}}(-2d_2+2d_5+d_6-d_7+4d_8)$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}^0\Omega^-_b$	$\sqrt{2}(d_3+d_4+2(d_6+d_7))$
$T^-_{bb\bar{u}\bar{d}} \to \bar{p}\Xi'^0_b$	$\frac{2d_2-d_3-d_4-2d_5-3d_6-d_7+12d_8}{\sqrt{2}}$	$T^-_{bb\bar{u}\bar{d}} \to \bar{n}\Xi'^-_b$	$\frac{2d_2+d_3+d_4-2d_5+d_6+3d_7-4d_8}{\sqrt{2}}$

Class III	Amplitude		
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^-\Lambda^0_b$	$-\frac{2(a_1-2a_2)}{\sqrt{3}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^-\Xi^0_b$	$\frac{8a_2}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^0\Xi^-_b$	$\sqrt{\frac{2}{3}}(a_1+2a_2)$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^0\Lambda^0_b$	$-\frac{2(a_1-2a_2)}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0\Xi^0_b$	$-\sqrt{\frac{2}{3}}(a_1-2a_2)$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^-\Xi^0_b$	$\frac{2(a_1+2a_2)}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^0\Xi^-_b$	$\frac{2(a_1+2a_2)}{\sqrt{3}}$		

Class IV	Amplitude		
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^{--}\Sigma^+_b$	$-b_1+2b_2+b_3-3b_4$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^-\Sigma^0_b$	$-\sqrt{\frac{2}{3}}(b_1-2b_2+b_4)$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^0\Sigma^-_b$	$-\frac{b_1-2b_2+b_3-b_4}{\sqrt{3}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^-\Xi'^0_b$	$\sqrt{\frac{2}{3}}(-b_1+2b_2+b_3+b_4)$
$T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^0\Xi'^-_b$	$\frac{-b_1+2b_2+3b_4}{\sqrt{3}}$	$T^-_{bb\bar{u}\bar{s}} \to \bar{\Xi}'^0\Omega^-_b$	$\frac{-b_1+2b_2+b_3+5b_4}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^-\Sigma^+_b$	$-\frac{b_1+2b_2-b_3+3b_4}{\sqrt{3}}$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^0\Sigma^0_b$	$-\sqrt{\frac{2}{3}}(b_1+2b_2+b_4)$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^+\Sigma^-_b$	$-b_1-2b_2-b_3+b_4$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0\Xi'^0_b$	$-\frac{b_1+2b_2+b_4}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^+\Xi'^-_b$	$-\sqrt{\frac{2}{3}}(b_1+2b_2+b_3-b_4)$	$T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^+\Omega^-_b$	$-\frac{b_1+2b_2+b_3-b_4}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^-\Xi'^0_b$	$\sqrt{\frac{2}{3}}(b_3+2b_4)$	$T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^0\Xi'^-_b$	$\sqrt{\frac{2}{3}}(b_3+2b_4)$
$T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^0\Omega^-_b$	$\sqrt{\frac{2}{3}}(b_3+2b_4)$		

There are no relations of decay widths for class I.

The relations of decay widths for class II become

$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^+ \Xi'^-_b) = \frac{1}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}^+ \Omega^-_b).$$

The relations of decay widths for class III are

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^- \Lambda^0_b) = 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0 \Xi^0_b), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^0 \Lambda^0_b) = 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}^0 \Xi^0_b),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^- \Xi^0_b) = 2\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^0 \Xi^-_b), \qquad \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^- \Xi^0_b) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^0 \Xi^-_b),$$
$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Delta}^- \Lambda^0_b) = \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^0 \Lambda^0_b), \qquad \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^0 \Xi^-_b) = 2\Gamma(T^-_{bb\bar{u}\bar{s}} \to \bar{\Sigma}'^0 \Xi^-_b).$$

The relations of decay widths for class IV become

$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^0 \Sigma^0_b) = 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^0 \Xi'^0_b), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^+ \Sigma^-_b) = \frac{3}{2}\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^+ \Xi'^-_b),$$
$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Delta}^+ \Sigma^-_b) = 3\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^+ \Omega^-_b), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Sigma}'^+ \Xi'^-_b) = 2\Gamma(T^0_{bb\bar{d}\bar{s}} \to \bar{\Xi}'^+ \Omega^-_b),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^- \Xi'^0_b) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^0 \Xi'^-_b), \qquad \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^- \Xi'^0_b) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^0 \Omega^-_b),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Delta}^0 \Xi'^-_b) = \Gamma(T^-_{bb\bar{u}\bar{d}} \to \bar{\Sigma}'^0 \Omega^-_b).$$

3. Decays into a bottom meson and two light mesons

The Hamiltonian for decays into a bottom meson and two light mesons is

$$\begin{aligned}
\mathcal{H}_{\rm eff} = {}& c_1 (T_{bb3})_{[ij]} (\overline{B})^i M^k_l M^l_k (H_3)^j + c_2 (T_{bb3})_{[ij]} (\overline{B})^i M^j_k M^k_l (H_3)^l \\
&+ c_3 (T_{bb3})_{[ij]} (\overline{B})^l M^j_k M^k_l (H_3)^i + c_4 (T_{bb3})_{[ij]} (\overline{B})^k M^i_k M^j_l (H_3)^l \\
&+ c_5 (T_{bb3})_{[ij]} (\overline{B})^m M^i_k M^j_l (H_{\bar{6}})^{[kl]}_m + c_6 (T_{bb3})_{[ij]} (\overline{B})^l M^i_m M^m_k (H_{\bar{6}})^{[jk]}_l \\
&+ c_7 (T_{bb3})_{[ij]} (\overline{B})^m M^i_k M^l_m (H_{\bar{6}})^{[jk]}_l + c_8 (T_{bb3})_{[ij]} (\overline{B})^m M^i_m M^l_k (H_{\bar{6}})^{[jk]}_l \\
&+ c_9 (T_{bb3})_{[ij]} (\overline{B})^k M^l_m M^m_l (H_{\bar{6}})^{[ij]}_k + c_{10} (T_{bb3})_{[ij]} (\overline{B})^m M^k_l M^l_m (H_{\bar{6}})^{[ij]}_k \\
&+ c_{11} (T_{bb3})_{[ij]} (\overline{B})^i M^l_m M^m_k (H_{\bar{6}})^{[jk]}_l + c_{12} (T_{bb3})_{[ij]} (\overline{B})^i M^j_k M^m_l (H_{15})^{\{kl\}}_m \\
&+ c_{13} (T_{bb3})_{[ij]} (\overline{B})^i M^l_m M^m_k (H_{15})^{\{jk\}}_l + c_{14} (T_{bb3})_{[ij]} (\overline{B})^l M^i_m M^m_k (H_{15})^{\{jk\}}_l \\
&+ c_{15} (T_{bb3})_{[ij]} (\overline{B})^m M^i_k M^l_m (H_{15})^{\{jk\}}_l + c_{16} (T_{bb3})_{[ij]} (\overline{B})^m M^i_m M^l_k (H_{15})^{\{jk\}}_l \\
&+ \overline{c_{11}} (T_{bb3})_{[ij]} (\overline{B})^i M^j_k M^m_l (H_{\bar{6}})^{[kl]}_m.
\end{aligned} \tag{37}$$

The $\overline{c_{11}}$ and c_{11} terms give the same contribution which always contains the factor $c_{11} - \overline{c_{11}}$. We remove the $\overline{c_{11}}$ term in the expanded amplitudes. The amplitudes are given in Table XXIV for the transition $b \to d$ and Table XXV for the transition $b \to s$. The relations are

$$\Gamma(T^-_{bb\bar{u}\bar{s}} \to B^- \pi^0 K^0) = 3\Gamma(T^-_{bb\bar{u}\bar{s}} \to B^- K^0 \eta), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 \pi^0 K^0) = 3\Gamma(T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0 K^0 \eta).$$
$$\Gamma(T^0_{bb\bar{d}\bar{s}} \to B^- \pi^+ \pi^0) = \Gamma(T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0 \pi^0 \pi^-), \qquad \Gamma(T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s \pi^0 \overline{K}^0) = 3\Gamma(T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s \overline{K}^0 \eta),$$
$$\Gamma(T^-_{bb\bar{u}\bar{d}} \to B^- \pi^0 \overline{K}^0) = 3\Gamma(T^-_{bb\bar{u}\bar{d}} \to B^- \overline{K}^0 \eta).$$

VI. NONLEPTONIC $T_{bc\bar{q}\bar{q}}$ DECAYS

The charm decays or (and) bottom decays can be present in the decays of $T_{bc\bar{q}\bar{q}}$. For the bottom decay, there is a new decay channel in which two heavy quarks of tetraquark interact by a virtual W-boson. The others can be obtained from those

TABLE XXIV. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a bottom meson and two light mesons induced by the charmless $b \to d$ transition.

Channel	Amplitude
$T^-_{bb\bar{u}\bar{s}} \to B^-\pi^0K^0$	$\frac{-c_2+c_4+2c_5+c_6-c_7-c_8+c_{11}+5c_{12}+c_{13}+3c_{14}-3c_{15}-c_{16}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to B^-\pi^-K^+$	$c_2-c_4-2c_5-c_6-c_7-c_8-c_{11}+3c_{12}-c_{13}-3c_{14}-c_{15}-3c_{16}$
$T^-_{bb\bar{u}\bar{s}} \to B^-K^0\eta$	$\frac{-c_2+c_4+2c_5+c_6-c_7-c_8+c_{11}+5c_{12}+c_{13}+3c_{14}-3c_{15}-c_{16}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0\pi^-K^0$	$-2(c_7+c_8+2(c_{15}+c_{16}))$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s\pi^0\pi^-$	$-4\sqrt{2}c_{12}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s\pi^-\eta$	$-\sqrt{\frac{2}{3}}(c_2-c_4+2c_5+c_6-c_7-c_8+c_{11}+3c_{12}+3c_{13}+c_{14}-c_{15}-3c_{16})$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_sK^0K^-$	$-c_2+c_4-2c_5-c_6-c_7-c_8-c_{11}+c_{12}-3c_{13}-c_{14}-3c_{15}-c_{16}$
$T^0_{bb\bar{d}\bar{s}} \to B^-\pi^+K^0$	$c_3+c_4+2c_5+c_6-c_8+2c_{10}-3c_{14}+2c_{15}-c_{16}$
$T^0_{bb\bar{d}\bar{s}} \to B^-\pi^0K^+$	$\frac{c_3+c_4+2c_5+c_6+2c_7+c_8+2c_{10}-3c_{14}-2c_{15}-5c_{16}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to B^-K^+\eta$	$-\frac{c_3+c_4+2c_5+c_6-2c_7-3c_8+2c_{10}-3c_{14}+6c_{15}+3c_{16}}{\sqrt{6}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0\pi^0K^0$	$\frac{-c_2-c_3+c_7+2c_8-2c_{10}+c_{11}+5c_{12}+c_{13}-2c_{14}-c_{15}-4c_{16}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0\pi^-K^+$	$c_2+c_3+c_7+2c_{10}-c_{11}+3c_{12}-c_{13}+2c_{14}-3c_{15}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0K^0\eta$	$\frac{-c_2-c_3+c_7+2c_8-2c_{10}+c_{11}+5c_{12}+c_{13}-2c_{14}-c_{15}-4c_{16}}{\sqrt{6}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\pi^+\pi^-$	$-2c_1-c_2-c_6+4c_9+c_{11}-3c_{12}-c_{13}-c_{14}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\pi^0\pi^0$	$-2c_1-c_2-c_6+4c_9+c_{11}+5c_{12}-c_{13}-c_{14}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\pi^0\eta$	$\frac{c_2-c_4+2c_5+c_6-c_7-c_8+c_{11}-c_{12}-5c_{13}+c_{14}-c_{15}+5c_{16}}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_sK^+K^-$	$-2c_1+c_3-c_6+c_7+4c_9+2c_{10}-2c_{13}+c_{14}-3c_{15}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_sK^0\overline{K}^0$	$-2c_1-c_2+c_3+c_4-2c_5-2c_6-c_8+4c_9+2c_{10}-c_{11}+c_{12}+3c_{13}+2c_{15}-c_{16}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\eta\eta$	$\frac{-6c_1-c_2+4c_3+2c_4-4c_5-5c_6-2c_7-6c_8+12c_9+8c_{10}-3c_{11}-3c_{12}+3c_{13}+3c_{14}+6c_{15}+6c_{16}}{3}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\pi^+\pi^-$	$2c_1+c_2-c_3-c_4-2c_5-2c_6-c_8+4c_9+2c_{10}-c_{11}+3c_{12}+c_{13}-2c_{15}-3c_{16}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\pi^0\pi^0$	$2c_1+c_2-c_3-c_4-2c_5-2c_6-c_8+4c_9+2c_{10}-c_{11}-5c_{12}+c_{13}+6c_{15}+5c_{16}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\pi^0\eta$	$-\frac{c_2+c_3+c_7+2c_8-2c_{10}+c_{11}-c_{12}-5c_{13}-6c_{14}-3c_{15}-4c_{16}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{d}} \to B^-K^+K^-$	$2c_1-c_3-c_6+c_7+4c_9+2c_{10}+2c_{13}+3c_{14}-c_{15}$
$T^-_{bb\bar{u}\bar{d}} \to B^-K^0\overline{K}^0$	$2c_1+c_2-c_6+4c_9+c_{11}-c_{12}-3c_{13}-3c_{14}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\eta\eta$	$\frac{6c_1+c_2-c_3+c_4+2c_5-2c_6-2c_7-3c_8+12c_9+2c_{10}+3c_{11}+3c_{12}-3c_{13}+3c_{16}}{3}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0\pi^0\pi^-$	$4\sqrt{2}(-c_{12}+c_{15}+c_{16})$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0\pi^-\eta$	$-\sqrt{\frac{2}{3}}(c_2+c_3+c_7+2c_8-2c_{10}+c_{11}+3c_{12}+3c_{13}+2c_{14}+c_{15})$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0K^0K^-$	$-c_2-c_3+c_7+2c_{10}-c_{11}+c_{12}-3c_{13}-2c_{14}-c_{15}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0_s\pi^0K^-$	$-\frac{c_3+c_4-2c_5-c_6+c_8-2c_{10}+c_{14}-6c_{15}-5c_{16}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0_s\pi^-\overline{K}^0$	$-c_3-c_4+2c_5+c_6-c_8+2c_{10}-c_{14}-2c_{15}-3c_{16}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0_sK^-\eta$	$\frac{c_3+c_4-2c_5-c_6-4c_7-3c_8-2c_{10}+c_{14}+2c_{15}+3c_{16}}{\sqrt{6}}$

for $T_{bb\bar{q}\bar{q}}$ decays with $B \to D$ and $B_c \to J/\psi$. For the charm decays, the decay amplitudes can be obtained from those for $T_{cc\bar{q}\bar{q}}$ decays with the replacement of $D \to B$ and $J/\psi \to B_c$. Thus, we do not present those results again.

A. $bc \to ud/s$ or $bc \to cd/s$ transition

1. Decays into two mesons

The transitions $bc \to cd/s$ and $bc \to ud/s$ can lead to the tetraquark to decays to two mesons, whose corresponding Feynman diagrams are given in Fig. 8(a). The Hamiltonian becomes

$$\begin{aligned}\mathcal{H}_{\rm eff} = {}& f_1(T_{bc3})_{[ij]}(\overline{D})^iM^j_k(H_3)^k + f_2(T_{bc3})_{[ij]}(\overline{D})^kM^j_k(H_3)^i + f_3(T_{bc3})_{[ij]}M^i_kM^k_l(H_{\bar{3}})^{[jl]} + f_4(T_{bc3})_{[ij]}M^i_kM^j_l(H_{\bar{3}})^{[kl]} \\ & + f_5(T_{bc3})_{[ij]}M^l_kM^k_l(H_{\bar{3}})^{[ij]} + f_6(T_{bc3})_{[ij]}M^i_kM^k_l(H_6)^{\{jl\}}. \end{aligned} \tag{38}$$

TABLE XXV. Doubly bottomed tetraquark $T_{bb\bar{q}\bar{q}}$ decays into a bottom meson and two light mesons induced by the charmless $b \to s$ transition.

Channel	Amplitude
$T^-_{bb\bar{u}\bar{s}} \to B^-\pi^+\pi^-$	$2c_1 - c_3 - c_6 + c_7 + 4c_9 + 2c_{10} + 2c_{13} + 3c_{14} - c_{15}$
$T^-_{bb\bar{u}\bar{s}} \to B^-\pi^0\pi^0$	$2c_1 - c_3 - c_6 - c_7 - 2c_8 + 4c_9 + 2c_{10} + 2c_{13} + 3c_{14} + 3c_{15} + 4c_{16}$
$T^-_{bb\bar{u}\bar{s}} \to B^-\pi^0\eta$	$-\frac{c_3+c_4+2c_5+c_6+c_8-2c_{10}+2c_{11}+4c_{12}-4c_{13}-3c_{14}-6c_{15}-5c_{16}}{\sqrt{3}}$
$T^-_{bb\bar{u}\bar{s}} \to B^-K^+K^-$	$2c_1 + c_2 - c_3 - c_4 - 2c_5 - 2c_6 - c_8 + 4c_9 + 2c_{10} - c_{11} + 3c_{12} + c_{13} - 2c_{15} - 3c_{16}$
$T^-_{bb\bar{u}\bar{s}} \to B^-K^0\overline{K}^0$	$2c_1 + c_2 - c_6 + 4c_9 + c_{11} - c_{12} - 3c_{13} - 3c_{14}$
$T^-_{bb\bar{u}\bar{s}} \to B^-\eta\eta$	$\frac{6c_1+4c_2-c_3-2c_4-4c_5-5c_6+c_7+12c_9+2c_{10}-12c_{12}-6c_{13}-9c_{14}+9c_{15}+6c_{16}}{3}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0\pi^0\pi^-$	$-\sqrt{2}(c_7 + c_8 - 2(c_{15} + c_{16}))$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0\pi^-\eta$	$\sqrt{\frac{2}{3}}(-c_3 - c_4 + 2c_5 + c_6 + c_7 + 2c_{10} - c_{14} + 4c_{15} + 3c_{16})$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0K^0K^-$	$-c_3 - c_4 + 2c_5 + c_6 - c_8 + 2c_{10} - c_{14} - 2c_{15} - 3c_{16}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s\pi^0K^-$	$-\frac{c_2+c_3+c_7+2c_8-2c_{10}+c_{11}+7c_{12}+3c_{13}+2c_{14}-3c_{15}-4c_{16}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_s\pi^-\overline{K}^0$	$-c_2 - c_3 + c_7 + 2c_{10} - c_{11} + c_{12} - 3c_{13} - 2c_{14} - c_{15}$
$T^-_{bb\bar{u}\bar{s}} \to \overline{B}^0_sK^-\eta$	$\frac{c_2+c_3+c_7+2c_8-2c_{10}+c_{11}-9c_{12}+3c_{13}+2c_{14}+13c_{15}+12c_{16}}{\sqrt{6}}$
$T^0_{bb\bar{d}\bar{s}} \to B^-\pi^+\pi^0$	$-\sqrt{2}(c_7 + c_8 - 2(c_{15} + c_{16}))$
$T^0_{bb\bar{d}\bar{s}} \to B^-\pi^+\eta$	$-\sqrt{\frac{2}{3}}(c_3 + c_4 + 2c_5 + c_6 + c_7 + 2c_{10} - 3c_{14} - 3c_{16})$
$T^0_{bb\bar{d}\bar{s}} \to B^-K^+\overline{K}^0$	$-c_3 - c_4 - 2c_5 - c_6 + c_8 - 2c_{10} + 3c_{14} - 2c_{15} + c_{16}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0\pi^+\pi^-$	$2c_1 - c_3 + c_6 - c_7 - 4c_9 - 2c_{10} + 2c_{13} - c_{14} + 3c_{15}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0\pi^0\pi^0$	$2c_1 - c_3 + c_6 + c_7 + 2c_8 - 4c_9 - 2c_{10} + 2c_{13} - c_{14} - c_{15} - 4c_{16}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0\pi^0\eta$	$\frac{c_3+c_4-2c_5-c_6-c_8+2c_{10}-2c_{11}-4c_{12}+4c_{13}+c_{14}+2c_{15}-c_{16}}{\sqrt{3}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0K^+K^-$	$2c_1 + c_2 + c_6 - 4c_9 - c_{11} + 3c_{12} + c_{13} + c_{14}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0K^0\overline{K}^0$	$2c_1 + c_2 - c_3 - c_4 + 2c_5 + 2c_6 + c_8 - 4c_9 - 2c_{10} + c_{11} - c_{12} - 3c_{13} - 2c_{15} + c_{16}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0\eta\eta$	$\frac{6c_1+4c_2-c_3-2c_4+4c_5+5c_6-c_7-12c_9-2c_{10}-12c_{12}-6c_{13}+3c_{14}-3c_{15}+6c_{16}}{3}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\pi^+K^-$	$-c_2 - c_3 - c_7 - 2c_{10} + c_{11} - 3c_{12} + c_{13} - 2c_{14} + 3c_{15}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\pi^0\overline{K}^0$	$\frac{c_2+c_3-c_7-2c_8+2c_{10}-c_{11}-5c_{12}-c_{13}+2c_{14}+c_{15}+4c_{16}}{\sqrt{2}}$
$T^0_{bb\bar{d}\bar{s}} \to \overline{B}^0_s\overline{K}^0\eta$	$\frac{c_2+c_3-c_7-2c_8+2c_{10}-c_{11}-5c_{12}-c_{13}+2c_{14}+c_{15}+4c_{16}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\pi^+K^-$	$c_2 - c_4 - 2c_5 - c_6 - c_7 - c_8 - c_{11} + 3c_{12} - c_{13} - 3c_{14} - c_{15} - 3c_{16}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\pi^0\overline{K}^0$	$\frac{-c_2+c_4+2c_5+c_6-c_7-c_8+c_{11}+5c_{12}+c_{13}+3c_{14}-3c_{15}-c_{16}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to B^-\overline{K}^0\eta$	$\frac{-c_2+c_4+2c_5+c_6-c_7-c_8+c_{11}+5c_{12}+c_{13}+3c_{14}-3c_{15}-c_{16}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0\pi^0K^-$	$-\frac{c_2-c_4+2c_5+c_6-c_7-c_8+c_{11}+7c_{12}+3c_{13}+c_{14}-c_{15}-3c_{16}}{\sqrt{2}}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0\pi^-\overline{K}^0$	$-c_2 + c_4 - 2c_5 - c_6 - c_7 - c_8 - c_{11} + c_{12} - 3c_{13} - c_{14} - 3c_{15} - c_{16}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0K^-\eta$	$\frac{c_2-c_4+2c_5+c_6-c_7-c_8+c_{11}-9c_{12}+3c_{13}+c_{14}-c_{15}-3c_{16}}{\sqrt{6}}$
$T^-_{bb\bar{u}\bar{d}} \to \overline{B}^0_s\overline{K}^0K^-$	$-2(c_7 + c_8 + 2(c_{15} + c_{16}))$

The decay amplitudes for T_{bc3} decays into a charmed meson and a light meson are given in Table XXVI. We have the relations:

$$\Gamma(T^0_{bc\bar{u}\bar{s}} \to D^0\pi^0) = \frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{s}} \to D^+\pi^-) = \frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}} \to D^0\pi^+) = \Gamma(T^+_{bc\bar{d}\bar{s}} \to D^+\pi^0),$$
$$\Gamma(T^+_{bc\bar{d}\bar{s}} \to D^0K^+) = \Gamma(T^0_{bc\bar{u}\bar{d}} \to D^+_sK^-), \qquad \Gamma(T^+_{bc\bar{d}\bar{s}} \to D^+_s\overline{K}^0) = \Gamma(T^0_{bc\bar{u}\bar{s}} \to D^+_sK^-),$$
$$\Gamma(T^0_{bc\bar{u}\bar{d}} \to D^0\overline{K}^0) = \Gamma(T^0_{bc\bar{u}\bar{d}} \to D^+K^-), \qquad \Gamma(T^0_{bc\bar{u}\bar{s}} \to D^0\eta) = \Gamma(T^+_{bc\bar{d}\bar{s}} \to D^+\eta),$$
$$\Gamma(T^0_{bc\bar{u}\bar{s}} \to D^+_s\pi^-) = \Gamma(T^0_{bc\bar{u}\bar{s}} \to D^0K^0) = 2\Gamma(T^+_{bc\bar{d}\bar{s}} \to D^+_s\pi^0),$$
$$\Gamma(T^0_{bc\bar{u}\bar{d}} \to D^0\pi^0) = \frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}} \to D^+K^0) = \frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{d}} \to D^+\pi^-).$$

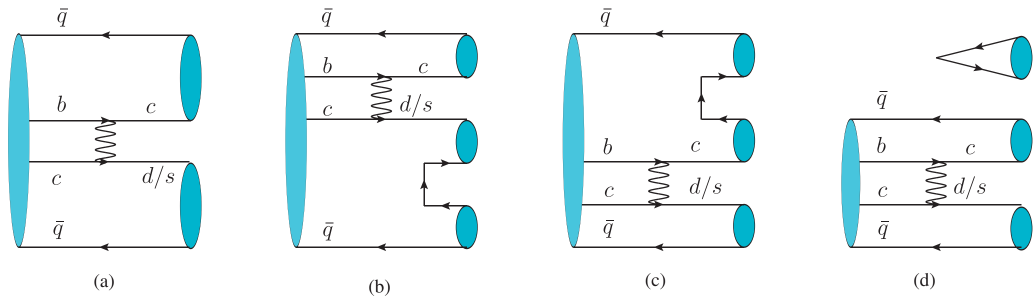

FIG. 8. Feynman diagrams for nonleptonic decays of doubly heavy tetraquark $T_{bc\bar{q}\,\bar{q}}$. Panel (a) corresponds with two mesons by the new W-exchange process. Panels (b),(c),(d) correspond with three mesons by the new W-exchange process.

The decay amplitudes for T_{bc3} decays into two light mesons are given in Table XXVII. The relations of decay widths are

$$\Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^0\overline{K}^0) = \frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^+K^-) = 3\Gamma(T^0_{bc\bar{u}\bar{d}} \to \eta\overline{K}^0),$$
$$\Gamma(T^+_{bc\bar{d}\bar{s}} \to \pi^0K^+) = \frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}} \to \pi^+K^0) = 3\Gamma(T^+_{bc\bar{d}\bar{s}} \to \eta K^+),$$
$$\Gamma(T^+_{bc\bar{d}\bar{s}} \to K^+\overline{K}^0) = \frac{3}{2}\Gamma(T^+_{bc\bar{d}\bar{s}} \to \pi^+\eta) = 3\Gamma(T^0_{bc\bar{u}\bar{s}} \to \pi^0\eta),$$
$$\Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^0\pi^0) = \frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^+\pi^-), \qquad \Gamma(T^0_{bc\bar{u}\bar{s}} \to \pi^0K^0) = 3\Gamma(T^0_{bc\bar{u}\bar{s}} \to \eta K^0),$$
$$\Gamma(T^0_{bc\bar{u}\bar{s}} \to \pi^0\pi^0) = \frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{s}} \to \pi^+\pi^-).$$

2. Decays into three mesons

The effective Hamiltonian which leads to the tetraquark to decays to three mesons is

$$\begin{aligned}\mathcal{H}_{\rm eff} = {} & d_1(T_{bc3})_{[ij]}(\overline{D})^i M^j_k M^k_l (H_3)^l + d_2(T_{bc3})_{[ij]}(\overline{D})^i M^k_l M^l_k (H_3)^j + d_3(T_{bc3})_{[ij]}(\overline{D})^k M^i_k M^j_l (H_3)^l \\ & + d_4(T_{bc3})_{[ij]}(\overline{D})^k M^i_l M^l_k (H_3)^j + d_5(T_{bc3})_{[ij]} M^i_k M^j_m M^k_l (H_{\bar{3}})^{[lm]} + d_6(T_{bc3})_{[ij]} M^i_k M^k_l M^l_m (H_{\bar{3}})^{[jm]} \\ & + d_7(T_{bc3})_{[ij]} M^i_m M^k_l M^l_k (H_{\bar{3}})^{[jm]} + d_8(T_{bc3})_{[ij]} M^k_m M^l_k M^m_l (H_{\bar{3}})^{[ij]} + d_9(T_{bc3})_{[ij]} M^i_k M^j_m M^k_l (H_6)^{\{lm\}} \\ & + d_{10}(T_{bc3})_{[ij]} M^i_k M^k_l M^l_m (H_6)^{\{jm\}} + d_{11}(T_{bc3})_{[ij]} M^i_m M^k_l M^l_k (H_6)^{\{jm\}}. \end{aligned} \tag{39}$$

TABLE XXVI. Doubly heavy tetraquark $T_{bc\bar{q}\bar{q}}$ decays into a charmed meson and a light meson.

Channel	Amplitude ($/V_{cb}$)	Channel	Amplitude ($/V_{cb}$)
$T^0_{bc\bar{u}\bar{s}} \to D^0\pi^0$	$-\frac{f_2V^*_{cs}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{s}} \to D^0K^0$	$f_1V^*_{cd}$
$T^0_{bc\bar{u}\bar{s}} \to D^0\eta$	$-\frac{(2f_1+f_2)V^*_{cs}}{\sqrt{6}}$	$T^0_{bc\bar{u}\bar{s}} \to D^+\pi^-$	$-f_2V^*_{cs}$
$T^0_{bc\bar{u}\bar{s}} \to D^+_s\pi^-$	$-f_1V^*_{cd}$	$T^0_{bc\bar{u}\bar{s}} \to D^+_sK^-$	$-(f_1+f_2)V^*_{cs}$
$T^+_{bc\bar{d}\bar{s}} \to D^0\pi^+$	$-f_2V^*_{cs}$	$T^+_{bc\bar{d}\bar{s}} \to D^0K^+$	$f_2V^*_{cd}$
$T^+_{bc\bar{d}\bar{s}} \to D^+\pi^0$	$\frac{f_2V^*_{cs}}{\sqrt{2}}$	$T^+_{bc\bar{d}\bar{s}} \to D^+K^0$	$(f_1+f_2)V^*_{cd}$
$T^+_{bc\bar{d}\bar{s}} \to D^+\eta$	$-\frac{(2f_1+f_2)V^*_{cs}}{\sqrt{6}}$	$T^+_{bc\bar{d}\bar{s}} \to D^+_s\pi^0$	$\frac{f_1V^*_{cd}}{\sqrt{2}}$
$T^+_{bc\bar{d}\bar{s}} \to D^+_s\overline{K}^0$	$-(f_1+f_2)V^*_{cs}$	$T^+_{bc\bar{d}\bar{s}} \to D^+_s\eta$	$-\frac{(f_1+2f_2)V^*_{cd}}{\sqrt{6}}$
$T^0_{bc\bar{u}\bar{d}} \to D^0\pi^0$	$-\frac{(f_1+f_2)V^*_{cd}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{d}} \to D^0\overline{K}^0$	$f_1V^*_{cs}$
$T^0_{bc\bar{u}\bar{d}} \to D^0\eta$	$\frac{(f_1-f_2)V^*_{cd}}{\sqrt{6}}$	$T^0_{bc\bar{u}\bar{d}} \to D^+\pi^-$	$-(f_1+f_2)V^*_{cd}$
$T^0_{bc\bar{u}\bar{d}} \to D^+K^-$	$-f_1V^*_{cs}$	$T^0_{bc\bar{u}\bar{d}} \to D^+_sK^-$	$-f_2V^*_{cd}$

TABLE XXVII. Doubly heavy tetraquark $T_{bc\bar{q}\bar{q}}$ decays into two light mesons.

Channel	Amplitude ($/V_{cb}$)	Channel	Amplitude ($/V_{cb}$)
$T^0_{bc\bar{u}\bar{s}} \to \pi^+\pi^-$	$(-f_3+4f_5+f_6)V^*_{cs}$	$T^0_{bc\bar{u}\bar{s}} \to \pi^0\pi^0$	$(-f_3+4f_5+f_6)V^*_{cs}$
$T^0_{bc\bar{u}\bar{s}} \to \pi^0 K^0$	$\frac{(f_3+2f_4+f_6)V^*_{cd}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{s}} \to \pi^0\eta$	$\frac{(-f_3-2f_4+f_6)V^*_{cs}}{\sqrt{3}}$
$T^0_{bc\bar{u}\bar{s}} \to \pi^- K^+$	$-(f_3+2f_4+f_6)V^*_{cd}$	$T^0_{bc\bar{u}\bar{s}} \to K^+K^-$	$-2(f_3+f_4-2f_5)V^*_{cs}$
$T^0_{bc\bar{u}\bar{s}} \to K^0\overline{K}^0$	$-(f_3-4f_5+f_6)V^*_{cs}$	$T^0_{bc\bar{u}\bar{s}} \to K^0\eta$	$\frac{(f_3+2f_4+f_6)V^*_{cd}}{\sqrt{6}}$
$T^0_{bc\bar{u}\bar{s}} \to \eta\eta$	$-\frac{1}{3}(5f_3+4f_4+3(f_6-4f_5))V^*_{cs}$	$T^+_{bc\bar{d}\bar{s}} \to \pi^+ K^0$	$(f_3+2f_4-f_6)V^*_{cd}$
$T^+_{bc\bar{d}\bar{s}} \to \pi^+\eta$	$\sqrt{\frac{2}{3}}(-f_3-2f_4+f_6)V^*_{cs}$	$T^+_{bc\bar{d}\bar{s}} \to \pi^0 K^+$	$\frac{(f_3+2f_4-f_6)V^*_{cd}}{\sqrt{2}}$
$T^+_{bc\bar{d}\bar{s}} \to K^+\overline{K}^0$	$(-f_3-2f_4+f_6)V^*_{cs}$	$T^+_{bc\bar{d}\bar{s}} \to K^+\eta$	$\frac{(-f_3-2f_4+f_6)V^*_{cd}}{\sqrt{6}}$
$T^0_{bc\bar{u}\bar{d}} \to \pi^+\pi^-$	$-2(f_3+f_4-2f_5)V^*_{cd}$	$T^0_{bc\bar{u}\bar{d}} \to \pi^+ K^-$	$-(f_3+2f_4+f_6)V^*_{cs}$
$T^0_{bc\bar{u}\bar{d}} \to \pi^0\pi^0$	$-2(f_3+f_4-2f_5)V^*_{cd}$	$T^0_{bc\bar{u}\bar{d}} \to \pi^0\overline{K}^0$	$\frac{(f_3+2f_4+f_6)V^*_{cs}}{\sqrt{2}}$
$T^0_{bc\bar{u}\bar{d}} \to \pi^0\eta$	$\frac{2f_6V^*_{cd}}{\sqrt{3}}$	$T^0_{bc\bar{u}\bar{d}} \to K^+K^-$	$(-f_3+4f_5+f_6)V^*_{cd}$
$T^0_{bc\bar{u}\bar{d}} \to K^0\overline{K}^0$	$-(f_3-4f_5+f_6)V^*_{cd}$	$T^0_{bc\bar{u}\bar{d}} \to \overline{K}^0\eta$	$\frac{(f_3+2f_4+f_6)V^*_{cs}}{\sqrt{6}}$
$T^0_{bc\bar{u}\bar{d}} \to \eta\eta$	$\frac{2}{3}(-f_3+f_4+6f_5)V^*_{cd}$		

The corresponding Feynman diagrams are given in Figs. 8(b), 8(c), and 8(d). We give the amplitudes in Table XXVIII for a D meson plus two light mesons, and Table XXIX for three light mesons. Then, the relations of decay widths for a D meson plus two light mesons are as follows:

$$\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^0\pi^0\pi^0)=\frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^0\pi^+\pi^-)=\frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+\pi^+\pi^-)=\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+\pi^0\pi^0),$$

$$\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^0\pi^0K^0)=3\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^0K^0\eta)=\frac{3}{4}\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^+_s\pi^-\eta)=\frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^+_sK^0K^-)$$
$$=\frac{3}{2}\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+_s\pi^0\eta)=\frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^0\pi^-K^+),$$

$$\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^+K^0K^-)=3\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^0\pi^0\eta)=\frac{3}{2}\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^+\pi^-\eta)=\frac{3}{2}\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^0\pi^+\eta)$$
$$=3\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+\pi^0\eta)=\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^0K^+\overline{K}^0),$$

$$\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^+_s\pi^0K^-)=3\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+_s\overline{K}^0\eta)=\frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^+_s\pi^-\overline{K}^0)=3\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^+_s\eta K^-)$$
$$=\frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+_s\pi^+K^-)=\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+_s\pi^0\overline{K}^0),$$

$$\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^0\pi^+K^0)=6\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^0K^+\eta)=2\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^+_s\pi^0K^-)=\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^+_s\pi^-\overline{K}^0)$$
$$=6\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^+_sK^-\eta)=2\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^0\pi^0K^+),$$

$$\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+\pi^0K^0)=3\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+K^0\eta)=\frac{3}{2}\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^0\pi^0\eta)=\frac{3}{4}\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^+\pi^-\eta)$$
$$=\frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^+K^0K^-)=\frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+\pi^-K^+),$$

$$\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^0\pi^+K^-)=6\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^0\overline{K}^0\eta)=6\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^+K^-\eta)=2\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^0\pi^0\overline{K}^0)$$
$$=2\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^+\pi^0K^-)=\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^+\pi^-\overline{K}^0),$$

$$\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+_s\pi^+\pi^-)=\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^0\overline{K}^0K^0)=2\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+_s\pi^0\pi^0),$$

$$\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^0\pi^+\pi^-)=\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+_s\overline{K}^0K^0)=2\Gamma(T^0_{bc\bar{u}\bar{d}}\to D^0\pi^0\pi^0),$$

$$\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^0\eta\eta)=\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+\eta\eta),\quad \Gamma(T^0_{bc\bar{u}\bar{d}}\to D^0K^+K^-)=\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+_sK^+K^-),$$

$$\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+K^0\overline{K}^0)=\Gamma(T^0_{bc\bar{u}\bar{s}}\to D^0K^+K^-),\quad \Gamma(T^0_{bc\bar{u}\bar{s}}\to D^0K^0\overline{K}^0)=\Gamma(T^+_{bc\bar{d}\bar{s}}\to D^+K^+K^-).$$

TABLE XXVIII. Doubly heavy tetraquark $T_{bc\bar{q}\bar{q}}$ decays into a charmed meson and two light mesons.

Channel	Amplitude ($/V_{cb}$)	Channel	Amplitude ($/V_{cb}$)
$T^0_{bc\bar{u}\bar{s}} \to D^0\pi^+\pi^-$	$(2d_2+d_4)V^*_{\rm cs}$	$T^0_{bc\bar{u}\bar{s}} \to D^0\pi^0\pi^0$	$(2d_2+d_4)V^*_{\rm cs}$
$T^0_{bc\bar{u}\bar{s}} \to D^0\pi^0K^0$	$\frac{(d_3-d_1)V^*_{\rm cd}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{s}} \to D^0\pi^0\eta$	$\frac{(d_4-d_3)V^*_{\rm cs}}{\sqrt{3}}$
$T^0_{bc\bar{u}\bar{s}} \to D^0\pi^-K^+$	$(d_1-d_3)V^*_{\rm cd}$	$T^0_{bc\bar{u}\bar{s}} \to D^0K^+K^-$	$(d_1+2d_2-d_3+d_4)V^*_{\rm cs}$
$T^0_{bc\bar{u}\bar{s}} \to D^0K^0\overline{K}^0$	$(d_1+2d_2)V^*_{\rm cs}$	$T^0_{bc\bar{u}\bar{s}} \to D^0K^0\eta$	$\frac{(d_3-d_1)V^*_{\rm cd}}{\sqrt{6}}$
$T^0_{bc\bar{u}\bar{s}} \to D^0\eta\eta$	$\frac{1}{3}(4d_1+6d_2-2d_3+d_4)V^*_{\rm cs}$	$T^0_{bc\bar{u}\bar{s}} \to D^+\pi^-\eta$	$\sqrt{\frac{2}{3}}(d_4-d_3)V^*_{\rm cs}$
$T^0_{bc\bar{u}\bar{s}} \to D^+K^0K^-$	$(d_4-d_3)V^*_{\rm cs}$	$T^0_{bc\bar{u}\bar{s}} \to D^+_s\pi^0K^-$	$\frac{(d_4-d_1)V^*_{\rm cs}}{\sqrt{2}}$
$T^0_{bc\bar{u}\bar{s}} \to D^+_s\pi^-\overline{K}^0$	$(d_4-d_1)V^*_{\rm cs}$	$T^0_{bc\bar{u}\bar{s}} \to D^+_s\pi^-\eta$	$\sqrt{\frac{2}{3}}(d_3-d_1)V^*_{\rm cd}$
$T^0_{bc\bar{u}\bar{s}} \to D^+_sK^0K^-$	$(d_3-d_1)V^*_{\rm cd}$	$T^0_{bc\bar{u}\bar{s}} \to D^+_sK^-\eta$	$\frac{(d_1-d_4)V^*_{\rm cs}}{\sqrt{6}}$
$T^+_{bc\bar{d}\bar{s}} \to D^0\pi^+K^0$	$(d_3-d_4)V^*_{\rm cd}$	$T^+_{bc\bar{d}\bar{s}} \to D^0\pi^+\eta$	$\sqrt{\frac{2}{3}}(d_4-d_3)V^*_{\rm cs}$
$T^+_{bc\bar{d}\bar{s}} \to D^0\pi^0K^+$	$\frac{(d_3-d_4)V^*_{\rm cd}}{\sqrt{2}}$	$T^+_{bc\bar{d}\bar{s}} \to D^0K^+\overline{K}^0$	$(d_4-d_3)V^*_{\rm cs}$
$T^+_{bc\bar{d}\bar{s}} \to D^0K^+\eta$	$\frac{(d_4-d_3)V^*_{\rm cd}}{\sqrt{6}}$	$T^+_{bc\bar{d}\bar{s}} \to D^+\pi^+\pi^-$	$(2d_2+d_4)V^*_{\rm cs}$
$T^+_{bc\bar{d}\bar{s}} \to D^+\pi^0\pi^0$	$(2d_2+d_4)V^*_{\rm cs}$	$T^+_{bc\bar{d}\bar{s}} \to D^+\pi^0K^0$	$\frac{(d_4-d_1)V^*_{\rm cd}}{\sqrt{2}}$
$T^+_{bc\bar{d}\bar{s}} \to D^+\pi^0\eta$	$\frac{(d_3-d_4)V^*_{\rm cs}}{\sqrt{3}}$	$T^+_{bc\bar{d}\bar{s}} \to D^+\pi^-K^+$	$(d_1-d_4)V^*_{\rm cd}$
$T^+_{bc\bar{d}\bar{s}} \to D^+K^+K^-$	$(d_1+2d_2)V^*_{\rm cs}$	$T^+_{bc\bar{d}\bar{s}} \to D^+K^0\overline{K}^0$	$(d_1+2d_2-d_3+d_4)V^*_{\rm cs}$
$T^+_{bc\bar{d}\bar{s}} \to D^+K^0\eta$	$\frac{(d_4-d_1)V^*_{\rm cd}}{\sqrt{6}}$	$T^+_{bc\bar{d}\bar{s}} \to D^+\eta\eta$	$\frac{1}{3}(4d_1+6d_2-2d_3+d_4)V^*_{\rm cs}$
$T^+_{bc\bar{d}\bar{s}} \to D^+_s\pi^+\pi^-$	$-(d_1+2d_2)V^*_{\rm cd}$	$T^+_{bc\bar{d}\bar{s}} \to D^+_s\pi^+K^-$	$(d_4-d_1)V^*_{\rm cs}$
$T^+_{bc\bar{d}\bar{s}} \to D^+_s\pi^0\pi^0$	$-(d_1+2d_2)V^*_{\rm cd}$	$T^+_{bc\bar{d}\bar{s}} \to D^+_s\pi^0\overline{K}^0$	$\frac{(d_1-d_4)V^*_{\rm cs}}{\sqrt{2}}$
$T^+_{bc\bar{d}\bar{s}} \to D^+_s\pi^0\eta$	$\frac{(d_1-d_3)V^*_{\rm cd}}{\sqrt{3}}$	$T^+_{bc\bar{d}\bar{s}} \to D^+_sK^+K^-$	$-(2d_2+d_4)V^*_{\rm cd}$
$T^+_{bc\bar{d}\bar{s}} \to D^+_sK^0\overline{K}^0$	$-(d_1+2d_2-d_3+d_4)V^*_{\rm cd}$	$T^+_{bc\bar{d}\bar{s}} \to D^+_s\overline{K}^0\eta$	$\frac{(d_1-d_4)V^*_{\rm cs}}{\sqrt{6}}$
$T^+_{bc\bar{d}\bar{s}} \to D^+_s\eta\eta$	$-\frac{1}{3}(d_1+6d_2-2d_3+4d_4)V^*_{\rm cd}$	$T^0_{bc\bar{u}\bar{d}} \to D^0\pi^+\pi^-$	$(d_1+2d_2-d_3+d_4)V^*_{\rm cd}$
$T^0_{bc\bar{u}\bar{d}} \to D^0\pi^+K^-$	$(d_1-d_3)V^*_{\rm cs}$	$T^0_{bc\bar{u}\bar{d}} \to D^0\pi^0\pi^0$	$(d_1+2d_2-d_3+d_4)V^*_{\rm cd}$
$T^0_{bc\bar{u}\bar{d}} \to D^0\pi^0\overline{K}^0$	$\frac{(d_3-d_1)V^*_{\rm cs}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{d}} \to D^0\pi^0\eta$	$\frac{(d_4-d_1)V^*_{\rm cd}}{\sqrt{3}}$
$T^0_{bc\bar{u}\bar{d}} \to D^0K^+K^-$	$(2d_2+d_4)V^*_{\rm cd}$	$T^0_{bc\bar{u}\bar{d}} \to D^0K^0\overline{K}^0$	$(d_1+2d_2)V^*_{\rm cd}$
$T^0_{bc\bar{u}\bar{d}} \to D^0\overline{K}^0\eta$	$\frac{(d_3-d_1)V^*_{\rm cs}}{\sqrt{6}}$	$T^0_{bc\bar{u}\bar{d}} \to D^0\eta\eta$	$\frac{1}{3}(d_1+6d_2+d_3+d_4)V^*_{\rm cd}$
$T^0_{bc\bar{u}\bar{d}} \to D^+\pi^0K^-$	$\frac{(d_3-d_1)V^*_{\rm cs}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{d}} \to D^+\pi^-\overline{K}^0$	$(d_3-d_1)V^*_{\rm cs}$
$T^0_{bc\bar{u}\bar{d}} \to D^+\pi^-\eta$	$\sqrt{\frac{2}{3}}(d_4-d_1)V^*_{\rm cd}$	$T^0_{bc\bar{u}\bar{d}} \to D^+K^0K^-$	$(d_4-d_1)V^*_{\rm cd}$
$T^0_{bc\bar{u}\bar{d}} \to D^+K^-\eta$	$\frac{(d_1-d_3)V^*_{\rm cs}}{\sqrt{6}}$	$T^0_{bc\bar{u}\bar{d}} \to D^+_s\pi^0K^-$	$\frac{(d_4-d_3)V^*_{\rm cd}}{\sqrt{2}}$
$T^0_{bc\bar{u}\bar{d}} \to D^+_s\pi^-\overline{K}^0$	$(d_4-d_3)V^*_{\rm cd}$	$T^0_{bc\bar{u}\bar{d}} \to D^+_sK^-\eta$	$\frac{(d_3-d_4)V^*_{\rm cd}}{\sqrt{6}}$

The relations of decay widths for three light mesons can be written as

$$
\begin{aligned}
2\Gamma(T^0_{bc\bar{u}\bar{s}}\to\pi^+\pi^0\pi^-) &= \frac{4}{3}\Gamma(T^0_{bc\bar{u}\bar{s}}\to\pi^0\pi^0\pi^0) = \frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}}\to\pi^+\pi^+\pi^-) = 2\Gamma(T^+_{bc\bar{d}\bar{s}}\to\pi^+\pi^0\pi^0) \\
&= \Gamma(T^+_{bc\bar{d}\bar{s}}\to\pi^+K^+K^-), \\
\Gamma(T^0_{bc\bar{u}\bar{s}}\to\pi^0\pi^-K^+) &= \frac{3}{2}\Gamma(T^0_{bc\bar{u}\bar{s}}\to\pi^0K^0\eta) = 3\Gamma(T^0_{bc\bar{u}\bar{s}}\to\pi^-K^+\eta) = \Gamma(T^+_{bc\bar{d}\bar{s}}\to\pi^+\pi^0K^0) \\
&= 3\Gamma(T^+_{bc\bar{d}\bar{s}}\to\pi^+K^0\eta) = \frac{3}{2}\Gamma(T^+_{bc\bar{d}\bar{s}}\to\pi^0K^+\eta), \\
\Gamma(T^+_{bc\bar{d}\bar{s}}\to\pi^0K^+\overline{K}^0) &= 3\Gamma(T^+_{bc\bar{d}\bar{s}}\to K^+\overline{K}^0\eta) = \frac{3}{4}\Gamma(T^0_{bc\bar{u}\bar{d}}\to\pi^+K^-\eta) = \frac{3}{2}\Gamma(T^0_{bc\bar{u}\bar{d}}\to\pi^0\overline{K}^0\eta), \\
\Gamma(T^+_{bc\bar{d}\bar{s}}\to\pi^+\pi^-K^+) &= \frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}}\to K^+K^+K^-), \qquad \Gamma(T^0_{bc\bar{u}\bar{s}}\to\pi^+\pi^-K^0) = \frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{s}}\to K^0K^0\overline{K}^0),
\end{aligned}
$$

TABLE XXIX. Doubly heavy tetraquark $T_{bc\bar{q}\bar{q}}$ decays into three light mesons.

Channel	Amplitude ($/V_{cb}$)	Channel	Amplitude ($/V_{cb}$)
$T^0_{bc\bar{u}\bar{s}} \to \pi^+\pi^0\pi^-$	$\frac{(-d_6-2d_7+d_{10}+2d_{11})V^*_{cs}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{s}} \to \pi^+\pi^-K^0$	$-(d_6+2d_7+d_{10}+2d_{11})V^*_{cd}$
$T^0_{bc\bar{u}\bar{s}} \to \pi^+\pi^-\eta$	$\frac{(-2d_5-3d_6+2d_7+12d_8-2d_9+3d_{10}+6d_{11})V^*_{cs}}{\sqrt{6}}$	$T^0_{bc\bar{u}\bar{s}} \to \pi^+K^0K^-$	$-(d_5+2d_6-6d_8+d_9)V^*_{cs}$
$T^0_{bc\bar{u}\bar{s}} \to \pi^0\pi^0\pi^0$	$\frac{3(-d_6-2d_7+d_{10}+2d_{11})V^*_{cs}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{s}} \to \pi^0\pi^0K^0$	$-(d_6+2d_7-2d_9+d_{10}+2d_{11})V^*_{cd}$
$T^0_{bc\bar{u}\bar{s}} \to \pi^0\pi^0\eta$	$\frac{(-2d_5-3d_6+2d_7+12d_8-2d_9+3d_{10}+6d_{11})V^*_{cs}}{\sqrt{6}}$	$T^0_{bc\bar{u}\bar{s}} \to \pi^0\pi^-K^+$	$-\sqrt{2}d_9V^*_{cd}$
$T^0_{bc\bar{u}\bar{s}} \to \pi^0K^+K^-$	$-\frac{(d_5+3d_6+2d_7-6d_8+d_9-d_{10}-2d_{11})V^*_{cs}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{s}} \to \pi^0K^0\overline{K}^0$	$\frac{(d_5+d_6-2d_7-6d_8-d_9+d_{10}+2d_{11})V^*_{cs}}{\sqrt{2}}$
$T^0_{bc\bar{u}\bar{s}} \to \pi^0K^0\eta$	$\frac{2d_9V^*_{cd}}{\sqrt{3}}$	$T^0_{bc\bar{u}\bar{s}} \to \pi^0\eta\eta$	$-\frac{(3d_6+6d_7+8d_9-3d_{10}-6d_{11})V^*_{cs}}{3\sqrt{2}}$
$T^0_{bc\bar{u}\bar{s}} \to \pi^-K^+\overline{K}^0$	$(-d_5-2d_6+6d_8+d_9)V^*_{cs}$	$T^0_{bc\bar{u}\bar{s}} \to \pi^-K^+\eta$	$\sqrt{\frac{2}{3}}d_9V^*_{cd}$
$T^0_{bc\bar{u}\bar{s}} \to K^+K^0K^-$	$-(d_6+2d_7-2d_9+d_{10}+2d_{11})V^*_{cd}$	$T^0_{bc\bar{u}\bar{s}} \to K^+K^-\eta$	$\frac{(d_5+3d_6+2d_7-6d_8-3d_9+3d_{10}+6d_{11})V^*_{cs}}{\sqrt{6}}$
$T^0_{bc\bar{u}\bar{s}} \to K^0K^0\overline{K}^0$	$-2(d_6+2d_7+d_{10}+2d_{11})V^*_{cd}$	$T^0_{bc\bar{u}\bar{s}} \to K^0\overline{K}^0\eta$	$\frac{(d_5+3d_6+2d_7-6d_8-d_9+3d_{10}+6d_{11})V^*_{cs}}{\sqrt{6}}$
$T^0_{bc\bar{u}\bar{s}} \to K^0\eta\eta$	$-\frac{(3d_6+6d_7-2d_9+3d_{10}+6d_{11})V^*_{cd}}{3}$	$T^0_{bc\bar{u}\bar{s}} \to \eta\eta\eta$	$\frac{(2d_5+7d_6+6d_7-12d_8-6d_9+9d_{10}+18d_{11})V^*_{cs}}{\sqrt{6}}$
$T^+_{bc\bar{d}\bar{s}} \to \pi^+\pi^+\pi^-$	$2(-d_6-2d_7+d_{10}+2d_{11})V^*_{cs}$	$T^+_{bc\bar{d}\bar{s}} \to \pi^+\pi^0\pi^0$	$(-d_6-2d_7+d_{10}+2d_{11})V^*_{cs}$
$T^+_{bc\bar{d}\bar{s}} \to \pi^+\pi^0K^0$	$\sqrt{2}d_9V^*_{cd}$	$T^+_{bc\bar{d}\bar{s}} \to \pi^+\pi^-K^+$	$(d_6+2d_7-d_{10}-2d_{11})V^*_{cd}$
$T^+_{bc\bar{d}\bar{s}} \to \pi^+K^+K^-$	$(-d_6-2d_7+d_{10}+2d_{11})V^*_{cs}$	$T^+_{bc\bar{d}\bar{s}} \to \pi^+K^0\overline{K}^0$	$(-d_6-2d_7-2d_9+d_{10}+2d_{11})V^*_{cs}$
$T^+_{bc\bar{d}\bar{s}} \to \pi^+K^0\eta$	$\sqrt{\frac{2}{3}}d_9V^*_{cd}$	$T^+_{bc\bar{d}\bar{s}} \to \pi^+\eta\eta$	$-\frac{(3d_6+6d_7+8d_9-3d_{10}-6d_{11})V^*_{cs}}{3}$
$T^+_{bc\bar{d}\bar{s}} \to \pi^0\pi^0K^+$	$(d_6+2d_7+2d_9-d_{10}-2d_{11})V^*_{cd}$	$T^+_{bc\bar{d}\bar{s}} \to \pi^0K^+\overline{K}^0$	$-\sqrt{2}d_9V^*_{cs}$
$T^+_{bc\bar{d}\bar{s}} \to \pi^0K^+\eta$	$-\frac{2d_9V^*_{cd}}{\sqrt{3}}$	$T^+_{bc\bar{d}\bar{s}} \to K^+K^+K^-$	$2(d_6+2d_7-d_{10}-2d_{11})V^*_{cd}$
$T^+_{bc\bar{d}\bar{s}} \to K^+K^0\overline{K}^0$	$(d_6+2d_7+2d_9-d_{10}-2d_{11})V^*_{cd}$	$T^+_{bc\bar{d}\bar{s}} \to K^+\overline{K}^0\eta$	$-\sqrt{\frac{2}{3}}d_9V^*_{cs}$
$T^+_{bc\bar{d}\bar{s}} \to K^+\eta\eta$	$\frac{(3d_6+6d_7+2d_9-3d_{10}-6d_{11})V^*_{cd}}{3}$	$T^0_{bc\bar{u}\bar{d}} \to \pi^+\pi^0\pi^-$	$\sqrt{2}(-d_9+d_{10}+2d_{11})V^*_{cd}$
$T^0_{bc\bar{u}\bar{d}} \to \pi^+\pi^-\overline{K}^0$	$-(d_6+2d_7-2d_9+d_{10}+2d_{11})V^*_{cs}$	$T^0_{bc\bar{u}\bar{d}} \to \pi^+\pi^-\eta$	$-\sqrt{\frac{2}{3}}(d_5+3d_6+2d_7-6d_8)V^*_{cd}$
$T^0_{bc\bar{u}\bar{d}} \to \pi^+K^0K^-$	$(-d_5-2d_6+6d_8+d_9)V^*_{cd}$	$T^0_{bc\bar{u}\bar{d}} \to \pi^+K^-\eta$	$-2\sqrt{\frac{2}{3}}d_9V^*_{cs}$
$T^0_{bc\bar{u}\bar{d}} \to \pi^0\pi^0\pi^0$	$3\sqrt{2}(-d_9+d_{10}+2d_{11})V^*_{cd}$	$T^0_{bc\bar{u}\bar{d}} \to \pi^0\pi^0\overline{K}^0$	$-(d_6+2d_7-2d_9+d_{10}+2d_{11})V^*_{cs}$
$T^0_{bc\bar{u}\bar{d}} \to \pi^0\pi^0\eta$	$-\sqrt{\frac{2}{3}}(d_5+3d_6+2d_7-6d_8)V^*_{cd}$	$T^0_{bc\bar{u}\bar{d}} \to \pi^0K^+K^-$	$-\frac{(d_5+2d_6-6d_8+d_9-2d_{10}-4d_{11})V^*_{cd}}{\sqrt{2}}$
$T^0_{bc\bar{u}\bar{d}} \to \pi^0K^0\overline{K}^0$	$\frac{(d_5+2d_6-6d_8-d_9+2d_{10}+4d_{11})V^*_{cd}}{\sqrt{2}}$	$T^0_{bc\bar{u}\bar{d}} \to \pi^0\overline{K}^0\eta$	$\frac{2d_9V^*_{cs}}{\sqrt{3}}$
$T^0_{bc\bar{u}\bar{d}} \to \pi^0\eta\eta$	$\frac{\sqrt{2}(d_9+3d_{10}+6d_{11})V^*_{cd}}{3}$	$T^0_{bc\bar{u}\bar{d}} \to \pi^-K^+\overline{K}^0$	$-(d_5+2d_6-6d_8+d_9)V^*_{cd}$
$T^0_{bc\bar{u}\bar{d}} \to K^+\overline{K}^0K^-$	$-(d_6+2d_7+d_{10}+2d_{11})V^*_{cs}$	$T^0_{bc\bar{u}\bar{d}} \to K^+K^-\eta$	$\frac{(d_5-4d_7-6d_8+d_9)V^*_{cd}}{\sqrt{6}}$
$T^0_{bc\bar{u}\bar{d}} \to K^0\overline{K}^0\overline{K}^0$	$-2(d_6+2d_7+d_{10}+2d_{11})V^*_{cs}$	$T^0_{bc\bar{u}\bar{d}} \to K^0\overline{K}^0\eta$	$\frac{(d_5-4d_7-6d_8-d_9)V^*_{cd}}{\sqrt{6}}$
$T^0_{bc\bar{u}\bar{d}} \to \overline{K}^0\eta\eta$	$-\frac{(3d_6+6d_7-2d_9+3d_{10}+6d_{11})V^*_{cs}}{3}$	$T^0_{bc\bar{u}\bar{d}} \to \eta\eta\eta$	$\sqrt{\frac{2}{3}}(d_5-d_6-6(d_7+d_8))V^*_{cd}$

$$\Gamma(T^0_{bc\bar{u}\bar{d}} \to K^+\overline{K}^0K^-) = \frac{1}{2}\Gamma(T^0_{bc\bar{u}\bar{d}} \to K^0\overline{K}^0\overline{K}^0), \qquad \Gamma(T^0_{bc\bar{u}\bar{s}} \to \pi^+\pi^-\eta) = 2\Gamma(T^0_{bc\bar{u}\bar{s}} \to \pi^0\pi^0\eta),$$
$$2\Gamma(T^0_{bc\bar{u}\bar{s}} \to \pi^0\pi^0K^0) = \Gamma(T^0_{bc\bar{u}\bar{s}} \to K^+K^0K^-), \qquad \Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^+\pi^-\overline{K}^0) = 2\Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^0\pi^0\overline{K}^0),$$
$$\Gamma(T^0_{bc\bar{u}\bar{s}} \to \pi^0\eta\eta) = \frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}} \to \pi^+\eta\eta), \qquad \Gamma(T^+_{bc\bar{d}\bar{s}} \to \pi^0\pi^0K^+) = \frac{1}{2}\Gamma(T^+_{bc\bar{d}\bar{s}} \to K^+K^0\overline{K}^0),$$
$$\Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^+\pi^0\pi^-) = \frac{2}{3}\Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^0\pi^0\pi^0), \qquad \Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^+\pi^-\eta) = 2\Gamma(T^0_{bc\bar{u}\bar{d}} \to \pi^0\pi^0\eta),$$
$$\Gamma(T^0_{bc\bar{u}\bar{d}} \to K^+K^-\eta) = \Gamma(T^0_{bc\bar{u}\bar{d}} \to K^0\overline{K}^0\eta).$$

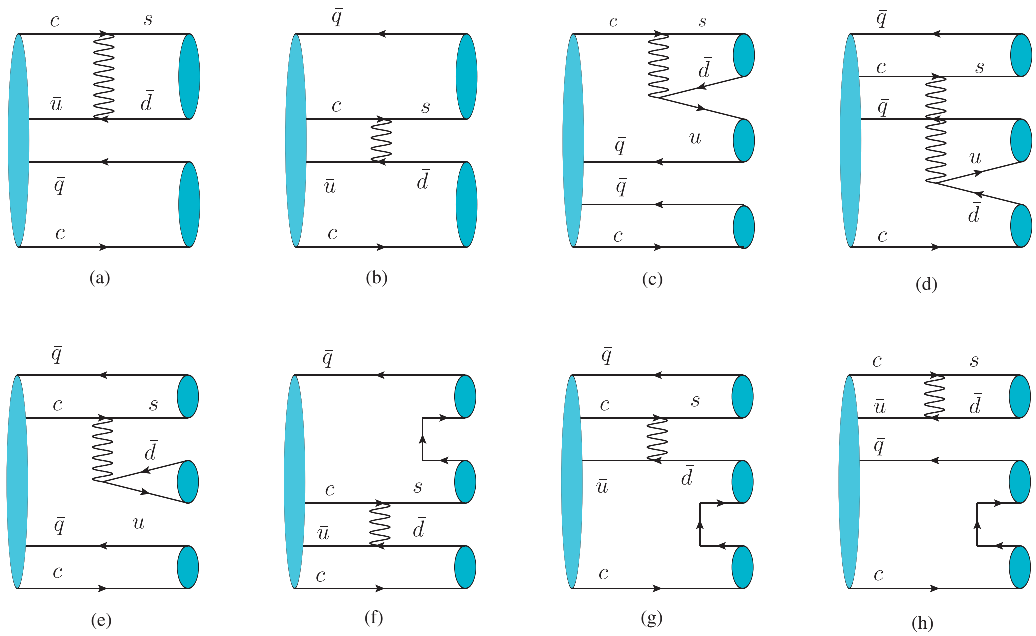

FIG. 9. Feynman diagrams for nonleptonic decays of doubly charmed tetraquark $T_{cc\bar{q}\bar{q}}$. Panels (a),(b) correspond with a charmed meson and light meson final states by the W-exchange process. Panels (c),(d),(e),(f),(g),(h) correspond with a charmed meson and two light meson final state processes.

VII. NONLEPTONIC $T_{cc\bar{q}\bar{q}}$ DECAYS

For the charm quark decays, we classified them into the following: Cabibbo allowed, singly Cabibbo suppressed, and doubly Cabibbo suppressed, i.e.,

$$c \to s\bar{d}u, \qquad c \to u\bar{d}d/\bar{s}s, \qquad c \to d\bar{s}u. \tag{40}$$

For the Cabibbo allowed decays, the nonzero components are

$$(H_{\bar{6}})^{31}_2 = -(H_{\bar{6}})^{13}_2 = 1, \quad (H_{15})^{31}_2 = (H_{15})^{13}_2 = 1. \tag{41}$$

For the singly Cabibbo-suppressed decays, the nonzero components become

$$\begin{aligned}(H_{\bar{6}})^{31}_3 &= -(H_{\bar{6}})^{13}_3 = (H_{\bar{6}})^{12}_2 = -(H_{\bar{6}})^{21}_2 = \sin(\theta_C),\\ (H_{15})^{31}_3 &= (H_{15})^{13}_3 = -(H_{15})^{12}_2 = -(H_{15})^{21}_2 = \sin(\theta_C).\end{aligned} \tag{42}$$

TABLE XXX. Doubly charmed tetraquark $T_{cc\bar{q}\bar{q}}$ decays into a charmed meson and a light meson. sC is the abbreviation of $\sin(\theta_C)$.

Channel	Amplitude	Channel	Amplitude
$T^{+}_{cc\bar{u}\bar{s}} \to D^0\pi^+$	$f_1 - 2f_2 + f_4$	$T^{+}_{cc\bar{u}\bar{s}} \to D^0K^+$	$(f_1 - 2f_2 + f_4)\mathrm{sC}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+\pi^0$	$\frac{2f_2-f_3-f_5}{\sqrt{2}}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+K^0$	$(-2f_2 + f_3 - f_5)\mathrm{sC}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+\eta$	$\frac{-2f_2+f_3-3f_5}{\sqrt{6}}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\pi^0$	$\frac{(f_1-f_3-f_4-f_5)\mathrm{sC}}{\sqrt{2}}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\overline{K}^0$	$f_1 - 2f_2 - f_4$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\eta$	$\frac{(-3f_1+4f_2+f_3+3f_4-3f_5)\mathrm{sC}}{\sqrt{6}}$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+\pi^+$	$f_1 - f_3 + f_4 - f_5$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+K^+$	$(f_1 - f_3 + f_4 - f_5)\mathrm{sC}$
$T^{+}_{cc\bar{u}\bar{d}} \to D^+\overline{K}^0$	$f_1 - f_3 - f_4 + f_5$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\pi^+$	$(f_1 - f_3 + f_4 - f_5)\mathrm{sC}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+_sK^0$	$(f_1 - f_3 - f_4 + f_5)\mathrm{sC}^2$	$T^{+}_{cc\bar{u}\bar{d}} \to D^0\pi^+$	$(f_1 - 2f_2 + f_4)(-\mathrm{sC})$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+_sK^+$	$(-f_1 + f_3 - f_4 + f_5)\mathrm{sC}^2$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+\pi^0$	$\frac{(f_1-2f_2-f_4+2f_5)\mathrm{sC}}{\sqrt{2}}$
$T^{+}_{cc\bar{u}\bar{d}} \to D^0K^+$	$(f_1 - 2f_2 + f_4)\mathrm{sC}^2$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+\eta$	$\frac{(-3f_1+2f_2+2f_3+3f_4)\mathrm{sC}}{\sqrt{6}}$
$T^{+}_{cc\bar{u}\bar{d}} \to D^+K^0$	$(f_1 - 2f_2 - f_4)\mathrm{sC}^2$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\overline{K}^0$	$(2f_2 - f_3 + f_5)\mathrm{sC}$
$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\pi^0$	$-\sqrt{2}f_5\mathrm{sC}^2$		
$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\eta$	$\sqrt{\frac{2}{3}}(2f_2 - f_3)\mathrm{sC}^2$		

At last, for the doubly Cabibbo-suppressed decays, the nonzero formulas become

$$(H_{\bar{6}})^{21}_{3} = -(H_{\bar{6}})^{12}_{3} = \sin^2\theta_C, \qquad (H_{15})^{21}_{3} = (H_{15})^{12}_{3} = \sin^2\theta_C. \tag{43}$$

A. Decays into a charmed meson and a light meson by the W-exchange process

Following the above formulas, the effective Hamiltonian for decays involving a charmed meson and a light meson is

$$\begin{aligned}\mathcal{H}_{\rm eff} = {} & f_1(T_{cc3})_{[ij]}(\overline{D})^i M^k_l (H_{\bar{6}})^{[jl]}_k + f_2(T_{cc3})_{[ij]}(\overline{D})^l M^k_l (H_{\bar{6}})^{[ij]}_k \\ & + f_3(T_{cc3})_{[ij]}(\overline{D})^k M^j_l (H_{\bar{6}})^{[il]}_k + f_4(T_{cc3})_{[ij]}(\overline{D})^i M^k_l (H_{15})^{\{jl\}}_k \\ & + f_5(T_{cc3})_{[ij]}(\overline{D})^k M^j_l (H_{15})^{\{il\}}_k.\end{aligned} \tag{44}$$

The corresponding Feynman diagrams are shown in Figs. 9(a) and 9(b). We expand the Hamiltonian to obtain the decay amplitudes which are given in Table XXX. The relations of decay widths become

$$\Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^+K^0) = \Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\overline{K}^0), \qquad \Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\pi^+) = \Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+K^+),$$
$$\Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^0\pi^+) = \Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^0K^+).$$

TABLE XXXI. Doubly charmed tetraquark $T_{cc\bar{q}\bar{q}}$ decays into a charmed meson and two light mesons. sC is the abbreviation of $\sin(\theta_C)$.

Channel	Amplitude	Channel	Amplitude
$T^{+}_{cc\bar{u}\bar{s}} \to D^0\pi^+\pi^0$	$\frac{b_4+b_5+b_{11}+b_{12}}{\sqrt{2}}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^0K^+K^0$	$(b_4+b_5+2b_8-b_{11}-b_{12})\mathrm{sC}^2$
$T^{+}_{cc\bar{u}\bar{s}} \to D^0\pi^+\eta$	$\frac{2b_1-b_4+b_5-4b_7-2b_8+2b_9+3b_{11}+b_{12}}{\sqrt{6}}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+K^0K^0$	$2(b_4+b_5-b_{11}-b_{12})\mathrm{sC}^2$
$T^{+}_{cc\bar{u}\bar{s}} \to D^0K^+\overline{K}^0$	$b_1+b_5-2b_7+b_8+b_9-b_{12}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\pi^0K^0$	$-\frac{(b_1+2b_2+b_3+b_8-b_9-b_{10})\mathrm{sC}^2}{\sqrt{2}}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+\pi^+\pi^-$	$b_3+b_5-4b_6-2b_7+b_{10}+b_{12}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\pi^-K^+$	$(b_1+2b_2+b_3-b_8-b_9-b_{10})\mathrm{sC}^2$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+\pi^0\pi^0$	$b_3-b_4-4b_6-2b_7+b_{10}-b_{11}$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+_sK^0\eta$	$-\frac{(b_1+2b_2+b_3+2b_4+2b_5+b_8-b_9-b_{10}-2b_{11}-2b_{12})\mathrm{sC}^2}{\sqrt{6}}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+\pi^0\eta$	$\frac{2b_2+b_3+b_4+2b_7+b_{10}-b_{11}}{\sqrt{3}}$	$T^{++}_{cc\bar{d}\bar{s}} \to D^0K^+K^+$	$-2(b_4+b_5+b_{11}+b_{12})\mathrm{sC}^2$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+K^+K^-$	$2(b_2+b_3-2b_6)$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+K^+K^0$	$(b_4+b_5-2b_8+b_{11}+b_{12})(-\mathrm{sC}^2)$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+K^0\overline{K}^0$	$b_3+b_5-4b_6-2b_7-b_{10}-b_{12}$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\pi^+K^0$	$(b_1+2b_2+b_3+b_8+b_9+b_{10})(-\mathrm{sC}^2)$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+\eta\eta$	$\frac{4b_2+5b_3-b_4-12b_6-2b_7-3b_{10}+3b_{11}}{3}$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\pi^0K^+$	$-\frac{(b_1+2b_2+b_3-b_8+b_9+b_{10})\mathrm{sC}^2}{\sqrt{2}}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\pi^+K^-$	$b_1+b_5-2b_7-b_8-b_9+b_{12}$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+_sK^+\eta$	$\frac{(b_1+2b_2+b_3+2b_4+2b_5-b_8+b_9+b_{10}+2b_{11}+2b_{12})\mathrm{sC}^2}{\sqrt{6}}$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\pi^0\overline{K}^0$	$\frac{-b_1+b_4+2b_7-b_8+b_9+b_{11}}{\sqrt{2}}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^0\pi^+K^0$	$(b_1+b_5-2b_7+b_8+b_9-b_{12})\mathrm{sC}^2$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\overline{K}^0\eta$	$-\frac{b_1+b_4+2b_5-2b_7+b_8-b_9-3b_{11}-2b_{12}}{\sqrt{6}}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^0\pi^0K^+$	$\frac{(b_1+b_5-2b_7-b_8+b_9+2b_{11}+b_{12})\mathrm{sC}^2}{\sqrt{2}}$
$T^{++}_{cc\bar{d}\bar{s}} \to D^0\pi^+\pi^+$	$2(b_4+b_5+b_{11}+b_{12})$	$T^{+}_{cc\bar{u}\bar{d}} \to D^0K^+\eta$	$-\frac{(b_1-2b_4-b_5-2b_7-b_8+b_9-b_{12})\mathrm{sC}^2}{\sqrt{6}}$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+\pi^+\pi^0$	$-\frac{b_4+b_5+b_{11}+b_{12}}{\sqrt{2}}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+\pi^0K^0$	$-\frac{(b_1+b_5-2b_7+b_8-b_9-2b_{11}-b_{12})\mathrm{sC}^2}{\sqrt{2}}$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+\pi^+\eta$	$\frac{2b_1+4b_2+2b_3+b_4+b_5-2b_8+2b_9+2b_{10}+b_{11}+b_{12}}{\sqrt{6}}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+\pi^-K^+$	$(b_1+b_5-2b_7-b_8-b_9+b_{12})\mathrm{sC}^2$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+K^+\overline{K}^0$	$b_1+2b_2+b_3+b_8+b_9+b_{10}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+K^0\eta$	$-\frac{(b_1-2b_4-b_5-2b_7+b_8-b_9+b_{12})\mathrm{sC}^2}{\sqrt{6}}$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\pi^+\overline{K}^0$	$b_4+b_5-2b_8+b_{11}+b_{12}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\pi^+\pi^-$	$2(b_2+b_3-2b_6)\mathrm{sC}^2$
$T^{+}_{cc\bar{u}\bar{d}} \to D^0\pi^+\overline{K}^0$	$b_4+b_5+2b_8-b_{11}-b_{12}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\pi^0\pi^0$	$2(b_2+b_3-2b_6)\mathrm{sC}^2$
$T^{+}_{cc\bar{u}\bar{d}} \to D^+\pi^+K^-$	$b_1+2b_2+b_3-b_8-b_9-b_{10}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\pi^0\eta$	$\frac{2(b_{10}-b_{11})\mathrm{sC}^2}{\sqrt{3}}$
$T^{+}_{cc\bar{u}\bar{d}} \to D^+\pi^0\overline{K}^0$	$-\frac{b_1+2b_2+b_3+b_4+b_5+b_8-b_9-b_{10}-b_{11}-b_{12}}{\sqrt{2}}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+_sK^+K^-$	$(b_3+b_5-4b_6-2b_7+b_{10}+b_{12})\mathrm{sC}^2$
$T^{+}_{cc\bar{u}\bar{d}} \to D^+\overline{K}^0\eta$	$-\frac{b_1+2b_2+b_3-b_4-b_5+b_8-b_9-b_{10}+b_{11}+b_{12}}{\sqrt{6}}$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+_sK^0\overline{K}^0$	$(b_3+b_5-4b_6-2b_7-b_{10}-b_{12})\mathrm{sC}^2$
$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\overline{K}^0\overline{K}^0$	$2(b_4+b_5-b_{11}-b_{12})$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\eta\eta$	$-\frac{2}{3}(b_2-b_3+2b_4+6b_6+4b_7)\mathrm{sC}^2$

B. Decays into a charmed meson and two light mesons

For decays involving a charmed meson and two light mesons, the effective Hamiltonian is

$$\begin{aligned}\mathcal{H}_{\rm eff} &= b_1(T_{cc3})_{[ij]}(\overline{D})^i M^l_m M^m_k (H_{\bar 6})^{[jk]}_l + b_2(T_{cc3})_{[ij]}(\overline{D})^m M^i_k M^j_l (H_{\bar 6})^{[kl]}_m + b_3(T_{cc3})_{[ij]}(\overline{D})^l M^i_m M^m_k (H_{\bar 6})^{[jk]}_l \\ &+ b_4(T_{cc3})_{[ij]}(\overline{D})^m M^i_k M^l_m (H_{\bar 6})^{[jk]}_l + b_5(T_{cc3})_{[ij]}(\overline{D})^m M^i_m M^l_k (H_{\bar 6})^{[jk]}_l + b_6(T_{cc3})_{[ij]}(\overline{D})^k M^l_m M^m_l (H_{\bar 6})^{[ij]}_k \\ &+ b_7(T_{cc3})_{[ij]}(\overline{D})^m M^k_l M^l_m (H_{\bar 6})^{[ij]}_k + b_8(T_{cc3})_{[ij]}(\overline{D})^i M^j_k M^m_l (H_{15})^{\{kl\}}_m + b_9(T_{cc3})_{[ij]}(\overline{D})^i M^l_m M^m_k (H_{15})^{\{jk\}}_l \\ &+ b_{10}(T_{cc3})_{[ij]}(\overline{D})^l M^i_m M^m_k (H_{15})^{\{jk\}}_l + b_{11}(T_{cc3})_{[ij]}(\overline{D})^m M^i_k M^l_m (H_{15})^{\{jk\}}_l + b_{12}(T_{cc3})_{[ij]}(\overline{D})^m M^i_m M^l_k (H_{15})^{\{jk\}}_l \\ &+ \bar{b_1}(T_{cc3})_{[ij]}(\overline{D})^i M^j_k M^m_l (H_{\bar 6})^{[kl]}_m. \end{aligned} \tag{45}$$

These related Feynman diagrams with the three final states are shown in Fig. 9. For the production of two light mesons, some terms contain one QCD coupling while the others contain two QCD couplings. We found that b_1 and $\bar{b_1}$ terms give the

TABLE XXXII. Doubly charmed tetraquark $T_{cc\bar q\bar q}$ decays into a charmed meson and two light mesons. sC is the abbreviation of $\sin(\theta_C)$.

Channel	Amplitude
$T^+_{cc\bar u\bar s} \to D^0\pi^+K^0$	$(b_1-b_4-2b_7-b_8+b_9+b_{11})\mathrm{sC}$
$T^+_{cc\bar u\bar s} \to D^0\pi^0K^+$	$\frac{(b_1+b_4+2b_5-2b_7+b_8+b_9+b_{11})\mathrm{sC}}{\sqrt{2}}$
$T^+_{cc\bar u\bar s} \to D^0K^+\eta$	$-\frac{(b_1+b_4+2b_5-2b_7+5b_8+b_9-3b_{11}-4b_{12})\mathrm{sC}}{\sqrt{6}}$
$T^+_{cc\bar u\bar s} \to D^+\pi^0K^0$	$\frac{(2b_2+b_3+2b_4+b_5+2b_7-b_{10}-b_{12})\mathrm{sC}}{\sqrt{2}}$
$T^+_{cc\bar u\bar s} \to D^+\pi^-K^+$	$(2b_2+b_3-b_5+2b_7-b_{10}-b_{12})(-\mathrm{sC})$
$T^+_{cc\bar u\bar s} \to D^+K^0\eta$	$\frac{(2b_2+b_3-2b_4-3b_5+2b_7-b_{10}+4b_{11}+3b_{12})\mathrm{sC}}{\sqrt{6}}$
$T^+_{cc\bar u\bar s} \to D^+_s\pi^+\pi^-$	$(-b_1+b_3-4b_6+b_8+b_9+b_{10})\mathrm{sC}$
$T^+_{cc\bar u\bar s} \to D^+_s\pi^0\pi^0$	$(b_1-b_3+4b_6+b_8-b_9-b_{10})(-\mathrm{sC})$
$T^+_{cc\bar u\bar s} \to D^+_s\pi^0\eta$	$\frac{(b_1+2b_2+b_3-b_4-b_5+b_8-b_9+b_{10}-b_{11}+b_{12})\mathrm{sC}}{\sqrt{3}}$
$T^+_{cc\bar u\bar s} \to D^+_sK^+K^-$	$(b_1+2b_2+2b_3+b_5-4b_6-2b_7-b_8-b_9+b_{12})\mathrm{sC}$
$T^+_{cc\bar u\bar s} \to D^+_sK^0\overline{K}^0$	$(b_3-b_4-4b_6-2b_7-b_{10}+b_{11})\mathrm{sC}$
$T^+_{cc\bar u\bar s} \to D^+_s\eta\eta$	$\frac{1}{3}(3b_1+4b_2+5b_3+2b_4+6b_5-12b_6-8b_7+3b_8-3b_9-3b_{10}-6b_{11}-6b_{12})\mathrm{sC}$
$T^{++}_{cc\bar d\bar s} \to D^0\pi^+K^+$	$2(b_4+b_5+b_{11}+b_{12})\mathrm{sC}$
$T^{++}_{cc\bar d\bar s} \to D^+\pi^+K^0$	$(b_1+2b_2+b_3+b_4+b_5-b_8+b_9+b_{10}+b_{11}+b_{12})\mathrm{sC}$
$T^{++}_{cc\bar d\bar s} \to D^+\pi^0K^+$	$\frac{(b_1+2b_2+b_3-b_4-b_5+b_8+b_9+b_{10}-b_{11}-b_{12})\mathrm{sC}}{\sqrt{2}}$
$T^{++}_{cc\bar d\bar s} \to D^+K^+\eta$	$-\frac{(b_1+2b_2+b_3-b_4-b_5+5b_8+b_9+b_{10}-b_{11}-b_{12})\mathrm{sC}}{\sqrt{6}}$
$T^{++}_{cc\bar d\bar s} \to D^+_s\pi^+\pi^0$	$-\sqrt{2}b_8\mathrm{sC}$
$T^{++}_{cc\bar d\bar s} \to D^+_s\pi^+\eta$	$\sqrt{\frac{2}{3}}(b_1+2b_2+b_3-b_4-b_5+2b_8+b_9+b_{10}-b_{11}-b_{12})\mathrm{sC}$
$T^{++}_{cc\bar d\bar s} \to D^+_sK^+\overline{K}^0$	$(b_1+2b_2+b_3+b_4+b_5-b_8+b_9+b_{10}+b_{11}+b_{12})\mathrm{sC}$
$T^+_{cc\bar u\bar d} \to D^0\pi^+\pi^0$	$\sqrt{2}(b_8-b_{11}-b_{12})\mathrm{sC}$
$T^+_{cc\bar u\bar d} \to D^0\pi^+\eta$	$-\sqrt{\frac{2}{3}}(b_1+b_4+2b_5-2b_7+2b_8+b_9-b_{12})\mathrm{sC}$
$T^+_{cc\bar u\bar d} \to D^0K^+\overline{K}^0$	$(b_1-b_4-2b_7-b_8+b_9+b_{11})(-\mathrm{sC})$
$T^+_{cc\bar u\bar d} \to D^+\pi^+\pi^-$	$(b_1+2b_2+2b_3+b_5-4b_6-2b_7-b_8-b_9+b_{12})(-\mathrm{sC})$
$T^+_{cc\bar u\bar d} \to D^+\pi^0\pi^0$	$(b_1+2b_2+2b_3+b_5-4b_6-2b_7+b_8-b_9-2b_{11}-b_{12})(-\mathrm{sC})$
$T^+_{cc\bar u\bar d} \to D^+\pi^0\eta$	$\frac{(b_1+b_4+2b_5-2b_7+b_8-b_9-2b_{10}-b_{11}-2b_{12})\mathrm{sC}}{\sqrt{3}}$
$T^+_{cc\bar u\bar d} \to D^+K^+K^-$	$(b_1-b_3+4b_6-b_8-b_9-b_{10})\mathrm{sC}$
$T^+_{cc\bar u\bar d} \to D^+K^0\overline{K}^0$	$(b_3-b_4-4b_6-2b_7-b_{10}+b_{11})(-\mathrm{sC})$
$T^+_{cc\bar u\bar d} \to D^+\eta\eta$	$\frac{1}{3}(3b_1+2b_2-2b_3-2b_4-3b_5+12b_6+2b_7+3b_8-3b_9+3b_{12})\mathrm{sC}$
$T^+_{cc\bar u\bar d} \to D^+_s\pi^+K^-$	$(2b_2+b_3-b_5+2b_7-b_{10}-b_{12})\mathrm{sC}$
$T^+_{cc\bar u\bar d} \to D^+_s\pi^0\overline{K}^0$	$-\frac{(2b_2+b_3-b_5+2b_7-b_{10}+2b_{11}+b_{12})\mathrm{sC}}{\sqrt{2}}$
$T^+_{cc\bar u\bar d} \to D^+_s\overline{K}^0\eta$	$-\frac{(2b_2+b_3+4b_4+3b_5+2b_7-b_{10}-2b_{11}-3b_{12})\mathrm{sC}}{\sqrt{6}}$

same contribution which always contains the factor $b_1 - \bar{b}_1$. Then $\bar{b}_1$ terms are removed in the final results. The decay amplitudes are given in Table XXXI and Table XXXII. Based on the expanded amplitudes, the relations become

$$\Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^0\pi^+K^0) = \Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^0K^+\overline{K}^0), \qquad \Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^+\pi^-K^+) = \Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\pi^+K^-),$$
$$\Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\pi^+\pi^-) = \Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+K^+K^-), \qquad \Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+\pi^+K^0) = \Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+_sK^+\overline{K}^0),$$
$$\Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+\pi^+\pi^-) = \Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^+_sK^+K^-), \qquad \Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\pi^0\pi^0) = \frac{1}{2}\Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\pi^+\pi^-),$$
$$\Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^0\pi^+\pi^+) = 4\Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^0\pi^+\pi^0) = 4\Gamma(T^{++}_{cc\bar{d}\bar{s}} \to D^+\pi^+\pi^0),$$
$$\Gamma(T^{+}_{cc\bar{u}\bar{d}} \to D^+K^0\overline{K}^0) = \Gamma(T^{+}_{cc\bar{u}\bar{s}} \to D^+_sK^0\overline{K}^0).$$

VIII. GOLDEN DECAY CHANNELS

In order to hunt for the doubly heavy tetraquarks, the golden decay channels are very useful [50]. So we list them and give an estimation of the decay branching fractions in this section. In the following lists, a hadron is a general particle and can be replaced by the states with the identical quark contents. The light pseudoscalar may be replaced by the light vector meson such as replacing the $\overline{K}^0$ by $\overline{K}^{*0}$ which decays into $K^-\pi^+$. The modes involving the neutral mesons π^0, η, ρ^0, ω are removed because a neutral meson is difficult to be reconstructed at hadron-hadron colliders.

A. $T_{cc\bar{q}\bar{q}}$

For the $T_{cc\bar{q}\bar{q}}$ decays, we collected Cabibbo allowed decays in Table XXXIII. From the data of the D meson decays, we conclude that these Cabibbo allowed decay channels in Table XXXIII may lead to branching fractions at a few percent level. Note that to reconstruct the final charm meson, another factor of 10^{-3} is required.

B. $T_{bc\bar{q}\bar{q}}$

To reconstruct the $T_{bc\bar{q}\bar{q}}$ tetraquark, lists of possible modes are given in Table XXXIV. The decay width is dominant by the charm quark decay from the estimation of the magnitudes of CKM matrix elements. For the charm quark decay, the typical branching fractions in Table XXXIV are estimated to be a few percents. If the bottom quark decays, the branching fraction might be smaller than 10^{-3}. Note that to reconstruct the final charm or bottom meson, another factor of 10^{-3} is also required.

C. $T_{bb\bar{q}\bar{q}}$

To reconstruct the $T_{bb\bar{q}\bar{q}}$, we listed the gold decay channels in Table XXXV. The branching fractions are estimated at the order 10^{-3}. In these decay channels, the reconstruction of the final bottom meson or bottom baryon state requires another factor of 10^{-3}. For the reconstruction of J/ψ or D or charmed baryons, the corresponding factor becomes to be 10^{-2}. Thus, to reconstruct the doubly

TABLE XXXIII. Cabibbo allowed $T_{cc\bar{q}\bar{q}}$ decays. $\overline{K}^0$ can be replaced by vector meson $\overline{K}^{*0}$.

Two-Body Decays		
$T^{+}_{cc\bar{u}\bar{s}} \to D^0l^+\nu$	$T^{+}_{cc\bar{u}\bar{s}} \to D^0\pi^+$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\overline{K}^0$
$T^{++}_{cc\bar{d}\bar{s}} \to D^+l^+\nu$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+\pi^+$	
$T^{+}_{cc\bar{u}\bar{d}} \to D^+\overline{K}^0$		

Three-Body Decays				
$T^{+}_{cc\bar{u}\bar{s}} \to D^+\pi^-l^+\nu$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+_sK^-l^+\nu$	$T^{+}_{cc\bar{u}\bar{s}} \to D^0K^+\overline{K}^0$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+\pi^+\pi^-$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+K^+K^-$
$T^{+}_{cc\bar{u}\bar{s}} \to D^+K^0\overline{K}^0$	$T^{+}_{cc\bar{u}\bar{s}} \to D^+_s\pi^+K^-$			
$T^{++}_{cc\bar{d}\bar{s}} \to D^0\pi^+l^+\nu$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\overline{K}^0l^+\nu$	$T^{++}_{cc\bar{d}\bar{s}} \to D^0\pi^+\pi^+$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+K^+\overline{K}^0$	$T^{++}_{cc\bar{d}\bar{s}} \to D^+_s\pi^+\overline{K}^0$
$T^{+}_{cc\bar{u}\bar{d}} \to D^0\overline{K}^0l^+\nu$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+K^-l^+\nu$	$T^{+}_{cc\bar{u}\bar{d}} \to D^0\pi^+\overline{K}^0$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+\pi^+K^-$	$T^{+}_{cc\bar{u}\bar{d}} \to D^+_s\overline{K}^0\overline{K}^0$

TABLE XXXIV. Cabibbo allowed $T_{bc\bar{q}\,\bar{q}}$ decays. $\overline{K}^0$ can be replaced by vector meson $\overline{K}^{*0}$.

Two-Body Decays (Charm Decays)

$T^0_{bc\bar{u}\,\bar{s}} \to B^- l^+ \nu$		$T^0_{bc\bar{u}\,\bar{s}} \to B^- \pi^+$		$T^0_{bc\bar{u}\,\bar{s}} \to \overline{B}{}^0_s \overline{K}{}^0$
$T^+_{bc\bar{d}\,\bar{s}} \to \overline{B}{}^0 l^+ \nu$		$T^+_{bc\bar{d}\,\bar{s}} \to \overline{B}{}^0 \pi^+$		
$T^0_{bc\bar{u}\,\bar{d}} \to \overline{B}{}^0 \overline{K}{}^0$				

Three-Body Decays (Charm Decays)

$T^0_{bc\bar{u}\,\bar{s}} \to \overline{B}{}^0 \pi^- l^+ \nu$	$T^0_{bc\bar{u}\,\bar{s}} \to \overline{B}{}^0_s K^- l^+ \nu$	$T^0_{bc\bar{u}\,\bar{s}} \to B^- K^+ \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{s}} \to \overline{B}{}^0 \pi^+ \pi^-$	$T^0_{bc\bar{u}\,\bar{s}} \to \overline{B}{}^0 K^+ K^-$
$T^0_{bc\bar{u}\,\bar{s}} \to \overline{B}{}^0 K^0 \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{s}} \to \overline{B}{}^0_s \pi^+ K^-$			
$T^+_{bc\bar{d}\,\bar{s}} \to B^- \pi^+ l^+ \nu$	$T^+_{bc\bar{d}\,\bar{s}} \to \overline{B}{}^0_s \overline{K}{}^0 l^+ \nu$	$T^+_{bc\bar{d}\,\bar{s}} \to B^- \pi^+ \pi^+$	$T^+_{bc\bar{d}\,\bar{s}} \to \overline{B}{}^0 K^+ \overline{K}{}^0$	$T^+_{bc\bar{d}\,\bar{s}} \to \overline{B}{}^0_s \pi^+ \overline{K}{}^0$
$T^0_{bc\bar{u}\,\bar{d}} \to \overline{B}{}^0 K^- l^+ \nu$	$T^0_{bc\bar{u}\,\bar{d}} \to B^- \overline{K}{}^0 l^+ \nu$	$T^0_{bc\bar{u}\,\bar{d}} \to B^- \pi^+ \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{d}} \to \overline{B}{}^0 \pi^+ K^-$	$T^0_{bc\bar{u}\,\bar{d}} \to \overline{B}{}^0_s \overline{K}{}^0 \overline{K}{}^0$

Two-Body Decays (Bottom Decays)

$T^0_{bc\bar{u}\,\bar{s}} \to D^0 J/\psi$	$T^0_{bc\bar{u}\,\bar{s}} \to \overline{D}{}^0 D^0$	$T^0_{bc\bar{u}\,\bar{s}} \to D^+ D^-$	$T^0_{bc\bar{u}\,\bar{s}} \to D^+_s D^-_s$	$T^0_{bc\bar{u}\,\bar{s}} \to \Xi^+_{cc} \Lambda^-_{\bar{c}}$
$T^0_{bc\bar{u}\,\bar{s}} \to \Lambda^-_{\bar{c}} \Lambda^+_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \bar{\Xi}'^0_{\bar{c}} \Xi'^0_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \Xi^+_{cc} \Sigma^-_{\bar{c}}$	$T^0_{bc\bar{u}\,\bar{s}} \to \Omega^+_{cc} \Xi^-_{\bar{c}}$	$T^0_{bc\bar{u}\,\bar{s}} \to \Omega^+_{cc} \Xi'^-_{\bar{c}}$
$T^0_{bc\bar{u}\,\bar{s}} \to \Xi^{++}_{cc} \Sigma^{--}_{\bar{c}}$	$T^0_{bc\bar{u}\,\bar{s}} \to \bar{\Sigma}^- \Xi^+_{cc}$	$T^0_{bc\bar{u}\,\bar{s}} \to \Lambda^-_{\bar{c}} \Sigma^+_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \Xi^-_{\bar{c}} \Xi^+_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \Xi^-_{\bar{c}} \Xi'^+_c$
$T^0_{bc\bar{u}\,\bar{s}} \to \bar{\Xi}^0_{\bar{c}} \Xi^0_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \bar{\Xi}^0_{\bar{c}} \Xi'^0_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \Sigma^-_{\bar{c}} \Lambda^+_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \Sigma^{--}_{\bar{c}} \Sigma^{++}_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \Sigma^-_{\bar{c}} \Sigma^+_c$
$T^0_{bc\bar{u}\,\bar{s}} \to \Xi'^-_{\bar{c}} \Xi^+_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \bar{\Sigma}^0_{\bar{c}} \Sigma^-_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \bar{\Xi}'^0_{\bar{c}} \Xi^0_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \Xi'^-_{\bar{c}} \Xi'^+_c$	$T^0_{bc\bar{u}\,\bar{s}} \to \bar{\Omega}^0_{\bar{c}} \Omega^0_c$
$T^+_{bc\bar{d}\,\bar{s}} \to D^+ J/\psi$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+_s D^0$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+ \overline{D}{}^0$	$T^+_{bc\bar{d}\,\bar{s}} \to J/\psi \pi^+$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Lambda}^0 \Xi^+_{cc}$
$T^+_{bc\bar{d}\,\bar{s}} \to \Omega^+_{cc} \bar{\Xi}'^0_{\bar{c}}$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Xi}'^0_{\bar{c}} \Xi^+_c$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Sigma}^- \Xi^{++}_{cc}$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Sigma}'^- \Xi^{++}_{cc}$	$T^+_{bc\bar{d}\,\bar{s}} \to \Omega^+_{cc} \bar{\Xi}^0_{\bar{c}}$
$T^+_{bc\bar{d}\,\bar{s}} \to \Xi^+_{cc} \bar{\Sigma}^0_{\bar{c}}$	$T^+_{bc\bar{d}\,\bar{s}} \to \Xi^{++}_{cc} \Lambda^-_{\bar{c}}$	$T^+_{bc\bar{d}\,\bar{s}} \to \Xi^{++}_{cc} \Sigma^-_{\bar{c}}$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Sigma}^0 \Xi^+_{cc}$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Sigma}'^0 \Xi^+_{cc}$
$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Xi}'^0 \Omega^+_{cc}$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Xi}^0 \Omega^+_{cc}$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Xi}^0_{\bar{c}} \Xi^+_c$	$T^+_{bc\bar{d}\,\bar{s}} \to \Lambda^-_{\bar{c}} \Sigma^{++}_c$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Xi}^0_{\bar{c}} \Xi'^+_c$
$T^+_{bc\bar{d}\,\bar{s}} \to \Sigma^-_{\bar{c}} \Sigma^{++}_c$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Sigma}^0_{\bar{c}} \Sigma^+_c$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Xi}'^0_{\bar{c}} \Xi'^+_c$	$T^+_{bc\bar{d}\,\bar{s}} \to \bar{\Sigma}^0_{\bar{c}} \Lambda^+_c$	
$T^0_{bc\bar{u}\,\bar{d}} \to D^+ D^-_s$	$T^0_{bc\bar{u}\,\bar{d}} \to J/\psi \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{d}} \to D^0 D^0$	$T^0_{bc\bar{u}\,\bar{d}} \to \bar{\Delta}^{--} \Xi^{++}_{cc}$	$T^0_{bc\bar{u}\,\bar{d}} \to \bar{\Delta}^- \Xi^+_{cc}$
$T^0_{bc\bar{u}\,\bar{d}} \to \bar{\Sigma}^- \Omega^+_{cc}$	$T^0_{bc\bar{u}\,\bar{d}} \to \bar{\Sigma}'^- \Omega^+_{cc}$	$T^0_{bc\bar{u}\,\bar{d}} \to \bar{p} \Xi^+_{cc}$	$T^0_{bc\bar{u}\,\bar{d}} \to \Omega^+_{cc} \Lambda^-_{\bar{c}}$	$T^0_{bc\bar{u}\,\bar{d}} \to \Lambda^-_{\bar{c}} \Xi'^+_c$
$T^0_{bc\bar{u}\,\bar{d}} \to \bar{\Xi}^0_{\bar{c}} \Omega^0_c$	$T^0_{bc\bar{u}\,\bar{d}} \to \Sigma^-_{\bar{c}} \Xi^+_c$	$T^0_{bc\bar{u}\,\bar{d}} \to \bar{\Sigma}^0_{\bar{c}} \Xi^0_c$	$T^0_{bc\bar{u}\,\bar{d}} \to \Sigma^-_{\bar{c}} \Xi'^+_c$	$T^0_{bc\bar{u}\,\bar{d}} \to \bar{\Sigma}^0_{\bar{c}} \Xi'^0_c$
$T^0_{bc\bar{u}\,\bar{d}} \to \bar{\Xi}'^0_{\bar{c}} \Omega^0_c$				

Three-Body Decays (Bottom Decays)

$T^0_{bc\bar{u}\,\bar{s}} \to D^+ \pi^- J/\psi$	$T^0_{bc\bar{u}\,\bar{s}} \to D^+_s K^- J/\psi$	$T^0_{bc\bar{u}\,\bar{s}} \to D^0 \overline{D}{}^0 D^0$	$T^0_{bc\bar{u}\,\bar{s}} \to D^0 D^- D^+$	$T^0_{bc\bar{u}\,\bar{s}} \to D^0 D^0 K^0$
$T^0_{bc\bar{u}\,\bar{s}} \to D^0 D^+_s \pi^-$	$T^0_{bc\bar{u}\,\bar{s}} \to D^0 D^- \pi^+$	$T^0_{bc\bar{u}\,\bar{s}} \to D^0 D^-_s D^+_s$	$T^0_{bc\bar{u}\,\bar{s}} \to D^0 D^-_s K^+$	$T^0_{bc\bar{u}\,\bar{s}} \to D^+ \overline{D}{}^0 \pi^-$
$T^0_{bc\bar{u}\,\bar{s}} \to D^+ D^-_s K^0$	$T^0_{bc\bar{u}\,\bar{s}} \to D^+_s \overline{D}{}^0 K^-$	$T^0_{bc\bar{u}\,\bar{s}} \to D^+_s D^- \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{s}} \to J/\psi \pi^+ \pi^-$	$T^0_{bc\bar{u}\,\bar{s}} \to J/\psi K^0 \overline{K}{}^0$
$T^0_{bc\bar{u}\,\bar{s}} \to J/\psi K^+ K^-$	$T^0_{bc\bar{u}\,\bar{s}} \to D^0 \pi^+ \pi^-$	$T^0_{bc\bar{u}\,\bar{s}} \to D^0 K^+ K^-$	$T^0_{bc\bar{u}\,\bar{s}} \to D^0 K^0 \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{s}} \to D^+ K^0 K^-$
$T^0_{bc\bar{u}\,\bar{s}} \to D^+_s \pi^- \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{s}} \to \pi^+ K^0 K^-$	$T^0_{bc\bar{u}\,\bar{s}} \to \pi^- K^+ \overline{K}{}^0$		
$T^+_{bc\bar{d}\,\bar{s}} \to D^0 \pi^+ J/\psi$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+_s \overline{K}{}^0 J/\psi$	$T^+_{bc\bar{d}\,\bar{s}} \to D^0 \overline{D}{}^0 D^+$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+ D^- D^+$	$T^+_{bc\bar{d}\,\bar{s}} \to D^0 D^0 K^+$
$T^+_{bc\bar{d}\,\bar{s}} \to D^0 D^+ K^0$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+ D^+_s \pi^-$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+ D^-_s D^+_s$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+_s D^+_s K^-$	$T^+_{bc\bar{d}\,\bar{s}} \to D^0 \overline{D}{}^0 \pi^+$
$T^+_{bc\bar{d}\,\bar{s}} \to D^+ D^- \pi^+$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+ D^-_s K^+$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+_s \overline{D}{}^0 \overline{K}{}^0$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+_s D^-_s \pi^+$	$T^+_{bc\bar{d}\,\bar{s}} \to J/\psi K^+ \overline{K}{}^0$
$T^+_{bc\bar{d}\,\bar{s}} \to D^0 K^+ \overline{K}{}^0$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+ \pi^+ \pi^-$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+ K^+ K^-$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+ K^0 \overline{K}{}^0$	$T^+_{bc\bar{d}\,\bar{s}} \to D^+_s \pi^+ K^-$
$T^+_{bc\bar{d}\,\bar{s}} \to \pi^+ \pi^+ \pi^-$	$T^+_{bc\bar{d}\,\bar{s}} \to \pi^+ K^+ K^-$	$T^+_{bc\bar{d}\,\bar{s}} \to \pi^+ K^0 \overline{K}{}^0$		
$T^0_{bc\bar{u}\,\bar{d}} \to D^0 \overline{K}{}^0 J/\psi$	$T^0_{bc\bar{u}\,\bar{d}} \to D^+ K^- J/\psi$	$T^0_{bc\bar{u}\,\bar{d}} \to D^0 D^-_s D^+$	$T^0_{bc\bar{u}\,\bar{d}} \to D^0 D^+ \pi^-$	$T^0_{bc\bar{u}\,\bar{d}} \to D^0 D^+_s K^-$
$T^0_{bc\bar{u}\,\bar{d}} \to D^0 \overline{D}{}^0 \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{d}} \to D^+ \overline{D}{}^0 K^-$	$T^0_{bc\bar{u}\,\bar{d}} \to D^+ D^- \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{d}} \to D^0 D^-_s \pi^+$	$T^0_{bc\bar{u}\,\bar{d}} \to D^+_s D^-_s \overline{K}{}^0$
$T^0_{bc\bar{u}\,\bar{d}} \to J/\psi \pi^+ K^-$	$T^0_{bc\bar{u}\,\bar{d}} \to D^0 \pi^+ K^-$	$T^0_{bc\bar{u}\,\bar{d}} \to D^+ \pi^- \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{d}} \to \pi^+ \pi^- \overline{K}{}^0$	$T^0_{bc\bar{u}\,\bar{d}} \to K^+ \overline{K}{}^0 K^-$
$T^0_{bc\bar{u}\,\bar{d}} \to K^0 \overline{K}{}^0 \overline{K}{}^0$				

TABLE XXXV. Cabibbo allowed $T_{bb\bar q\bar q}$ decays. $\overline{K}^0$ can be replaced by vector meson $\overline{K}^{*0}$.

Two-Body Decays				
$T^-_{bb\bar u\bar s}\to B^-J/\psi$	$T^-_{bb\bar u\bar s}\to D^0B_c^-$	$T^-_{bb\bar u\bar s}\to B^-\overline{D}^0$	$T^-_{bb\bar u\bar s}\to \overline{B}^0D^-$	$T^-_{bb\bar u\bar s}\to \overline{B}^0_sD_s^-$
$T^-_{bb\bar u\bar s}\to \Xi^0_{bc}\Lambda^-_{\bar c}$	$T^-_{bb\bar u\bar s}\to \Lambda^-_{\bar c}\Lambda^0_b$	$T^-_{bb\bar u\bar s}\to \Xi^0_{bc}\Sigma^-_{\bar c}$	$T^-_{bb\bar u\bar s}\to \Omega^0_{bc}\Xi^-_{\bar c}$	$T^-_{bb\bar u\bar s}\to \Omega^0_{bc}\Xi'^-_{\bar c}$
$T^-_{bb\bar u\bar s}\to \Xi^+_{bc}\Sigma^{--}_{\bar c}$	$T^-_{bb\bar u\bar s}\to \bar\Sigma^-\Xi^0_{bc}$	$T^-_{bb\bar u\bar s}\to \Lambda^-_{\bar c}\Sigma^0_b$	$T^-_{bb\bar u\bar s}\to \Xi^-_{\bar c}\Xi^0_b$	$T^-_{bb\bar u\bar s}\to \Xi^-_{\bar c}\Xi'^0_b$
$T^-_{bb\bar u\bar s}\to \bar\Xi^0_{\bar c}\Xi^-_b$	$T^-_{bb\bar u\bar s}\to \bar\Xi^0_{\bar c}\Xi'^-_b$	$T^-_{bb\bar u\bar s}\to \Sigma^-_{\bar c}\Lambda^0_b$	$T^-_{bb\bar u\bar s}\to \Sigma^{--}_{\bar c}\Sigma^+_b$	$T^-_{bb\bar u\bar s}\to \Sigma^-_{\bar c}\Sigma^0_b$
$T^-_{bb\bar u\bar s}\to \Xi'^-_{\bar c}\Xi^0_b$	$T^-_{bb\bar u\bar s}\to \bar\Sigma^0_{\bar c}\Sigma^-_b$	$T^-_{bb\bar u\bar s}\to \bar\Xi'^0_{\bar c}\Xi^-_b$	$T^-_{bb\bar u\bar s}\to \Xi'^-_{\bar c}\Xi'^0_b$	$T^-_{bb\bar u\bar s}\to \bar\Xi'^0_{\bar c}\Xi'^-_b$
$T^-_{bb\bar u\bar s}\to \bar\Omega^0_{\bar c}\Omega^-_b$				
$T^0_{bb\bar d\bar s}\to D^+B_c^-$	$T^0_{bb\bar d\bar s}\to \overline{B}^0J/\psi$	$T^0_{bb\bar d\bar s}\to B^-D_s^+$	$T^0_{bb\bar d\bar s}\to \overline{B}^0_sD^0$	$T^0_{bb\bar d\bar s}\to \overline{B}^0\overline{D}^0$
$T^0_{bb\bar d\bar s}\to B_c^-\pi^+$	$T^0_{bb\bar d\bar s}\to \bar\Lambda^0\Xi^0_{bc}$	$T^0_{bb\bar d\bar s}\to \bar\Sigma^-\Xi^+_{bc}$	$T^0_{bb\bar d\bar s}\to \bar\Sigma'^-\Xi^+_{bc}$	$T^0_{bb\bar d\bar s}\to \Omega^0_{bc}\bar\Xi^0_{\bar c}$
$T^0_{bb\bar d\bar s}\to \Xi^0_{bc}\bar\Sigma^0_{\bar c}$	$T^0_{bb\bar d\bar s}\to \Xi^+_{bc}\Lambda^-_{\bar c}$	$T^0_{bb\bar d\bar s}\to \Xi^+_{bc}\Sigma^-_{\bar c}$	$T^0_{bb\bar d\bar s}\to \bar\Sigma^0\Xi^0_{bc}$	$T^0_{bb\bar d\bar s}\to \bar\Sigma'^0\Xi^0_{bc}$
$T^0_{bb\bar d\bar s}\to \bar\Xi'^0\Omega^0_{bc}$	$T^0_{bb\bar d\bar s}\to \bar\Xi^0\Omega^0_{bc}$	$T^0_{bb\bar d\bar s}\to \bar\Xi^0_{\bar c}\Xi^0_b$	$T^0_{bb\bar d\bar s}\to \Lambda^-_{\bar c}\Sigma^+_b$	$T^0_{bb\bar d\bar s}\to \bar\Xi^0_{\bar c}\Xi'^0_b$
$T^0_{bb\bar d\bar s}\to \Sigma^-_{\bar c}\Sigma^+_b$	$T^0_{bb\bar d\bar s}\to \bar\Sigma^0_{\bar c}\Sigma^0_b$	$T^0_{bb\bar d\bar s}\to \bar\Xi'^0_{\bar c}\Xi'^0_b$	$T^0_{bb\bar d\bar s}\to \bar\Sigma^0_{\bar c}\Lambda^0_b$	$T^0_{bb\bar d\bar s}\to \bar\Xi'^0_{\bar c}\Xi^0_b$
$T^0_{bb\bar d\bar s}\to \Omega^0_{bc}\bar\Xi'^0_{\bar c}$				
$T^-_{bb\bar u\bar d}\to \overline{B}^0D_s^-$	$T^-_{bb\bar u\bar d}\to B_c^-\overline{K}^0$	$T^-_{bb\bar u\bar d}\to B^-D^0$	$T^-_{bb\bar u\bar d}\to \bar\Delta^{--}\Xi^+_{bc}$	$T^-_{bb\bar u\bar d}\to \bar\Delta^-\Xi^0_{bc}$
$T^-_{bb\bar u\bar d}\to \bar\Sigma^-\Omega^0_{bc}$	$T^-_{bb\bar u\bar d}\to \bar\Sigma'^-\Omega^0_{bc}$	$T^-_{bb\bar u\bar d}\to \bar p\Xi^0_{bc}$	$T^-_{bb\bar u\bar d}\to \Omega^0_{bc}\Lambda^-_{\bar c}$	$T^-_{bb\bar u\bar d}\to \Lambda^-_{\bar c}\Xi'^0_b$
$T^-_{bb\bar u\bar d}\to \bar\Xi^0_{\bar c}\Omega^-_b$	$T^-_{bb\bar u\bar d}\to \Sigma^-_{\bar c}\Xi^0_b$	$T^-_{bb\bar u\bar d}\to \bar\Sigma^0_{\bar c}\Xi^-_b$	$T^-_{bb\bar u\bar d}\to \Sigma^-_{\bar c}\Xi'^0_b$	$T^-_{bb\bar u\bar d}\to \bar\Sigma^0_{\bar c}\Xi'^-_b$
$T^-_{bb\bar u\bar d}\to \bar\Xi'^0_{\bar c}\Omega^-_b$				
Three-Body Decays				
$T^-_{bb\bar u\bar s}\to \overline{B}^0\pi^-J/\psi$	$T^-_{bb\bar u\bar s}\to \overline{B}^0_sK^-J/\psi$	$T^-_{bb\bar u\bar s}\to D^+\pi^-B_c^-$	$T^-_{bb\bar u\bar s}\to D_s^+K^-B_c^-$	$T^-_{bb\bar u\bar s}\to B^-D^0K^0$
$T^-_{bb\bar u\bar s}\to B^-D_s^+\pi^-$	$T^-_{bb\bar u\bar s}\to \overline{B}^0_sD^0\pi^-$	$T^-_{bb\bar u\bar s}\to B^-D^-\pi^+$	$T^-_{bb\bar u\bar s}\to B^-D_s^-K^+$	$T^-_{bb\bar u\bar s}\to \overline{B}^0\overline{D}^0\pi^-$
$T^-_{bb\bar u\bar s}\to \overline{B}^0D_s^-K^0$	$T^-_{bb\bar u\bar s}\to \overline{B}^0_s\overline{D}^0K^-$	$T^-_{bb\bar u\bar s}\to \overline{B}^0_sD^-\overline{K}^0$	$T^-_{bb\bar u\bar s}\to B_c^-\pi^+\pi^-$	$T^-_{bb\bar u\bar s}\to B_c^-K^0\overline{K}^0$
$T^-_{bb\bar u\bar s}\to B_c^-K^+K^-$	$T^-_{bb\bar u\bar s}\to D^0\overline{D}^0B^-$	$T^-_{bb\bar u\bar s}\to D^0D^-\overline{B}^0$	$T^-_{bb\bar u\bar s}\to D^0D_s^-\overline{B}^0_s$	$T^-_{bb\bar u\bar s}\to D^+D^-B^-$
$T^-_{bb\bar u\bar s}\to D_s^+D_s^-B^-$				
$T^0_{bb\bar d\bar s}\to B^-\pi^+J/\psi$	$T^0_{bb\bar d\bar s}\to \overline{B}^0_s\overline{K}^0J/\psi$	$T^0_{bb\bar d\bar s}\to D^0\pi^+B_c^-$	$T^0_{bb\bar d\bar s}\to D_s^+\overline{K}^0B_c^-$	$T^0_{bb\bar d\bar s}\to B^-D^0K^+$
$T^0_{bb\bar d\bar s}\to B^-D^+K^0$	$T^0_{bb\bar d\bar s}\to \overline{B}^0D^0K^0$	$T^0_{bb\bar d\bar s}\to \overline{B}^0D_s^+\pi^-$	$T^0_{bb\bar d\bar s}\to \overline{B}^0_sD^+\pi^-$	$T^0_{bb\bar d\bar s}\to \overline{B}^0_sD_s^+K^-$
$T^0_{bb\bar d\bar s}\to B^-\overline{D}^0\pi^+$	$T^0_{bb\bar d\bar s}\to \overline{B}^0D^-\pi^+$	$T^0_{bb\bar d\bar s}\to \overline{B}^0D_s^-K^+$	$T^0_{bb\bar d\bar s}\to \overline{B}^0_s\overline{D}^0\overline{K}^0$	$T^0_{bb\bar d\bar s}\to \overline{B}^0_sD_s^-\pi^+$
$T^0_{bb\bar d\bar s}\to B_c^-K^+\overline{K}^0$	$T^0_{bb\bar d\bar s}\to D^0\overline{D}^0\overline{B}^0$	$T^0_{bb\bar d\bar s}\to D^+\overline{D}^0B^-$	$T^0_{bb\bar d\bar s}\to D^+D^-\overline{B}^0$	$T^0_{bb\bar d\bar s}\to D^+D_s^-\overline{B}^0_s$
$T^0_{bb\bar d\bar s}\to D_s^+D_s^-\overline{B}^0$				
$T^-_{bb\bar u\bar d}\to B^-\overline{K}^0J/\psi$	$T^-_{bb\bar u\bar d}\to \overline{B}^0K^-J/\psi$	$T^-_{bb\bar u\bar d}\to D^0\overline{K}^0B_c^-$	$T^-_{bb\bar u\bar d}\to D^+K^-B_c^-$	$T^-_{bb\bar u\bar d}\to B^-D^+\pi^-$
$T^-_{bb\bar u\bar d}\to B^-D_s^+K^-$	$T^-_{bb\bar u\bar d}\to \overline{B}^0D^0\pi^-$	$T^-_{bb\bar u\bar d}\to \overline{B}^0_sD^0K^-$	$T^-_{bb\bar u\bar d}\to B^-\overline{D}^0\overline{K}^0$	$T^-_{bb\bar u\bar d}\to \overline{B}^0\overline{D}^0K^-$
$T^-_{bb\bar u\bar d}\to \overline{B}^0D^-\overline{K}^0$	$T^-_{bb\bar u\bar d}\to B^-D_s^-\pi^+$	$T^-_{bb\bar u\bar d}\to \overline{B}^0_sD_s^-\overline{K}^0$	$T^-_{bb\bar u\bar d}\to B_c^-\pi^+K^-$	$T^-_{bb\bar u\bar d}\to D^0D_s^-\overline{B}^0$
$T^-_{bb\bar u\bar d}\to D^+D_s^-B^-$				

bottomed tetraquark, the channel $T^-_{bb\bar u\bar d}\to\bar p\Xi^0_{bc}(\to p\Sigma+B^-)$ is a wonderful tool whose branching fraction is the order of 10^{-6}.

IX. CONCLUSIONS

Most theoretical works including the lattice QCD simulations supported the possibility of the stable doubly heavy tetraquarks. In this work, we have given the spectra of the doubly heavy tetraquarks by the Sakharov-Zeldovich formula. We found that $T^-_{bb\bar u\bar d}(\mathbf{3})$ is about 73 MeV below the BB^* threshold. In order to hunt for these stable doubly heavy tetraquarks, we investigated systematically the semileptonic and nonleptonic weak decay amplitudes of the stable doubly heavy tetraquarks under the flavor SU(3) symmetry which is a powerful tool to analyze the general decay properties. The ratios between decay widths of different channels were also given. We have given the Cabibbo allowed two-body and three-body decay channels of the stable doubly heavy tetraquarks, which have large branching ratios and shall be employed as the "discovery channels" in the reconstructions at future LHCb and Bell II experiments.

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