Residues of Λ_Q -type and Σ_Q -type baryons in the Bethe-Salpeter equation approach

Qi-Xin Yu,^{1,*} Zhen-Yang Wang,² Jing-Juan Qi,¹ and Xin-Heng Guo^{1,†}

¹*College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China*

²*Physics Department, Ningbo University, Zhejiang 315211, China*

(Received 19 June 2018; published 20 August 2018)

We study the residues of Λ_Q -type baryons (Λ_Q and Ξ_Q^A) ($Q = b, c$) and Σ_Q -type baryons ($\Sigma_Q^{(*)}$, $\Xi_Q^{S(*)}$ and $\Omega_Q^{(*)}$) in the quark-diquark model within the Bethe-Salpeter (BS) formalism. These residues can be used, e.g., in the calculations of the amplitudes in the scattering processes. After constructing the baryonic currents in the BS formalism, we derive the relations between the BS wave functions and the residues for these baryons. The BS equations are solved numerically with the kernel including the scalar confinement and the one-gluon-exchange terms and with the covariant instantaneous approximation being employed in the calculations. Finally, we obtain the numerical values of the residues 0.103 GeV $\sim$ 0.224 GeV for Λ_Q , 0.143 GeV $\sim$ 0.215 GeV for Ξ_Q^A , 0.262 GeV $\sim$ 0.361 GeV for $\Sigma_Q^{(*)}$, 0.313 GeV $\sim$ 0.460 GeV for $\Xi_Q^{S(*)}$ and 0.473 GeV $\sim$ 0.571 GeV for $\Omega_Q^{(*)}$ in the ranges of the parameters in our model.

DOI: 10.1103/PhysRevD.98.036013

I. INTRODUCTION

The residues of baryons are important parameters in studying their hadronic properties; e.g., in QCD sum rules, the residues are needed as one of the main inputs for further calculating decay constants and coupling constants of baryons [1–3]. In this paper, we will present the calculations of the residues for Λ_Q -type ($Q = b, c$) baryons which include Λ_Q and Ξ_Q^A , and Σ_Q -type baryons which are $\Sigma_Q^{(*)}$, $\Xi_Q^{S(*)}$ and $\Omega_Q^{(*)}$ [4,5]. In the heavy quark effective theory (HQET), baryons are categorized based on the angular momentum and parity (j^P) of the light part. For the ground states of heavy baryons, j^P of the light part can be either 0^+ or 1^+ , which naturally gives the Λ_Q -type and Σ_Q -type baryons, respectively. Furthermore, after adding the heavy quark in these baryons, $j^P = \frac{1}{2}^+$ for Λ_Q and Ξ_Q^A , and the degenerate states $j^P = \frac{1}{2}^+$, $j^P = \frac{3}{2}^+$ for Σ_Q (Ξ_Q^S , Ω_Q), Σ_Q^* (Ξ_Q^{S*} , Ω_Q^*) can be obtained.

The properties of baryons which contain a single heavy quark can be simplified due to the fact that the mass (or the flavor) and spin of the heavy quark become irrelevant to the

leading order in the $1/m_Q$ expansion. Since the light part has good spin and isospin numbers, these baryons can be investigated in the quark-diquark picture [1,4,6–10]. The residues of baryons were mainly investigated with the method of QCD sum rules during the past few decades [2,3,11–14]. The constructed baryonic currents employed in QCD sum rules are not based on the quark-diquark model, which can be seen from the dimension of the residues [12]. In this work, we will construct the baryonic currents for Λ_Q -type and Σ_Q -type baryons respectively. Then the Bethe-Salpeter (BS) equation will be employed to calculate the residues of Λ_Q -type and Σ_Q -type baryons in the quark-diquark picture in the heavy-quark limit $m_Q \rightarrow \infty$. On top of that, in order to see how large the $1/m_c$ corrections are, the residue for Λ_c is also calculated as an example without taking the heavy-quark limit considering the ratio of m_D/m_c (m_D stands for the mass of the diquark) is not very small. These residues reflect the inner structures of heavy baryons in the quark-diquark picture.

In order to obtain further prospects of Λ_Q -type and Σ_Q -type baryons for experimental study and future colliders, it is important to have accurate predictions for the production rates of heavy baryons, and that is where we will need the residues of these baryons. In Refs. [15,16], the authors established the connection between the baryonic current in the quark-diquark model and the scattering amplitude of a diquark and a heavy quark to a heavy baryon $\Omega_{Q'Q''Q}$, where baryons containing a heavy scalar diquark and a heavy quark were considered. In our work, we will study the baryons containing one heavy quark and a light diquark. Following the formalism in Ref. [16], the production

*yuqx@mail.bnu.edu.cn

†Corresponding author.

xhguo@bnu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

FIG. 1. The example of the production of $\Omega_{q_1q_2Q}$ from a scalar diquark $D(q_1q_2)$ and a heavy quark Q through the exchange of the gluon g [16].

amplitude of $\Omega_{q_1q_2Q}$ from a scalar diquark D and a heavy quark Q is (see Fig. 1; q_1 and q_2 are light quarks which compose of the diquark D)

$$T \sim g_s^2 \frac{1}{q^2} F_S(q^2) \langle \Omega_{q_1q_2Q} | J | 0 \rangle, \quad (1)$$

where J is the baryonic current, q is the momentum transfer ($q = k - k'$), g_s is the strong coupling constant, and F_S is the form factor of the diquark which is introduced to describe the interaction between the diquark and the gluon [16]. It can be seen from Eq. (1) that the residue of the baryon, which is related to the matrix element $\langle \Omega_{q_1q_2Q} | J | 0 \rangle$, is needed in the calculation. Conversely, although there has been no direct experimental measurement of the residues for Λ_Q -type and Σ_Q -type baryons, one can make use of the relation between the residue in the quark-diquark model and the scattering amplitude for the production of the heavy baryon in Eq. (1) to obtain the values of the residues of heavy baryons from experimental data. Obviously, the residues of heavy baryons can also be used to calculate fragmentation functions of heavy baryons.

The BS equations for the Λ_Q -type and the Σ_Q -type baryons have been established in the quark-diquark picture in the leading order of the $1/m_Q$ expansion [4,5]. They were applied to study the properties of the Λ_Q -type and the Σ_Q -type baryons, and theoretical predictions were found to be consistent with the available experimental data [10,17]. The kernel for the BS equation is motivated by the potential model, which consists of the scalar confinement and the one-gluon-exchange terms [4,10,17–20]. We will construct the formalism for the residue calculations in the BS equation approach [21]. After that, the BS equations will be solved using the covariant instantaneous approximation [18,22–24], which is appropriate since the energy exchange between constituents inside the heavy baryons is expected to be small when we use the constituent quark masses in the BS equation. Thus we employ this approximation in this

work for the purpose of simplifying our calculations. It should be noted that solving the BS equation in the Minkowski space has been done before by other researchers [25,26], and we will also try to accomplish that by modifying our model in our future work. Finally, the obtained BS wave functions will be applied to calculate the residues of Λ_Q , Ξ_Q^A , $\Sigma_Q^{(*)}$, $\Xi_Q^{S(*)}$ and $\Omega_Q^{(*)}$.

This paper is organized as follows. In Sec. II, we construct the baryonic currents for Λ_Q -type and Σ_Q -type baryons, and define the residues of the heavy baryons in the quark-diquark picture. In Sec. III, the BS equations for Λ_Q -type and Σ_Q -type baryons are presented in the heavy-quark limit. In Sec. IV, we give the numerical results for the BS wave functions of Λ_Q -type and Σ_Q -type baryons and their corresponding residues. Finally, the summary is given in Sec. V.

II. BARYONIC CURRENTS FOR Λ_Q -TYPE AND Σ_Q -TYPE BARYONS

In this section, we present the baryonic currents for the heavy baryons. Generally, for a baryon composed of a heavy quark Q and two light quarks q_1 and q_2 , the baryonic current has the form [3,12,14]

$$J = [q_1^{iT} C \Gamma \tau q_2^j] \Gamma' Q^k \epsilon_{ijk}, \quad (2)$$

where i, j, k are the color indices, the index T indicates the transposition of the matrix, C stands for the charge-conjugation matrix, τ is the matrix in the flavor space and Γ and Γ' represent the light-side and heavy-side Dirac structures of current vertices, respectively. Neglecting color indices and τ , the current could be abbreviated as

$$J = [q^T C \Gamma q] \Gamma' Q. \quad (3)$$

Then, the current for the Λ_Q -type baryon including the scalar diquark D ($q^T C \gamma^5 q$) composed of two light quarks can be written in the following form:

$$J_{\Lambda_Q} = DQ. \quad (4)$$

In the rest frame, Σ_Q -type and Σ_Q^* -type baryons are described by the current $\vec{J} = \vec{A}Q$ with $\vec{A}$ being the isovector diquark field [12,27,28]. $\vec{J}$ can be decomposed as

$$\vec{J} = \vec{A}Q = \left(\vec{J} + \frac{1}{3} \vec{\gamma} \vec{\gamma} \cdot \vec{J} \right) - \frac{1}{3} \vec{\gamma} \vec{\gamma} \cdot \vec{J}, \quad (5)$$

where the part $\vec{J} + \frac{1}{3} \vec{\gamma} \vec{\gamma} \cdot \vec{J}$ satisfies the relation $\vec{\gamma} \cdot \vec{J} = 0$, and thus has spin 3/2 corresponding to the Σ_Q^* -type baryon, and the other part can be written in the following form:

$$-\frac{1}{3}\vec{\gamma}\vec{\gamma}\cdot\vec{J} = -\frac{1}{3}\vec{\gamma}\gamma_5\gamma_5\vec{\gamma}\cdot\vec{A}Q = \frac{1}{3}\vec{\gamma}\gamma_5 J_{1/2}, \quad (6)$$

where $J_{1/2} = \vec{A} \cdot \vec{\gamma}\gamma_5 Q$ satisfies the condition $\gamma_0 \vec{J}_{1/2} = \vec{J}_{1/2}$, indicating $\vec{J}_{1/2}$ has spin 1/2 and corresponds to Σ_Q . The above currents can be generalized to the general frame by substituting γ_0 with $\not{v}$ and $\vec{\gamma}$ with $\gamma_\mu^\perp = \gamma_\mu - \not{v}v^\mu$ (v is the velocity of the Σ_Q -type or Σ_Q^* -type baryon). With these substitutions, we arrive at the current

$$J_{1/2} = A_\mu(\gamma^\mu - \not{v}v^\mu)\gamma_5 Q. \quad (7)$$

Since A_μ satisfies the condition $v^\mu A_\mu = 0$, we have the following form for the baryonic current of a Σ_Q -type baryon:

$$J_{\Sigma_Q} = J_{1/2} = A_\mu \gamma^\mu \gamma_5 Q. \quad (8)$$

Similarly, we have the following form for the current of a Σ_Q^* -type baryon:

$$J_{\Sigma_Q^*}^\mu = A^\mu Q + \frac{1}{3}(\gamma^\mu - \not{v}v^\mu)\not{A}Q. \quad (9)$$

We can define the residues for Λ_Q -type and $\Sigma_Q^{(*)}$ -type baryons in the quark-diquark picture as follows:

$$\langle 0 | J_{\Lambda_Q} | \Lambda_Q \rangle = f_{\Lambda_Q} u_{\Lambda_Q}, \quad (10)$$

$$\langle 0 | J_{\Sigma_Q} | \Sigma_Q \rangle = f_{\Sigma_Q} u_{\Sigma_Q}, \quad (11)$$

$$\langle 0 | J_{\Sigma_Q^*}^\mu | \Sigma_Q^* \rangle = \frac{1}{\sqrt{3}} f_{\Sigma_Q^*} u_{\Sigma_Q^*}^\mu, \quad (12)$$

where f_{Λ_Q} , f_{Σ_Q} and $f_{\Sigma_Q^*}$ are residues for Λ_Q -type, Σ_Q -type and Σ_Q^* -type baryons, respectively, u_{Λ_Q} , u_{Σ_Q} and $u_{\Sigma_Q^*}^\mu$ are their corresponding spinors. We will show later that in the leading order of HQET the residues for the spin doublet, e.g., $\{\Sigma_Q, \Sigma_Q^*\}$, are the same.

III. THE BETHE-SALPETER FORMALISM FOR Λ_Q -TYPE AND Σ_Q -TYPE BARYONS

A. The BS equations for Λ_Q -type baryons

In this subsection, we take Λ_Q as an example for the Λ_Q -type baryons. Λ_Q can be treated as the combination of a light scalar diquark and a heavy quark. As mentioned before, in this quark-diquark picture, the BS wave function for Λ_Q is defined as [5]

$$\chi(x_1, x_2, P) = \langle 0 | T\psi(x_1)\phi(x_2) | \Lambda_Q(P) \rangle, \quad (13)$$

where $P = m_{\Lambda_Q} v$ is the momentum of Λ_Q , v is its velocity, and $\phi(x_1)$ and $\psi(x_2)$ are the field operators of the scalar

diquark and the heavy quark, respectively. Define $\lambda_1 = \frac{m_Q}{m_Q + m_D}$, $\lambda_2 = \frac{m_D}{m_Q + m_D}$, where m_Q and m_D are the masses of the heavy quark and the light diquark, respectively. The BS equation in momentum space can be expressed as

$$\chi(x_1, x_2, P) = e^{-iPX} \int \frac{d^4 p}{(2\pi)^4} e^{-ipx} \chi_P(p), \quad (14)$$

where $X = \lambda_1 x_1 + \lambda_2 x_2$ is the coordinator of the center of mass, $x = x_1 - x_2$ is the relative coordinator, and p is the corresponding relative momentum. Then the momentum of the heavy quark is $p_1 = \lambda_1 P + p$, and $p_2 = \lambda_2 P - p$ for the light diquark.

The BS equation for Λ_Q has the following form [5]:

$$\chi_P(p) = S_F(p_1) \int \frac{d^4 q}{(2\pi)^4} G(P, p, q) \chi_P(q) S_D(p_2), \quad (15)$$

where $G(P, p, q)$ is the kernel which includes the one-gluon-exchange term and the confinement term [4,18],

$$-iG = I \otimes IV_1 + v_\mu \otimes (p_2 + p_2')^\mu F(Q^2) V_2, \quad (16)$$

where the first term corresponds to the scalar confinement while the second term is from the one-gluon-exchange diagram, and the vertex describing the interaction between the gluon and two scalar diquarks is $(p_2 + p_2')^\mu F(Q^2)$ where $F(Q^2)$ is the form factor due to the structure of the scalar diquark.

It has been shown that in the leading order of the $1/m_Q$ expansion we only need one scalar function, $f_P(p)$, to describe the BS wave function of Λ_Q [5]. $f_P(p)$ has the following relation with $\chi_P(p)$:

$$\chi_P(p) = f_P(p) u_{\Lambda_Q}, \quad (17)$$

where u_{Λ_Q} is the Dirac spinor of Λ_Q and $f_P(p)$ is a scalar function of p_l and p_t^2 , which are defined as $p_l = v \cdot p - \lambda_2 m_{\Lambda_Q}$, $p_t = p - (v \cdot p)v$. Substituting the explicit forms of propagators of the scalar diquark and the heavy quark into Eq. (15), and completing the integral $\int \frac{dq_l}{2\pi}$, we obtain the following equation for the scalar function $\tilde{f}_P(p_t) (= \int \frac{dq_l}{2\pi} f_P(p))$:

$$\begin{aligned} \tilde{f}_P(p_t) = & -\frac{1}{2W_p(\lambda_1 m_{\Lambda_Q} - \omega_Q - W_p)} \\ & \times \int \frac{d^3 q_t}{(2\pi)^3} (\tilde{V}_1 - 2W_p \tilde{V}_2) \tilde{f}_P(q_t), \end{aligned} \quad (18)$$

where $\omega_Q = \sqrt{m_Q^2 - p_t^2}$, $W_p = \sqrt{m_D^2 - p_t^2}$ and $\tilde{V}_{1(2)} = \int \frac{dq_l}{2\pi} V_{1(2)}$. The covariant instantaneous approximation is applied in obtaining Eq. (18). Meanwhile, we have the explicit forms for $\tilde{V}_1$,

$$\tilde{V}_1 = \frac{8\pi\kappa}{[(p_t - q_t)^2 + \mu^2]^2} - (2\pi)^3 \delta^3(p_t - q_t) \times \int \frac{d^3k}{(2\pi)^3} \frac{8\pi\kappa}{(k^2 + \mu^2)^2}, \quad (19)$$

and $\tilde{V}_2$ combining the form factor $F(Q^2) = \frac{\alpha_{\text{seff}} Q_0^2}{Q^2 + Q_0^2}$ [29],

$$\tilde{V}_2 = -\frac{16\pi}{3} \frac{\alpha_{\text{seff}}^2 Q_0^2}{[(p_t - q_t)^2 + \mu^2][(p_t - q_t)^2 + Q_0^2]}, \quad (20)$$

where κ and α_{seff} are coupling parameters related to the confinement and one-gluon-exchange interactions, and μ is introduced to avoid the infrared divergence in numerical calculations; we will apply the limit $\mu \rightarrow 0$ in the end. Q_0^2 is a parameter which is used to freeze $F(Q^2)$ when Q^2 is small. From the analysis of the electromagnetic form factor for the proton, Q_0^2 was found to be consistent with the experimental data when $Q_0^2 = 3.2 \text{ GeV}^2$ [17,29].

From the fact that the Isgur-Wise function is normalized to one at the zero-recoil point, we can determine the normalization constant for Λ_Q [5]. In the heavy-quark limit, the Isgur-Wise function $\xi(\omega)$ ($\omega = v \cdot v'$ is the velocity transfer) is related to the matrix $\langle \Lambda_c(v') | \bar{c} \gamma_\mu b | \Lambda_b(v) \rangle$, which has the following relation with the BS wave function of Λ_Q [5]:

$$\langle \Lambda_c(v') | \bar{c} \gamma_\mu b | \Lambda_b(v) \rangle = \int \frac{d^4p}{(2\pi)^4} \bar{\chi}_{P'}(p') \gamma_\mu \chi_P(p) S_D^{-1}(p_2), \quad (21)$$

where v and v' are the velocities of Λ_b and Λ_c , respectively. Then, we have

$$\xi(\omega) = \int \frac{d^4p}{(2\pi)^4} f_{P'}(p') f_P(p) S_D^{-1}(p_2). \quad (22)$$

Since a c quark is not as heavy as a b quark, $1/m_c$ corrections, which are of order m_D/m_c , may not be small. Thus, we also establish the BS equation for Λ_c without taking the limit $m_Q \rightarrow \infty$. In this case, Eq. (17) takes the following form:

$$\chi'_P(p) = (f_1 + \not{p}_t f_2) u_{\Lambda_Q}, \quad (23)$$

where f_1 and f_2 are scalar functions of p_t and p_t^2 . We rewrite the full propagator of the heavy quark in the following form:

$$S'_F(p_1) = \frac{i}{2\omega_Q} \left[\frac{\not{p}\omega_Q + \not{p}_t + m_Q}{\lambda_1 m_{\Lambda_Q} + p_t - \omega_Q + i\varepsilon} + \frac{\not{p}\omega_Q - \not{p}_t - m_Q}{\lambda_1 m_{\Lambda_Q} + p_t + \omega_Q - i\varepsilon} \right]. \quad (24)$$

Following the same routine for $f_P(p)$, we obtain the following coupled integral equations for $\tilde{f}_1(p_t)$ and $\tilde{f}_2(p_t)$:

$$\begin{aligned} \tilde{f}_1(p_t) = & \frac{1}{4W_p \omega_Q [(W_p + \omega_Q)^2 - m_{\Lambda_Q}^2]} \\ & \times \{ [(2m_{\Lambda_Q} \omega_Q + 2m_Q(W_p + \omega_Q)) \tilde{V}_1 + (4m_{\Lambda_Q} m_Q W_p + 3m_{\Lambda_Q} (k_t^2 + k_t \cdot q_t)) \\ & + (W_p + \omega_Q)(4W_p \omega_Q - 5(p_t^2 - p_t \cdot q_t))] \tilde{V}_2 \} \tilde{f}_1(q_t) + [2(W_p + \omega_Q)(p_t \cdot q_t) \tilde{V}_1 \\ & + 2((m_Q(W_p + \omega_Q) + \omega_Q m_{\Lambda_Q}) q_t^2 - (p_t \cdot q_t)(m_{\Lambda_Q}(2W_p + \omega_Q) + m_Q(W_p + \omega_Q))) \tilde{V}_2] \tilde{f}_2(q_t) \}, \end{aligned} \quad (25)$$

$$\begin{aligned} \tilde{f}_2(p_t) = & \frac{1}{4W_p \omega_Q [(W_p + \omega_Q)^2 - m_{\Lambda_Q}^2]} \\ & \times \{ [2(W_p + \omega_Q) \tilde{V}_1 + \left(2m_{\Lambda_Q}(2W_p + \omega_Q) + 2m_{\Lambda_Q} \frac{p_t \cdot q_t}{p_t^2} \omega_Q - 2m_Q \frac{p_t^2 + p_t \cdot q_t}{p_t^2} (W_p + \omega_Q) \right) \tilde{V}_2] \tilde{f}_1(q_t) \\ & + \left[2 \left(m_{\Lambda_Q} \omega_Q \frac{p_t \cdot q_t}{p_t^2} - m_Q(W_p + \omega_Q) \frac{p_t \cdot q_t}{p_t^2} \right) \tilde{V}_1 \right. \\ & \left. + 2 \left(-m_{\Lambda_Q} W_p \omega_Q \frac{p_t \cdot q_t}{p_t^2} + m_Q W_p (W_p + \omega_Q) \frac{p_t \cdot q_t}{p_t^2} - (W_p + \omega_Q)(p_t \cdot q_t + q_t^2) \right) \tilde{V}_2 \right] \tilde{f}_2(q_t) \}. \end{aligned} \quad (26)$$

The normalization of $\chi'_P(p)$ can be accomplished by employing the same normalization scheme as in Ref. [30].

B. The BS equations for Σ_Q -type baryons

In this subsection, we will take $\Sigma_Q^{(*)}$ as an example for Σ_Q -type baryons. $\Sigma_Q^{(*)}$ is regarded as a bound state of an axial-vector diquark A_μ and a heavy quark ψ_Q . We can use $B_\mu = u_Q \epsilon_\mu$ to represent the heavy baryon state, where u_Q stands for the Dirac spinor of Q , and ϵ_μ is the polarization vector of A_μ . The state B_μ could be split into spin-1/2 and spin-3/2 degenerate states in the heavy-quark limit corresponding to Σ_Q and Σ_Q^* , respectively [4,28,31,32]. According to Refs. [31–33], the doublet $\{\Sigma_Q, \Sigma_Q^*\}$ could be represented by B_μ^m , where $m = 1$ (2) stands for spin-1/2 (3/2). The explicit forms of B_μ^1 and B_μ^2 are [28]

$$B_\mu^1(v) = \frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5 u(v), \quad (27)$$

$$B_\mu^2(v) = u_\mu(v), \quad (28)$$

where v is the velocity of $\Sigma_Q^{(*)}$, $u(v)$ is the Dirac spinor and $u_\mu(v)$ is the Rarita-Schwinger vector spinor [28]. Besides, $B_\mu^m(v)$ satisfies the conditions [4]

$$\not{v} B_\mu^m = B_\mu^m(v), \quad v^\mu B_\mu^m(v) = 0, \quad \gamma^\mu B_\mu^2 = 0. \quad (29)$$

The BS wave function of $\Sigma_Q^{(*)}$ is defined as

$$\chi_\mu(x_1, x_2, P) = \langle 0 | T \psi_Q(x_1) A_\mu(x_2) | \Sigma_Q^{(*)}(P) \rangle. \quad (30)$$

Define $\eta_1 = \frac{m_Q}{m_Q + m_A}$, $\eta_2 = \frac{m_A}{m_Q + m_A}$, where m_Q and m_A denote the masses of the heavy quark and the axial-vector diquark, respectively. Then, $\Sigma_Q^{(*)}$ follows a similar BS equation as for Λ_Q :

$$\chi_{Pm}^\mu(p) = S_F(p_1) \int \frac{d^4 q}{(2\pi)^4} G_{\rho\nu}(P, p, q) \chi_{Pm}^\nu(q) S_D^{\mu\rho}(-p_2), \quad (31)$$

where χ_{Pm}^μ is the BS wave function of $\Sigma_Q^{(*)}$ in momentum space, $G_{\rho\nu}(P, p, q)$ is the kernel in the $\Sigma_Q^{(*)}$ case, which is similar to the form used in the Λ_Q case [4,5], and $S_F(p_1)$ and $S_D^{\mu\rho}$ are the propagators of the heavy quark and the axial-vector diquark, respectively.

Due to the flavor and spin symmetries in the heavy-quark limit, and taking into account Lorentz transformation, P -parity transformation and spin rotation, we have the following form for the BS wave function of $\Sigma_Q^{(*)}$ [4,28,34]:

$$\chi_{Pm}^\mu = A B_m^\mu(v) + C v^\mu p_\nu B_m^\nu(v) + D p_\nu^\mu p_\nu B_m^\nu(v), \quad (32)$$

where A , C and D are Lorentz scalar functions. After substituting the explicit forms of the propagators and the kernel $G_{\rho\nu}(P, p, q)$ into Eq. (31), we obtain three coupled integral equations,

$$\begin{aligned} \tilde{A}(p_i^2) = & -\frac{1}{2w_p(\eta_1 m_{\Sigma_Q^{(*)}} - \omega_Q - w_p)} \int \frac{d^3 q_t}{(2\pi)^3} \left[\tilde{A}(q_t^2)(\tilde{V}_1 - 2w_p \tilde{V}_2) - \tilde{C}(q_t^2) \tilde{V}_2 \frac{p_i^2 q_t^2 - (p_t \cdot q_t)^2}{2p_i^2} \right. \\ & \left. + \tilde{D}(q_t^2)(\tilde{V}_1 - 2w_p \tilde{V}_2) \frac{p_i^2 q_t^2 - (p_t \cdot q_t)^2}{2p_i^2} \right], \end{aligned} \quad (33)$$

$$\begin{aligned} \tilde{C}(p_i^2) = & -\frac{1}{2m_D^2 w_p(\eta_1 m_{\Sigma_Q^{(*)}} - \omega_Q - w_p)} \int \frac{d^3 q_t}{(2\pi)^3} \left\{ \tilde{A}(q_t^2)[w_p \tilde{V}_1 - (m_D^2 - w_p(\eta_1 m_{\Sigma_Q^{(*)}} - \omega_Q)) \tilde{V}_2] \right. \\ & + \tilde{C}(q_t^2) \left[\frac{(p_t \cdot q_t)}{p_i^2} (m_D^2 - w_p(\eta_1 m_{\Sigma_Q^{(*)}} - \omega_Q)) \tilde{V}_1 - \frac{(p_t \cdot q_t)^2}{p_i^2} w_p \tilde{V}_2 \right] \\ & \left. + \tilde{D}(q_t^2) \left[\frac{(p_t \cdot q_t)^2}{p_i^2} w_p \tilde{V}_1 - \frac{(p_t \cdot q_t)^2}{p_i^2} (m_D^2 + w_p(\eta_1 m_{\Sigma_Q^{(*)}} - \omega_Q)) \tilde{V}_2 \right] \right\}, \end{aligned} \quad (34)$$

$$\begin{aligned} \tilde{D}(p_i^2) = & \frac{1}{2m_D^2 w_p(\eta_1 m_{\Sigma_Q^{(*)}} - \omega_Q - w_p)} \int \frac{d^3 q_t}{(2\pi)^3} \left\{ \tilde{A}(q_t^2)(\tilde{V}_1 - w_p \tilde{V}_2) \right. \\ & + \tilde{C}(q_t^2) \left[-\frac{p_t \cdot q_t}{p_i^2} w_p \tilde{V}_1 + \frac{m_D^2(3(p_t \cdot q_t)^2 - p_i^2 q_t^2) - 2p_i^2(p_t \cdot q_t)^2}{2p_i^4} \tilde{V}_2 \right] \\ & \left. + \tilde{D}(q_t^2) \left[\frac{-m_D^2(3(p_t \cdot q_t)^2 - p_i^2 q_t^2) + 2p_i^2(p_t \cdot q_t)^2}{2p_i^4} \tilde{V}_1 - \frac{p_i^2(p_t \cdot q_t)^2 - m_D^2(3(p_t \cdot q_t)^2 - p_i^2 q_t^2)}{p_i^4} w_p \tilde{V}_2 \right] \right\}, \end{aligned} \quad (35)$$

where $w_p = \sqrt{m_A^2 - p_t^2}$, and terms like $\int \frac{d^4 q}{(2\pi)^4} q^\mu f$ and $\int \frac{d^4 q}{(2\pi)^4} q_t^\mu q_t^\nu f$ can be expanded to terms including p_t^μ and v^μ on the grounds of Lorentz invariance [4].

For the normalization of the BS wave function of $\Sigma_Q^{(*)}$, we can use a similar technique as in the Λ_Q case. The Isgur-Wise function for $\Sigma_Q^{(*)}$ has the following relation with the transition matrix $\Sigma_b^{(*)} \rightarrow \Sigma_c^{(*)}$ at the zero-recoil point:

$$\langle \Sigma_c^{(*)}(v) | \bar{c} \Gamma b | \Sigma_b^{(*)}(v) \rangle = \xi(1) \bar{B}_{m' \mu}(v) \Gamma B_m^\mu(v), \quad (36)$$

where $\xi(1)$ is the Isgur-Wise function for $\Sigma_Q^{(*)}$ at the zero-recoil point. Meanwhile, the above transition matrix can be expressed with the BS wave function of $\Sigma_Q^{(*)}$ as follows:

$$\langle \Sigma_c^{(*)}(v) | \bar{c} \Gamma b | \Sigma_b^{(*)}(v) \rangle = \int \frac{d^4 p}{(2\pi)^4} \frac{i}{\eta_1 m_{\Sigma_Q^{(*)}} - \omega_Q + p_l + i\epsilon} \bar{\chi}_{Pm'}^\mu(p) \Gamma \int \frac{d^4 q}{(2\pi)^4} G_{\mu\nu}(P, p, q) \chi_{Pm}^\nu(p). \quad (37)$$

Substituting Eq. (37) into Eq. (36), we arrive at

$$\xi(1) = \int \frac{p_t^2 dp_t}{4\pi^2} \frac{2}{\eta_1 m_{\Sigma_Q^{(*)}} - \omega_Q - w_p} \left[\tilde{A}(p_t^2) h_1(|p_t|) - \frac{1}{3} p_t^2 \tilde{C}(p_t^2) h_2(|p_t|) - \frac{1}{3} p_t^2 \tilde{D}(p_t^2) h_3(|p_t|) + \frac{1}{6m_A^2} p_t^2 h_2(|p_t|) h_4(|p_t|) \right], \quad (38)$$

where $h_i(|p_t|)$ ($i = 1, 2, 3, 4$) are of the forms:

$$h_1(|p_t|) = \int \frac{q_t^2 dq_t}{4\pi^2} \left\{ 8\pi\kappa F_1 [\tilde{A}(q_t^2) - \frac{1}{3} q_t^2 \tilde{D}(q_t^2)] + F_2 \left[2w_p \tilde{A}(q_t^2) - \frac{1}{3} q_t^2 \tilde{C}(q_t^2) - \frac{2}{3} w_p q_t^2 \tilde{D}(q_t^2) \right] - 8\pi\kappa F_1 \left[\tilde{A}(p_t^2) - \frac{1}{3} p_t^2 \tilde{D}(p_t^2) \right] \right\}, \quad (39)$$

$$h_2(|p_t|) = \int \frac{q_t^2 dq_t}{4\pi^2} \left[F_2 \tilde{A}(q_t^2) + 8\pi\kappa F_3 \frac{1}{p_t^2} \tilde{C}(q_t^2) + F_4 \frac{1}{p_t^2} \tilde{D}(q_t^2) - 8\pi\kappa F_1 \tilde{C}(p_t^2) \right], \quad (40)$$

$$h_3(|p_t|) = \int \frac{q_t^2 dq_t}{4\pi^2} \left\{ 8\pi\kappa F_1 \tilde{A}(q_t^2) + 2w_p F_2 \tilde{A}(q_t^2) - 8\pi\kappa F_5 \frac{1}{p_t^2} \tilde{D}(q_t^2) + F_4 \frac{1}{p_t^2} [\tilde{C}(q_t^2 + 2w_p \tilde{D}(q_t^2))] - 8\pi\kappa F_1 [p_t^2 \tilde{D}(p_t^2) - \tilde{A}(p_t^2)] \right\}, \quad (41)$$

$$h_4(|p_t|) = \int \frac{q_t^2 dq_t}{4\pi^2} \left\{ -F_2 \tilde{A}(q_t^2) + 8\pi\kappa F_3 \frac{1}{p_t^2} \tilde{C}(q_t^2) - F_4 \frac{1}{p_t^2} \tilde{D}(q_t^2) - 8\pi\kappa F_1 \tilde{C}(p_t^2) \right\}, \quad (42)$$

where F_i ($i = 1, 2, 3, 4, 5$) are functions of $|p_t|$ and $|q_t|$, the explicit forms of which can be found in Ref. [4], and $\beta = \alpha_{\text{seff}}^2 Q_0^2$. In this way, the scalar functions $\tilde{A}(p_t^2)$, $\tilde{C}(p_t^2)$ and $\tilde{D}(p_t^2)$ can be normalized with the aid of Eqs. (38)–(42).

IV. CALCULATIONS OF RESIDUES FOR Λ_Q -TYPE AND Σ_Q -TYPE BARYONS

A. The numerical solutions for the BS wave functions of Λ_Q -type and Σ_Q -type baryons

In this section, we will solve the BS equations numerically and then apply the results to calculate the residues of Λ_Q -type and Σ_Q -type baryons, which are defined in

Eqs. (10)–(12). To solve Eq. (18) and the two groups of coupled integral equations (25), (26), and (33)–(35) numerically, we discretize the integration regions into n pieces with n being sufficiently large. Then the equations become eigenvalue equations and can be solved with the Gaussian integration method.

There are several parameters in our model for Λ_Q -type and Σ_Q -type baryons: α_{seff} , κ , Q_0^2 , m_D , and m_A . The coupling parameter κ is taken to be in the range $0.02 \text{ GeV}^3 \sim 0.08 \text{ GeV}^3$ in the baryon case [35]. Once κ is settled, α_{seff} can be determined by solving the eigenvalue equations because of the constraint between κ and α_{seff} . $Q_0^2 = 3.2 \text{ GeV}^2$ extracted from the electromagnetic form factor of the proton is taken in both Λ_Q -type and Σ_Q -type

cases, as it is associated with the light diquark [4,5,35,36]. For the heavy quark masses, we will take the values that have been used in many papers [5,23,37], $m_b = 5.02$ GeV, $m_c = 1.58$ GeV, because the predictions they lead to are consistent with experimental data. Besides, we will let the diquark masses vary in reasonable regions: 0.60–0.80 GeV for the light scalar diquark ud [5,17], 0.90–1.00 GeV for the light scalar diquark us , 0.90–1.10 GeV for the light axial-vector diquark uu , ud or dd , 1.05–1.15 GeV for the axial-vector diquark us or ds and 1.15–1.25 GeV for the axial-vector diquark ss . On top of that, the parameter μ , which is introduced here to cancel the infrared divergence, will be taken to approach 0 at the end of the calculations.

For the masses of Λ_Q -type and Σ_Q -type baryons, we have the following expressions in the leading order of the $1/m_Q$ expansion:

$$m_i = m_Q + m_{D_i} + E_i \quad (i = \Lambda_Q, \Xi_Q^A), \quad (43)$$

$$m_i = m_Q + m_{A_i} + E_i \quad (i = \Sigma_Q^{(*)}, \Xi_Q^{S(*)}, \Omega_Q^{(*)}), \quad (44)$$

where E_i is the binding energy. It can be seen that the parameters $m_{D_i(A_i)}$ and E_i are constrained by the relation $m_{D_i(A_i)} + E_i = m_i - m_Q$, where we omit the $1/m_Q$ corrections. Thus, we will have $m_{D_i} + E_i$ as 0.60 GeV for $i = \Lambda_Q$ and 0.77 GeV for $i = \Xi_Q^A$, and $m_{A_i} + E_i$ as 0.79 GeV for $i = \Sigma_Q^{(*)}$, 0.93 GeV for $i = \Xi_Q^{S(*)}$ and 1.03 GeV for $i = \Omega_Q^{(*)}$ [38]. In Table I, we give the values of α_{seff} we obtain while solving the BS equation for $m_D = 0.60, 0.70, 0.80$ GeV with different values of κ ranging from 0.02 to 0.08 GeV³ for Λ_Q [17].

On top of that, we have to pay extra attention to the baryons that include a c quark because the ratio m_D/m_c is not very small. Here we take Λ_c as an example to see how the $1/m_c$ corrections will influence the residue's value if we do not take the heavy-quark limit in the calculations. In this case, the expansion for the mass of Λ_c becomes

$$m_{\Lambda_c} = m_c + m_D + E_{\Lambda_c} + \mathcal{O}(1/m_c) + \dots \quad (45)$$

We take $m_{\Lambda_c} = 2.28$ GeV, $m_c = 1.58$ GeV, and let m_D in Eq. (45) vary in the region 0.70–0.90 GeV, which is different from the one in the heavy-quark limit, to effectively include $1/m_c$ corrections in Eq. (45). We find that in this region Eqs. (25) and (26) have numerical solutions and obtain the value of α_{seff} in Λ_c , which is listed in Table II.

As we can see from the data in Table I, once the parameters m_D and Q_0^2 are fixed, α_{seff} for Λ_Q will undergo some notable change as κ increases, and for each κ , α_{seff} also increases as m_D increases. When the confinement parameter κ is relatively small, the value of m_D will have a larger influence on α_{seff} . On top of that, we also find that when κ and the mass for the diquark we choose are small enough, there will not be a numerical solution for the BS wave function of Λ_Q , which suggests that the value of m_D should be at least 0.60 GeV. In general, it can be seen from the data in Table III that α_{seff} in Ξ_Q^A follows a similar pattern to the change of α_{seff} in Λ_Q , but the increase in α_{seff} for Ξ_Q^A as κ increases is relatively small compared to the case in Λ_Q when the mass of the diquark is small.

For the case of Σ_Q -type baryons, we have to solve the three coupled integral equations (33)–(35) to obtain the numerical solutions of $\tilde{A}(p_i^2)$, $\tilde{C}(p_i^2)$ and $\tilde{D}(p_i^2)$. After discretization, these three equations can be changed to three matrix equations,

$$\tilde{A} = Z_1 \tilde{A} + Z_2 \tilde{C} + Z_3 \tilde{D}, \quad (46)$$

$$0 = R_1 \tilde{A} + R_2 \tilde{C} + R_3 \tilde{D}, \quad (47)$$

$$0 = T_1 \tilde{A} + T_2 \tilde{C} + T_3 \tilde{D}, \quad (48)$$

where Z_i , R_i and T_i ($i = 1, 2, 3$) are $n \times n$ matrices. With the help of Eqs. (47) and (48), $\tilde{C}$ and $\tilde{D}$ can be eliminated from Eq. (46). Thus, we can solve the wave function $\tilde{A}$ first and then we can obtain $\tilde{C}$ and $\tilde{D}$ by using the same technique. In Table IV, we give the values of α_{seff} for $\Sigma_Q^{(*)}$ in the ranges of the parameters in our model.

TABLE I. The values of α_{seff} for Λ_Q with different values of κ and m_D in the heavy-quark limit.

$\kappa(\text{GeV}^3)$	0.02	0.04	0.06	0.08
$m_D = 0.60$ GeV	0.58	0.65	0.68	0.71
$m_D = 0.70$ GeV	0.69	0.72	0.75	0.77
$m_D = 0.80$ GeV	0.77	0.79	0.81	0.82

TABLE II. The values of α_{seff} for Λ_c with different values of κ and m_D without taking the limit $m_c \rightarrow \infty$.

$\kappa(\text{GeV}^3)$	0.02	0.04	0.06	0.08
$m_D = 0.70$ GeV	0.62	0.67	0.71	0.73
$m_D = 0.80$ GeV	0.70	0.73	0.75	0.77
$m_D = 0.90$ GeV	0.80	0.82	0.83	0.84

TABLE III. The values of α_{seff} for Ξ_Q^A with different values of κ and m_D in the heavy-quark limit.

$\kappa(\text{GeV}^3)$	0.02	0.04	0.06	0.08
$m_D = 0.90$ GeV	0.69	0.71	0.73	0.74
$m_D = 0.95$ GeV	0.73	0.75	0.76	0.77
$m_D = 1.00$ GeV	0.77	0.78	0.79	0.80

TABLE IV. The values of α_{seff} for $\Sigma_Q^{(*)}$ with different values of κ and m_D in the heavy-quark limit.

$\kappa(\text{GeV}^3)$	0.02	0.04	0.06	0.08
$m_A = 0.90$ GeV	0.62	0.63	0.63	0.64
$m_A = 1.00$ GeV	0.69	0.70	0.70	0.73
$m_A = 1.06$ GeV	0.75	0.76	...	...

TABLE V. The values of α_{seff} for $\Xi_Q^{(*)}$ with different values of κ and m_D in the heavy-quark limit.

$\kappa(\text{GeV}^3)$	0.02	0.04	0.06	0.08
$m_A = 1.05 \text{ GeV}$	0.64	0.65	0.66	0.67
$m_A = 1.10 \text{ GeV}$	0.69	0.69	0.70	0.71
$m_A = 1.15 \text{ GeV}$	0.72	0.73	0.73	0.74

As shown in Table IV there are small changes in the values of α_{seff} as κ varies. We find no numerical solutions for α_{seff} when the mass of the axial-vector diquark is 1.10 GeV, which suggests that the maximum value for the mass of the light axial-vector diquark in $\Sigma_Q^{(*)}$ should be below 1.10 GeV, and we find that the maximum mass value for m_A allowed in our model is 1.06 GeV.

The values of α_{seff} are given in Tables V and VI for $\Xi_Q^{(*)}$ and $\Omega_Q^{(*)}$ respectively, where we can see that α_{seff} for $\Xi_Q^{(*)}$ and $\Omega_Q^{(*)}$ increases as κ and the mass of the diquark increase. Generally, the changes of α_{seff} in these three baryons ($\Sigma_Q^{(*)}$, $\Xi_Q^{(*)}$ and $\Omega_Q^{(*)}$) with different values of κ and m_A are quite similar due to the approximate $SU(3)$ flavor symmetry.

In Fig. 2, we plot the BS wave function for Λ_Q -type baryons with respect to $|p_t|$. It can be seen from these curves that the shapes of all of these BS wave functions are similar, and clearly, the BS wave functions for Λ_Q are steeper than those for Ξ_Q^A , which can be attributed to the heavier diquark in Ξ_Q^A . Similarly, we obtain four BS wave functions for Λ_Q and Ξ_Q^A in different scenarios in Fig. 3, where we κ fix at 0.06 GeV^3 , and allow the mass of the diquarks of Λ_Q and Ξ_Q^A to vary in their regions. It can be seen that the results in Figs. 2 and 3 are similar.

In Fig. 4, we plot the BS wave function for Λ_c without taking the heavy-quark limit $m_c \rightarrow \infty$. It can be seen that the magnitude of $\tilde{f}_2$ is smaller compared to that of $\tilde{f}_1$, indicating that the contribution from the $\not{p}_t$ term is relatively small. Thus we expect the $1/m_c$ corrections to the residue of Λ_c to be small.

We present the solutions of the BS wave functions for Σ_Q -type baryon in Figs. 5–7, where we give the numerical results of BS wave functions for $\Sigma_Q^{(*)}$, $\Xi_Q^{(*)}$ and $\Omega_Q^{(*)}$, with κ in the range 0.02 – 0.08 GeV^3 for all of the Σ_Q -type baryons in this work. For example, the BS wave function of $\Sigma_Q^{(*)}$ is

TABLE VI. The values of α_{seff} for $\Omega_Q^{(*)}$ with different values of κ and m_D in the heavy-quark limit.

$\kappa(\text{GeV}^3)$	0.02	0.04	0.06	0.08
$m_A = 1.15 \text{ GeV}$	0.58	0.60	0.63	0.64
$m_A = 1.20 \text{ GeV}$	0.62	0.63	0.65	0.67
$m_A = 1.25 \text{ GeV}$	0.65	0.67	0.67	0.69

FIG. 2. The normalized BS wave functions for Λ_Q -type baryons, Λ_Q ($m_D = 0.70 \text{ GeV}$) and Ξ_Q^A ($m_D = 0.95 \text{ GeV}$), with $\kappa = 0.02 \text{ GeV}^3$ and $\kappa = 0.08 \text{ GeV}^3$ respectively. FIG. 3. The normalized BS wave functions for Λ_Q -type baryons, Λ_Q ($m_D = 0.60$ – 0.80 GeV) and Ξ_Q^A ($m_D = 0.90$ – 1.00 GeV) when $\kappa = 0.06 \text{ GeV}^3$ for both baryons. FIG. 4. The normalized BS wave functions of Λ_c when $m_D = 0.80 \text{ GeV}$ without taking the heavy-quark limit.

solved with different values of κ when $m_D = 1.00 \text{ GeV}$. In comparison, we also illustrate the other two Σ_Q -type baryons, and we can see that curves become smoother and wider as the mass m_A increases. Besides, it can be

FIG. 5. The normalized BS wave functions of $\Sigma_Q^{(*)}$ with $\kappa = 0.02 \text{ GeV}^3$ and $\kappa = 0.08 \text{ GeV}^3$ when $\kappa = 0.90 \text{ GeV}^3$.

FIG. 6. The normalized BS wave functions of $\Xi_Q^{(*)}$ with $\kappa = 0.02 \text{ GeV}^3$ and $\kappa = 0.08 \text{ GeV}^3$ when $m_A = 1.05 \text{ GeV}$.

FIG. 7. The normalized BS wave functions of $\Omega_Q^{(*)}$ with $\kappa = 0.02 \text{ GeV}^3$ and $\kappa = 0.08 \text{ GeV}^3$ when $m_A = 1.20 \text{ GeV}$.

easily seen from these figures that the BS wave functions for $\Sigma_Q^{(*)}$ follow the same pattern as those for Λ_Q .

B. Calculations of residues of Λ_Q -type and Σ_Q -type baryons

From the baryonic currents for Λ_Q -type and Σ_Q -type baryons constructed in the quark-diquark model in Sec. II, we can obtain the relations between the residues and the BS wave functions for Λ_Q -type and Σ_Q -type baryons as follows:

$$\langle 0 | J_{\Lambda_Q} | \Lambda_Q \rangle = \langle 0 | D \psi | \Lambda_Q \rangle = \int \frac{d^4 p}{(2\pi)^4} \chi_P(p), \quad (49)$$

$$\begin{aligned} \langle 0 | J_{\Sigma_Q} | \Sigma_Q \rangle &= \langle 0 | A^\mu \gamma_\mu \gamma_5 \psi | \Sigma_Q \rangle \\ &= \int \frac{d^4 p}{(2\pi)^4} \gamma_\mu \gamma_5 \chi_{Pm}^\mu(p) \quad (m = 1). \end{aligned} \quad (50)$$

Explicitly, the residues of Λ_Q -type baryons (e.g., Λ_Q) can be expressed as

$$f_{\Lambda_Q} = \int \frac{dp_t}{2\pi^2} \tilde{f}_P(|p_t|) p_t^2. \quad (51)$$

For Σ_Q -type baryons, we have $B_1^\mu = \frac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\gamma^5 u(v)$. Substituting this equation into Eq. (50), we arrive at

$$\begin{aligned} f_{\Sigma_Q} u_{\Sigma_Q} &= \int \frac{d^4 p}{(2\pi)^4} \frac{1}{\sqrt{3}} \\ &\times [A(4 - \not{p}) - C \not{p} \gamma_5 \not{p} \gamma_5 - D \not{p} \gamma_5 \not{p} \gamma_5] u_{\Sigma_Q}. \end{aligned} \quad (52)$$

Since the Dirac spinor u_{Σ_Q} satisfies the condition $\not{p} u_{\Sigma_Q} = u_{\Sigma_Q}$, and $\int \frac{d^4 p}{(2\pi)^4} p_t^\mu f(|p|^2) = 0$ [$f(|p|^2)$ stands for a function of $|p|^2$], we have

$$f_{\Sigma_Q} = \frac{1}{\sqrt{3}} \int \frac{d^4 p}{(2\pi)^4} [3A(p) - |p_t|^2 D(p)]. \quad (53)$$

Finally, completing the integral $\int \frac{dp_t}{2\pi}$ in Eq. (53) and integrating over θ and ϕ in the spherical coordinate system, one immediately obtains

$$f_{\Sigma_Q} = \frac{1}{\sqrt{3}} \int \frac{dp_t}{2\pi^2} |p_t|^2 [3\tilde{A}(|p_t|^2) - |p_t|^2 \tilde{D}(|p_t|^2)]. \quad (54)$$

For Σ_Q^* -type baryons (e.g., Σ_Q^*), it can be shown that in the leading order of HQET, we have

$$\langle 0 | J_{\Sigma_Q^*}^\mu | \Sigma_Q^* \rangle = \frac{1}{\sqrt{3}} f_{\Sigma_Q^*} u_{\Sigma_Q^*}^\mu. \quad (55)$$

Substituting the explicit form of $J_{\Sigma_Q^*}$ into Eq. (55) and multiplying $\sqrt{3} p_{t\mu}$ on both sides of it, we have

TABLE VII. In the second and third lines, the residues of Λ_Q and Ξ_Q^A are shown with different values of κ and in the ranges of m_D in our model in the heavy-quark limit. In the fourth line, the residues of Λ_c are shown without taking the heavy-quark limit.

$\kappa(\text{GeV}^3)$	0.02	0.04	0.06	0.08
$f_{\Lambda_Q}(\text{GeV})$	0.103–0.182	0.137–0.198	0.163–0.212	0.182–0.224
$f_{\Xi_Q^A}(\text{GeV})$	0.143–0.181	0.161–0.194	0.176–0.205	0.189–0.215
$f_{\Lambda_c}(\text{GeV})$	0.090–0.195	0.128–0.213	0.152–0.229	0.174–0.243

$$\int \frac{d^4 p}{(2\pi)^4} \sqrt{3} \left[A(p) + \frac{1}{3} D(p) p_t^2 + \frac{1}{3} C(p) \not{p}_t \right] = f_{\Sigma_Q^*}. \quad (56)$$

Following the same routine used in the calculation of f_{Σ_Q} , we obtain

$$f_{\Sigma_Q^*} = \frac{1}{\sqrt{3}} \int \frac{d p_t}{2\pi^2} |p_t|^2 [3\tilde{A}(|p_t|^2) - |p_t|^2 \tilde{D}(|p_t|^2)], \quad (57)$$

so the residues of Σ_Q^* -type baryons are the same as those of Σ_Q -type baryons, which is consistent with the heavy quark symmetry.

In Table VII, we give the numerical results for the residues of Λ_Q -type baryons Λ_Q and Ξ_Q^A with m_D ranging from 0.60 to 0.80 GeV and 0.90 to 1.00 GeV respectively when $\kappa = 0.02, 0.04, 0.06, 0.08 \text{ GeV}^3$. We also give the results for the residues of Λ_c when the limit $m_c \rightarrow \infty$ is not taken so that we can see how $1/m_c$ corrections can influence the residue of Λ_c . It can be seen from the table that residues for Λ_c increase slightly in this case, and the deviation from those in the heavy-quark limit is basically less than 10%. In general, we find that both f_{Λ_Q} and $f_{\Xi_Q^A}$ increase as m_D and κ increase. Besides, it can be seen that the value of f_{Λ_Q} changes from 0.103 to 0.224 GeV in the ranges of the model parameters κ and m_D , and $f_{\Xi_Q^A}$ changes from 0.143 to 0.215 GeV with the ranges of κ and m_D . Furthermore, the dependence of the residue f_{Λ_Q} on m_D is stronger than that on κ . For example, the variation caused by the change of m_D is about 0.079 GeV when κ is 0.02 GeV^3 ; by contrast, the variation is 0.042 GeV as κ changes from 0.02 to 0.08 GeV^3 when m_D is 0.80 GeV. We find the same pattern for the dependence of $f_{\Xi_Q^A}$ on κ and m_D . On top of that, we find that when κ is fixed, the variation caused by the mass of the diquark is larger for Λ_Q than for Ξ_Q^A ; e.g., when $\kappa = 0.04 \text{ GeV}^3$, the variation of the residue of Λ_Q is 0.061 GeV, but only 0.033 GeV for Ξ_Q^A .

The results for the residues of Σ_Q -type baryons, Σ_Q^* , $\Xi_Q^{S(*)}$ and $\Omega_Q^{(*)}$, with different values of κ and in the ranges of m_A are given in Table VIII. We find that the residue of Σ_Q^* is in the range 0.262–0.361 GeV, with the parameters κ and m_D varying in their regions. The residue of $\Xi_Q^{S(*)}$ is in

TABLE VIII. The residues of $\Sigma_Q^{(*)}$, $\Xi_Q^{S(*)}$ and $\Omega_Q^{(*)}$ with different values of κ and in the ranges of m_A in the heavy-quark limit.

$\kappa(\text{GeV}^3)$	0.02	0.04	0.06	0.08
$f_{\Sigma_Q^*}(\text{GeV})$	0.262–0.288	0.295–0.299	0.305–0.336	0.317–0.361
$f_{\Xi_Q^{S(*)}}(\text{GeV})$	0.313–0.403	0.381–0.422	0.428–0.442	0.452–0.460
$f_{\Omega_Q^{(*)}}(\text{GeV})$	0.350–0.507	0.423–0.542	0.452–0.559	0.473–0.571

the range 0.313–0.460 GeV, which is generally larger than that of Σ_Q^* , and the residue of $\Omega_Q^{(*)}$ in the range 0.350–0.571 GeV is larger than those of both Σ_Q^* and $\Xi_Q^{S(*)}$. The variations of the residues for $\Xi_Q^{S(*)}$ and $\Omega_Q^{(*)}$ become smaller as κ increases. Furthermore, when κ is fixed, e.g., at 0.04 GeV^3 , the variations of the residues increase as the masses of Σ_Q -type baryons increase. Besides, we find that the variation due to the mass of the diquark is larger for $\Xi_Q^{S(*)}$ than for Σ_Q^* ; e.g., the variation is 0.026 GeV for Σ_Q^* , but 0.090 GeV for $\Xi_Q^{S(*)}$ when $\kappa = 0.02 \text{ GeV}^3$. However, when $\kappa = 0.08 \text{ GeV}^3$, the variation of the residue is only 0.008 GeV for $\Xi_Q^{S(*)}$, but 0.044 GeV for Σ_Q^* . It suggests that different values of κ have different influences on the residues of Σ_Q -type baryons. It can be seen in general that the residues of Σ_Q -type baryons are larger than those of Λ_Q -type baryons, which is consistent with the residues from QCD sum rules [14]. The changes in the residues of Σ_Q -type baryons with κ and m_A follow a similar pattern as those of Λ_Q -type baryons.

Generally, the parameters in our model can be determined by comparing the theoretical predictions and experimental data for Λ_Q -type or Σ_Q -type baryons. In Refs. [4,5,10,36,39], some phenomenological predictions for Λ_Q -type and Σ_Q -type baryons such as semileptonic, nonleptonic and strong decay widths were given in the BS equation approach. Furthermore, because the heavy quark is not infinite in reality, the $1/m_Q$ corrections to the BS equation for Λ_Q -type baryons were analyzed in Ref. [36] to obtain more accurate predictions. From the analyses of Refs. [4,5,10,36,39], we find that the uncertainties of the predictions caused by the parameter κ are bigger than those of the diquark mass. However, from the predictions for the residues of Λ_Q -type and Σ_Q -type baryons, we find that the uncertainties from the masses of the diquarks (m_D and m_A) are larger than those of κ .

V. SUMMARY AND DISCUSSION

The residues of Λ_Q -type baryons (Λ_Q and Ξ_Q^A) and Σ_Q -type baryons (Σ_Q^* , $\Xi_Q^{S(*)}$ and $\Omega_Q^{(*)}$) are interesting quantities both theoretically and experimentally, since they play an important role in calculating the production

amplitudes of heavy baryons and can be used to calculate fragmentation functions of heavy baryons. In this work, we constructed the baryonic currents for Λ_Q -type and Σ_Q -type baryons in the quark-diquark picture, and then defined the residues of Λ_Q -type and Σ_Q -type baryons in this picture. After that, we derived the relations between the residues and corresponding BS wave functions of Λ_Q -type and Σ_Q -type baryons.

The BS equations of Λ_Q and $\Sigma_Q^{(*)}$ were solved numerically based on the interaction kernel including confinement and one-gluon-exchange terms in the covariant instantaneous approximation. For the doublet states in Σ_Q -type baryons (e.g., Σ_Q and Σ_Q^*), we proved that their residues defined in our model are the same in the heavy-quark limit, which is consistent with the heavy quark symmetry. Using the numerical solutions for the BS wave functions, we obtained numerical results for the residues: 0.103–0.224 GeV for Λ_Q , 0.143–0.215 GeV for Ξ_Q^A ,

0.262–0.361 GeV for $\Sigma_Q^{(*)}$, 0.313–0.460 GeV for $\Xi_Q^{S(*)}$ and 0.350–0.571 GeV for $\Omega_Q^{(*)}$. Since the ratio of m_D/m_c is not very small, we also calculated the residue of Λ_c as an example without taking the heavy-quark limit and found that the results increase by less than 10%. The uncertainties of the residues come from κ and the masses of the diquarks (m_D and m_A). Much more data will be available from future experimental measurements, which can help us constrain the model parameters more accurately by comparing experimental data and the BS equation predictions.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Projects No. 11775024 and No. 11575023).

-
- [1] D. Ebert, T. Feldmann, C. Kettner, and H. Reinhardt, *Z. Phys. C* **71**, 329 (1996).
 - [2] Z. G. Wang, *Eur. Phys. J. C* **75**, 359 (2015).
 - [3] S. Groote, J. G. Korner, and O. I. Yakovlev, *Phys. Rev. D* **55**, 3016 (1997).
 - [4] X. H. Guo, A. W. Thomas, and A. G. Williams, *Phys. Rev. D* **59**, 116007 (1999).
 - [5] X. H. Guo and T. Muta, *Phys. Rev. D* **54**, 4629 (1996).
 - [6] X. Artru and M. Mekhfi, *Z. Phys. C* **45**, 669 (1990).
 - [7] H. Meyer and P. J. Mulders, *Nucl. Phys. A* **528**, 589 (1991).
 - [8] S. P. Tong, Y. B. Ding, X. H. Guo, H. Y. Jin, X. Q. Li, P. N. Shen, and R. Zhang, *Phys. Rev. D* **62**, 054024 (2000).
 - [9] X. H. Guo and P. Kroll, *Z. Phys. C* **59**, 567 (1993).
 - [10] X. H. Guo, K. W. Wei, and X. H. Wu, *Phys. Rev. D* **77**, 036003 (2008).
 - [11] E. V. Shuryak, *Nucl. Phys. B* **198**, 83 (1982).
 - [12] A. G. Grozin and O. I. Yakovlev, *Phys. Lett. B* **285**, 254 (1992).
 - [13] O. I. Yakovlev, R. Ruckl, and S. Weinzierl, *arXiv:hep-ph/0007344*.
 - [14] S. Groote, in *Progress in Heavy Quark Physics: Proceedings of the IVth International Workshop, Rostock, 1997*, edited by M. Beyer, T. Mannel, and H. Schröder (University of Rostock, Rostock, Germany, 1998), p. 195.
 - [15] M. A. Gomshi Nobary, T. Osati, and Z. Bahadori, *Nucl. Phys. A* **821**, 210 (2009).
 - [16] M. A. Gomshi Nobary, B. Nikoobakht, and J. Naji, *Nucl. Phys. A* **789**, 243 (2007).
 - [17] L. L. Liu, C. Wang, Y. Liu, and X. H. Guo, *Phys. Rev. D* **95**, 054001 (2017).
 - [18] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, *Phys. Rev. D* **17**, 3090 (1978); **21**, 313(E) (1980).
 - [19] K. W. Wei, B. Chen, and X. H. Guo, *Phys. Rev. D* **92**, 076008 (2015).
 - [20] E. E. Salpeter and H. A. Bethe, *Phys. Rev.* **84**, 1232 (1951).
 - [21] D. Lurie, *Particles and Fields* (John Wiley & Sons, New York, 1968).
 - [22] H. Y. Jin, C. S. Huang, and Y. B. Dai, *Z. Phys. C* **56**, 707 (1992).
 - [23] Y. B. Dai, C. S. Huang, and H. Y. Jin, *Phys. Lett. B* **331**, 174 (1994).
 - [24] Y. B. Dai, C. S. Huang, and H. Y. Jin, *Z. Phys. C* **60**, 527 (1993).
 - [25] J. Carbonell and V. A. Karmanov, *Eur. Phys. J. A* **46**, 387 (2010).
 - [26] W. de Paula, T. Frederico, G. Salme, and M. Viviani, *Phys. Rev. D* **94**, 071901 (2016).
 - [27] H. Georgi, *Nucl. Phys. B* **348**, 293 (1991).
 - [28] A. F. Falk, *Nucl. Phys. B* **378**, 79 (1992).
 - [29] M. Anselmino, P. Kroll, and B. Pire, *Z. Phys. C* **36**, 89 (1987).
 - [30] M.-H. Weng, X.-H. Guo, and A. W. Thomas, *Phys. Rev. D* **83**, 056006 (2011).
 - [31] T. Mannel, W. Roberts, and Z. Ryzak, *Phys. Lett. B* **271**, 421 (1991).
 - [32] W. Roberts, *Nucl. Phys. B* **389**, 549 (1993).
 - [33] F. Hussain, G. Thompson, and J. G. Korner, *arXiv:hep-ph/9311309*.
 - [34] Y. B. Dai, X. H. Guo, and C. S. Huang, *Nucl. Phys. B* **412**, 277 (1994).
 - [35] X. H. Guo and H. K. Wu, *Phys. Lett. B* **654**, 97 (2007).
 - [36] X. H. Guo, A. W. Thomas, and A. G. Williams, *Phys. Rev. D* **61**, 116015 (2000).
 - [37] X. H. Guo, H. Y. Jin, and X. Q. Li, *Phys. Rev. D* **53**, 1153 (1996).
 - [38] C. Patrignani *et al.* (Particle Data Group), *Chin. Phys. C* **40**, 100001 (2016).
 - [39] X. H. Guo, *Mod. Phys. Lett. A* **13**, 2265 (1998).