Extended MSSM in supersymmetric SO(10) grand unification

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We apply the perturbative grand unification due to renormalization to distinguish TeV-scale relics of supersymmetric SO(10) scenarios. With rational theoretical constraints taken into account, we find that for the breaking pattern of either SU(5) or Pati-Salam, only extra matter 16 supermultiplet of SO(10) can appear at the TeV scale, apart from the MSSM spectrum.

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I. INTRODUCTION

The discovery of the standard model (SM)-like Higgs [1,2] provides a new portal to TeV-scale new physics at the LHC in the forthcoming years. Among other things, such new physics models may reveal the “nature” of the SM-like Higgs and offer a novel mechanism to stabilize divergence involving the SM Higgs. For those interesting scenarios in the literature, in this paper we are restricted to the idea of supersymmetry (SUSY). Specifically, we will utilize the grand unification (GUT) [3], which is one of the most beautiful features delivered by SUSY, to distinguish TeV-scale relics of SUSY GUT models. For reviews on this subject, see e.g., [4,5].

In the viewpoint of unification, the minimal supersymmetric standard model (MSSM) can be embedded into conventional SU(5) [6–8], SO(10) [9,10], or other GUT models with gauge groups of higher ranks. In light of our previous study on SU(5) [11], we will continue to explore the TeV-scale relics of SUSY SO(10) unification. Comparing with SU(5), the low-energy effective theories of SO(10) are more complex. The first major reason is that there may be multiple intermediate scales between the weak and GUT scale. The second reason is that since a lot of higher-dimensional representations of SO(10) trivially satisfy gauge anomaly free condition, the constraint imposed by this condition is much weaker in SO(10). Earlier studies on low-energy effective theory which is consistent with perturbative SUSY SO(10) unification are based on specific motivations such as the Higgs mass [12] and neutrino physics [13–15].

Instead of particular phenomenological concerns, we will take a systematic analysis on the low-energy effective

theory. In order to simplify the analysis on extra matter beyond the MSSM spectrum, we will explore SO(10) scenarios with the following theoretical features:

- (i) The SO(10) unification is strictly perturbative.
- (ii) In the chain of gauge symmetry breaking,

$$\text{SO}(10) \xrightarrow{H_1} G_1 \xrightarrow{H_2} G_2 \cdots \xrightarrow{H_n} G_{\text{SM}}, \quad (1)$$

where G_{SM} refers to the SM gauge group. When the Higgs component fields responsible for two nearby steps of gauge symmetry breaking can be contained in a single Higgs supermultiplet, these two Higgs supermultiplets will be identified as the same one. Otherwise, they differ from each other¹.

- (iii) In order to avoid dangerous mixings among Higgs vevs $\langle H_i \rangle$, all of H_i are forbidden to directly couple to each other.
- (iv) In order to avoid dangerous masses or mixing effects, neither the MSSM fields nor extra matters are allowed to directly couple to any Higgs supermultiplets H_i in Eq. (1).²

The main reason for the last point is that the vacuum expectations (vevs) of $\langle H_i \rangle$ would result in large matter masses or large mixing effects if they were directly coupled to either MSSM fields or extra matters, which would lead to them playing no role at weak scale. For example, coupling a **54**, which can break SU(5) to the SM gauge group, to MSSM Higgs **10_H** through interaction $\mathbf{54} \times \mathbf{10}_H \times \mathbf{10}_H$, yields an unfavorable Higgsino mass for a large vev of SM singlet in $\langle \mathbf{54} \rangle$.

Theoretical constraints above have been partially imposed in the literature to our knowledge. However, they

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¹In this situation, the splitting between two nearby scales of gauge symmetry breaking is generally large. We will impose the relatively strong constraint that the two Higgs supermultiplets responsible for such large splitting are forbidden to directly couple to each other.

²There is a loophole. When the vev of the last Higgs $\langle H_n \rangle$ is of the order of the TeV scale, they are allowed to couple to H_n .

have never been combined together to derive a systematic analysis on the low-energy effective theory. The paper is organized as follows. In Sec. II, we discuss the extra matter supermultiplets which are consistent with our starting points in two well known patterns of gauge symmetry breaking. In Sec. III, we examine the perturbative unification with these representations. Finally, we conclude in Sec. IV.

II. REPRESENTATIONS

According to our starting points, in this section we investigate the representation of extra matter which can directly couple to the SM Higgs $\mathbf{10}_H$ in the following two patterns of gauge symmetry breaking,

$$\text{A: } \text{SO}(10) \xrightarrow{H_1} \text{SU}(5) \xrightarrow{H_2} G_{\text{SM}},$$

$$\begin{aligned} \text{B: } \text{SO}(10) &\xrightarrow{H_1} \text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \\ &\xrightarrow{H_2} \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \\ &\xrightarrow{H_3} G_{\text{SM}}. \end{aligned}$$

Pattern A [16–19] is a two-step breaking with SU(5) subgroup, and pattern B [20–22] is a three-step breaking referred to as the Pati-Salam model [21].

Note, in the MSSM the SM fermion matters are described by $\mathbf{16}_i$ of SO(10) with index $i = 1-3$, and the SM Higgs is contained in the $\mathbf{10}_H$ of SO(10). In particular, $\mathbf{16}_i$ contains three-generation right-hand neutrinos, whereas $\mathbf{10}_H$ is composed of $\mathbf{5}_H$ and $\bar{\mathbf{5}}_H$ of SU(5) which contain the two Higgs doublets of MSSM and two color-triplets.

A. SU(5)

In this pattern of symmetry breaking, H_1 should contain an SU(5) singlet, there are two candidates $H_1 = \{\mathbf{16}, \mathbf{126}\}$. The second Higgs H_2 should contain a 24 of SU(5), which corresponds to three potential choices $H_2 = \{\mathbf{45}, \mathbf{54}, \mathbf{210}\}$. Since $H_1 \neq H_2$, we take the rationale that the splitting between these two broken scales is large.

With potential assignments on H_1 and H_2 above, there are six sets of combinations. In each case, there may exist four types of dangerous gauge-invariant superpotentials which violate the last two starting points in Sec. I,

$$\begin{aligned} H_1 \times \mathcal{N} \times \mathcal{N}; \\ H_2 \times \mathcal{N} \times \mathcal{N}; \\ H_1 \times H_2 \times \mathcal{N}; \\ H_1 \times H_1 \times H_2, H_2 \times H_2 \times H_1, \end{aligned} \quad (2)$$

where MSSM matter field $\mathcal{N} = \{\mathbf{16}_i, \mathbf{10}_H\}$. In Eq. (2), the first three types of superpotentials tender to seed Dirac or Majorana masses to MSSM matters or MSSM Higgs

doublets; and the last type of gauge invariant superpotentials yields dangerous mixings between vevs of H_1 and H_2 .

First, one finds a dangerous operator $\mathbf{16} \times \mathbf{16}_i \times \mathbf{10}_H$ of the first type in Eq. (2) which disfavors the choice $H_1 = \mathbf{16}$. Second, due to dangerous operator $\mathbf{126} \times \mathbf{210} \times \mathbf{10}_H$ the set of $H_1 = \mathbf{126}$ and $H_2 = \mathbf{210}$ is also disfavored. Therefore, we are left with two combinations $H_1 = \mathbf{126}$ and $H_2 = \{\mathbf{45}, \mathbf{54}\}$. In the case $(H_1, H_2) = (\mathbf{126}, \mathbf{54})$, there exists a dangerous operator $\mathbf{126} \times \mathbf{126} \times \mathbf{54}$. In the last case, $(H_1, H_2) = (\mathbf{126}, \mathbf{45})$ denotes the new matter supermultiplets with $\mathcal{M}$, and we find that unsafe superpotentials exclude $\mathcal{M} = \{\mathbf{10}, \bar{\mathbf{16}}, \mathbf{120}, \mathbf{144}, \bar{\mathbf{144}}, \mathbf{210}\}$, leaving us only two possibilities,

$$\mathbf{16}_M \times \mathbf{16}_M \times \mathbf{10}_H, \quad \mathbf{16}_M \times \mathbf{16}_i \times \mathbf{10}_H. \quad (3)$$

Compared with breaking pattern A, there is another pattern of two-step breaking,

$$\text{SO}(10) \xrightarrow{H_1} G_{\text{SM}} \times \text{U}(1) \xrightarrow{H_2} G_{\text{SM}}.$$

In this case, the potential choices are $H_1 = \{\mathbf{45}, \mathbf{210}\}$ and $H_2 = \{\mathbf{16}, \mathbf{126}\}$. According to Eq. (2), dangerous operator $\mathbf{16} \times \mathbf{16}_i \times \mathbf{10}_H$ excludes the case $H_2 = \mathbf{16}$. Moreover, a dangerous operator $\mathbf{210} \times \mathbf{126} \times \mathbf{10}_H$ excludes $H_1 = \mathbf{210}$. Therefore, there is only a viable combination $(H_1, H_2) = (\mathbf{45}, \mathbf{126})$, in which case the extra matter $\mathcal{M}$ is similar to those of SU(5) subgroup.

B. Pati-Salam

In this pattern of symmetry breaking, H_1 should contain a singlet of $\text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$, which has two choices $H_1 = \{\mathbf{54}, \mathbf{210}\}$. H_2 should contain a singlet of $\text{SU}(3)_c$ and $\text{U}(1)_{B-L}$, which is a 15 of $\text{SU}(4)_c$. There are two representations $H_2 = \{\mathbf{45}, \mathbf{210}\}$ of SO(10) which include such a 15. Finally, $H_3 = \mathbf{16}$ offers the breaking of $\text{SU}(2)_R \times \text{U}(1)_{B-L} \rightarrow \text{U}(1)_Y$.

Since 210 contains both a singlet of $\text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$ and a $\mathbf{15}$ of $\text{SU}(4)_c$, according to the second starting point both H_1 and H_2 are identified as 210. In this case, the three-step breaking is approximately two-step. With $H_1 = H_2 = \mathbf{210}$, we are left with a single choice $(H_1, H_2, H_3) = (\mathbf{210}, \mathbf{210}, \mathbf{16})$, where similar to previous discussions about SU(5) there are four viable choices for extra matters,

$$\begin{aligned} \mathbf{16}_M \times \mathbf{16}_M \times \mathbf{10}_H, \quad \mathbf{10}_M \times \mathbf{54}_M \times \mathbf{10}_H, \\ \mathbf{16}_M \times \mathbf{144}_M \times \mathbf{10}_H, \quad \mathbf{144}_M \times \bar{\mathbf{144}}_M \times \mathbf{10}_H. \end{aligned} \quad (4)$$

Note, unlike in pattern A, extra matter supermultiplets in Eq. (4) are allowed to directly couple to H_3 because the broken scale of $\text{SU}(2)_R \times \text{U}(1)_{B-L}$ [21,23,24] can be close to the TeV scale (see, e.g., [25–30]).

III. PERTURBATIVE UNIFICATION

With the theoretical constraints in the Introduction, we have clarified that a single or two **16** supermultiplets are allowed in pattern A, whereas two **16**s, a **10** with **54**, a **16** with **144**, or a pair of vectorlike **144** may appear in the pattern B. Now, we examine whether any of them are consistent with the first constraint—perturbative unification.

We start with the one-loop renormalization group equations (RGEs) for SM gauge coupling constant,

$$\frac{d}{dt} \alpha_i^{-1} = -\frac{b_i}{2\pi}, \quad (5)$$

where RG scale $t = \ln \mu$ and coefficients $b_i = (b_{U(1)_Y}, b_{SU(2)_L}, b_{SU(3)_c})$ are determined by [31,32]

$$b_i = \begin{cases} (41/10, -19/6, -7), & \mu \in [M_Z, \Lambda_{\text{SUSY}}] \\ (33/5 + \delta b_1(\mathbf{M}), 1 + \delta b_{2L}(\mathbf{M}), -3 + \delta b_3(\mathbf{M})), & \mu \in [\Lambda_{\text{SUSY}}, \Lambda_5] \\ b_5 = \delta b_5(\mathbf{M}), & \mu \in [\Lambda_5, \Lambda_{10}], \end{cases} \quad (7)$$

where b_5 denotes the b coefficient of SU(5) subgroup, and $\delta b_i(\mathbf{M})$ refer to contributions to b coefficients due to extra matter. In particular, $\delta b_i(\mathbf{16}_M) = (2, 2, 2)$ and $\delta b_i(\mathbf{16}_M + \mathbf{16}_M) = (4, 4, 4)$, respectively, in Eq. (7). Figure 1 shows the plots of RG running of SM gauge coupling constants according to Eq. (7). It reveals that for $\Lambda_{\text{SUSY}} = 1$ TeV the SU(5) unification occurs at $\Lambda_5 \simeq 10^{16.3}$ GeV. Moreover, the SO(10) unification in both cases can occur at Λ_{10} large than 10^{18} GeV. Comparing Λ_5

$$b_i = -\left\{ \frac{11}{3} C_2^i(G) - \frac{4}{3} \cdot \kappa \cdot T(r_{f_i}) - \frac{1}{6} T(r_{s_i}) \right\}. \quad (6)$$

Here, $C_2(G)$ is the quadratic Casimir invariant, and $T(r)$ refers to dynkin index that depends on details of the representation [5].

A. SU(5)

In the case of SU(5) subgroup there are two intermediate scales Λ_{SUSY} and Λ_5 between M_Z and Λ_{10} , corresponding to SUSY and SU(5) breaking, respectively. The b_i coefficients are given by³

with Λ_{10} , one finds that there is indeed sufficient splitting between them, which verifies previous arguments.

B. Pati-Salam

In the case of Pati-Salam model there are two intermediate scales Λ_{SUSY} and Λ_R between M_Z and Λ_{10} , which denotes SUSY and $SU(2)_R \times U(1)_{B-L}$ breaking scale, respectively. In this case, the coefficients b_i are given by

$$b_i = \begin{cases} (41/10, -19/6, -7), & \mu \in [M_Z, \Lambda_{\text{SUSY}}] \\ (33/5 + \delta b_1(\mathbf{M}), 1 + \delta b_2(\mathbf{M}), -3 + \delta b_3(\mathbf{M})), & \mu \in [\Lambda_{\text{SUSY}}, \Lambda_R] \end{cases} \quad (8)$$

for the RG scale between M_Z and Λ_R , and

$$b_i = (6 + 2 + \delta b_{B-L}(\mathbf{M}), 0 + 2 + \delta b_{2L}(\mathbf{M}), 0 + 2 + \delta b_{2R}(\mathbf{M}), -3 + 2 + \delta b_3(\mathbf{M})), \quad \mu \in [\Lambda_R, \Lambda_{10}] \quad (9)$$

for the RG scale between Λ_R and Λ_{10} . Above the RG scale Λ_R , MSSM matters and Higgs field $H_3 = \mathbf{16}$ contributes to $\delta b_i = (6, 0, 0, -3)$ and $\delta b_i = (2, 2, 2, 2)$ in Eq. (9), respectively. Regardless of what extra matter appears above Λ_{SUSY} and what kind of Higgs $H_{1,2,3}$ above Λ_R , SO(10)

unification at the one-loop level yields $\ln(\Lambda_{10}/\Lambda_R) \simeq 2.53$ [33,34], which uniquely determines Λ_R once the content of extra matters is identified.

Take a pair of $\mathbf{16}_M$ e.g., they contribute to $\delta b_i(\mathbf{M}) = (4, 4, 4)$ and $\delta b_i(\mathbf{M}) = (4, 4, 4, 4)$ in Eq. (8) and Eq. (9), respectively, which gives rise to $\Lambda_R \simeq 10^{15.5}$ GeV, unified gauge coupling $\alpha^{-1} \simeq 5.45$, and $\Lambda_{10} \simeq 10^{16.7}$ GeV. Figure 2 shows the RG running of SM gauge coupling constant, which offers us perturbative SO(10) unification. Note that the RG running of the $SU(2)_R$ gauge coupling

³In order to eliminate the triplet fields in $\mathbf{10}_H$ some additional matter representations may be required as well. Nevertheless, they are usually decoupled below the scale Λ_5 , which don't affect the RG analysis between M_Z and Λ_5 .

FIG. 1. One-loop RGEs for SM gauge coupling α_1^{-1} (green), α_2^{-1} (blue) and α_3^{-1} (red) for extra matter $\mathbf{16}_M$ (dashed) and $\mathbf{16}_M + \mathbf{16}_M$ (solid), respectively. Here SUSY breaking scale $\Lambda_{\text{SUSY}} = 1$ TeV.

FIG. 2. One-loop RGEs for SM gauge coupling α_1^{-1} (green), α_{2L}^{-1} (blue), α_3^{-1} (red) and α_{B-L}^{-1} (black) for extra matter $\mathbf{16}_M + \mathbf{16}_M$. For $\Lambda_{\text{SUSY}} = 1$ TeV, one obtains $\Lambda_R \simeq 10^{15.5}$ GeV and $\Lambda_{10} \simeq 10^{16.7}$ GeV. The RG running of $\text{SU}(2)_R$ gauge coupling constant between Λ_R and Λ_{10} coincides with that of $\text{SU}(2)_L$.

constant between Λ_R and Λ_{10} coincides with that of $\text{SU}(2)_L$, and as required α_Y^{-1} is equal to $\frac{3}{5}\alpha_{2R}^{-1} + \frac{2}{5}\alpha_{B-L}^{-1}$ at RG scale Λ_R .

We repeat the analysis for other choices on extra matters in Eq. (4). We find that, in these cases, b coefficients such as $\delta b_i(\mathbf{10}_M + \mathbf{54}_M) = (11.5, 11.5, 11.5, 11.5)$ are always too large to support the idea of perturbative unification.

IV. CONCLUSION

In the forthcoming years, we will enter into a new era of precise Higgs physics, which means that studying new physics through the Higgs portal will become very interesting. In this paper, we have utilized perturbative unification due to renormalization to explore the low energy effective theory of SUSY $\text{SO}(10)$ scenarios. With the rational theoretical constraints taken into account, we find that for the breaking pattern of either $\text{SU}(5)$ or Pati-Salam only $\mathbf{16}$ supermultiplet can appear at the TeV scale apart from the MSSM spectrum.

The quarks or leptons in the $\mathbf{16}$ supermultiplet(s) can be either chiral or vectorlike. Note, the vectorlike fermion mass requires addition of SM singlet (with vev of order TeV) which does not affect our discussions. While the chiral case has been excluded, the vectorlike quarks or leptons are smoking guns in these SUSY $\text{SO}(10)$ scenarios. Moreover, the neutral fermions of singlet or doublets of the $\mathbf{16}$ supermultiplet can serve as dark matter totally, or partially with the neutralinos of the MSSM.

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