Light pseudo-Goldstone Higgs boson from $SO(10)$ GUT with realistic phenomenology

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Within the supersymmetric $SO(10)$ grand unified theory (GUT), a new mechanism giving the light Higgs doublet as a pseudo-Goldstone mode is suggested. Realizing this mechanism, we present an explicit model with fully realistic phenomenology. In particular, desirable symmetry breaking and natural all-order hierarchy are achieved. The constructed model allows one to have a realistic fermion pattern, nucleon stability, and successful gauge coupling unification. The suggested mechanism opens prospects in the field for a novel $SO(10)$ GUT model building and for further investigations.

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I. INTRODUCTION

Despite enormous success, the Standard Model (SM), failing to explain several observations, needs to be extended. Making steps beyond the SM, the grand unification theory (GUT) [1] is a highly motivated and elegant idea. The best candidate for GUT model building is the $SO(10)$ group [2], whose spinorial 16_i -plets ($i = 1, 2, 3$), together with SM matter, include right-handed neutrino ν^c states (one per generation)— $16_i = \{q, u^c, d^c, l, e^c, \nu^c\}_i$. Thus, within the $SO(10)$ GUT, the most compelling seesaw neutrino mass generation mechanism [3] finds a natural realization, accommodating the data from various neutrino oscillation experiments. As shown, the same mass generation mechanism can generate the observed baryon asymmetry through leptogenesis [4]. Towards circumventing other problems, the supersymmetric (SUSY) extensions turn out to be a relief. SUSY ensures the stability of the weak scale. The SM's minimal SUSY extension (MSSM) leads to successful gauge coupling unification [5] (a necessary ingredient for GUTs) and includes the viable dark matter candidate—the neutralino.

Although there are numerous merits of the SUSY $SO(10)$ GUT (we study henceforth), the most challenging problem—the doublet-triplet (DT) splitting problem—(together with other puzzles we address below) needs to be taken seriously [6]. The missing vacuum expectation value (VEV) mechanism, proposed earlier [7], enabling

one to have quite realistic constructions [8–13], triggered an intense investigation in the field [13]. An alternative, and perhaps most elegant, idea for resolving the DT problem is the pseudo-Goldstone boson (PGB) mechanism, which proposes the light Higgs as the PGB [14,15]. However, so far, attempts to realize this idea within GUTs, have been successful only within $SU(6)$ gauge symmetry [15–17]. Obviously, it would be very exciting to find a possibility for the PGB mechanism realization within the $SO(10)$ GUT.

In this paper, the first time within the SUSY $SO(10)$ GUT, I propose the possibility for the Higgs doublet supermultiplets emerging as light PGBs. The model I present turns out to be fully realistic. Consistent GUT symmetry breaking is achieved, and the MSSM Higgs doublets are light pseudo-Goldstones, even after taking into account all allowed high-order operators. The model I construct offers a realistic fermion pattern, nucleon stability, and successful gauge coupling unification. Because of these issues (usually turning out to be severe problems for various GUTs), I find the presented $SO(10)$ model to be quite successful.

The mechanism proposed in this paper opens the door for future work to build varieties of PGB $SO(10)$ models and investigate their phenomenological implications.

II. THE SETUP AND THE MECHANISM

First, I describe the PGB mechanism within the SUSY $SO(10)$ GUT and then, on a concrete model, demonstrate its natural realization. The minimal superfield content, ensuring $SO(10) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \equiv G_{\text{SM}}$ breaking, is $\{45_H, 16_H, \overline{16}_H\}$ [and possibly some additional $SO(10)$ singlet states]. Thus, the symmetry-breaking scalar superpotential W_H consists of three parts:

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$$W_H = W_H^{(45)} + W_H^{(16)} + W_H^{(45,16)}, \quad (1)$$

where $W_H^{(45)}$ and $W_H^{(16)}$ involve separately 45_H and $\{16_H, \overline{16}_H\}$ -plets, respectively. The $W_H^{(45)}$, including some powers of 45_H (such as $45_H^2, 45_H^4, \dots$), is responsible for fixing the VEV $\langle 45_H \rangle$ towards the desirable direction. The $W_H^{(16)}$ fixes the VEVs of 16_H and $\overline{16}_H$ in their $\nu_H^c, \bar{\nu}_H^c$ scalar components and can be $W_H^{(16)} = S(i\overline{16}_H 16_H + v_R^2)$, where S is the $SO(10)$ singlet. The condition $F_S = 0$, together with D -flatness, would fix $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle = v_R$. For us, it is essential that the terms like $(\overline{16}_H 16_H)^2$ and $\overline{16}_H 16_H 45_H^{2n+1}$ are absent or are properly suppressed (will be explained below). The superpotential $W_H^{(45,16)}$ includes couplings of $\overline{16}_H 16_H$ with *only even* powers of 45_H (this can be guaranteed by discrete symmetries), such as $\overline{16}_H 45_H^2 16_H$. Without $W_H^{(45,16)}$, the W_H has global symmetry $SO(10)_{45} \times U(16)$ [e.g., $W_H^{(45)}$ possesses $SO(10)_{45}$ symmetry, while the $W_H^{(16)}$ has $U(16)$ global symmetry]. With the VEVs $\langle \nu_H^c \rangle$ and $\langle \bar{\nu}_H^c \rangle$, the breaking $U(16) \rightarrow U(15)$ happens and pseudo-Goldstone states emerge. Clearly, those modes (at least some of those) would gain masses by including the terms of $W_H^{(45,16)}$. Our goal is to see if it is possible to arrange the couplings such that MSSM Higgs doublets emerge as light pseudo-Goldstones while all remaining states acquire large masses. It turns out that in this setup this is indeed possible.

Assume that the 45_H has a VEV towards the $B-L$ direction $\langle 45_H \rangle \sim V_{B-L}$. Thus, with $SO(10) \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R$ decomposition

$$45_H = \Sigma(15, 1, 1) + \Sigma_R(1, 1, 3) + \Sigma_L(1, 3, 1) + (6, 2, 2), \quad (2)$$

the $SU(4)_c$'s adjoint $\Sigma(15, 1, 1)$ have the VEV $\langle \Sigma \rangle = V_{B-L} \cdot \text{Diag}(1, 1, 1, -3)$, causing the breaking of $SO(10)$ gauge symmetry down to $G_{3221} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. With this breaking, the 30 states (from 45_H) absorbed by superheavy gauge fields become genuine Goldstones. These are the fragment $(6, 2, 2)$ [given in (2)] plus $SU(3)_c$'s triplet-antitriplet pair $3_\Sigma, \bar{3}_\Sigma$ coming from $\Sigma(15, 1, 1)$. The remaining fragments of 45_H should gain large masses. Moreover, with $W_H^{(16)}$'s terms, such as $\lambda_S S(i\overline{16}_H 16_H + v_R^2)$, one can have $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle = v_R$, inducing the breaking $G_{3221} \rightarrow G_{SM}$. The spontaneous breaking $W_H^{(16)}$'s global $U(16)$ would give unwanted pseudo-Goldstone modes. However, the superpotential $W_H^{(45,16)} \supset \frac{\lambda i}{4! M_*} \overline{16}_H 45_H^2 16_H$ coupling will break $U(16)$ but, due to the $\langle 45_H \rangle$ configuration, render some global symmetries. In particular, using in this coupling the VEV $\langle 45_H \rangle = \langle \Sigma \rangle \propto Q_{B-L}$ [see Eq. (2) and the comment below it], the bilinear terms with respect to the states $16_H = \{f_H\}, \overline{16}_H = \{\bar{f}_H\}$, where $f_H = \{\nu_H^c, e_H^c, l_H, u_H^c, d_H^c, q_H\}$, will be

$$\begin{aligned} \frac{\lambda}{M_*} \overline{16}_H 45_H^2 16_H &\rightarrow \lambda \frac{V_{B-L}^2}{M_*} \sum_f \bar{f}_H (Q_{B-L}^f)^2 f_H + \dots \\ &= \lambda \frac{V_{B-L}^2}{M_*} (\bar{\nu}_H^c \nu_H^c + \bar{e}_H^c e_H^c + \bar{l}_H l_H) \\ &\quad + \lambda \frac{V_{B-L}^2}{M_*} \frac{1}{9} (\bar{u}_H^c u_H^c + \bar{d}_H^c d_H^c + \bar{q}_H q_H) + \dots, \end{aligned} \quad (3)$$

where I have displayed terms not respecting the $U(16)$ global symmetry. However, due to the relation $M_{f_H \bar{f}_H} \propto (Q_{B-L}^f)^2$, as seen from (3), the second and last lines possess $U(4)$ and $U(12)$ symmetries, respectively. The $U(4)$ global symmetry, which has leptonic states, is spontaneously broken down to $U(3)$ by the VEVs $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle = v_R$. Because of $U(4) \rightarrow U(3)$ breaking, the seven Goldstones emerge. Among them, three—one SM singlet and the pair $e_H^c, \bar{e}_H^c$ —are genuine Goldstones absorbed by the coset $[SU(2)_R \times U(1)_{B-L}]/U(1)_Y$ gauge fields. The states l_H and $\bar{l}_H$, having the quantum numbers of the MSSM Higgs superfields, emerge as light (as desired) pseudo-Goldstone modes. The remaining fragments, given in the last line in (3), will be heavy.

However, for this mechanism to be realized, some care needs to be exercised. The λ coupling term, with nonzero $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle = v_R$, would also trigger the VEV of 45_H in the I_{3R} direction, i.e., the VEV of $\Sigma_R(1, 1, 3)$ of (2) with the $\langle \Sigma_R \rangle = V_R \cdot \text{Diag}(1, -1)$ configuration in $SU(2)_R$ space. As expected, $V_R \sim \lambda v_R^2 / V_{B-L}$. This, in (3), would introduce $U(4)$ symmetry-breaking mass terms $\sim \lambda^2 v_R^2 / M_*$. Thus, the PGB's mass—the μ term—will be $\mu \sim \lambda^2 v_R^2 / M_*$. For its proper suppression, for $v_R = 10^{16-17} \text{ GeV}$, $M_* \simeq 2.4 \times 10^{18} \text{ GeV}$, we need $\lambda \lesssim 10^{-6}$. Besides this, terms such as $\overline{16}_H 45_H^{2n+1} 16_H$ and $(16_H \overline{16}_H)^{k+2}$ ($n, k = 0, 1, 2, \dots$) should be absent or adequately suppressed. Below, I present the model where all this is naturally realized; i.e., smallness or absence of any coupling will follow from the symmetries (we will invoke).

III. MODEL WITH ALL-ORDER NATURAL HIERARCHY

Realizing the mechanism described above, we augment the SUSY $SO(10)$ GUT by $\mathcal{U}(1)_A \times \mathcal{Z}_4$ symmetry, where $\mathcal{U}(1)_A$ is an anomalous gauge symmetry and $\mathcal{Z}_4$ is discrete R symmetry. For symmetry breaking and all-order DT hierarchy, the anomalous $\mathcal{U}(1)_A$ was first applied in Ref. [17] [within the PGB $SU(6)$ GUT] and was proven to be very efficient for realistic $SO(10)$ model building [9, 12]. Besides the states $45_H, 16_H$, and $\overline{16}_H$, we introduce two $SO(10)$ singlet superfields S and S_1 , which will be used in superpotential W_H of Eq. (1). In Table I, I display the $\mathcal{U}(1)_A \times \mathcal{Z}_4$ transformation properties of these fields together with other states (introduced and discussed later on). Note that, under $\mathcal{Z}_4$ symmetry, the whole

TABLE I. $\mathcal{U}(1)_A$ and $\mathcal{Z}_4$ charges Q_i and ω_i of the superpotential and superfield ϕ_i . The transformations under $\mathcal{U}(1)_A$ and $\mathcal{Z}_4$ are, respectively, $\phi_i \rightarrow e^{iQ_i} \phi_i$ and $\phi_i \rightarrow e^{i\omega_i} \phi_i$; $\omega = \frac{2\pi}{4}$.

	W	45_H	16_H	$\overline{16}_H$	S	S_1	10_H	$10'_H$	16_i
Q_i	0	0	-5	1	0	4	-6	6	3
ω_i	2ω	ω	$-\omega$	ω	2ω	0	0	ω	ω

superpotential transforms as $W \rightarrow -W$. Thus, the relevant superpotential couplings are

$$\begin{aligned}
 W_H^{(45)} &= \frac{1}{2} M_{45} \text{tr}(45_H^2) + \frac{\lambda_1}{M_*^3} [\text{tr}(45_H^2)]^3 \\
 &\quad + \frac{\lambda_2}{M_*^3} \text{tr}(45_H^2) \text{tr}(45_H^4) + \frac{\lambda_3}{M_*^3} \text{tr}(45_H^6), \\
 W_H^{(16)} &= -\lambda_S S \left(i \frac{S_1}{M_*} \overline{16}_H 16_H + \Lambda^2 \right), \\
 W_H^{(45,16)} &= \frac{\bar{\lambda} i S_1}{4! M_*^2} \overline{16}_H 45_H^2 16_H.
 \end{aligned} \quad (4)$$

With anomalous $\mathcal{U}(1)_A$, having the string origin, the Fayet-Iliopoulos term $\int d^4\theta \xi V_A$ is always generated [18], and the corresponding D -term potential is

$$V_D = \frac{g_A^2}{8} \left(\xi + \sum_i Q_i |\phi_i|^2 \right)^2 \rightarrow \frac{g_A^2}{8} (\xi - 4v_R^2 + 4|\langle S_1 \rangle|^2)^2. \quad (5)$$

At the last stage in (5), we have taken into account that $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle \equiv v_R$, which ensures the vanishing of all D terms of $SO(10)$. With $\xi > 0$ and looking for the solution of $\langle S_1 \rangle \ll v_R$, the condition $\langle D_A \rangle = 0$ from (5) gives

$$v_R \simeq 0.5 \sqrt{\xi}. \quad (6)$$

Next, we investigate the symmetry-breaking pattern from the superpotential couplings given in (4). With the branching $\overline{16} \times 16 = 1 + 45 + 210$ and noting that 45_H^2 identically vanishes in the antisymmetric 45 channel, the remaining relevant contraction [breaking global $U(16)$ symmetry] is in the 210 channel. Thus, under the $\overline{16}_H 45_H^2 16_H$ term, we assume the invariant $(\overline{16}_H 16_H)_{210} \cdot (45_H^2)_{210}$ (see the Appendix for more details).

Writing 45_H 's VEV in $SO(10)$ group space as

$$\langle 45_H \rangle = i\sigma_2 \otimes \text{Diag}(V_{B-L}, V_{B-L}, V_{B-L}, V_R, V_R) \quad (7)$$

and imposing $F_{\phi_i} = 0$, from the superpotential couplings (4), taking into account that v_R is fixed as (6), we obtain

$$\begin{aligned}
 \langle S_1 \rangle &\simeq \frac{\Lambda^2}{v_R^2} M_*, \\
 \langle S \rangle &= -\frac{3\bar{\lambda}}{\lambda_S M_*} V_{B-L}^2 \left(1 + \frac{2V_R}{V_{B-L}} + \frac{V_R^2}{3V_{B-L}^2} \right),
 \end{aligned} \quad (8)$$

$$\begin{aligned}
 &6(36\lambda_1 + 6\lambda_2 + \lambda_3) V_{B-L}^4 + 16(18\lambda_1 + \lambda_2) V_{B-L}^2 V_R^2 \\
 &\quad + 8(12\lambda_1 + \lambda_2) V_R^4 \\
 &= -M_{45} M_*^3 + \bar{\lambda} \langle S_1 \rangle v_R^2 M_* \left(1 + \frac{V_R}{V_{B-L}} \right),
 \end{aligned} \quad (9)$$

$$\begin{aligned}
 &(16\lambda_1 + 4\lambda_2 + \lambda_3) V_R^5 + 4(12\lambda_1 + \lambda_2) V_{B-L}^2 V_R^3 \\
 &\quad + 2(18\lambda_1 + \lambda_2) V_{B-L}^4 V_R + \frac{1}{6} M_{45} M_*^3 V_R \\
 &= \frac{1}{4} \bar{\lambda} \langle S_1 \rangle v_R^2 V_{B-L} M_* \left(1 + \frac{V_R}{3V_{B-L}} \right).
 \end{aligned} \quad (10)$$

From (8)–(10), one can see that the desirable VEV configuration is obtained. Indeed, from (8), with $v_R \sim 10^{17}$ GeV [fixed from (6)] and $\Lambda = (10^{13} - 10^{14})$ GeV, we obtain $\frac{\langle S_1 \rangle}{M_*} \simeq 10^{-8} - 10^{-6}$. Thus, an effective, and suppressed, λ coupling (ensuring suppressed Higgs mass) is generated. Small $\langle S_1 \rangle$ also ensures naturally suppressed V_R (in a limit $\langle S_1 \rangle \rightarrow 0$, one has the solution $V_R \rightarrow 0$). At the leading order, in powers of $\frac{\langle S_1 \rangle}{M_*}$, from (8)–(10) we get

$$\begin{aligned}
 \text{with } \frac{\Lambda}{v_R} \ll 1: \quad &V_{B-L} \simeq \left(\frac{-M_{45} M_*^3}{6(36\lambda_1 + 6\lambda_2 + \lambda_3)} \right)^{1/4}, \\
 &\frac{\langle S_1 \rangle}{M_*} \simeq \frac{\Lambda^2}{v_R^2}, \\
 &V_R \simeq -\bar{\lambda} \frac{\langle S_1 \rangle}{M_*} \frac{v_R^2 M_*^2}{4(4\lambda_2 + \lambda_3) V_{B-L}^3}, \\
 &\langle S \rangle \simeq -\frac{3\bar{\lambda}}{\lambda_S} \frac{V_{B-L}^2}{M_*}.
 \end{aligned} \quad (11)$$

For the masses of l_H and $\bar{l}_H$, coming from the 16_H and $\overline{16}_H$, we have (in the Appendix, I present the decomposition helping to compute the masses of this and remaining fragments)

$$\begin{aligned}
 M_{l_H l_H} &= \frac{-6\bar{\lambda} \langle S_1 \rangle V_{B-L} V_R}{M_*^2} \left(1 - \frac{V_R}{3V_{B-L}} \right) \\
 &\simeq \left(\frac{\Lambda}{v_R} \right)^4 \frac{3\bar{\lambda}^2 v_R^2 M_*^2}{2(4\lambda_2 + \lambda_3) V_{B-L}^2}.
 \end{aligned} \quad (12)$$

The latter's value, i.e., the μ term, with $v_R \sim V_{B-L}$ and for $\frac{\Lambda}{v_R} \sim (1-2) \times 10^{-4}$ will be $\sim 1-10$ TeV—of desirable magnitude. Allowed higher-order operator $\frac{1}{M_*^2} (S_1 \overline{16}_H 45_H 16_H)^2$ is harmless, because it contributes to the μ term by the

amount $\sim \frac{\langle S_1 \rangle^2 V_{B-L}^2 v_R^2}{M_*^5} \lesssim 10^{-2}$ GeV (for $V_{B-L}, v_R \sim M_*/30$).

The masses of colored modes (from $16_H, \bar{16}_H$) are

$$M_{\bar{q}_H q_H} \simeq M_{\bar{u}_H^c u_H^c} \simeq M_{\bar{d}_H^c d_H^c} \equiv M_{T_1} \simeq -4\bar{\lambda} \left(\frac{\Lambda}{v_R} \right)^2 \frac{V_{B-L}^2}{M_*}, \quad (13)$$

around $\sim 10^8$ GeV.

Below, I show how MSSM Higgs doublets can have desirable couplings to the MSSM matter and how nucleon stability is ensured [19]. Before discussing these, I give the scalar sector's extension, which ensures desirable Yukawa couplings and realistic phenomenology. I introduce two scalar superfields in the fundamental representation (10-plets) of $SO(10)$: 10_H and $10'_H$ with $\mathcal{U}(1)_A \times \mathcal{Z}_4$ charges given in Table I. Thus, the relevant superpotential couplings are

$$W_H^{(16,10)} = \alpha_1 \frac{S_1}{M_*} \bar{16}_H \bar{16}_H 10_H + \lambda' 10_H 45_H 10'_H + \alpha_2 \frac{(\bar{16}_H 16_H)^3}{M_*^5} 10'_H 10'_H \quad (14)$$

($\alpha_{1,2}$ and λ' are dimensionless couplings), which, with notations $\{l_H, \bar{l}_H\} \equiv \{h_d^{16_H}, h_u^{16_H}\}$, give the doublet mass matrix:

$$M_D = \begin{pmatrix} h_d^{16_H} & h_d^{10_H} & h_d^{10'_H} \\ h_u^{16_H} & 0 & \lambda' V_R \\ h_u^{10_H} & 0 & -\lambda' V_R \\ h_u^{10'_H} & 0 & M' \end{pmatrix} + \mathcal{O}(\text{TeV}), \quad (15)$$

$$\text{with } \bar{M} = \alpha_1 \frac{\langle S_1 \rangle v_R}{M_*}, \quad M' = \alpha_2 \frac{v_R^6}{M_*^5}. \quad (16)$$

In (15), the $\mathcal{O}(\text{TeV})$ stand for possible corrections of the order of (1–10) TeV (I will comment shortly) and are harmless.

From (15), we see that one doublet pair, identified with MSSM Higgs superfields h_u and h_d , is light. The remaining doublets (denoted as $D_{1,2}$ and $\bar{D}_{1,2}$) are heavy. Analyzing the matrix (15), the distribution of h_u and h_d in original states can be found:

$$h_u^{10_H} = e^{i\phi_x} |x| h_u + \dots, \quad h_d^{16_H} = h_d, \quad \text{with } \frac{1}{|x|} = \frac{1}{|M'|} \left(|M'|^2 + |\lambda' V_R|^2 + \frac{|\lambda' V_R|^4}{|\bar{M}|^2} \right)^{1/2}, \quad (17)$$

the composition crucial for building a realistic fermion pattern (especially for large top Yukawa coupling λ_t).

As far as other operators, allowed by symmetries, are concerned, the couplings $\frac{S S_1 45_H}{M_*^2} 16_H 16_H 10'_H$ and $\frac{S 45_H}{M_*^6} (\bar{16}_H 16_H)^2 \bar{16}_H 16_H 10'_H$ with $\frac{\langle S \rangle}{M_*} \sim 10^{-4}$, $\frac{\langle S_1 \rangle}{M_*} = \frac{\Lambda^2}{v_R^2} \sim 10^{-8}$, and $v_R, V_{B-L} \sim \frac{M_*}{30}$ will contribute to the μ term by an amount of $\sim$ few TeV and therefore are harmless [20].

IV. YUKAWA COUPLINGS AND PROTON STABILITY

All MSSM matter is embedded in $SO(10)$'s 16_i -plets ($i = 1, 2, 3$). Their $\mathcal{U}(1)_A \times \mathcal{Z}_4$ charges are given in Table I. The effective operators, generating up- and down-type quark and charged lepton masses, are

$$W_Y = Y_U^{ij} 16_i 16_j 10_H + \frac{45_H^2}{\langle \bar{16}_H 16_H \rangle M_*} Y^{ij} 16_i 16_j 16_H 16_H. \quad (18)$$

The first term is responsible for the up-type quark Yukawa couplings. According to (17), the 10_H -plet includes the h_u by the weight $|x|$, which with $|\lambda' V_R| \sim |\bar{M}| \sim |M'|$ can naturally be $|x| \approx 0.5$ –1. Therefore, the λ_t coupling can naturally have a desirable value. On the other hand, the h_d entirely resides in the 16_H -plet, and down quark and charged lepton masses are emerging from the second term of (18), which can be obtained by integrating out heavy vectorlike fermion superfields. For instance, with additional 10_f states [with $\mathcal{U}(1)_A \times \mathcal{Z}_4$ charges $Q_{10_f} = 2$, $\omega_{10_f} = \omega$] and couplings $[\frac{y^{i\alpha}}{M_*} 16_H 45_H 16_i 10_{f\alpha} - \frac{\kappa^{\alpha\beta}}{4M_*} \bar{16}_H 16_H 10_{f\alpha} 10_{f\beta}]$, one can see that integration of 10_f states (gaining masses after substitution of the VEV $\langle \bar{16}_H 16_H \rangle$) leads to the second term of (18) with $Y^{ij} \simeq (y \kappa^{-1} y^T)^{ij}$. Note that the 45_H^2 can couple to the third family by the contraction $\text{tr}(45_H^2)$ [i.e., $\frac{\text{tr}(45_H^2)}{\langle \bar{16}_H 16_H \rangle} 16_3 16_3 16_H 16_H$] and, therefore, one can have $\lambda_b = \lambda_\tau$ —the bottom-tau unification—at the GUT scale. The couplings $y^{i\alpha}$ and $\kappa^{\alpha\beta}$ can also be obtained via renormalizable interactions. For example, having additional 16_f and $\bar{16}_f$ states [with $Q_{16_f} = -Q_{\bar{16}_f} = 3$ and $\omega_{16_f} = 2\omega$, $\omega_{\bar{16}_f} = 0$] and superpotential terms $16_i 45_H \bar{16}_f + 16_H 10_f 16_f + M_f \bar{16}_f 16_f$, decoupling of 16_f and $\bar{16}_f$ generates a term with $y^{i\alpha}$ coupling above. I will not discuss here more details (should be pursued elsewhere) but emphasize that after the GUT symmetry breaking some fragments from 10_f , 16_f , and $\bar{16}_f$ can have masses below the GUT scale and their contribution, as thresholds, may play an important role for the gauge coupling unification (discussed at the end).

The right-handed neutrinos ν_i^c are gaining masses ($\equiv M_R$) via the effective operator $(16_i \bar{16}_H)^2 \frac{(\bar{16}_H 16_H)^2}{\langle S \rangle M_*^4}$, which can be generated by decoupling the $SO(10)$ singlets with masses $\sim \langle S \rangle$. With $\frac{\langle S \rangle}{M_*} \sim 10^{-4}$, $M_R \sim \frac{v_R^6}{\langle S \rangle M_*^4} \approx 10^{14}$ GeV indeed of desirable values, generating the light neutrino masses ~ 0.1 eV via the seesaw mechanism [3]. Before

discussing $d = 5$ proton decay, note that $10'_H$'s couplings with matter $\frac{1}{M_*^8}(\overline{16}_H 16_H)^3 S 45_H 16_i 16_j 10'_H \rightarrow \sim 10^{-14} 16_i 16_j 10'_H$ are so suppressed that they should be completely ignored.

The couplings in (18), after substituting appropriate VEVs, besides the Yukawa interactions

$$W_Y \rightarrow (qY_U u^c + lY_U \nu^c)h_u + (qY_D d^c + e^c Y_E l)h_d, \quad (19)$$

also generate couplings

$$\frac{1}{2} qY_U qT_{10_H} + qY_U l\bar{T}_{10_H} + qY_{ql} l\bar{T}_{16_H} \quad (20)$$

(with notation $\bar{T}_{16_H} = d_H^c$). Integration of the triplet states, forming the mass matrix coupling $T^T M_T \bar{T}$, leads to a baryon number-violating $LLLL$ $d = 5$ operator:

$$\mathcal{O}_{d=5}^{LLLL} = \frac{1}{2} [qY_{ql} l(M_T^{-1})_{12} + qY_U l(M_T^{-1})_{22}] qY_U q. \quad (21)$$

The triplet mass matrix, derived from the couplings (14) and taking into account (13), has the form

$$M_T = \begin{matrix} & \bar{T}_{16_H} & \bar{T}_{10_H} & \bar{T}_{10'_H} \\ \begin{matrix} T_{\overline{16}_H} \\ T_{10_H} \\ T_{10'_H} \end{matrix} & \begin{pmatrix} M_{T_1} & \bar{M} & 0 \\ 0 & 0 & \lambda' V_{B-L} \\ 0 & -\lambda' V_{B-L} & M' \end{pmatrix} \end{matrix}, \quad (22)$$

from which $(M_T^{-1})_{12} = \frac{-\bar{M}M'}{M_{T_1}(\lambda' V_{B-L})^2}$ and $(M_T^{-1})_{22} = \frac{M'}{(\lambda' V_{B-L})^2}$. For $\lambda' \simeq 1$, the mass scales $M_{T_1} \simeq 10^8$ GeV, $\bar{M} \simeq 3 \times 10^{11}$ GeV, and $M' \simeq 8 \times 10^{11}$ GeV, whose values are dictated from (13), (16), and desirable phenomenology [remember, the value $V_R \simeq 5 \times 10^{11}$ GeV obtained from (11) and the need for $|x| \approx 0.5$ –1 of (17) giving the needed value of λ_t , so we take $|\lambda' V_R| = 5 \times 10^{11}$ GeV], and with $V_{B-L} \simeq 8 \times 10^{16}$ GeV, we obtain $(M_T^{-1})_{12} \simeq \frac{1}{2.7 \times 10^{18} \text{ GeV}}$ and $(M_T^{-1})_{22} \simeq \frac{1}{8 \times 10^{21} \text{ GeV}}$. These ensure the adequate suppression of $d = 5$ operators in (21) [21]. Remarkably, although the color triplets from 16_H and $\overline{16}_H$ have intermediate mass [$\sim 10^8$ GeV; see Eq. (13)], the model allows one to have a stable nucleon and therefore be fully self-consistent.

V. GAUGE COUPLING UNIFICATION

Finally, let us discuss the impact of some intermediate states, populating the “desert” between the electroweak and GUT scales. From (22), one triplet pair has mass M_{T_1} given

in (13) (together with the remaining colored states). We denote the masses of the remaining two triplet-antitriplet pairs by $M_{T_{2,3}}$. These and heavy doublets' masses $M_{D_{1,2}}$ are found from (22) and (15): $M_{T_{2,3}} \simeq |\lambda' V_{B-L}|$ and $2M_{D_{1,2}}^2 = |\bar{M}|^2 + |M'|^2 + 2|\lambda' V_R|^2 \pm [(|\bar{M}|^2 - |M'|^2)^2 + 4|\lambda' V_R M'|^2]^{1/2}$. Moreover, from 45_H come a colored octet (with mass M_8) and $SU(2)_L$ triplet (with mass M_3). One physical $e^c + \bar{e}^c$ pair, with mass $M_{e_H^c}$, comes from the superposition of the corresponding fragments from 45_H and $16_H, \overline{16}_H$ (the orthogonal combination is an absorbed Goldstone). A similar thing happens with the scalar supermultiplets having the quantum numbers of $u_H^c, q_H, \bar{u}_H^c$, and $\bar{q}_H$. We will denote the masses of the corresponding physical states by $M_{u_s^c}$ and M_{q_s} . From Eqs. (4), (11), and (13), we have $\frac{M_8}{8\lambda_2 + 4\lambda_3} \simeq \frac{M_{3(e_H^c)}}{4\lambda_2 + \lambda_3} \simeq 6 \frac{V_{B-L}^4}{M_*^2}$ and $M_{u_s^c(q_s)} \simeq M_{\bar{u}_H^c \bar{q}_H^c} \simeq M_{T_1}$. From the vectorlike heavy matter $16_f, \overline{16}_f, 10_f$ [discussed after Eq. (18)], we assume that n_f pairs of $u_f^c + \bar{u}_f^c$ and $l_f + \bar{l}_f$ happen to have masses $M_{u_f^c}$ and M_{l_f} , respectively, below the GUT scale M_G . Thus, their thresholds will also contribute to the gauge coupling running. With all this, imposing the unification condition for the gauge couplings at scale M_G , from renormalisation group equations we find $\alpha_3^{-1}(M_Z) = [\alpha_3^{-1}(M_Z)]_{\text{MSSM}} + \Delta_3$, where $[\alpha_3^{-1}(M_Z)]_{\text{MSSM}}$ corresponds to the value one would have obtained within the MSSM. Δ_3 includes corrections from additional states (below the M_G scale) of our model:

$$\Delta_3 = \frac{3}{14\pi} \ln \left(\frac{M_{D_1} M_{D_2}}{M_{T_2} M_{T_3}} \right)^3 \frac{M_3^8 M_G^2}{M_{T_1} M_{e_H^c}^2 M_8^7} \left(\frac{M_G}{M_{u_f^c}} \right)^{5n_f} \left(\frac{M_G}{M_{l_f}} \right)^{3n_f} + \delta_3^{(2)}. \quad (23)$$

The $\delta_3^{(2)}$ stands for a two-loop correction, taken into account upon numerical calculations. Between the thresholds' masses, the reasonable (and fairly natural) balance, giving successful gauge coupling unification, can be found. For instance, with the choice

FIG. 1. Gauge coupling unification within the presented $SO(10)$ model. The spectrum of states is given in Eq. (24).

$$\begin{aligned}
M_{D_1} &\simeq 3.6 \times 10^{11}, \quad M_{D_2} \simeq 1.05 \times 10^{12}, \quad M_{T_1} = 10^8, \\
M_{T_{2,3}} &\simeq M_G = 8 \times 10^{16}, \quad M_8 \simeq 2.29 \times 10^{14}, \quad M_{3(e_H^c)} = 10^{13}, \\
M_{u_f^c} &\simeq 7.38 \times 10^{13}, \quad M_{l_f} \simeq 1.04 \times 10^{15}, \quad n_f = 3 \quad (24)
\end{aligned}$$

(all masses are given in GeV), the gauge couplings meet at the scale $M_G \simeq 8 \times 10^{16}$ GeV, as displayed in Fig. 1, and the unified gauge coupling is $\alpha_G(M_G) \simeq 0.214$ [22]. This shows how the presented scenario gives consistent gauge coupling unification.

VI. SUMMARY

The main result of this paper is the novel mechanism suggested within the $SO(10)$ GUT for the Higgs particle being the light pseudo-Goldstone mode. Moreover, realizing and demonstrating the proposed mechanism, an explicit model with a fully realistic phenomenology has been constructed.

Within a scenario by the proposed mechanism, it is tempting to address in detail more phenomenological issues. Also, it would be interesting to investigate the origin of various exploited higher-order operators through renormalizable interactions, together with possibilities of suppressing the V_R (the 45_H 's VEV in the I_{3R} direction). For this, the proposal of Ref. [10] can be applied, which as shown in Ref. [12] (within a different construction) can be very efficient when supported by anomalous or/and discrete symmetries. The latter, instead of being global, can arise from discrete gauge symmetry [23] having a stringy origin. It is tempting to use such symmetry as a flavor symmetry (gaining more motivation) and study related issues in more detail. In this paper, I preferred to stay with the minimal setup, so I have not pursued such possibilities. All these, supported by the mechanism discussed here, open wide prospect towards novel $SO(10)$ GUT model building and motivate one to study related phenomenology, such as the fermion mass pattern, neutrino oscillations, leptogenesis, and proton decay rates. The latter still needs to be probed by ongoing nucleon decay search experiments [24].

APPENDIX DECOMPOSITION OF $W_H^{(45,16)}$

The invariant $(\bar{16}_H 16_H)_{210} \cdot (45_H^2)_{210}$, which we assume to appear in the superpotential coupling $W_H^{(45,16)}$ of Eq. (4), in explicit $SO(10)$ index contractions, is

$$\begin{aligned}
(\bar{16}_H 16_H)_{210} \cdot (45_H^2)_{210} &= \langle \bar{16}_H^T | B \Gamma_i \Gamma_j \Gamma_m \Gamma_n | 16_H \rangle \Phi^{[ijmn]} \\
\text{with } \Phi^{[ijmn]} &= 45_H^{ij} 45_H^{mn} - 45_H^{mj} 45_H^{in} + 45_H^{im} 45_H^{jn}. \quad (A1)
\end{aligned}$$

Here $i, j, m, n = 1-10$ and Γ 's are gamma matrices in $SO(10)$'s spinorial representation and B is the analog of the charge conjugation operator for $SO(10)$.

With $SO(10) \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R$ decomposition of (2) and

$$\begin{aligned}
16_H &= F_H(4, 2, 1) + F_H^c(\bar{4}, 1, 2), \\
\bar{16}_H &= \bar{F}_H(\bar{4}, 2, 1) + \bar{F}_H^c(4, 1, 2), \quad (A2)
\end{aligned}$$

the relevant couplings, extracted from the operator of $W_H^{(45,16)}$ in terms of $SU(4)_c \times SU(2)_L \times SU(2)_R$'s fragments, will be

$$\begin{aligned}
\frac{i}{4!} (\bar{16}_H 16_H)_{210} \cdot (45_H^2)_{210} &\rightarrow \frac{1}{2} (\bar{F}_H \Sigma^2 F_H + \bar{F}_H^c \Sigma^2 F_H^c) \\
&- 2 \bar{F}_H^c \Sigma \Sigma_R F_H^c + 2 \bar{F}_H^c \Sigma_R^2 F_H^c \\
&- \left(\frac{1}{8} \text{tr} \Sigma^2 + \frac{1}{2} \text{tr} \Sigma_R^2 \right) (\bar{F}_H F_H + \bar{F}_H^c F_H^c). \quad (A3)
\end{aligned}$$

Factors in (A3) correspond to the normalization of the VEVs:

$$\begin{aligned}
\langle \Sigma \rangle &= V_{B-L} \cdot \text{Diag}(1, 1, 1, -3), \\
\langle \Sigma_R \rangle &= V_R \cdot \text{Diag}(1, -1), \quad (A4)
\end{aligned}$$

where $\langle \Sigma \rangle$ and $\langle \Sigma_R \rangle$ are given in $SU(4)_c$ and $SU(2)_R$ group spaces, respectively. From these, one can easily obtain the results presented in Eqs. (8)–(10), (12), and (13).

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