Interpretation of the newly discovered $\Omega(2012)$

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Very recently, Belle Collaboration reported observation of a narrow state called $\Omega(2012)$ with mass $2012.4 \pm 0.7(\mathrm{stat}) \pm 0.6(\mathrm{syst})$ MeV and width $6.4^{+2.5}_{-2.0}(\mathrm{stat}) \pm 1.6(\mathrm{syst})$ MeV. We calculate the mass and residue of the $\Omega(2012)$ state by employing the QCD sum rule method. Comparison of the obtained results with the experimental data allows us to interpret this state as a $1P$ orbital excitation of the ground state Ω baryon, i.e., with quantum numbers $J^P = \frac{3}{2}^-$.

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The theoretical calculations of different parameters of hadrons and comparison of the obtained results with existing experimental data not only test our present knowledge on these states but also provide insights into the search for new states. Despite the fact that, in the hadronic sector, many particles have been observed and their properties intensively studied, there is still much work to do. Even for the hadrons containing only the light quarks, their excited states require more investigation. The quark model predicts some baryonic excited states that have not yet been observed in the experiment. Searching for these missing baryon resonances attracts attention of not only the experimentalists but also the theoreticians. To understand and identify such states, it is necessary to broaden the studies on these baryons.

As a result of these circumstances, the recent observation of the Belle Collaboration has attracted much attention. They reported observation of $\Omega(2012)$ with mass $2012.4 \pm 0.7(\mathrm{stat}) \pm 0.6(\mathrm{syst})$ MeV and width $6.4^{+2.5}_{-2.0}(\mathrm{stat}) \pm 1.6(\mathrm{syst})$ MeV [1] with a conclusion that it has more likely a spin-parity $J^P = 3/2^-$. To date, there are a few Ω baryons listed in the Particle Data Group (PDG) [2]. Only one of them, which is the ground state $\Omega(1672)$, is well established; we lack certain knowledge of the nature of the others. To identify the spectrum of the Ω states, the first orbital excitation of the $\Omega(1672)$ state was investigated with different models. The quark model [3–11], lattice gauge theory [12,13], and Skyrme model [14] are among those studies whose predictions gave mass values that are consistent with the experimental result of the Belle Collaboration. This may be taken as support for $\Omega(2012)$ being an orbital excitation of $\Omega(1672)$. To identify the properties of the $\Omega(2012)$ baryon, it would also be helpful to investigate its other properties besides the mass. Its strong decay was studied recently in [15], using the chiral quark model, and as a result, the possibility of $\Omega(2012)$ being $J^P = 3/2^-$ was underlined; however, it also stated that the results obtained for the possibility of it being $J^P = 1/2^-$ or $J^P = 3/2^+$ are consistent with the results of the Belle Collaboration within the uncertainties. There are also some studies on the radiative decays [16] and magnetic moments of negative parity baryons [17,18].

References [3,19] present the predictions on the mass of radially excited decuplet baryons. The prediction of [3] for the $2S$ state is 2065 MeV and it is close to the mass of $\Omega(2012)$. On the other hand, the prediction given by Ref. [19] is 2176 ± 219 MeV obtained using the QCD sum rule method. If we consider the central value of this result, it is larger than the observed mass of $\Omega(2012)$. Therefore, in order to get new information about the identification of the nature of $\Omega(2012)$, it is necessary to investigate the mass of the orbital excitation of $\Omega(1672)$, which we represent as an excited Ω state in the remaining part of the discussion. Taking this motivation in hand in the present study, we make a QCD sum rule calculation for the mass of the excited Ω state. The QCD sum rule method [20–22] is among the powerful nonperturbative methods used in the literature extensively with considerable success. To make the calculations in this method, one follows three steps. First, one is the calculation of a given correlation function in terms of hadronic degrees of freedom (hadronic side). The next step is the calculation of the same correlator in terms of QCD degrees of freedom (theoretical or QCD side). The final step is comprised of the match of the results of these previous two steps considering the coefficients of the same Lorentz structure from both sides.

For the present calculation, the mentioned correlation function is as follows:

$$\Pi_{\mu\nu}(q) = i\int d^4x e^{iq\cdot x}\langle 0|\mathcal{T}\{J_\mu(x)J^\dagger{}_\nu(0)\}|0\rangle. \quad (1)$$

Here, J_μ is the interpolating current for the state of interest written in terms of quark fields. In the calculations, we use two forms of the interpolating current,

$$J_{\mu(+)} = \epsilon^{abc}(s^{aT}C\gamma_\mu s^b)s^c, \quad (2)$$

$$J_{\mu(-)} = \epsilon^{abc}(s^{aT}C\gamma_\mu s^b)\gamma_5 s^c, \quad (3)$$

where a, b, and c represent the color indices and C is the charge conjugation operator. The subscripts $(+)$ and $(-)$ denote the parities of the corresponding interpolating currents. The main peculiarity of these currents is that they interact with both parities.

The hadronic representation of the correlator is obtained by inserting a complete set of hadronic states in the correlator given in Eq. (1). For positive parity current, by isolating the ground and first orbital excitation, we get

$$\Pi^{\mathrm{Had}}_{\mu\nu(+)}(q) = \frac{\langle 0|J_{\mu(+)}|+(q,s)\rangle\langle +(q,s)|J^\dagger_{\nu(+)}|0\rangle}{q^2-m_+^2} + \frac{\langle 0|J_{\mu(+)}|-(q,s)\rangle\langle -(q,s)|J^\dagger_{\nu(+)}|0\rangle}{q^2-m_-^2} + \cdots, \quad (4)$$

where the $|+(q,s)\rangle$ and $|-(q,s)\rangle$ represent the ground state $\Omega(1672)$ and its first orbital excitation $\Omega(2012)$, respectively, and m_+ and m_- are the corresponding masses. The dots are used to represent the contributions coming from higher states and continuum. The matrix elements between vacuum and one-particle states present in Eq. (4) are parametrized as

$$\langle 0|J_{\mu(+)}|+(q,s)\rangle = \lambda_+ u_\mu(q,s),$$
$$\langle 0|J_{\mu(+)}|-(q,s)\rangle = \lambda_-\gamma_5 u_\mu(q,s), \quad (5)$$

where $\lambda_+(\lambda_-)$ stands for the residue of the corresponding baryon. Performing the summation over spins of spin-$\frac{3}{2}$ baryons with the help of the formula

$$\sum_s u_\mu(q,s)\bar{u}_\nu(q,s) = -(\not{q}+m_B)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2q_\mu q_\nu}{3m_B^2} + \frac{q_\mu\gamma_\nu - q_\nu\gamma_\mu}{3m_B}\right], \quad (6)$$

the result of the physical part takes the form

$$\Pi^{\mathrm{Had}}_{\mu\nu(+)}(q) = -\frac{\lambda_+^2}{q^2-m_+^2}(\not{q}+m_+)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2q_\mu q_\nu}{3m_+^2} + \frac{q_\mu\gamma_\nu - q_\nu\gamma_\mu}{3m_+}\right] - \frac{\lambda_-^2}{q^2-m_-^2}(\not{q}-m_-)\left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2q_\mu q_\nu}{3m_-^2} + \frac{q_\mu\gamma_\nu - q_\nu\gamma_\mu}{3m_-}\right] + \cdots. \quad (7)$$

It should be noted that the current $J_{\mu(+)}$ couples not only to spin-3/2 baryons but also to spin-1/2 states. To remove the contribution of unwanted states having spin-1/2, we will choose the proper Lorentz structure which is free from the spin-1/2 pollution. The contribution of the spin-1/2 states is determined by the matrix element

$$\langle 0|J_\mu\left|\frac{1}{2}(q)\right\rangle = A\left(\gamma_\mu - \frac{4q_\mu}{m_{\frac{1}{2}}}\right)u(q). \quad (8)$$

From here, we see that the terms proportional to γ_μ or q_μ contain spin-1/2 contributions. To avoid this pollution, we chose the structures $\not{q}g_{\mu\nu}$ and $g_{\mu\nu}$, which solely contain contributions coming from spin-3/2 states. With this consideration, the result becomes

$$\Pi^{\mathrm{Had}}_{\mu\nu(+)}(q) = -\frac{\lambda_+^2}{q^2-m_+^2}(\not{q}g_{\mu\nu} + m_+ g_{\mu\nu}) - \frac{\lambda_-^2}{q^2-m_-^2}(\not{q}g_{\mu\nu} - m_- g_{\mu\nu}) + \cdots. \quad (9)$$

The hadronic side of the correlation function for the second current given in Eq. (3) can be obtained from Eq. (9) with the following replacements: $\lambda_+ \to \lambda'_-$, $\lambda_- \to \lambda'_+$, $m_+ \to m_-$ and $m_- \to m_+$. Applying the Borel transformation with respect to $(-q^2)$ in order to suppress the contributions coming from higher states and continuum, finally we get the following results for the hadronic sides:

$$\hat{\mathcal{B}}\Pi^{\mathrm{Had}}_{\mu\nu(+)}(q) = \lambda_+^2 e^{-\frac{m_+^2}{M^2}}(\not{q}g_{\mu\nu} + m_+ g_{\mu\nu}) + \lambda_-^2 e^{-\frac{m_-^2}{M^2}}(\not{q}g_{\mu\nu} - m_- g_{\mu\nu}) + \cdots, \quad (10)$$

$$\hat{\mathcal{B}}\Pi^{\mathrm{Had}}_{\mu\nu(-)}(q) = \lambda_-'^2 e^{-\frac{m_-^2}{M^2}}(\not{q}g_{\mu\nu} + m_- g_{\mu\nu}) + \lambda_+'^2 e^{-\frac{m_+^2}{M^2}}(\not{q}g_{\mu\nu} - m_+ g_{\mu\nu}) + \cdots. \quad (11)$$

In the next part of the discussion, we denote the coefficient of the Lorentz structure $\not{q}g_{\mu\nu}$ as Π_1 and that of Lorentz structure $g_{\mu\nu}$ as Π_2, correspondingly.

After completing the calculations in the hadronic side, now we turn our attention to calculate the correlation function from the QCD side using operator product expansion. As an example, we present its expression by using the interpolating current given in Eq. (2). The calculation leads to the result

$$\Pi^{\mathrm{OPE}}_{\mu\nu(+)}(q) = \epsilon_{abc}\epsilon_{a'b'c'}\int d^4x e^{iqx}\langle 0|\{S_s^{ca'}(x)\gamma_\nu \tilde{S}_s^{ab'}(x)\gamma_\mu S_s^{bc'}(x) - S_s^{ca'}(x)\gamma_\nu \tilde{S}_s^{bb'}(x)\gamma_\mu S_s^{ac'}(x) - S_s^{cb'}(x)\gamma_\nu \tilde{S}_s^{aa'}(x)\gamma_\mu S_s^{bc'}(x) + S_s^{cb'}(x)\gamma_\nu \tilde{S}_s^{ba'}(x)\gamma_\mu S_s^{ac'}(x) - S_s^{cc'}(x)\mathrm{Tr}[S_s^{ba'}(x)\gamma_\nu \tilde{S}_s^{ab'}(x)\gamma_\mu] + S_s^{cc'}(x)\mathrm{Tr}[S_s^{bb'}(x)\gamma_\nu \tilde{S}_s^{aa'}(x)\gamma_\mu]\}|0\rangle, \tag{12}$$

with $\tilde{S}(x) = CS^T(x)C$ and the $S_s^{ab}(x)$ is the light quark propagator which is given as

$$S_q^{ab}(x) = i\frac{\not{x}}{2\pi^2x^4}\delta_{ab} - \frac{m_q}{4\pi^2x^2}\delta_{ab} - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q}{4}\not{x}\right)\delta_{ab} - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\left(1 - i\frac{m_q}{6}\not{x}\right)\delta_{ab} - \frac{ig_sG_{ab}^{\theta\eta}}{32\pi^2x^2}[\not{x}\sigma_{\theta\eta} + \sigma_{\theta\eta}\not{x}] - \frac{\not{x}x^2g_s^2}{7776}\langle\bar{q}q\rangle^2\delta_{ab} - \frac{x^4\langle\bar{q}q\rangle\langle g_s^2G^2\rangle}{27648}\delta_{ab} + \frac{m_q}{32\pi^2}\left[\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E\right]g_sG_{ab}^{\theta\eta}\sigma_{\theta\eta} + \cdots, \tag{13}$$

in x space with the Euler constant, $\gamma_E \simeq 0.577$. The parameter Λ is a scale parameter separating the perturbative and nonperturbative regions. After the insertion of the propagator into Eq. (12) and performing the Fourier and Borel transformations as well as continuum subtraction, for the QCD side of the correlation function, corresponding to the coefficients of the selected structures, we get

$$\Pi^B_{1(+)} = -\Pi^B_{1(-)} = \frac{1}{\pi^2}\int_0^{s_0} ds e^{-\frac{s}{M^2}}\left\{\frac{s^2}{5\times 2^5\pi^2} - \frac{3\langle\bar{s}s\rangle m_s}{2^2} - \frac{5\langle g_s^2G^2\rangle}{3^2\times 2^6\pi^2} + \frac{\langle g_s^2G^2\rangle\langle\bar{s}s\rangle m_s}{3^2M^4}\mathrm{Log}\left[\frac{s}{\Lambda^2}\right]\right\} + \frac{3m_0^2\langle\bar{s}s\rangle m_s}{2^3\pi^2} + \frac{4\langle\bar{s}s\rangle^2}{3} - \frac{7m_0^2\langle\bar{s}s\rangle^2}{3^2M^2} + \frac{\langle g_s^2G^2\rangle\langle\bar{s}s\rangle m_s}{3\times 2^3\pi^2M^2} - \frac{53\langle g_s^2G^2\rangle m_0^2\langle\bar{s}s\rangle m_s}{3^3\times 2^6\pi^2M^4} + \frac{\langle g_s^2G^2\rangle\langle\bar{s}s\rangle m_s}{3^2\pi^2s_0M^2}\left[M^2 + s_0\mathrm{Log}\left[\frac{s}{\Lambda^2}\right]\right]e^{-\frac{s_0}{M^2}}, \tag{14}$$

and

$$\Pi^B_{2(+)} = \Pi^B_{2(-)} = \frac{1}{\pi^2}\int_0^{s_0} ds e^{-\frac{s}{M^2}}\left\{\frac{3s^2m_s}{2^6\pi^2} - \frac{s\langle\bar{s}s\rangle}{3} + \frac{m_0^2\langle\bar{s}s\rangle}{2\times 3} + \frac{5\langle g_s^2G^2\rangle m_s}{3\times 2^7\pi^2}\left[(2\gamma_E - 1) - 2\mathrm{Log}\left[\frac{s}{\Lambda^2}\right]\right] + \frac{\langle g_s^2G^2\rangle^2m_s}{3^2\times 2^8\pi^2M^4}\mathrm{Log}\left[\frac{s}{\Lambda^2}\right]\right\} - \frac{5\gamma_E\langle g_s^2G^2\rangle M^2m_s}{3\times 2^6\pi^4} + 2\langle\bar{s}s\rangle^2m_s + \frac{\langle\bar{s}s\rangle\langle g_s^2G^2\rangle}{3^2\times 2^2\pi^2} + \frac{5\langle g_s^2G^2\rangle^2m_s}{3^3\times 2^8\pi^2M^2} - \frac{11m_0^2\langle\bar{s}s\rangle^2m_s}{M^2} - \frac{m_0^2\langle\bar{s}s\rangle\langle g_s^2G^2\rangle}{3^2\times 2^5\pi^2M^2} + \frac{\langle\bar{s}s\rangle^2\langle g_s^2G^2\rangle m_s}{3\times 2^2M^4} + \frac{m_0^2\langle\bar{s}s\rangle^2\langle g_s^2G^2\rangle m_s}{3\times 2^2M^6} + \left[\frac{5\gamma_E\langle g_s^2G^2\rangle M^2m_s}{3\times 2^6\pi^4} + \frac{\langle g_s^2G^2\rangle^2m_s}{3^2\times 2^8M^2s_0}\left(M^2 + s_0\mathrm{Log}\left[\frac{s_0}{\Lambda^2}\right]\right)\right]e^{-\frac{s_0}{M^2}}. \tag{15}$$

At this stage, the calculations of the physical and QCD sides are completed and we need to match the coefficients of the same structures obtained from both sides which give us the following sum rules:

$$\lambda_+^2e^{-\frac{m_+^2}{M^2}} + \lambda_-^2e^{-\frac{m_-^2}{M^2}} = \Pi^B_{1(+)}, \qquad m_+\lambda_+^2e^{-\frac{m_+^2}{M^2}} - m_-\lambda_-^2e^{-\frac{m_-^2}{M^2}} = \Pi^B_{2(+)}. \tag{16}$$

$$\lambda_+'^2e^{-\frac{m_+^2}{M^2}} + \lambda_-'^2e^{-\frac{m_-^2}{M^2}} = \Pi^B_{1(-)}, \qquad -m_+\lambda_+'^2e^{-\frac{m_+^2}{M^2}} + m_-\lambda_-'^2e^{-\frac{m_-^2}{M^2}} = \Pi^B_{2(-)}. \tag{17}$$

In Eq. (16), there are three unknown parameters which are the mass and residue of the orbitally excited state Ω as well as the residue of the ground state Ω. For determination of these three parameters, we need at least three equations. Therefore, the third equation is obtained from the first one given in Eq. (16) by performing derivative with respect to $(-\frac{1}{M^2})$ variable. After some calculations for the mass and the residue of the excited state Ω from Eq. (16), we get

TABLE I. Some input parameters.

Parameters	Values
m_Ω	1672.45 ± 0.29 MeV [2]
m_s	128^{+12}_{-4} MeV [2]
$\langle \bar{q}q \rangle (1\ \text{GeV})$	$(-0.24 \pm 0.01)^3\ \text{GeV}^3$ [23]
$\langle \bar{s}s \rangle$	$0.8\langle \bar{q}q \rangle$ [23]
m_0^2	$(0.8 \pm 0.1)\ \text{GeV}^2$ [23]
$\langle g_s^2 G^2 \rangle$	$4\pi^2(0.012 \pm 0.004)\ \text{GeV}^4$ [24]
Λ	(0.5 ± 0.1) GeV [25]

$$m_-^2 = \frac{\frac{d}{d(-\frac{1}{M^2})}(\Pi^B_{2(+)} - m_+\Pi^B_{1(+)})}{\Pi^B_{2(+)} - m_+\Pi^B_{1(+)}}$$

$$\lambda_-^2 = \frac{\Pi^B_{2(+)} - m_+\Pi^B_{1(+)}}{m_+ + m_-} e^{\frac{m_-}{M^2}}. \tag{18}$$

Together with the mass of the ground state Ω, some of the input parameters that we need to perform the numerical analysis are given in Table I. Note that, in Table I, the mass of the s quark is presented, rescaling it to the normalization point $\mu_0^2 = 1\ \text{GeV}^2$. In addition to these parameters, sum rules contain two auxiliary parameters, Borel mass parameter M^2 and continuum threshold s_0. The physical quantities should be practically independent of these parameters. To obtain a working window for M^2, we require the pole dominance over the contributions of higher states and continuum, and also the results coming from higher-dimensional operators should contribute less than the lower-dimensional ones, since OPE should be convergent. These requirements lead to the following working window for the Borel parameter:

$$3.0\ \text{GeV}^2 \leq M^2 \leq 4.0\ \text{GeV}^2. \tag{19}$$

For the threshold parameter, we choose the interval

$$7.3\ \text{GeV}^2 \leq s_0 \leq 8.4\ \text{GeV}^2, \tag{20}$$

which leads to a relatively weak dependence of the results on the threshold parameter.

Using the above working windows of auxiliary parameters, we show the dependencies of the mass and residue of the $\Omega(2012)$ state given in Eq. (18) as a function of M^2 at fixed values of s_0 and as a function of s_0 at fixed values of M^2 in Figs. 1 and 2, respectively. We observe that the dependencies of the mass and residue of the excited state of

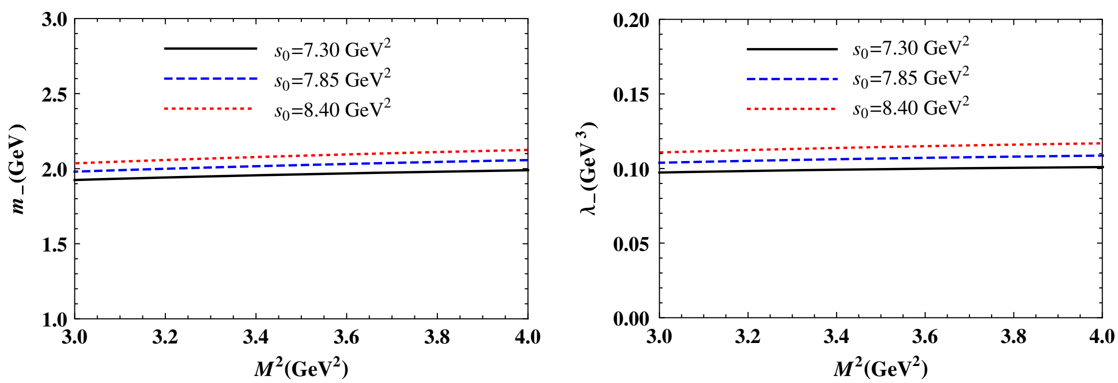

FIG. 1. Left: The mass of the orbitally excited Ω baryon vs Borel parameter M^2. Right: The residue of the orbitally excited Ω baryon vs Borel parameter M^2.

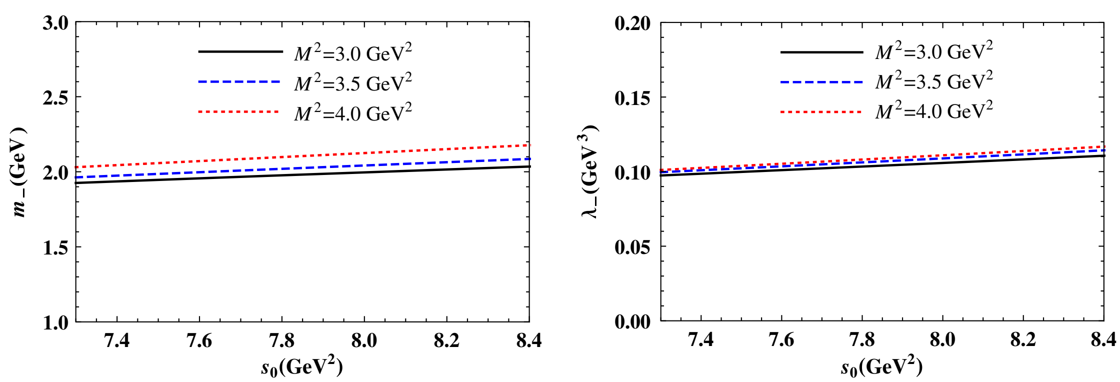

FIG. 2. Left: The mass of the orbitally excited Ω baryon vs threshold parameter s_0. Right: The residue of the orbitally excited Ω baryon vs threshold parameter s_0.

Ω on the auxiliary parameters are rather weak in their working intervals.

Using the positive parity current, we get our final results of mass and residue for the excited Ω state as

$$m_- = 2019^{+17}_{-29}\ \mathrm{MeV} \qquad \lambda_- = 0.108^{+0.004}_{-0.005}\ \mathrm{GeV}^3. \tag{21}$$

We perform a similar analysis for the mass and residue of the excited Ω state using the negative parity current and Eq. (17). Our final predictions in this case are

$$m_- = 2020^{+19}_{-28}\ \mathrm{MeV} \qquad \lambda'_- = 0.094^{+0.003}_{-0.004}\ \mathrm{GeV}^3. \tag{22}$$

The errors in the results are due to the uncertainties carried by the input parameters as well as those coming from the working windows for auxiliary parameters. As is seen, the obtained results for the mass predicted from the positive and negative parity currents are nicely consistent with the experimental value $2012.4 \pm 0.7(\mathrm{stat}) \pm 0.6(\mathrm{syst})$ MeV measured by the Belle Collaboration.

In summary, inspired by the recent discovery of the Belle Collaboration, we calculated the mass and residue of the $\Omega(2012)$ state by using two different forms of interpolating current within the QCD sum rule approach. We found that the mass prediction is insensitive to the choice of the interpolating current. We compared the obtained result for the mass of this state with the experimental value, which allowed us to assign the quantum numbers $J^P = \frac{3}{2}^-$ for the $\Omega(2012)$ state. The result obtained for the residue of this state can be used in determinations of its electromagnetic properties as well as many parameters related to different decays of this particle.

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