Heavy quark spin multiplet structure of $\bar{P}^{(*)}\Sigma_Q^{(*)}$ molecular states

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We study the structure of heavy quark spin (HQS) multiplets for heavy meson-baryon molecular states in a coupled system of $\bar{P}^{(*)}\Sigma_Q^{(*)}$, constructing the one-pion exchange potential with S-wave orbital angular momentum. Using the light cloud spin basis, we find that there are four types of HQS multiplets classified by the structure of heavy quark spin and light cloud spin. The multiplets which have attractive potential are determined by the sign of the coupling constant for the heavy meson-pion interactions. Furthermore, the difference in the structure of the light cloud spin gives the restrictions of the decay channel, which implies that the partial decay width has the information for the structure of HQS multiplets. This behavior is more likely to appear in the hidden-bottom sector than in the hidden-charm sector.

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I. INTRODUCTION

The exotic hadrons are the very interesting research subjects in hadron and nuclear physics. In 2015, the Large Hadron Collider beauty experiment (LHCb) Collaboration announced the observation of two hidden-charm pentaquarks, $P_c^+(4380)$ and $P_c^+(4450)$, in the decay of $\Lambda_b^0 \rightarrow J/\psi K^- p$ [1–3]. Their masses are $M_{4380} = 4380 \pm 8 \pm 28$ MeV and $M_{4450} = 4449.8 \pm 1.7 \pm 2.5$ MeV, and decay widths are $\Gamma_{4380} = 205 \pm 18 \pm 86$ MeV and $\Gamma_{4450} = 39 \pm 5 \pm 19$ MeV. The spin and parity J^P of them are not well determined. The one state is $J = 3/2$ and the other state is $J = 5/2$, and they have opposite parity.

Before the LHCb observation, some theoretical studies of hidden-charm pentaquarks were done [4–7]. After the LHCb announcement, there were many analyses based on the hadronic molecular state [8–24], compact pentaquark state [25–33], quark-cluster model [34], baryocharmonium model [35], hadroquarkonia model [36], topological soliton model [37], and meson-baryon molecules coupled with five-quark states [38]. The kinematical rescattering effects are also discussed in Refs. [39–43].

There are many theoretical descriptions for P_c^+ pentaquarks. Among those pictures, the hadronic molecular one has been used for several other exotic hadrons, especially near the thresholds. For example, since the mass of $X(3872)$ is close to the $D\bar{D}^*$ threshold, $X(3872)$ includes the $D\bar{D}^*$ molecule structure [44]. The masses of $P_c^+(4380)$ and $P_c^+(4450)$ are slightly below the thresholds of $\bar{D}\Sigma_c^*$ and $\bar{D}^*\Sigma_c$, respectively. They can be considered as the loosely bound state of the heavy meson and heavy baryon.

Charm quarks are included in P_c^+ pentaquarks. The masses of the heavy quarks, charm and bottom, are much larger than the typical scale of low-energy QCD, $\Lambda_{\text{QCD}} \sim 200$ MeV. For the heavy quark region, there is a characteristic property in the quark interaction. The spin-dependent interaction of the heavy quark is suppressed by the inverse of the heavy quark mass, $1/m_Q$. By this suppression, the heavy quark spin symmetry (HQS) appeared in the heavy quark limit [45–49]. As a result, we can decompose the total spin $\vec{J}$ to heavy quark spin $\vec{S}$ and the other spin $\vec{j}$:

$$\vec{J} = \vec{S} + \vec{j}. \quad (1)$$

The total spin is conserved and the heavy quark spin is also conserved in the heavy quark limit because of the suppression of the spin-dependent force. Thus, the other spin part is also conserved. This conservation leads to the mass degeneracy of heavy hadrons. Let us consider the heavy meson $q\bar{Q}$ with a light quark q and a heavy quark Q . For $j \geq 1/2$, there are two degenerate states with total spin:

$$J_{\pm} = j \pm 1/2. \quad (2)$$

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These two states are called the HQS doublet. There is only the $J = 1/2$ state for $j = 0$; hence, it is called the HQS singlet.

Such an HQS multiplet structure is seen in the charm and bottom hadron mass spectrum. For example, the small mass difference is obtained between the heavy-light pseudoscalar ($J = 0$) and vector ($J = 1$) mesons, 140 MeV between D and D^* , and 45 MeV between B and B^* . These mass splittings are much smaller than those in the light quark sectors, 600 MeV between π and ρ , and 400 MeV between K and K^* . This observation indicates that the approximate heavy quark spin symmetry is realized in the charm and bottom quark sectors, and these two mesons with $J = 0, 1$ belong to the HQS doublet having the heavy spin $S = 1/2$ and the other spin $j = 1/2$.

The approximate mass degeneracy is also observed in the heavy-light baryons. The mass splitting between Σ_c ($J = 1/2$) and Σ_c^* ($J = 3/2$) (Σ_b and Σ_b^*) is about 65 MeV (20 MeV). They are the HQS doublet state with the heavy spin $S = 1/2$ and the other spin $j = 1$. On the other hand, the heavy-light baryons Λ_c and Λ_b with the light diquark spin 0 are a HQS singlet state.

In this paper, we study the structure of HQS multiplets of $Q\bar{Q}qq$ -type pentaquarks regarding them as molecular states of $\bar{P}^{(*)}\Sigma_Q^{(*)}$. Here, $\bar{P}^{(*)}$ means a HQS doublet meson with an anti-heavy-quark-like $\bar{D}(B)$ and $\bar{D}^*(B^*)$ and $\Sigma_Q^{(*)}$ stands for a HQS doublet baryon with a heavy-quark-like $\Sigma_c(\Sigma_b)$ and $\Sigma_c^*(\Sigma_b^*)$.

The HQS doublet structures of the $\bar{P}^{(*)}$ meson and $\Sigma_Q^{(*)}$ baryon which have one heavy quark are well known. The HQS multiplet structure of the $\bar{P}^{(*)}N$ molecular state with a single heavy quark is discussed in Refs. [50–52]. They showed that the degeneracy of $j \pm 1/2$ states can be expanded to the multihadron system. In this paper, we study the HQS multiplet structure of P_c -like pentaquarks as a doubly heavy quarks system. The appearance of the HQS multiplet for $\bar{P}^{(*)}\Sigma_Q^{(*)}$ molecules is demonstrated by introducing the one-pion exchange potential (OPEP) which is derived from the heavy hadron effective theory respecting the heavy quark symmetry. We focus on the $\bar{P}^{(*)}\Sigma_Q^{(*)}$ molecules with S-wave orbital angular momentum for simplicity. The effect of the tensor force by the S-D mixing is important for OPEP. However, we do not include the D-wave states complicating the system because the S-wave channel is enough to see the spin decomposition to the heavy quark spin and the other spin of the $\bar{P}^{(*)}\Sigma_Q^{(*)}$ molecules. Our purpose in this paper is to demonstrate the HQS multiplet of $\bar{P}^{(*)}\Sigma_Q^{(*)}$. Thus, we study the simple S-wave case in the present study. Since the heavy quark spin and the other spin are separately conserved by the heavy quark spin symmetry, the heavy meson-baryon molecular basis is not suitable to discuss

the structure of HQS multiplet. It is convenient to deal with the corresponding spin structure with appropriate basis. Thus, we define the light cloud spin (LCS) basis as a suitable basis to study the HQS multiplet structure and discuss that what types of HQS multiplets can exist under the OPEP.

This paper is organized as follows. In Sec. II, we construct the one-pion exchange potential in the hadronic molecular (HM) basis. The basis transformation from the HM basis to the LCS basis is discussed in Sec. III. We show the numerical result in Sec. IV. Finally, we summarize the work in this paper and discuss the results in Sec. V.

II. POTENTIAL

In this section, we construct the OPEP for $\bar{P}^{(*)}\Sigma_Q^{(*)}$ molecular states based on the heavy quark symmetry and the chiral symmetry. The $\bar{P}^{(*)}$ mesons and pion interaction Lagrangian is given by [53–57]

$$\mathcal{L}_{HH\pi} = g\text{Tr}[\bar{H}H\gamma_\mu\gamma_5 A^\mu]. \quad (3)$$

The heavy meson doublet field H is

$$H = \frac{1 + \not{p}}{2} [P_\mu^* \gamma^\mu + i P \gamma_5]. \quad (4)$$

P and P^* are pseudoscalar meson and vector meson fields in the HQS doublet. The axial vector current for the pion is given by

$$A_\mu = \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \quad (5)$$

where $\xi = \exp(i\hat{\pi}/\sqrt{2}f_\pi)$. The pion decay constant is $f_\pi = 92.4$ MeV and the pion field $\hat{\pi}$ is defined by

$$\hat{\pi} = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}. \quad (6)$$

The coupling constant g is determined as $|g| = 0.59$ from the decay of $D^* \rightarrow D\pi$ [58].

The $\Sigma_Q^{(*)}$ baryons and pion interaction Lagrangian is given by [56,59]

$$\mathcal{L}_{BB\pi} = \frac{3}{2} g_1 i v_\sigma \epsilon^{\mu\nu\rho\sigma} \text{Tr}[\bar{S}_\mu A_\nu S_\rho]. \quad (7)$$

The superfield S_μ for Σ_Q and Σ_Q^* is represented as

$$S_\mu = \hat{\Sigma}_{Q\mu}^* - \sqrt{\frac{1}{3}}(\gamma_\mu + v_\mu)\gamma_5\hat{\Sigma}_Q. \quad (8)$$

The heavy baryon fields $\hat{\Sigma}_{Q(\mu)}^{(*)}$ are defined by

$$\hat{\Sigma}_{Q(\mu)}^{(*)} = \begin{pmatrix} \Sigma_{Q(\mu)}^{(*)++} & \frac{1}{\sqrt{2}}\Sigma_{Q(\mu)}^{(*)+} \\ \frac{1}{\sqrt{2}}\Sigma_{Q(\mu)}^{(*)+} & \Sigma_{Q(\mu)}^{(*)0} \end{pmatrix}. \quad (9)$$

Σ_Q and $\Sigma_{Q\mu}^*$ are spin 1/2 and 3/2 baryon fields in the HQS doublet. For the coupling constant g_1 , we use $g_1 = (\sqrt{8}/3)g_4$ and $g_4 = 0.999$ estimated in Ref. [59]. The coupling g_4 is determined by the decay of $\Sigma_c^* \rightarrow \Lambda_c \pi$ and its sign follows the quark model estimation.

We construct the one pion exchange potential using the above Lagrangians. At each vertex, we introduce a cutoff parameter Λ via the monopole type form factor,

$$F(q) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + |\vec{q}|^2}, \quad (10)$$

where m_π is a mass of the exchanging pion, and $\vec{q}$ is its momentum. We use the same cutoff for the $\bar{P}^{(*)}\bar{P}^{(*)}\pi$ and $\Sigma_Q^{(*)}\Sigma_Q^{(*)}\pi$ vertices for simplicity, and fix the value of the cutoff at 1000 and 1500 MeV.

In the present analysis, we concentrate on the S-wave $\bar{P}^{(*)}\Sigma_Q^{(*)}$ molecular states to clarify their HQS multiplet structures. In the hadronic molecule (HM) basis, the spin structures of molecular states are described by the product of meson-baryon spins. Then, the possible spins of the $\bar{P}^{(*)}\Sigma_Q^{(*)}$ states are

$$\bar{P}\Sigma_Q = [\bar{Q}q]_0 \otimes [Q[d]_1]_{1/2} = \frac{1}{2}, \quad (11)$$

$$\bar{P}\Sigma_Q^* = [\bar{Q}q]_0 \otimes [Q[d]_1]_{3/2} = \frac{3}{2}, \quad (12)$$

$$\bar{P}^*\Sigma_Q = [\bar{Q}q]_1 \otimes [Q[d]_1]_{1/2} = \frac{1}{2} \oplus \frac{3}{2}, \quad (13)$$

$$\bar{P}^*\Sigma_Q^* = [\bar{Q}q]_1 \otimes [Q[d]_1]_{3/2} = \frac{1}{2} \oplus \frac{3}{2} \oplus \frac{5}{2}, \quad (14)$$

where Q , $\bar{Q}$, q , and d stand for a heavy quark, heavy antiquark, light quark, and diquark in the $\Sigma_Q^{(*)}$ baryon, respectively, and the index j of $[\alpha]_j$ means the spin of α .

The wave functions and OPEPs for each spin state are

$$\psi_{1/2^-}^{\text{HM}} = \begin{pmatrix} |\bar{P}\Sigma_Q\rangle_{1/2^-} \\ |\bar{P}^*\Sigma_Q\rangle_{1/2^-} \end{pmatrix}, \quad (15)$$

$$V_{\pi,1/2^-}^{\text{HM}}(r) = \frac{gg_1}{f_\pi^2} \begin{pmatrix} 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{3}} & \frac{2}{3} & \frac{1}{3\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{3\sqrt{2}} & \frac{5}{6} \end{pmatrix} C_\pi(r), \quad (16)$$

$$\psi_{3/2^-}^{\text{HM}} = \begin{pmatrix} |\bar{P}\Sigma_Q^*\rangle_{3/2^-} \\ |\bar{P}^*\Sigma_Q\rangle_{3/2^-} \\ |\bar{P}^*\Sigma_Q^*\rangle_{3/2^-} \end{pmatrix}, \quad (17)$$

$$V_{\pi,3/2^-}^{\text{HM}}(r) = \frac{gg_1}{f_\pi^2} \begin{pmatrix} 0 & -\frac{1}{2\sqrt{3}} & -\frac{5}{2\sqrt{15}} \\ -\frac{1}{2\sqrt{3}} & -\frac{1}{3} & \frac{5}{6\sqrt{5}} \\ -\frac{5}{2\sqrt{15}} & \frac{5}{6\sqrt{5}} & \frac{1}{3} \end{pmatrix} C_\pi(r), \quad (18)$$

$$\psi_{5/2^-}^{\text{HM}} = \begin{pmatrix} |\bar{P}^*\Sigma_Q^*\rangle_{5/2^-} \end{pmatrix}, \quad (19)$$

$$V_{\pi,5/2^-}^{\text{HM}}(r) = -\frac{gg_1}{2f_\pi^2} C_\pi(r). \quad (20)$$

The function $C_\pi(r)$ is defined as

$$C_\pi(r) = \frac{m_\pi^2}{4\pi} \left[\frac{e^{-m_\pi r} - e^{-\Lambda r}}{r} - \frac{\Lambda^2 - m_\pi^2}{2\Lambda} e^{-\Lambda r} \right]. \quad (21)$$

It should be noted that we subtract the contact terms from the potential.

III. HQS MULTIPLY STRUCTURE OF $\bar{P}^{(*)}\Sigma_Q^{(*)}$

We construct the OPEP for the HM base in Sec. II. However, it is inconvenient to see the structure of the HQS multiplet. In this section, we introduce the light cloud spin (LCS) basis, where the spin structure of the $Q\bar{Q}qd$ states is divided into the heavy quark spin $[Q\bar{Q}]_S$ and light cloud spin $[q[d]_1]_j$. It is a natural spin description in the heavy hadron systems, because the heavy quark spin and light cloud spin are separately conserved in the heavy quark effective theory.

Here, we treat the HQS structure of the doubly heavy system in the following manner: The pentaquark as a bound state of $\bar{P}^{(*)}$ and $\Sigma_Q^{(*)}$ is labeled by the velocity v of the pentaquark. It is natural to assume that both $\bar{P}^{(*)}$ and $\Sigma_Q^{(*)}$ have the same velocity v .

The spin structures of the $\bar{P}^{(*)}\Sigma_Q^{(*)}$ molecular states in the LCS basis are given by

$$[\bar{Q}Q]_0 \otimes [q[d]_1]_{1/2} = \frac{1}{2} \quad (\text{singlet}), \quad (22)$$

$$[\bar{Q}Q]_0 \otimes [q[d]_1]_{3/2} = \frac{3}{2} \quad (\text{singlet}), \quad (23)$$

$$[\bar{Q}Q]_1 \otimes [q[d]_1]_{1/2} = \frac{1}{2} \oplus \frac{3}{2} \quad (\text{doublet}), \quad (24)$$

$$[\bar{Q}Q]_1 \otimes [q[d]_1]_{3/2} = \frac{1}{2} \oplus \frac{3}{2} \oplus \frac{5}{2} \quad (\text{triplet}). \quad (25)$$

There are four types of HQS multiplets—the spin 1/2 singlet, spin 3/2 singlet, spin (1/2, 3/2) doublet, and spin (1/2, 3/2, 5/2) triplet which are classified by the heavy quark spin $S = 0, 1$ and the light cloud spin $j = 1/2, 3/2$.

Using unitary transformation matrices, we translate the basis from the HM basis to the LCS basis. For the spin 1/2,

$$\begin{aligned} \psi_{1/2}^{\text{LCS}} &= U_{1/2}^{-1} \psi_{1/2}^{\text{HM}} \\ &= \begin{pmatrix} |[\bar{Q}Q]_0 \otimes [q[d]_1]_{1/2}\rangle_{1/2}^{\text{singlet}} \\ |[\bar{Q}Q]_1 \otimes [q[d]_1]_{1/2}\rangle_{1/2}^{\text{doublet}} \\ |[\bar{Q}Q]_1 \otimes [q[d]_1]_{3/2}\rangle_{1/2}^{\text{triplet}} \end{pmatrix}, \end{aligned} \quad (26)$$

$$\begin{aligned} V_{\pi,1/2}^{\text{LCS}}(r) &= U_{1/2}^{-1} V_{1/2}^{\text{HM}} U_{1/2} \\ &= \frac{gg_1}{f_\pi^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix} C_\pi(r). \end{aligned} \quad (27)$$

For the spin 3/2,

$$\begin{aligned} \psi_{3/2}^{\text{LCS}} &= U_{3/2}^{-1} \psi_{3/2}^{\text{HM}} \\ &= \begin{pmatrix} |[\bar{Q}Q]_0 \otimes [q[d]_1]_{3/2}\rangle_{3/2}^{\text{singlet}} \\ |[\bar{Q}Q]_1 \otimes [q[d]_1]_{1/2}\rangle_{3/2}^{\text{doublet}} \\ |[\bar{Q}Q]_1 \otimes [q[d]_1]_{3/2}\rangle_{3/2}^{\text{triplet}} \end{pmatrix}, \end{aligned} \quad (28)$$

$$\begin{aligned} V_{\pi,3/2}^{\text{LCS}}(r) &= U_{3/2}^{-1} V_{3/2}^{\text{HM}} U_{3/2} \\ &= \frac{gg_1}{f_\pi^2} \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix} C_\pi(r). \end{aligned} \quad (29)$$

For the spin 5/2,

$$\begin{aligned} \psi_{5/2}^{\text{LCS}} &= U_{5/2}^{-1} \psi_{5/2}^{\text{HM}} \\ &= (|[\bar{Q}Q]_1 \otimes [q[d]_1]_{3/2}\rangle_{5/2}^{\text{triplet}}), \end{aligned} \quad (30)$$

$$\begin{aligned} V_{\pi,5/2}^{\text{LCS}}(r) &= U_{5/2}^{-1} V_{5/2}^{\text{HM}} U_{5/2} \\ &= -\frac{gg_1}{2f_\pi^2} C_\pi(r). \end{aligned} \quad (31)$$

Here, we call the components labeled by (singlet, doublet, or triplet) of $\psi_{J_p}^{\text{LCS}}$ the spin J (singlet, doublet, or triplet) state. For instance, the first component of $\psi_{1/2}^{\text{LCS}}$ in Eq. (26), i.e., $|[\bar{Q}Q]_0 \otimes [q[d]_1]_{1/2}\rangle_{1/2}^{\text{singlet}}$, is called the spin 1/2 singlet state. The transformation matrices determined by the Clebsch-Gordan coefficient of spin reconstruction are given by

$$U_{1/2} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2\sqrt{3}} & \frac{2}{\sqrt{6}} \\ -\frac{1}{2\sqrt{3}} & \frac{5}{6} & \frac{2}{3\sqrt{2}} \\ \frac{2}{\sqrt{6}} & \frac{2}{3\sqrt{2}} & -\frac{1}{3} \end{pmatrix}, \quad (32)$$

$$U_{3/2} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{\sqrt{3}} & \frac{\sqrt{15}}{6} \\ -\frac{1}{\sqrt{3}} & \frac{1}{3} & \frac{\sqrt{5}}{3} \\ \frac{\sqrt{15}}{6} & \frac{\sqrt{5}}{3} & \frac{1}{6} \end{pmatrix}, \quad (33)$$

$$U_{5/2} = 1. \quad (34)$$

The potential matrices in the LCS basis are diagonalized corresponding to the HQS multiplet components. We find the particular values of the matrix elements of the OPEP; +1 for $[\bar{Q}Q]_0 \otimes [q[d]_1]_{1/2}$ and $[\bar{Q}Q]_1 \otimes [q[d]_1]_{1/2}$, and −2 for $[\bar{Q}Q]_0 \otimes [q[d]_1]_{3/2}$ and $[\bar{Q}Q]_1 \otimes [q[d]_1]_{3/2}$. Hence, these components play a different role, either an attraction or a repulsion, depending on the whole sign of the potential.

IV. NUMERICAL RESULTS

Before solving the coupled channel Schrödinger equations under the LCS basis potential, let us discuss the sign assignment of a coupling constant of the heavy meson-pion interaction, $|g| = 0.59$. In the usual case, its sign is taken as a plus following the quark models. However, only the absolute value is determined by the decay of $D^* \rightarrow D\pi$ [58], and the sign of g is not determined.¹ Since the sign ambiguity of the D - D^* - π coupling is absorbed into a redefinition of the relative phase of the D and D^* meson fields, the sign of the off-diagonal component of the potential matrix is absorbed by the redefinition. This is consistent with the fact that the signs of off-diagonal components are irrelevant for the eigenvalues. On the other

¹It could be argued that the relative sign of g and g_1 is not determined.

FIG. 1. Attractive potential $V(r) = -\frac{gg_1}{2f_\pi^2} C_\pi$ for $\Lambda = 1000$ (purple solid curve) and 1500 MeV (green dotted curve), where $g = 0.59$, $g_1 = 0.942$, and $f_\pi = 92.4$ MeV.

hand, signs of diagonal components, which originate from the sign of the $D^*-D^*\pi$ coupling, affect the eigenvalues. The signs of the diagonal components depend on the sign of the coupling constant. Hence, we examined both signs to clarify the sign dependence.

This sign assignment is important in the present study. For example, the coefficients of the HQS singlet and doublet component are $+1$ in the spin $1/2$ potential of LCS basis in Eq. (27). Thus, these potentials play as a repulsive one when we assign $g = +0.59$, but they are the attractive potentials when we choose $g = -0.59$. On the other hand, the HQS triplet with the coefficient $-1/2$ has the attractive potential for $g = +0.59$ and repulsive potential for $g = -0.59$. In other words, the sign of the coupling constant (the interaction models in general) determines which multiplets have the attractive potential. We calculate the cases with both signs of g to study the behavior of the attractive multiplets in this section.

A. Results in case of $g = +0.59$

When we assign as $g = +0.59$, the HQS multiplets which have attractive potential are $J^P = 3/2^-$ singlet and $J^P = (1/2^-, 3/2^-, 5/2^-)$ triplet. The potential is written as

$$V(r) = -\frac{gg_1}{2f_\pi^2} C_\pi(r), \quad (35)$$

and we show in Fig. 1.

First, we show the results obtained by solving the Schrödinger equation preserving the heavy quark spin

TABLE I. Masses of relevant charmed and bottomed hadrons [58].

	$\bar{D}$	$\bar{D}^*$	B	B^*
Mass[MeV]	1867.21	2008.56	5279.48	5324.65
	Σ_c	Σ_c^*	Σ_b	Σ_b^*
Mass[MeV]	2453.54	2518.13	5813.4	5833.6

FIG. 2. Obtained binding energies for $\Lambda = 1000$ (purple solid curve) and 1500 MeV (green dotted curve) with $g = +0.59$. The energy is measured from the threshold of $\bar{P}_{\text{ave}}\Sigma_{Q\text{ave}}$. The mass parameter μ_{ave} is changed from 1 GeV to 100 GeV.

symmetry. We define the spin-averaged mass for the $\bar{P}^{(*)}$ mesons and $\Sigma_Q^{(*)}$ baryons as

$$M_{\bar{P}\text{ave}} = \frac{M_{\bar{P}} + 3M_{\bar{P}^*}}{4}, \quad (36)$$

$$M_{\Sigma_Q\text{ave}} = \frac{2M_{\Sigma_Q} + 4M_{\Sigma_Q^*}}{6}, \quad (37)$$

to deal with the degeneracy of the HQS doublet meson and baryon, respectively. The masses of the relevant charmed and bottomed hadrons are shown in Table I. The spin-averaged reduced mass is defined as

$$\mu_{\text{ave}} = \frac{M_{\bar{P}\text{ave}}M_{\Sigma_Q\text{ave}}}{M_{\bar{P}\text{ave}} + M_{\Sigma_Q\text{ave}}}. \quad (38)$$

When $\mu_{\text{ave}} = 1.102, 1.474, 1.699$, and 2.779 GeV, the spin-averaged masses of $\bar{D}^{(*)}\Sigma_c^{(*)}$, $\bar{D}^{(*)}\Sigma_b^{(*)}$, $B^{(*)}\Sigma_c^{(*)}$, and $B^{(*)}\Sigma_b^{(*)}$ are reproduced, respectively. We solve the Schrödinger equation keeping the heavy quark spin symmetry by changing the mass parameter μ_{ave} from 1 to 100 GeV. To obtain the bound state solutions, we use the Gaussian expansion method [60].

The results of $\Lambda = 1000$ and 1500 MeV are shown in Fig. 2. All four states, spin $3/2$ singlet and spin $(1/2, 3/2, 5/2)$ triplet, are degenerate because of the heavy quark spin symmetry and their bound state solutions are obtained for all range of μ_{ave} .

Next, we show the results including the effect of the heavy quark spin symmetry breaking. The breaking is introduced by the nonzero mass difference between the HQS multiplet, namely $\bar{P}$ and $\bar{P}^*$, and Σ_Q and Σ_Q^* .² To see

²The higher-order terms of the effective Lagrangians also break the heavy quark symmetry. However, we employ the leading term of Lagrangians in this study.

TABLE II. Values of parameters to include the effect of heavy quark spin symmetry breaking in Eqs. (39)–(42).

$a[\text{GeV}^2]$	$b[\text{GeV}^2]$	$c[\text{GeV}^2]$	$d[\text{GeV}^2]$	$w[\text{GeV}^3]$	$x[\text{GeV}^3]$	$y[\text{GeV}^3]$	$z[\text{GeV}^3]$
-2.0798	-1.8685	1.9889	2.0814	2.9468	3.1677	-3.1729	-3.0629

the mass dependence of a binding energy, the heavy hadron masses are parametrized as follows:

$$M_{\bar{P}} = 2\mu + \frac{a}{2\mu} + \frac{w}{(2\mu)^2}, \quad (39)$$

$$M_{\bar{P}^*} = 2\mu + \frac{b}{2\mu} + \frac{x}{(2\mu)^2}, \quad (40)$$

$$M_{\Sigma_Q} = 2\mu + \frac{c}{2\mu} + \frac{y}{(2\mu)^2}, \quad (41)$$

$$M_{\Sigma_Q^*} = 2\mu + \frac{d}{2\mu} + \frac{z}{(2\mu)^2}, \quad (42)$$

FIG. 3. Energies of the $\bar{P}^{(*)}\Sigma_Q^{(*)}$ states with heavy quark spin symmetry breaking effect, obtained for $\Lambda = 1000$ MeV (the upper figure) and 1500 MeV (the lower figure) with $g = +0.59$. These energies are measured from $\bar{P}\Sigma_Q$ threshold. The purple solid, green dotted and yellow dashed-dotted curves are the energies of spin (1/2, 3/2, 5/2) triplet states respectively and the light blue dashed curve is that of spin 3/2 singlet state. For the sake of reference, we show the result for the case of keeping the heavy quark spin symmetry by the red dashed-dotted-dotted curve (Common mass).

where μ is a parameter corresponding to the reduced mass of $\bar{P}^{(*)}\Sigma_Q^{(*)}$ state. The eight parameters of a, b, c, d, w, x, y and z are fixed to reproduce the eight hadron masses in Table I by taking $\mu = 1.102$ (2.779) GeV for charm (bottom) sector. The values of these parameters are shown in Table II. Note that the charm (bottom) hadron masses are reproduced when we take $\mu = 1.102$ (2.779) GeV, and the heavy quark spin symmetry is restored as the mass parameter μ increases. The energies obtained by solving the Schrödinger equations with the effect of heavy quark spin symmetry breaking are shown in Fig. 3. The labels in Fig. 3, e.g., spin 1/2 triplet, are named as being it at the heavy quark limit. For instance, the solid line named as spin 1/2 triplet displays the energy of the state which becomes the spin 1/2 triplet state at the heavy quark limit. We note that the components belonging to the same J^P state can be mixed in the finite hadron mass region as shown later, while they are not mixed at the heavy quark limit. The energies of the $\bar{D}^{(*)}\Sigma_c^{(*)}$, $\bar{D}^{(*)}\Sigma_b^{(*)}$, $B^{(*)}\Sigma_c^{(*)}$ and $B^{(*)}\Sigma_b^{(*)}$ states are correspond to the values at $\mu = 1.102, 1.474, 1.699$, and 2.779 GeV, respectively.

All four states are degenerate and the binding energy is -13.7 MeV for $\Lambda = 1000$ MeV and -22.3 MeV for $\Lambda = 1500$ MeV in the heavy quark limit. As μ becomes smaller, the degeneracy is solved. At $\mu = 1.102$ GeV, only two (three) states can be bound for $\Lambda = 1000$ MeV (1500 MeV).

For the spin 1/2 and 3/2 states, each component is completely separated in the heavy quark limit as shown in Eqs. (27) and (29). In the finite heavy hadron mass region, however, the kinetic term with the nonzero mass splitting of the HQS multiplets gives a mixing of the HQS singlet, doublet and triplet components. The percentage of (singlet, doublet, triplet) components in wave functions for $\Lambda = 1000$ MeV is shown in Table III. For $\mu \geq 3$ GeV, each component is perfectly separated. These ratios are hardly changed even in the case of $\Lambda = 1500$ MeV. We can see

TABLE III. Percentage of (singlet, doublet, triplet) components in wave functions of the spin 1/2 and 3/2 states in the case of $g = +0.59$ and $\Lambda = 1000$ MeV.

μ [GeV]	Spin 1/2 triplet	Spin 3/2 triplet	Spin 3/2 singlet
1	(0.8%, 0%, 99.2%)	(1.6%, 0%, 98.4%)	No bound state
2	(0%, 0%, 100%)	(0.9%, 0%, 99.1%)	No bound state
3	(0%, 0%, 100%)	(0%, 0%, 100%)	(100%, 0%, 0%)

FIG. 4. Attractive potential $V(r) = \frac{gg_1}{f_\pi^2} C_\pi$ for $\Lambda = 1000$ (purple solid curve) and 1500 MeV (green dotted curve), where $g = -0.59$, $g_1 = 0.942$ and $f_\pi = 92.4$ MeV.

FIG. 5. Obtained binding energies for $\Lambda = 1000$ (purple solid curve) and 1500 MeV (green dotted curve) with $g = -0.59$. The energy is measured from the threshold of $\bar{P}_{ave}\Sigma_{Qave}$. The mass parameter μ is changed from 1 GeV to 100 GeV.

that the effect of heavy quark spin symmetry breaking is small.

B. Result in case of $g = -0.59$

In the case of $g = -0.59$, the attractive multiplets are $J^P = 1/2^-$ singlet and $J^P = (1/2^-, 3/2^-)$ doublet. The potential is written as

$$V(r) = \frac{gg_1}{f_\pi^2} C_\pi(r), \quad (43)$$

and it is shown in Fig. 4. This potential is twice deeper than that of $g = +0.59$, and therefore we expect that the binding energy is larger.

FIG. 6. Energies obtained with heavy quark spin symmetry breaking for $\Lambda = 1000$ MeV (the upper figure) and 1500 MeV (the lower figure) with $g = -0.59$. These energies are measured from $\bar{P}\Sigma_Q$ threshold. The purple solid and light blue dashed curves are the energies of spin $(1/2, 3/2)$ doublet states and the green dotted curve is that of spin $1/2$ singlet state. For the sake of reference, we show the result for the case of keeping the heavy quark spin symmetry by the red dashed-dotted-dotted curve.

As in the case of $g = +0.59$, we show the result where the heavy quark spin symmetry is preserved in Fig. 5, and the result where it is broken in Fig. 6, for $g = -0.59$.

Figure 6 shows that all three states of spin $1/2$ singlet and $(1/2, 3/2)$ doublet are degenerate in the heavy quark limit and the binding energy is -29.5 MeV for $\Lambda = 1000$ MeV and -48.1 MeV for $\Lambda = 1500$ MeV, which agree with the binding energies in the heavy quark limit shown in Fig. 5. Unlike in the case of $g = +0.59$, all states are bound even at $\mu = 1.102$ GeV corresponding to $\bar{D}^{(*)}\Sigma_c^{(*)}$ state and their binding energies are a few MeV.

TABLE IV. Percentage of (singlet, doublet, triplet) components in wave functions in the case of $g = -0.59$ and $\Lambda = 1000$ MeV.

$\mu[\text{GeV}]$	Spin 1/2 singlet	Spin 1/2 doublet	Spin 3/2 doublet
1	(3.9%, 96.1%, 0%)	(96.5%, 3.4%, 0.1%)	(0%, 99.9%, 0.1%)
2	(0.2%, 99.8%, 0%)	(99.9%, 0.1%, 0%)	(0%, 100%, 0%)
3	(0%, 100%, 0%)	(100%, 0%, 0%)	(0%, 100%, 0%)

The mixing ratio of wave-function components for $\Lambda = 1000$ MeV is shown in Table IV. The mixing ratio of the minor components induced by the heavy quark symmetry breaking effect is slightly larger than the case of $g = +0.59$, however it is still small.

V. SUMMARY AND DISCUSSIONS

We showed that, in Sec. IV, the sign of the coupling constant of the heavy meson-pion interaction determines which multiplets have the attractive potential. In the case of $g = +0.59$, the spin 3/2 HQS singlet and spin (1/2, 3/2, 5/2) HQS triplet have the attractive potential. On the other hand, in the case of $g = -0.59$, the spin 1/2 HQS singlet and spin (1/2, 3/2) HQS doublet have the attractive potential.

This classification is explained by the light cloud spin structure in Eqs. (22)–(25). The light cloud spin of the spin 3/2 singlet and spin (1/2, 3/2, 5/2) triplet is $[q[d]_1]_{3/2}$ and that of the spin 1/2 singlet and spin (1/2, 3/2) doublet is $[q[d]_1]_{1/2}$. Because the pion exchange interaction³ is coupled to the light quark spin, the difference of the attractive multiplet comes from the difference of the light cloud spin structure. Moreover, we find the degeneracy of the HQS singlet and triplet (singlet and doublet) in the case of $g = +0.59$ (-0.59). It is a natural result because the OPEP does not depend on the heavy quark spin structure.

In the heavy quark limit, four (three) bound states exist for $g = +0.59$ (-0.59). However, the heavy quark symmetry is broken for real charm/bottom hadrons, so that all four (three) bound states may not exist in reality as demonstrated in Sec. IV. But we expect that there exist some HQS partners of P_c -like pentaquarks. In particular, for the bottom sector, the structure of HQS multiplet is more clearly than for charm sector, because the realization of the heavy quark symmetry is better. We expect the observation of the bottom pentaquarks to confirm the HQS multiplet structure of them.

The discussion in the LCS basis can be compared to the quark model calculations, treating the constituent quarks as degrees of freedom of the system. In Refs. [34,38], the short-range interaction in the P_c pentaquarks are studied, which is derived based on the quark cluster model. The contributions from the color magnetic interaction of $c\bar{c}uud$ are evaluated, and they find that the $c\bar{c}uud$ configurations having the other spin $j = 3/2$ are important to produce an attraction. On the other hand, the configurations with

$j = 1/2$ give a repulsion. In this study, we also obtain that the states with $j = 1/2$ and $j = 3/2$ have a different role as shown in Eqs. (27), (29) and (31), namely one is attractive and the other one is repulsive. Thus, we find that a role of the interaction is characterized by the light cloud spin in both of the quark model and the hadronic molecular model. It indicates that the discussion of the HQS multiplet structure can be applied not only to the molecules, but also to the compact multi-quark states. In Ref. [38], the $c\bar{c}uud$ potential for $J^P = 3/2^-$ with $(S, j) = (1, 3/2)$ is stronger than that with $(S, j) = (0, 3/2)$. This behavior also agrees with our results.

Focusing on the light cloud spin structure, there are constraints of the S-wave decay channel of the spin 3/2 HQS singlet and spin (1/2, 3/2, 5/2) HQS triplet. Since their light cloud spin is given by $[q[d]_1]_{3/2}$, they cannot couple to the S-wave $[Q\bar{Q}]N$ and $\bar{P}^{(*)}\Lambda_Q$ states. Here $[Q\bar{Q}]$, N and Λ_Q denote the heavy quarkonium, spin 1/2 nucleon and HQS singlet heavy baryonlike Λ_c , respectively.

Due to the heavy quark spin symmetry, heavy quark spin and light cloud spin are independently conserved. Therefore, $[q[d]_1]_{3/2}$ having light cloud spin 3/2 does not couple to the nucleon of spin 1/2. Moreover, $[q[d]_1]_{3/2}$ cannot construct the diquark spin 0 by the spin rearrangement. So, $[q[d]_1]_{3/2}$ cannot couple to Λ_Q with diquark spin 0 as well. As a result, the S-wave decay channels to $[Q\bar{Q}]N$ and $\bar{P}^{(*)}\Lambda_Q$ from the spin 3/2 HQS singlet and spin (1/2, 3/2, 5/2) HQS triplet are prohibited in the heavy quark limit. D-wave decay channels do exist; however, we expect that they are small.

On the other hand, there are no constraints on the S-wave decay to $[Q\bar{Q}]N$ and $\bar{P}^{(*)}\Lambda_Q$ for the spin 1/2 HQS singlet and spin (1/2, 3/2) HQS doublet which have the light cloud spin of $[q[d]_1]_{1/2}$ in the view of heavy quark spin symmetry. These restrictions are independent of the model and derived only from heavy quark symmetry. The difference in their S-wave decay channel restrictions should appear in the decay branching ratio of the $Q\bar{Q}qqq$ pentaquark state. We expect the measurement of the branching ratio to $[Q\bar{Q}]N$ and $\bar{P}^{(*)}\Lambda_Q$ to confirm the heavy quark symmetry in the P_c -like pentaquarks.

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³We do not consider the tensor force in this study, but it is also determined by the light cloud spin structure. Not only the pion interaction, but also the other light meson interactions depend on the light cloud spin.

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