$D\Xi$ and $D^*\Xi$ molecular states from one boson exchangeMing-Zhu Liu,¹ Tian-Wei Wu,¹ Ju-Jun Xie,² Manuel Pavon Valderrama,^{1,*} and Li-Sheng Geng^{1,†}¹*School of Physics and Nuclear Energy Engineering & International Research Center for Nuclei and Particles in the Cosmos & Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 100191, China*²*Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China*

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We explore the existence of $D\Xi$ and $D^*\Xi$ molecular states within the one boson exchange model. We regularize the potential derived in this model with a form factor and a cutoff of the order of 1 GeV. To determine the cutoff, we use the condition that the $X(3872)$ is reproduced as a pole in the $J^{PC} = 1^{++} D^*\bar{D}$ amplitude. From this, we find that the $J^P = \frac{1}{2}^- D^*\Xi$ system is on the verge of binding and has an unnaturally large scattering length. For the $J^P = \frac{1}{2}^- D\Xi$ and the $J^P = \frac{3}{2}^- D^*\Xi$ systems, the attraction is not enough to form a bound state. From heavy quark symmetry and the quark model, we can extend the previous model to the $P\Xi_{QQ}$ and $P^*\Xi_{QQ}$ systems, with $P = B, D$ and $\Xi_{QQ} = \Xi_{cc}, \Xi_{bb}$. In this case, we predict a series of triply heavy pentaquarklike molecules.

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I. INTRODUCTION

The discovery of the $X(3872)$ by the Belle Collaboration [1] 15 years ago represented the first hidden charm state that did not fit into the charmonium spectrum. Afterward, experiments have found a series of similar states, informally known as XYZ states. They cannot be easily accommodated in the naive quark model, and other components have to be invoked to explain their masses, decay widths, and production rates; see Ref. [2] for a recent review. A few are tetraquarklike, such as the $Z_c(3900)$ [3,4], $Z_c(4020)$ [5,6], $Z_b(10610)$, and $Z_b(10650)$ [7,8], while recently two pentaquarklike states have been observed, the $P_c(4380)$ and $P_c(4450)$ [9]. A few of them have the interesting feature of being close to an open flavor threshold. The most notable example is the $X(3872)$, located almost on top of the $D^0\bar{D}^{*0}$ threshold, but the list also includes the $Z_c(3900)$ ($D\bar{D}^*$) and $Z_c(4020)$ ($D^*\bar{D}^*$), the $Z_b(10610)$ ($B\bar{B}^*$) and $Z_b(10650)$ ($B^*\bar{B}^*$), and the $P_c(4450)$ near the $\bar{D}^*\Sigma_c$ threshold. This characteristic has led to the conjecture that the previous states might be hadronic molecules.

A hadronic molecule is a loosely bound state or resonance composed of hadrons. They were originally

proposed to explain the $\psi(4040)$ as a $D^*\bar{D}^*$ bound state [10,11]. Though the $\psi(4040)$ turned out to be a charmonium state at the end, the idea quickly caught the attention of theoreticians [12–15], and the later discovery of the $X(3872)$ showed that these speculations were indeed on the right track. Besides the $X(3872)$, the most prosaic example of a hadronic molecule is the deuteron, which also inspired the Weinberg compositeness condition [16]. Other strong molecular candidates include the $Z_b(10610)$, $Z_b(10650)$ [17–23], and $P_c(4450)$ [24–28].

Recently, the LHCb Collaboration observed five narrow states, $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, and $\Omega_c(3119)^0$ [29], four of which have also been recently confirmed from an analysis of the Belle data [30]. These states can be accommodated as excited Ω_c baryons [31,32], as compact baryons in which the ss strange quark pair forms a diquark [33], or as molecular states [34–38]. The LHCb data also hint at a structure around 3188 MeV, which is near the $D\Xi$ threshold (3179–3191 MeV) and could be a bound state of $D\Xi$. Wang *et al.* [39] used the Bethe-Salpeter formalism to study the S -wave $D\Xi$ interaction, and they found two bound states, one isoscalar and one isovector, which could contribute to the 3188 MeV structure near the five new narrow Ω_c states. Debastiani *et al.* [34] and Liang *et al.* [40] used an extension of the chiral unitary model to calculate the interactions between $D\Xi$, $D^*\Xi$, $\bar{B}\Xi$, $\bar{B}^*\Xi$ plus other channels and obtained one zero-width state with $J^P = 1/2^-, 3/2^-$, which couples mostly to $D^*\Xi$ and $\bar{B}^*\Xi$, and another state with $J^P = \frac{1}{2}^-$, which couples mostly to $D\Xi$ and $\bar{B}\Xi$.

In this work, we study possible bound states near the thresholds of $D\Xi$, $D^*\Xi$, $\bar{B}\Xi$, and $\bar{B}^*\Xi$ within the one boson

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exchange (OBE) model, in which we will also consider the replacement of Ξ by Ξ_{cc} or Ξ_{bb} to explore the existence of heavy molecules containing two and three charm/bottom quarks. The OBE model is an intuitive framework in which a few of the most quantitative successful descriptions of the nuclear force have been achieved [41,42]. Nowadays, it has been superseded by the effective field theory (EFT) approaches [43,44], which have two theoretical advantages over the OBE model: (i) the possibility of making *a priori* error estimates and (ii) the low-energy equivalence with quantum chromodynamics. However, the EFT approach requires a large number of data to fix the low-energy constants that substitute the exchange of light mesons such as the σ , ρ , and ω . This means that in situations in which hadron-hadron scattering data are poor, such as hadron molecules, the OBE potential has the advantage of providing a model of the short-range dynamics of these systems, at the price of sacrificing the systematicity of EFT. As a matter of fact, the seminal works that pioneered the idea of hadronic molecules [10,11] were indeed based on the OBE model, and a few modern explorations rely heavily on this model [18,19,24].

The manuscript is organized as follows. In Sec. II, we review the OBE model as applied to the $D\Xi$, $D^*\Xi$, plus analogous systems and derive the potentials in these systems. In Sec. III, we show the predictions we obtain for the molecular states. Finally, we present our conclusions in Sec. IV.

II. ONE BOSON EXCHANGE POTENTIAL

In this section, we explain the OBE model (see Ref. [42] for a review) and how it applies to the $D\Xi$ and $D^*\Xi$ systems. The OBE model is a generalization of the idea of Yukawa, namely, that nuclear forces arise from the exchange of pions, to shorter distances. For that, the exchange of other light mesons (besides the pion) is considered, in particular, the σ scalar meson and the ρ and ω vector mesons. The reason for the development of the OBE model was the failure of the original two-pion theories in the 1950s (which did not include chiral symmetry); see Ref. [45] for a historical perspective. The fundamental idea is that the bulk of the two-pion exchange potential can be described by the exchange of a heavier meson, which either couples strongly to pions or which can be interpreted as a multipion resonance. The idea is physically compelling and has been fairly successful at the phenomenological level, as illustrated by the nuclear potentials based on it [41,46]. There are limitations, however, such as the requirement of form factors to regularize the potentials or the requirement of fine tuning the coupling constants [47,48] (at least in the two-nucleon system). Yet, the OBE potential is still perfectly able to provide a good estimation of the plausibility of hadronic bound states and their location, as attested, for instance, in the pioneering work of Voloshin and Okun [10].

A. Lagrangian

We begin with the interaction of the D and D^* fields with the π , σ , ρ , and ω mesons. First, we group the fields of the D and D^* in the (heavy spin symmetric) superfield

$$H_v = \frac{1 + \not{v}}{2\sqrt{2}} (D^{*\mu} \gamma_\mu - D \gamma_5), \quad (1)$$

with v the velocity parameter and where we have used the convenient normalization of Falk and Luke [49]. In this normalization, we can write the interaction Lagrangian of the H superfield with the light mesons as

$$\mathcal{L}_{HH\pi} = \frac{g_1}{\sqrt{2}f_\pi} \text{Tr}[\bar{H}_v \gamma^\mu \gamma^5 \vec{\tau} \cdot \partial_\mu \vec{\pi} H_v], \quad (2)$$

$$\mathcal{L}_{HH\sigma} = -g_{\sigma 1} \text{Tr}[\bar{H}_v \sigma H_v], \quad (3)$$

$$\begin{aligned} \mathcal{L}_{HH\rho} = & g_{\rho 1} \text{Tr}[\bar{H}_v v_\mu \vec{\tau} \cdot \vec{\rho}^\mu H_v] \\ & + \frac{f_{\rho 1}}{4M_1} \text{Tr}[\bar{H}_v \sigma_{\mu\nu} \vec{\tau} \cdot (\partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu) H_v], \end{aligned} \quad (4)$$

$$\begin{aligned} \mathcal{L}_{HH\omega} = & -g_{\omega 1} \text{Tr}[\bar{H}_v v_\mu \omega^\mu H_v] \\ & - \frac{f_{\rho 1}}{4M_1} \text{Tr}[\bar{H}_v \sigma_{\mu\nu} \vec{\tau} \cdot (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) H_v], \end{aligned} \quad (5)$$

where the traces are in the 4×4 space spanned by the Dirac matrices when defining the superfield H . In the Lagrangian above, we take the normalization $f_\pi = 132$ MeV; g_1 , $g_{\sigma 1}$, $g_{\omega 1}$, $f_{\omega 1}$, $g_{\rho 1}$, and $f_{\rho 1}$ are the different couplings in the OBE model; and M_1 is a mass scale that we introduce for $f_{\omega 1}$ and $f_{\rho 1}$ to be dimensionless (M_1 is in principle arbitrary, but we will later take it to be the D -meson mass, i.e., $M_1 = m_D$). If we choose the velocity parameter to be $v = (1, \vec{0})$, we can substitute the superfield H_v by a nonrelativistic superfield H ,

$$H_v \rightarrow H = \frac{1}{\sqrt{2}} [P + \vec{P}^* \cdot \vec{\sigma}], \quad (6)$$

where H is a 2×2 matrix (instead of a 4×4 one). Now, the Lagrangian can be rewritten as

$$\mathcal{L}_{HH\pi} = -\frac{g_1}{\sqrt{2}f_\pi} \text{Tr}[H^\dagger \vec{\sigma} \cdot \nabla (\vec{\tau} \cdot \vec{\pi}) H], \quad (7)$$

$$\mathcal{L}_{HH\sigma} = g_{\sigma 1} \text{Tr}[H^\dagger \sigma H], \quad (8)$$

$$\begin{aligned} \mathcal{L}_{HH\rho} = & g_{\rho 1} \text{Tr}[H^\dagger \vec{\tau} \cdot \vec{\rho}^0 H] \\ & - \frac{f_{\rho 1}}{4M_1} \epsilon_{ijk} \text{Tr}[H^\dagger \sigma_k \vec{\tau} \cdot (\partial_i \vec{\rho}_j - \partial_j \vec{\rho}_i) H], \end{aligned} \quad (9)$$

$$\begin{aligned} \mathcal{L}_{HH\omega} = & -g_{\omega 1} \text{Tr}[H^\dagger \omega^0 H] \\ & + \frac{f_{\omega 1}}{4M_1} \epsilon_{ijk} \text{Tr}[H^\dagger \sigma_k (\partial_i \omega_j - \partial_j \omega_i) H]. \end{aligned} \quad (10)$$

Now, for the Ξ field, we can write the interaction Lagrangian,

$$\mathcal{L}_{\Xi\Xi\pi} = -\frac{g_2}{\sqrt{2}f_\pi} \bar{\psi}_\Xi \gamma^\mu \gamma^5 \vec{\tau} \cdot \partial_\mu \vec{\pi} \psi_\Xi, \quad (11)$$

$$\mathcal{L}_{\Xi\Xi\sigma} = g_{\sigma 2} \bar{\psi}_\Xi \sigma \psi_\Xi, \quad (12)$$

$$\begin{aligned} \mathcal{L}_{\Xi\Xi\rho} = & g_{\rho 2} \bar{\psi}_\Xi \gamma_\mu \vec{\tau} \cdot \vec{\rho}^\mu \psi_\Xi \\ & + \frac{f_{\rho 2}}{4M_2} \bar{\psi}_\Xi \sigma_{\mu\nu} \vec{\tau} \cdot (\partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu) \psi_\Xi, \end{aligned} \quad (13)$$

$$\begin{aligned} \mathcal{L}_{\Xi\Xi\omega} = & g_{\omega 2} \bar{\psi}_\Xi \gamma_\mu \omega^\mu \psi_\Xi \\ & + \frac{f_{\omega 2}}{4M_2} \bar{\psi}_\Xi \sigma_{\mu\nu} (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) \psi_\Xi, \end{aligned} \quad (14)$$

which is analogous to the one for the D and D^* (and it is identical in form to the one in the nucleon-nucleon case). In the Lagrangian, g_2 , $g_{\sigma 2}$, $g_{\omega 2}$, $f_{\omega 2}$, $g_{\rho 2}$, $f_{\rho 2}$, and M_2 denote the couplings and the mass scale for the cascade baryon. Here, we can use the heavy baryon formulation by making the field redefinition

$$\Xi = e^{iM_\Xi v \cdot x} \psi_\Xi. \quad (15)$$

If we choose again $v = (1, \vec{0})$, we arrive at the Lagrangian

$$\mathcal{L}_{\Xi\Xi\pi} = \frac{g_2}{\sqrt{2}f_\pi} \Xi^\dagger \vec{\sigma} \cdot \nabla (\vec{\tau} \cdot \vec{\pi}) \Xi, \quad (16)$$

$$\mathcal{L}_{\Xi\Xi\sigma} = g_{\sigma 2} \Xi^\dagger \sigma \Xi, \quad (17)$$

$$\begin{aligned} \mathcal{L}_{\Xi\Xi\rho} = & g_{\rho 2} \Xi^\dagger \vec{\tau} \cdot \vec{\rho}^0 \Xi \\ & - \frac{f_{\rho 2}}{4M} \epsilon_{ijk} \Xi^\dagger \sigma_k \vec{\tau} \cdot (\partial_i \vec{\rho}_j - \partial_j \vec{\rho}_i) \Xi, \end{aligned} \quad (18)$$

$$\begin{aligned} \mathcal{L}_{\Xi\Xi\omega} = & g_{\omega 2} \Xi^\dagger \omega^0 \Xi \\ & - \frac{f_{\omega 2}}{4M} \epsilon_{ijk} \Xi^\dagger \sigma_k (\partial_i \omega_j - \partial_j \omega_i) \Xi. \end{aligned} \quad (19)$$

B. OBE potential

The general form of the $P\Xi$ and $P^*\Xi$ OBE potential is

$$V = \zeta V_\pi + V_\sigma + V_\rho + \zeta V_\omega, \quad (20)$$

where the subscript indicates from which meson comes the contribution (π , σ , ρ , or ω) and with $\zeta = \pm 1$. We take the convention that

$$\zeta = +1 \quad \text{for } P\Xi \text{ and } P^*\Xi, \quad (21)$$

$$\zeta = -1 \quad \text{for } \bar{P}\Xi \text{ and } \bar{P}^*\Xi. \quad (22)$$

If the vector meson and the hadrons are pointlike, their vertices can be directly computed from the Lagrangian, and we end up with the following potentials in momentum space:

$$V_\pi(\vec{q}) = \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{g_1 g_2}{2f_\pi^2} \frac{a_1 \cdot \vec{q} \vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}, \quad (23)$$

$$V_\sigma(\vec{q}) = -\frac{g_{\sigma 1} g_{\sigma 2}}{q^2 + m_\sigma^2} \quad (24)$$

$$\begin{aligned} V_\rho(\vec{q}) = & \vec{\tau}_1 \cdot \vec{\tau}_2 \left[\frac{g_{\rho 1} g_{\rho 2}}{q^2 + m_\rho^2} \right. \\ & \left. - \frac{f_{\rho 1}}{2M_1} \frac{f_{\rho 2}}{2M_2} \frac{(a_1 \times \vec{q}) \cdot (\vec{\sigma}_2 \times \vec{q})}{q^2 + m_\rho^2} \right], \end{aligned} \quad (25)$$

$$\begin{aligned} V_\omega(\vec{q}) = & -\frac{g_{\omega 1} g_{\omega 2}}{q^2 + m_\omega^2} \\ & + \frac{f_{\omega 1}}{2M_1} \frac{f_{\omega 2}}{2M_2} \frac{(a_1 \times \vec{q}) \cdot (\vec{\sigma}_2 \times \vec{q})}{q^2 + m_\omega^2}. \end{aligned} \quad (26)$$

In the expressions, the subscript s1 and 2 are used for the P/P^* heavy meson and Ξ baryon, respectively. For $\vec{a}_1$, we take

$$\vec{a}_1 = 0 \quad \text{for } P, \quad (27)$$

$$\vec{a}_1 = \vec{S}_1 \quad \text{for } P^*, \quad (28)$$

with $\vec{S}$ the spin-1 angular momentum matrices. For the pion decay constant, we take the $f_\pi = 132$ MeV normalization. The choice of sign for the momentum space potential is such that the Lippmann-Schwinger equation reads $T = V + VG_0 T$, where the T matrix is in turn normalized so that for zero-energy scattering $T \rightarrow 2\pi a_0/\mu$, with a_0 the scattering length and μ the reduced mass of the system. That is, we are using the standard nonrelativistic normalization, which is also used in the two-nucleon system; see, for instance, Ref. [42].

We can take into account the finite size of hadrons by including form factors in the calculation, that is,

$$V_M(\vec{q}, \Lambda) = V_M(\vec{q}) F_1(q, m, \Lambda_1) F_2(q, m, \Lambda_1), \quad (29)$$

where F_1 and F_2 are the form factors corresponding to vertices 1 and 2, i.e., the P/P^* heavy meson and the Ξ baryon. The form factor can depend on the momentum transfer q , the mass of the exchanged meson m , and a cutoff Λ . Here, we will use a monopolar form factor of the type

$$F(q, m, \Lambda) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}, \quad (30)$$

for both vertices, where $q^2 = q_0^2 - \vec{q}^2$ is the 4-momentum of the exchanges meson.

In configuration space and for pointlike interactions, the components of the OBE potential take the form

$$V_\pi(\vec{r}) = -\vec{\tau}_1 \cdot \vec{\tau}_2 \frac{g_1 g_2}{6f_\pi^2} [-\vec{a}_1 \cdot \vec{\sigma}_2 \delta(\vec{r}) + \vec{a}_1 \cdot \vec{\sigma}_2 m_\pi^3 W_Y(m_\pi r) + S_{12}(\vec{r}) m_\pi^3 W_T(m_\pi r)], \quad (31)$$

$$V_\sigma(\vec{r}) = -g_{\sigma 1} g_{\sigma 2} m_\sigma W_Y(m_\sigma r), \quad (32)$$

$$V_\rho(\vec{r}) = \vec{\tau}_1 \cdot \vec{\tau}_2 \left[g_{\rho 1} g_{\rho 2} m_\rho W_Y(m_\rho r) + \frac{f_{\rho 1}}{2M_1} \frac{f_{\rho 2}}{2M_2} \left(-\frac{2}{3} \vec{a}_1 \cdot \vec{\sigma}_2 \delta(\vec{r}) + \frac{2}{3} \vec{a}_1 \cdot \vec{\sigma}_2 m_\rho^3 W_Y(m_\rho r) - \frac{1}{3} S_{12}(\hat{r}) m_\rho^3 W_T(m_\rho r) \right) \right], \quad (33)$$

$$V_\omega(\vec{r}) = -g_{\omega 1} g_{\omega 2} m_\omega W_Y(m_\omega r) - \frac{f_{\omega 1}}{2M_1} \frac{f_{\omega 2}}{2M_2} \left(-\frac{2}{3} \vec{a}_1 \cdot \vec{\sigma}_2 \delta(\vec{r}) + \frac{2}{3} \vec{a}_1 \cdot \vec{\sigma}_2 m_\omega^3 W_Y(m_\omega r) - \frac{1}{3} S_{12}(\hat{r}) m_\omega^3 W_T(m_\omega r) \right), \quad (34)$$

where the functions $W_Y(x)$ and $W_T(x)$ are defined as

$$W_Y(x) = \frac{e^{-x}}{4\pi x}, \quad (35)$$

$$W_T(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2} \right) \frac{e^{-x}}{4\pi x}. \quad (36)$$

The effects of the finite size of the hadrons are easy to take into account by making the changes

$$\delta(r) \rightarrow m^3 d(x, \lambda), \quad (37)$$

$$W_Y(x) \rightarrow W_Y(x, \lambda), \quad (38)$$

$$W_T(x) \rightarrow W_T(x, \lambda), \quad (39)$$

where $\lambda = \Lambda/m$. For the monopolar form factor of Eq. (30), we end up with

$$d(x, \lambda) = \frac{(\lambda^2 - 1)^2}{2\lambda} \frac{e^{-\lambda x}}{4\pi}, \quad (40)$$

$$W_Y(x, \lambda) = W_Y(x) - \lambda W_Y(\lambda x) - \frac{(\lambda^2 - 1)}{2\lambda} \frac{e^{-\lambda x}}{4\pi}, \quad (41)$$

$$W_T(x, \lambda) = W_T(x) - \lambda^3 W_T(\lambda x) - \frac{(\lambda^2 - 1)}{2\lambda} \lambda^2 \left(1 + \frac{1}{\lambda x} \right) \frac{e^{-\lambda x}}{4\pi}. \quad (42)$$

As a matter of fact, the structure of the OBE potential presented here is exceedingly simple. We can write it as a sum of a central, spin-spin, and tensor component,

$$V(\vec{r}) = V_C(r) + \vec{a}_1 \cdot \vec{\sigma}_2 V_S(r) + S_{12}(\hat{r}) V_T(r), \quad (43)$$

where for pointlike interactions we have

$$V_C(r) = -g_{\sigma 1} g_{\sigma 2} m_\sigma W_Y(m_\sigma r) + \vec{\tau}_1 \cdot \vec{\tau}_2 g_{\rho 1} g_{\rho 2} m_\rho W_Y(m_\rho r) - \zeta g_{\omega 1} g_{\omega 2} m_\omega W_Y(m_\omega r), \quad (44)$$

$$V_S(r) = -\zeta \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{g_1 g_2}{6f_\pi^2} [-\delta(\vec{r}) + m_\pi^3 W_Y(m_\pi r)] + \frac{2}{3} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{f_{\rho 1}}{2M_1} \frac{f_{\rho 2}}{2M_2} [-\delta(\vec{r}) + m_\rho^3 W_Y(m_\rho r)] - \frac{2}{3} \zeta \frac{f_{\omega 1}}{2M_1} \frac{f_{\omega 2}}{2M_2} [-\delta(\vec{r}) + m_\omega^3 W_Y(m_\omega r)], \quad (45)$$

$$V_T(r) = -\zeta \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{g_1 g_2}{6f_\pi^2} m_\pi^3 W_T(m_\pi r) - \frac{1}{3} \vec{\tau}_1 \cdot \vec{\tau}_2 \frac{f_{\rho 1}}{2M_1} \frac{f_{\rho 2}}{2M_2} m_\rho^3 W_T(m_\rho r) + \frac{1}{3} \zeta \frac{f_{\omega 1}}{2M_1} \frac{f_{\omega 2}}{2M_2} m_\omega^3 W_T(m_\omega r), \quad (46)$$

while for finite-size hadrons, we substitute $\delta(r)$, $W_Y(x)$, and $W_T(x)$ by their finite-size versions.

C. Couplings

For the axial coupling of the pion with the D and D^* mesons we take

$$g_1 = 0.60, \quad (47)$$

which is compatible with $g_1 = 0.59 \pm 0.01 \pm 0.07$ as extracted from the $D^* \rightarrow D\pi$ decay [50,51].

The coupling to the σ meson, in the case of the nucleon-nucleon interaction, can be determined from the nonlinear sigma model [52], yielding

$$g_{\sigma NN} = \sqrt{2} \frac{M_N}{f_\pi} \simeq 10.2. \quad (48)$$

For the case of the D and D^* mesons and Ξ baryons, we can estimate the coupling to the σ from the quark model. If we assume that σ only couples to the u and d quarks, we expect

$$g_{\sigma 1} = g_{\sigma 2} = \frac{g_{\sigma NN}}{3} \simeq 3.4. \quad (49)$$

We can also deduce from SU(3) flavor symmetry and the Okubo-Zweig-Iizuka (OZI) rule that

$$g_{\rho 1} = g_{\omega 1}, \quad (50)$$

$$g_{\rho 2} = g_{\omega 2}. \quad (51)$$

From the universality of the ρ couplings (Sakurai's universality [53]) and the Kawarabayashi-Suzuki-Fayyazuddin-Riazuddin (KSFR) relation [54,55], we expect

$$g_{\rho 1} = g_{\rho 2} = \frac{m_\rho}{2f_\pi^2} \simeq 2.9. \quad (52)$$

Yet, there might be deviations from this value. Regarding the ρ coupling to the heavy mesons, Casalbuoni *et al.* [56] indicate that

$$g_{\rho 1} = \beta \frac{m_\rho}{2f_\pi^2} \simeq 2.6, \quad (53)$$

where $\beta = 0.9$. The ρ coupling thus obtained actually coincides with lattice QCD calculations in the heavy quark limit [57], which yield $g_{\rho 1} = 2.6 \pm 0.1 \pm 0.4$. For the ρ and ω coupling to the cascade, there is also the possibility of obtaining it from the nucleon-nucleon case. In that case, the relevant relations are (see the Appendix)

$$g_{\rho 2} = g_{\rho NN}, \quad (54)$$

$$g_{\omega 2} = \frac{1}{3} g_{\omega NN}. \quad (55)$$

In the nucleon-nucleon case, the SU(3) + OZI relation is

$$g_{\omega NN} = 3g_{\rho NN}, \quad (56)$$

which is compatible with the analogous relation for the cascade baryon once we take into account the quark model. Yet, nuclear potentials usually violate the previous relation, requiring $g_{\omega NN} \sim 20$ or larger, a discrepancy that has been long known in OBE models and usually attributed to the fact that the $g_{\omega NN}$ used in nuclear potentials might indeed also account for some of the short-range quark-gluon exchanges [42]. Yet, this discrepancy can be understood in more modern terms within the renormalization ideas that have become commonplace after the advent of chiral EFT. The explanation lies in the fine-tuning nature of the nucleon-nucleon interaction, which translates into a fine-tuning of the ω coupling. In fact, the ω coupling provides a very important central contribution to the nuclear force, which is responsible in a large part for the concrete values of the scattering lengths of the singlet and triplet channels. By combining the OBE

model with modern renormalization techniques, this discrepancy disappears, and the SU(3) relation is recovered [48]. These findings indicate that the use of the SU(3) relations is the most judicious choice for determining the g_ω couplings, at least for exploratory studies of prospective hadron molecules in which we are not trying to fit fine-tuned systems.

For f_ρ and f_ω , the estimations in the case of the charmed mesons are as follows. SU(3) and the OZI rule imply that

$$f_{\rho 1} = f_{\omega 1}. \quad (57)$$

This relation appears automatically if the Lagrangians are written in terms of the vector meson nonet. Meanwhile, vector meson dominance applied to the weak decays of the charmed mesons [56] brings us to

$$\frac{f_{\rho 1}}{2M_1} = 2\lambda \frac{m_\rho}{2f_\pi} \quad (58)$$

$$= \kappa_{\rho 1} \frac{g_{\rho 1}}{2m_H}, \quad (59)$$

with $|\lambda| = 0.60 \pm 0.11 \text{ GeV}^{-1}$ and where in the second line we have written the coupling of the ρ in the normalization for which we take $M_1 = m_H$ with m_H the mass of the charmed meson. If we take $m_H = m_D$ and $g_{\rho 1} = 2.6$, and assume that λ is positive, we obtain

$$\kappa_{\rho 1} = 4.5 \pm 0.8. \quad (60)$$

For the cascade, the estimations are more involved. The reason is that the coupling of the ρ meson to the octet baryons depends in general on two coupling constants, the symmetric and antisymmetric octet couplings.¹ In the case of the nucleon-nucleon interaction, it is possible to use single vector meson dominance to obtain the relations

$$f_{\rho NN} = \kappa_{\rho NN} g_{\rho NN}, \quad (61)$$

$$f_{\omega NN} = \kappa_{\omega NN} g_{\omega NN}, \quad (62)$$

with $\kappa_{\rho NN} = \mu_p - \mu_n - 1$ and $\kappa_{\omega NN} = \mu_p + \mu_n - 1$, yielding $\kappa_{\rho NN} = 3.7$ and $\kappa_{\omega NN} = -0.1$. This idea can be adapted to the D and D^* charmed mesons and the Ξ baryon, in which case we have $\kappa_{\rho 2} = \mu_{\Xi^0} - \mu_{\Xi^-} - 1$ and $\kappa_{\omega 2} = \mu_{\Xi^0} + \mu_{\Xi^-} + 1$. The application of the previous idea implicitly assumes the convention $M_2 = m_\Xi$, which we will follow onward. From the experimental values $\mu_{\Xi^0} = -0.6507(25)$ and $\mu_{\Xi^-} = -1.250(14)$ listed in the Particle Data Group (PDG) [58], we obtain $\kappa_{\rho 2} = -0.401$ and $\kappa_{\omega 2} = -0.901$. Another possibility is to constrain them from the quark model, in which case we obtain

¹The electric-type coupling of the ρ meson—the g_ρ coupling—is an exception because of its universal character.

$$1 + \kappa_{\rho 2} = -\frac{1}{3} \frac{m_{\Xi}}{m_N} (1 + \kappa_{\rho NN}), \quad (63)$$

$$1 + \kappa_{\omega 2} = -\frac{1}{5} \frac{m_{\Xi}}{m_N} (1 + \kappa_{\rho NN}), \quad (64)$$

which yield $\kappa_{\rho 2} = -3.2$ and $\kappa_{\omega 2} = -1.3$, in stark contrast with the previous estimations. We can also consider the family of phenomenological soft-core potentials by the Nijmegen group [59–64], the extended soft core (ESC) potentials, which also contain estimations for the electric and magnetic couplings of the vector mesons with the cascade. In this case, we have $\kappa_{\rho 2} = -2.0, -0.7, -0.3$ $\kappa_{\omega 2} = -1.1, -1.9, -2.3$ for the ESC04a, ESC04d, and ESC08c potentials, respectively,² though it is worth noticing that $\kappa_{\omega 2}$ is obtained from a value of the omega coupling $g_{\omega 2} \sim (2-3)g_{\rho 2}$ that is considerably larger than the $SU(3) + OZI$ rule expectation. From the previous discussion, it is apparent that there is a considerable level of uncertainty in $\kappa_{\rho 2}$ and $\kappa_{\omega 2}$. For simplicity, we will use the values

$$\kappa_{\rho 2} = \kappa_{\omega 2} = -1.5. \quad (65)$$

This choice is similar to the geometric mean of the previous determinations ($\kappa_{\rho 2} = -1.3$ and $\kappa_{\omega 2} = -1.5$) and to the values we obtain when we compute the cascade magnetic moments at tree level in chiral perturbation theory ($\mu_{\Xi^0} = -1.60$ and $\mu_{\Xi^-} = -0.97$ [65] yielding $\kappa_{\rho 2} = -1.63$ and $\kappa_{\omega 2} = -1.57$). We review the set of parameters we use in the OBE potential in Tables I and II.

D. $D\Xi$ and $D^*\Xi$ wave function

For the molecules we are considering here, the total wave function is the product of the isospin and spin-spatial wave functions,

$$|\Psi\rangle = |IM_I\rangle |\psi_{JM}(\vec{r})\rangle, \quad (66)$$

where J refers to the total angular momentum of the molecule under consideration. The isospin wave function comes from the coupling of the isospin of the two hadrons in the molecule,

$$|IM_I\rangle = \sum_{M_1 M_2} \langle I_1 M_{I1} I_2 M_{I2} | IM \rangle |I_1 M_{I1}\rangle |I_2 M_{I2}\rangle. \quad (67)$$

The only subtlety in the isospin wave function is when the hadron contains a light antiquark $\bar{q}$, for which there will be a minus sign for one of the components of the isospin multiplet. For instance, if we consider the $\bar{D}$ ($\bar{c}q$) and the D ($c\bar{q}$), we have

²We have simply divided the magnetic and electric couplings $\kappa = f/g$ for these potentials.

TABLE I. Masses and quantum numbers of the hadrons from which we form molecules in the present work (plus the nucleon). For the N and heavy mesons, we use the isospin average of the masses listed in the PDG [58]. For the Ξ_{cc} , we use the experimental value of the Ξ_{cc}^{++} mass from the LHCb Collaboration [66], and for the Ξ_{bb} , we use the lattice QCD determination of Ref. [67]. We round the numbers at the MeV level.

Hadron	$I(J^P)$	M (MeV)
N	$\frac{1}{2}(\frac{1}{2}^+)$	938
Ξ	$\frac{1}{2}(\frac{1}{2}^+)$	1318
D	$\frac{1}{2}(0^-)$	1867
D^*	$\frac{1}{2}(1^-)$	2009
B	$\frac{1}{2}(0^-)$	5279
B^*	$\frac{1}{2}(1^-)$	5325
Ξ_{cc}	$\frac{1}{2}(\frac{1}{2}^+)$	3621
Ξ_{bb}	$\frac{1}{2}(\frac{1}{2}^+)$	10127

TABLE II. Masses, quantum numbers, and couplings of the light mesons of the OBE model ($\pi, \sigma, \rho, \omega$). For the magnetic-type coupling of the ρ and ω vector mesons, we have used the decomposition $f_{\rho(\omega)} = \kappa_{\rho(\omega)} g_{\rho(\omega)}$. M refers to the mass scale involved in the magnetic-type couplings.

Meson	$I^G(J^{PC})$	M (MeV)
π	$1^-(0^{-+})$	138
σ	$0^+(0^{++})$	600
ρ	$1^+(1^{--})$	770
ω	$0^-(1^{--})$	780

Coupling	D/D^*	Ξ
g	0.60	-0.25
g_{σ}	3.4	3.4
g_{ρ}	2.6	2.9
g_{ω}	2.6	2.9
κ_{ρ}	4.5	-1.5
κ_{ω}	4.5	-1.5
M	1867	1318

$$\bar{D} = \begin{pmatrix} \bar{D}^0 \\ D^- \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} D^+ \\ -\bar{D}^0 \end{pmatrix}, \quad (68)$$

where the upper and lower components represent the $|\frac{1}{2} + \frac{1}{2}\rangle$ and $|\frac{1}{2} - \frac{1}{2}\rangle$ isospinors, respectively. In constrast, for the cascade, we simply have

$$\Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \quad (69)$$

where we are using the same prescription for the isospinors.

For the part of the wave function containing the spatial and spin pieces, we can express it as a sum of different components with the same parity and total angular momentum, i.e., a partial wave expansion,

$$|\psi_{JM}(\vec{r})\rangle = \sum_{LS} \psi_{LSJ}(r) |^{2S+1}L_J\rangle, \quad (70)$$

where the sum over angular momentum only comprises even or odd L depending on the parity of the molecule $P = (-1)^L$. For the partial wave projection, we have adopted the spectroscopic notation $^{2S+1}L_J$, where S is the total spin, L is the orbital angular momentum, and J is the total angular momentum. We define the $|^{2S+1}L_J\rangle$ as

$$|^{2S+1}L_J\rangle = \sum_{M_S, M_L} \langle LM_L SM_S | JM \rangle |SM_S\rangle Y_{LM_L}(\hat{r}), \quad (71)$$

where $\langle LM_L SM_S | JM \rangle$ is a Clebsch-Gordan coefficient, $Y_{LM_L}(\hat{r})$ is a spherical harmonic, and $|SM_S\rangle$ is the spin wave function, which in turn can be defined as

$$|SM_S\rangle = \sum_{M_{S1}, M_{S2}} \langle S_1 M_{S1} S_2 M_{S2} | SM_S \rangle |S_1 M_{S1}\rangle |S_2 M_{S2}\rangle, \quad (72)$$

with $|S_1 M_{S1}\rangle$ and $|S_2 M_{S2}\rangle$ being the spin wave function of particles 1 and 2.

The mixing of partial waves with the same J but different S/L requires the tensor force. The coupling only happens in the $D^*\Xi$ and $\bar{D}^*\Xi$ cases because for $D\Xi$ and $\bar{D}\Xi$ the tensor force disappears. As molecular states are expected to be more probable for S waves, we will consider only the following partial waves:

$$\left| D\Xi \left(J = \frac{1}{2} \right) \right\rangle = |^2S_{\frac{1}{2}}\rangle, \quad (73)$$

$$\left| D^*\Xi \left(J = \frac{1}{2} \right) \right\rangle = \{|^2S_{\frac{1}{2}}\rangle, |^4D_{\frac{1}{2}}\rangle\}, \quad (74)$$

$$\left| D^*\Xi \left(J = \frac{3}{2} \right) \right\rangle = \{|^4S_{\frac{3}{2}}\rangle, |^2D_{\frac{3}{2}}\rangle, |^4D_{\frac{3}{2}}\rangle\}. \quad (75)$$

The evaluation of the spin-spin and tensor operators for these channels can be found in Table III.

TABLE III. Matrix elements of the spin-spin and tensor operators for the partial waves we are considering in this work; see Eqs. (73), (74), and (75) for the definitions.

	$\vec{a}_1 \cdot \vec{\sigma}_2$	S_{12}
$D\Xi(J = \frac{1}{2})$	0	0
$D^*\Xi(J = \frac{1}{2})$	$\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -\sqrt{2} \\ -\sqrt{2} & -2 \end{pmatrix}$
$D^*\Xi(J = \frac{3}{2})$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$

E. Extension to Ξ_{cc} and Ξ_{bb} baryons

We can extend the OBE model to the doubly heavy baryons in two ways. The first is the quark model, in which case we derive the interactions of the Ξ_{cc} and Ξ_{bb} with the light mesons from the ones for the Ξ . The second is heavy antiquark-diquark symmetry [68–70], in which case the derivation is from the heavy meson interactions.

In the quark model, we expect the strange quark to act as an expectator in what refers to the couplings to the π , σ , ρ , and ω light mesons. From this, the OBE Lagrangian for the Ξ_{cc} and Ξ_{bb} baryons is

$$\mathcal{L}_{\Xi_{QQ}\Xi_{QQ}\pi} = \frac{g_2}{\sqrt{2}f_\pi} \Xi_{QQ}^\dagger \vec{\sigma} \cdot \nabla(\vec{\tau} \cdot \vec{\pi}) \Xi_{QQ}, \quad (76)$$

$$\mathcal{L}_{\Xi_{QQ}\Xi_{QQ}\sigma} = g_{\sigma 2} \Xi_{QQ}^\dagger \sigma \Xi_{QQ}, \quad (77)$$

$$\begin{aligned} \mathcal{L}_{\Xi_{QQ}\Xi_{QQ}\rho} &= g_{\rho 2} \Xi_{QQ}^\dagger \vec{\tau} \cdot \vec{\rho}^0 \Xi_{QQ} \\ &\quad - \frac{f_{\rho 2}}{4M_2} \epsilon_{ijk} \Xi_{QQ}^\dagger \sigma_k \vec{\tau} \cdot (\partial_i \vec{\rho}_j - \partial_j \vec{\rho}_i) \Xi_{QQ}, \end{aligned} \quad (78)$$

$$\begin{aligned} \mathcal{L}_{\Xi_{QQ}\Xi_{QQ}\omega} &= g_{\omega 2} \Xi_{QQ}^\dagger \omega^0 \Xi_{QQ} \\ &\quad - \frac{f_{\omega 2}}{4M_2} \epsilon_{ijk} \Xi_{QQ}^\dagger \sigma_k (\partial_i \omega_j - \partial_j \omega_i) \Xi_{QQ}, \end{aligned} \quad (79)$$

where M_2 is the same as in the original Lagrangian for the cascade. In short, the couplings are the same as for the Ξ .

Heavy antiquark-diquark symmetry (HADS) is a manifestation of heavy quark symmetry, which states that a heavy quark pair behaves as a heavy antiquark [68]. According to this symmetry, the couplings of the Ξ_{cc} and Ξ_{bb} baryons can be deduced from those of the $\bar{D}$, $\bar{D}^*$ and $\bar{B}$, $\bar{B}^*$ heavy antimesons [69,70]. The application of HADS can actually be encapsulated in the two relations between the Lagrangian for the $\bar{P}$ and $\bar{P}^*$ heavy antimesons and the Ξ_{QQ} doubly heavy baryons,

$$\text{Tr}[\bar{H}^\dagger \bar{H}] \rightarrow \Xi_{QQ}^\dagger \Xi_{QQ}, \quad (80)$$

$$\text{Tr}[\bar{H}^\dagger \vec{\sigma} \bar{H}] \rightarrow -\frac{1}{3} \Xi_{QQ}^\dagger \vec{\sigma} \Xi_{QQ}, \quad (81)$$

where the bar over the H field indicates that we are dealing the heavy antimeson superfield. From this, the OBE Lagrangian for the heavy mesons, and changing the sign of the π and ω contributions to take into account their G parity, we arrive at

$$\mathcal{L}_{\Xi_{QQ}\Xi_{QQ}\pi} = -\frac{1}{3} \frac{g_1}{\sqrt{2}f_\pi} \Xi_{QQ}^\dagger \vec{\sigma} \cdot \nabla(\vec{\tau} \cdot \vec{\pi}) \Xi_{QQ}, \quad (82)$$

$$\mathcal{L}_{\Xi_{QQ}\Xi_{QQ}\sigma} = g_{\sigma 1} \Xi_{QQ}^\dagger \sigma \Xi_{QQ}, \quad (83)$$

$$\begin{aligned}\mathcal{L}_{\Xi_{QQ}\Xi_{QQ}\rho} &= g_{\rho 1} \Xi_{QQ}^\dagger \vec{\tau} \cdot \vec{\rho}^0 \Xi_{QQ} \\ &+ \frac{1}{3} \frac{f_{\rho 1}}{4M_1} \epsilon_{ijk} \Xi_{QQ}^\dagger \sigma_k \vec{\tau} \cdot (\partial_i \vec{\rho}_j - \partial_j \vec{\rho}_i) \Xi_{QQ},\end{aligned}\quad (84)$$

$$\begin{aligned}\mathcal{L}_{\Xi_{QQ}\Xi_{QQ}\omega} &= g_{\omega 1} \Xi_{QQ}^\dagger \omega^0 \Xi_{QQ} \\ &+ \frac{1}{3} \frac{f_{\omega 1}}{4M_1} \epsilon_{ijk} \Xi_{QQ}^\dagger \sigma_k (\partial_i \omega_j - \partial_j \omega_i) \Xi_{QQ}.\end{aligned}\quad (85)$$

The comparison with the Lagrangian derived from the quark model entails the HADS predictions

$$(g_2)_{\text{HADS}} = -\frac{g_1}{3}, \quad (86)$$

$$(g_{\sigma 2})_{\text{HADS}} = g_{\sigma 1}, \quad (87)$$

$$(g_{\rho 2})_{\text{HADS}} = g_{\rho 1}, \quad \left(\frac{f_{\rho 2}}{2M_2}\right)_{\text{HADS}} = -\frac{1}{3} \frac{f_{\rho 1}}{2M_1}, \quad (88)$$

$$(g_{\omega 2})_{\text{HADS}} = g_{\omega 1}, \quad \left(\frac{f_{\omega 2}}{2M_2}\right)_{\text{HADS}} = -\frac{1}{3} \frac{f_{\omega 1}}{2M_1}, \quad (89)$$

which can actually be checked against the quark model expectations. Owing to the choices of the couplings made before, we only have to compare five couplings: g_2 , $g_{\rho 2}$, $g_{\omega 2}$, $f_{\rho 2}$, and $f_{\omega 2}$. These comparisons are reduced to three as $g_{\rho 2} = g_{\omega 2}$ from $\text{SU}(3) + \text{OZI}$ and $f_{\rho 2} = f_{\omega 2}$ because of the choice we have made. For the axial coupling, we have

$$(g_2)_{\text{QM}} = -0.25 \text{ vs } (g_2)_{\text{HADS}} = -0.20, \quad (90)$$

which are actually very similar. For the ρ electric-type couplings, we have

$$(g_{\rho 2})_{\text{QM}} = 2.9 \quad \text{vs} \quad (g_{\rho 2})_{\text{HADS}} = 2.6, \quad (91)$$

which differ by 10% only. For the magnetic-type couplings, if we employ $M_2 = m_\Xi$ in the doubly heavy sector, the comparison can be directly made in terms of $\kappa_{\rho 2}$ and $\kappa_{\omega 2}$ instead of $f_{\rho 2}$ and $f_{\omega 2}$, yielding

$$(\kappa_{\rho 2})_{\text{QM}} = -1.5 \quad \text{vs} \quad (\kappa_{\rho 2})_{\text{HADS}} = -1.1 \pm 0.2, \quad (92)$$

plus identical predictions for $\kappa_{\omega 2}$. In this case, the difference is bigger, but both sets of values remain compatible. It is important to notice there that the HADS predictions are expected to be subjected to a sizeable error of $\Lambda_{\text{QCD}}/(m_Q v) \sim 30\% - 40\%$ in the charm sector (instead of the standard $\Lambda_{\text{QCD}}/m_Q \sim 10\% - 15\%$ for Heavy Quark Spin

Symmetry) [68,70], where v is the velocity of the heavy quark in the Ξ_{cc} baryon. For the quark model predictions, the situation is a bit more murky, owing to its status as a model (as they usually lack reliable error estimations). We warn, however, that the apparent similarity of both sets of predictions is not necessarily due to a compatibility between the two models; the choice that we have made for the couplings of the cascade has also played a role.

III. PREDICTIONS OF MOLECULAR STATES

Now, we solve the Schrödinger equation for the $H\Xi$ and $\bar{H}\Xi$ potentials with the coupling constant choices we have made in the previous section. For the cutoff in the form factor, we will fix Λ as to reproduce the $X(3872)$ in the isospin symmetric limit. In this limit, the $X(3872)$ is a $1^{++} D\bar{D}^*$ molecule with a binding energy of about 4 MeV, which corresponds to a binding energy of about 0 MeV if we consider isospin symmetry breaking in the masses of the charmed mesons. With the choices of the couplings previously made, we obtain the value $\Lambda = \Lambda_X \simeq 1.04$ GeV. For this cutoff, the charmed meson–cascade molecules do not bind but are pretty close to binding. The $J = \frac{1}{2}^- D^*\Xi$ system is the most attractive case. It binds for $\Lambda \geq 1.05$ GeV, which is just a tiny fraction above Λ_X . Concrete calculations indicate a scattering length of $a_0 = -18.7$ fm, which is indeed larger than any other scale in the system. For the other two configurations of the charmed meson–cascade system, we find that the $J = \frac{1}{2}^- D\Xi$ and $\frac{3}{2}^- D^*\Xi$ scattering length is $a_0 = -1.8$ fm in both cases, indicating a moderate degree of attraction.

This attraction will become able to bind if we increase the reduced mass of the system. For the $\frac{1}{2}^- \bar{B}^*\Xi$ molecule, the bottom counterpart of the $\frac{1}{2}^- D^*\Xi$, binding happens at $B = 2.9$ MeV. Meanwhile, for the $\frac{1}{2}^- \bar{B}\Xi$ and $\frac{1}{2}^- \bar{B}^*\Xi$ systems, the scattering lengths are expected to be large, $a_0 = -15.1$ fm and -7.2 fm, respectively. Owing to our choice of parameters for the standard cascade, we can basically extend the present calculation to doubly heavy baryons by simply changing the reduced mass in the calculations. Indeed, if we consider the charmed meson–doubly charmed baryon molecules, we find the binding energy of the $\frac{1}{2}^- D^*\Xi_{cc}$ molecule to be $B = 8.7$ MeV, while for the $\frac{1}{2}^- D\Xi_{cc}$ and $\frac{3}{2}^- D^*\Xi_{cc}$ molecules, $B = 0.3$ MeV and 0.2 MeV, respectively. As happened with the $\frac{1}{2}^- D^*\Xi$ system, the $\frac{1}{2}^- D\Xi_{cc}$ and $\frac{3}{2}^- D^*\Xi_{cc}$ molecules are close to the unitary limit, where their scattering lengths are $a_0 = 7.6$ fm and 10.1 fm, respectively. In Fig. 1, we plot the dependence of the binding energy on the reduced mass for the different J^P molecular configurations, where we also indicate the location of the thresholds. For comparison purposes, we also include the mean square radius of these molecules in Fig. 1, which is important to determine whether they are actual bound states (i.e., with a size

FIG. 1. Binding energies (red line) and root-mean-square radii (black line) for the $D\Xi$ and $D^*\Xi$ molecules depending on the reduced mass. The upper, middle, and lower panels correspond to the $\frac{1}{2}^- D\Xi/D\Xi_{QQ}$, $\frac{1}{2}^- D^*\Xi/D^*\Xi_{QQ}$, and $\frac{3}{2}^- D^*\Xi/D^*\Xi_{QQ}$ molecules, respectively. The vertical dotted lines indicate the reduced masses of the different molecules considered. The calculations are made with a form factor cutoff $\Lambda = \Lambda_X = 1.04$ GeV, which corresponds to the cutoff for which the $X(3872)$ is reproduced in the OBE model we use.

bigger than its components) or more compact objects requiring a different type of treatment.

The previous numbers are subject to theoretical errors, which are, however, not easy to estimate because the OBE

potential is a model after all. The coupling constants in the potential are in principle amenable to error estimations. The values we take for $g_{\omega 1}$ and $g_{\omega 2}$ are derived from SU(3) and the OZI rule and hence expected to have an uncertainty of about 20%. For the couplings derived from the quark model, $g_{\sigma 1}$ and $g_{\sigma 2}$, it is less clear which uncertainty to ascribe them, but probably 30% could be a good guess. For $g_{\rho 1}$ and $g_{\rho 2}$, the error depends on how much we expect the KSFR relation to fail, but probably 10% is enough. The axial coupling and its error g_1 are known experimentally, while for g_2 , the error is again determined from the quark model. However, the independent variation of each of the couplings followed by the subsequent addition of the errors in quadrature is cumbersome because of the large number of parameters to vary (not to mention that it is not so easy to determine the error of all of them). We instead find it much more convenient to simply assume a global uncertainty for the D/D^* and Ξ couplings with

$$g_{M1}(1 \pm \Delta_1) \quad \text{and} \quad g_{M2}(1 \pm \Delta_2), \quad (93)$$

where M stands for the π , σ , ρ , and ω mesons and with Δ_1 and Δ_2 the relative error we expect in each of the vertices. If we assume all the couplings to vary in the same direction, i.e., correlated errors, then the outcome is that there is an overall uncertainty in the $X(3872)$ and $D\Xi/D^*\Xi$ potentials. In this picture for a potential with a vertex of type i and j , we will assign the error

$$V_{ij}(1 \pm \Delta_i)(1 \pm \Delta_j), \quad (94)$$

where vertex type 1 refers to D/D^* and vertex type 2 refers to a cascade. We will assume the uncertainties on vertex types 1 and 2 to be uncorrelated. In the previous notation, the $X(3872)$ potential will be V_{11} , and the $D\Xi/D^*\Xi$ potential will be V_{12} . Besides, it is important to stress that the role of the couplings to the charmed mesons and the cascade play a fundamentally different role in the calculations. We are using the $X(3872)$ and hence the couplings of the charmed mesons as a way to fix the unknown parameter Λ in the OBE model, i.e. as a sort of *renormalization condition*. That is, a change in the charmed meson vertex piece of the potential entails a change in Λ_X from which to redo the predictions of the binding energy,

$$V'_{11} = V_{11}(1 \pm \Delta_1)^2 \rightarrow \Lambda'_X \rightarrow B'_{D\Xi}, \quad (95)$$

where $B'_{D\Xi}$ is the binding energy for Λ'_X , where the parameters for vertex 1 in V_{12} have to change congruently like how they change in V_{11} . After this, the error of the cascade baryon vertex $(1 \pm \Delta_2)$ should be added in quadrature. If we follow this procedure and assume a global $\Delta_1 = 0.15$, i.e., a 30% global error in the $X(3872)$ potential, we get $\Lambda_X = 1.04^{+0.18}_{-0.10}$ MeV. If we apply this idea to vertex 2 with $\Delta_2 = 0.15$, the predictions and

TABLE IV. Inverse of the scattering length for the $D\Xi$, $D^*\Xi$ systems and their bottom counterparts $\bar{B}\Xi$ and $\bar{B}^*\Xi$. They are deduced from the condition of reproducing the $X(3872)$ pole in the $D\bar{D}^*$ potential and by assuming a 15% error in the D/D^* and Ξ vertices (where the same applies to the $\bar{B}\Xi$ and $\bar{B}^*\Xi$ systems).

State	$I(J^P)$	$\frac{1}{a_0}$ (MeV)	State	$I(J^P)$	$\frac{1}{a_0}$ (MeV)
$D\Xi$	$0(\frac{1}{2}^-)$	-110^{+110}_{-120}	$\bar{B}\Xi$	$0(\frac{1}{2}^-)$	-10^{+80}_{-90}
$D^*\Xi$	$0(\frac{1}{2}^-)$	-10^{+70}_{-70}	$\bar{B}^*\Xi$	$0(\frac{1}{2}^-)$	$+70^{+60}_{-60}$
$D^*\Xi$	$0(\frac{3}{2}^-)$	-110^{+110}_{-120}	$\bar{B}^*\Xi$	$0(\frac{3}{2}^-)$	-20^{+100}_{-100}

uncertainties for the inverse of the $D\Xi$, $D^*\Xi$, $\bar{B}\Xi$, and $\bar{B}^*\Xi$ scattering lengths can be found in Table IV. Notice the choice of the inverse scattering length; the reason is that the scattering length diverges and then changes sign when there is a bound state. Its inverse, however, changes smoothly, hence the choice. Actually, the uncertainties are still compatible with the existence of bound states in the $D\Xi$ and $D^*\Xi$ systems. For the $P\Xi_{QQ}$ and $P^*\Xi_{QQ}$ bound states, their binding energies and uncertainties can be found in Table V.

We stress that the previous conclusions are derived from the hypothesis that the $X(3872)$ is molecular at the distances in which the OBE model applies. Besides the circumstantial fact that the $X(3872)$ is located close to the $D^0\bar{D}^{*0}$ threshold, the most convincing evidence that the X is molecular is the ratio of its isospin breaking decays $\Gamma(X \rightarrow J/\Psi 2\pi)$ and $\Gamma(X \rightarrow J/\Psi 3\pi)$ [71]. It is relatively easy to explain this branching ratio within the molecular picture [72,73], but not so if the $X(3872)$ is a compact charmoniumlike state [74]. However, the radiative decays $\Gamma(X \rightarrow J/\Psi \gamma)$ and $\Gamma(X \rightarrow \Psi(2S)\gamma)$ [75] offer a different perspective of the $X(3872)$, as they are difficult to explain without the existence of $c\bar{c}$ components in the X -wave function [76]. The $c\bar{c}$ component necessary to successfully explain the radiative components is small [77] but could nonetheless represents a source of systematic uncertainty for our predictions. In this regard, it has been shown that the $c\bar{c}$ short-range components do not necessarily affect the

long range picture of the $X(3872)$ as a hadronic molecule [78]. This means that, while these components are important at short distances, they are probably heavily suppressed at long distances. This is consistent with the approximation we are using here that the $X(3872)$ is molecular. Thus, we do not expect significant corrections to our predictions.

IV. SUMMARY

In this work, we have considered the $D\Xi$ and $D^*\Xi$ systems from the point of view of the OBE model and looked for possible molecular states and their locations. The OBE potential is traditionally regularized with a form factor and a cutoff. The cutoff is expected to be in the 1 GeV range, but there is a considerable uncertainty with respect to its value that translates into wildly different predictions. To reduce this uncertainty, we have determined the cutoff from the condition of reproducing the $X(3872)$ as a $1^{++} D^*\bar{D}$ molecular state, yielding $\Lambda = 1.04^{+0.18}_{-0.10}$ GeV. With this, we find that the $\frac{1}{2}^- D^*\Xi$ state is on the verge of binding, which translates into an unusually large scattering length of -18.7 fm. If we consider the theoretical uncertainties, it turns out that this molecule might very well bind, with the probability of binding being slightly smaller than the probability of not binding. This molecule has also been predicted in Ref. [34]. The $\frac{1}{2}^- D\Xi$ and $\frac{3}{2}^- D^*\Xi$ are unlikely to bind, though their interaction is indeed attractive as can be deduced from their scattering lengths. As a consequence, the interpretation of the $\Omega_c(3188)$ enhancement as a $D\Xi$ bound state [39] is disfavored. The conclusion about a possible $\frac{3}{2}^- D^*\Xi$ molecule is also different from Ref. [34], in which it is predicted, but the previous work includes a series of coupled channels that increase the binding by a small amount. Besides, there is the possibility that these two molecules bind within the uncertainties of our model.

The previous findings can be easily extended to systems in which the D and D^* are substituted by a B and B^* or in which instead of the cascade Ξ we have a doubly heavy baryon Ξ_{cc} or Ξ_{bb} . These systems have a large reduced mass and are thus more likely to bind. For the Ω_b -like molecular state, $\bar{B}\Xi$ and $\bar{B}^*\Xi$, we find that the $\frac{1}{2}^- \bar{B}^*\Xi$ binds within the errors of the present model. We note that this state have also been predicted in Ref. [40]. The other two configurations might bind as well, but this is contingent on the uncertainties. For the triply heavy molecules, we find the $\frac{1}{2}^- P^*\Xi_{QQ}$ system to be the most attractive, binding in all cases. The other two configurations— $\frac{1}{2}^- P\Xi_{QQ}$ and $\frac{3}{2}^- P^*\Xi_{QQ}$ —are less attractive. For the triply charmed pentaquark case, the previous two configurations are probably close to the unitary limit, where their central values for the binding energy are 0.3 and 0.2 MeV, respectively. For the triply bottom pentaquarks, all configurations bind within the theoretical uncertainties of the present model. Triply heavy pentaquarks have been considered previously in the literature. In Ref. [70], HADS is applied to the $X(3872)$ as a

TABLE V. Binding energies for the molecular states considered in this work from the condition that the form factor cutoff is chosen to reproduce the $X(3872)$ pole. The $\dagger$ symbol is used to indicate that the error is large enough to unbind the system.

State	$I(J^P)$	B (MeV)	State	$I(J^P)$	B (MeV)
$D\Xi_{cc}$	$0(\frac{1}{2}^-)$	$0^{+8}_{\dagger}$	$\bar{B}\Xi_{cc}$	$0(\frac{1}{2}^-)$	10^{+25}_{-9}
$D^*\Xi_{cc}$	$0(\frac{1}{2}^-)$	9^{+15}_{-8}	$\bar{B}^*\Xi_{cc}$	$0(\frac{1}{2}^-)$	32^{+27}_{-17}
$D^*\Xi_{cc}$	$0(\frac{3}{2}^-)$	$0^{+9}_{\dagger}$	$\bar{B}^*\Xi_{cc}$	$0(\frac{3}{2}^-)$	$8^{+28}_{\dagger}$
$D\Xi_{bb}$	$0(\frac{1}{2}^-)$	$3^{+15}_{\dagger}$	$\bar{B}\Xi_{cc}$	$0(\frac{1}{2}^-)$	25^{+39}_{-18}
$D^*\Xi_{bb}$	$0(\frac{1}{2}^-)$	19^{+21}_{-13}	$\bar{B}^*\Xi_{cc}$	$0(\frac{1}{2}^-)$	60^{+36}_{-25}
$D^*\Xi_{bb}$	$0(\frac{3}{2}^-)$	$3^{+16}_{\dagger}$	$\bar{B}^*\Xi_{cc}$	$0(\frac{3}{2}^-)$	21^{+47}_{-16}

$D^*\bar{D}$ molecule to deduce the existence of possible $P\Xi_{QQ}$, $P^*\Xi_{QQ}$, $P\Xi_{QQ}^*$, and $P^*\Xi_{QQ}^*$ bound states. From the $X(3872)$, the existence of isoscalar $\frac{3}{2}^-$ $P^*\Xi_{QQ}^*$ pentaquark-like molecules can indeed be deduced, while for the other J^P combinations, the information that can be obtained from the $X(3872)$ is insufficient to predict more states. In this context, the OBE model provides a phenomenological estimation of this missing dynamics, which allows us to fully explore the $P\Xi_{QQ}$ and $P^*\Xi_{QQ}$ cases.

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APPENDIX: COUPLINGS IN THE QUARK MODEL

Here, we present how to compute the couplings of the light mesons to different hadrons in the quark model. At the quark level, the Lagrangians describing quark interactions with light mesons can be written as [79]

$$\begin{aligned}\mathcal{L}_{Mqq} = & g_{\pi qq}(\bar{u}i\gamma_5 u \pi^0 - \bar{d}i\gamma_5 d \pi^0) \\ & + g_{\omega qq}(\bar{u}\gamma_\mu u \omega^\mu + \bar{d}\gamma_\mu d \omega^\mu) \\ & + g_{\rho qq}(\bar{u}\gamma_\mu u \rho^{0\mu} - \bar{d}\gamma_\mu d \rho^{0\mu}) \\ & + g_{\sigma qq}(\bar{u}u\sigma + \bar{d}d\sigma),\end{aligned}\quad (\text{A1})$$

where $g_{\pi qq}$, $g_{\rho qq}$, $g_{\omega qq}$, and $g_{\sigma qq}$ are the couplings of the light $q = u, d$ quarks to the light mesons. The Lagrangian above assumes a nonderivative coupling with the pion, which is nonchirally symmetric. This is inconsequential as the chiral derivative coupling leads to the same tree-level description, i.e., the same one pion exchange potential. To obtain the relations between g_{Mqq} and g_{Mhh} , where the latter refers to the couplings with an arbitrary hadron h , we require the interaction vertices calculated at the quark and hadron levels to be the same, i.e.,

$$\langle h, \vec{s} | \mathcal{L}_{Mhh} | h, \vec{s} \rangle \equiv \langle h, \vec{s} | \mathcal{L}_{Mqq} | h, \vec{s} \rangle, \quad (\text{A2})$$

where H denotes a hadron, $\vec{s}$ denotes its spin, and $\mathcal{L}_{MHH}$ is the OBE Lagrangian for the hadron H . For instance, let us consider the case of the coupling of the nucleon and the cascade to the pion,

$$\langle p \uparrow | \mathcal{L}_{\pi NN} | p \uparrow \rangle = \frac{g_{\pi NN}}{m_N} q_3, \quad (\text{A3})$$

$$\langle \Xi^0 \uparrow | \mathcal{L}_{\pi \Xi \Xi} | \Xi^0 \uparrow \rangle = \frac{g_{\pi \Xi \Xi}}{m_\Xi} q_3, \quad (\text{A4})$$

where q refers to the momentum of the pion. We can directly compare the previous matrix elements to the ones we obtain from the SU(6) quark model wave functions [79], yielding

$$\langle p \uparrow | \mathcal{L}_{\pi qq} | p \uparrow \rangle = \frac{5}{3} \frac{g_{\pi qq}}{m_q} q_3, \quad (\text{A5})$$

$$\langle \Xi^0 \uparrow | \mathcal{L}_{\pi qq} | \Xi^0 \uparrow \rangle = -\frac{1}{3} \frac{g_{\pi qq}}{m_q} q_3. \quad (\text{A6})$$

A direct comparison gives us the relation between $g_{\pi \Xi \Xi}$ and $g_{\pi NN}$:

$$g_{\pi \Xi \Xi} = -\frac{1}{5} \frac{m_\Xi}{m_N} g_{\pi NN}. \quad (\text{A7})$$

Repeating this procedure for the other light mesons, we obtain

$$g_{\sigma \Xi \Xi} = \frac{1}{3} g_{\sigma NN}, \quad (\text{A8})$$

$$g_{\omega \Xi \Xi} = \frac{1}{3} g_{\omega NN}, \quad (\text{A9})$$

$$(g_{\omega \Xi \Xi} + f_{\omega \Xi \Xi}) = -\frac{1}{3} \frac{m_\Xi}{m_N} (g_{\omega NN} + f_{\omega NN}), \quad (\text{A10})$$

$$g_{\rho \Xi \Xi} = g_{\rho NN}, \quad (\text{A11})$$

$$(g_{\rho \Xi \Xi} + f_{\rho \Xi \Xi}) = -\frac{1}{5} \frac{m_\Xi}{m_N} (g_{\rho NN} + f_{\rho NN}). \quad (\text{A12})$$

Thus, we can relate the nucleon coupling constants with the ones for the cascade or with the ones for other hadrons.

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