Froggatt-Nielsen mechanism in a model with $SU(3)_c \times SU(3)_L \times U(1)_X$ gauge group

Katri Huitu^{*} and Niko Koivunen[†]

Helsinki Institute of Physics and Department of Physics, University of Helsinki,
P.O. Box 64 (Gustaf Hållströmin katu 2), FI-00014 Helsinki, Finland

(Received 9 July 2017; revised manuscript received 13 November 2017; published 11 July 2018)

The models with the gauge group $SU(3)_c \times SU(3)_L \times U(1)_X$ (331 models) have been advocated to explain why there are three fermion generations in nature. As such, they can provide partial understanding of the flavor sector. The hierarchy of Yukawa couplings in the Standard Model is another puzzle that remains without a compelling explanation. We propose to use the Froggatt-Nielsen mechanism in a 331 model to explain both fundamental problems. It turns out that no additional representations in the scalar sector are needed to take care of this. The traditional 331 models predict unsuppressed scalar flavor-changing neutral currents at tree level. We show that they are strongly suppressed in our model.

DOI: 10.1103/PhysRevD.98.011701

I. INTRODUCTION

Understanding the number of generations has been attempted using models with an extended gauge group $SU(3)_c \times SU(3)_L \times U(1)_X$ [1–5]. In these so-called 331 models, the chiral anomalies are cancelled if the numbers of triplets and antitriplets are equal. This is possible only if the number of generations is 3. Although the 331 models give a possible explanation for the number of generations, they do not shed any light on vastly different masses among the generations. One possibility is to explain the masses dynamically within the Froggatt-Nielsen (FN) mechanism [6].

So far, no attempt to combine these two approaches has been made to the best of our knowledge. We propose here a model in which a combination of scalars emulates the flavon of the FN mechanism.¹ We emphasize that this requires special properties from the model, as explained later.

We construct here a model in which the gauge symmetry of a 331 model is broken by three scalar triplets, which is the minimum number of triplets to break the symmetry and generate tree-level masses for all the charged fermions. An attractive feature of our model is that the $SU(3)_L \times U(1)_X$ -breaking scale is at the same time the scale of flavor symmetry breaking.

II. PARTICLE CONTENT IN THE 331 MODEL

The symmetry breaking of a 331 model is more involved than in the case of the Standard Model (SM), since two-stage breaking is needed: $SU(3)_L \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$.

Using the three diagonal generators T_3 , T_8 , and X , the electric charge of particles can be defined as

$$Q = T_3 + \beta T_8 + X. \quad (1)$$

Different values of β have been considered in the literature. $\beta = \pm 1/\sqrt{3}$ [1–3] and $\beta = \pm\sqrt{3}$ [4,5] lead to integer charges for gauge bosons and scalars; see, e.g., Ref. [8]. The proposed mechanism will be found for $\beta = \pm \frac{1}{\sqrt{3}}$, with which there are no exotic electric charges.

With $\beta = -\frac{1}{\sqrt{3}}$, only two types of scalar triplets with neutral components can be formed. To generate tree-level masses for all the fermions, three scalar triplets are needed [2]. In our model, we use

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix}, \quad (2)$$

where $\eta \sim (1, 3, \frac{2}{3})$ and $\rho, \chi \sim (1, 3, -\frac{1}{3})$. The numbers in the parentheses label the transformation properties under the gauge group $SU(3)_c \times SU(3)_L \times U(1)_X$.

All the neutral fields can in general develop a nonzero vacuum expectation value (VEV). Degenerate minima are related to each other by $SU(3)_L$ rotation. Thus, the most general combination of VEVs can be written as

$$\langle \eta^0 \rangle = \frac{v'}{\sqrt{2}}, \quad \langle \rho^0 \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle \rho'^0 \rangle = \frac{v_2}{\sqrt{2}}, \quad \langle \chi'^0 \rangle = \frac{u}{\sqrt{2}}. \quad (3)$$

^{*}katri.huitu@helsinki.fi

[†]niko.koivunen@helsinki.fi

¹In frameworks with Higgs doublets, two electrically neutral fields have been used for a similar purpose; see Ref. [7].

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

Here, v_2 and u break the $SU(3)_L \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y$, and thus we expect $v_2, u \gg v', v_1$.

As required by anomaly cancellation, the same number of triplets and antitriplets is needed. Here, this is achieved by assigning the leptons and one of the quark families to $SU(3)_L$ triplets, while two quark families belong to antitriplets. The leptons are given by

$$L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ \nu'_i \end{pmatrix}_L \sim \left(1, 3, -\frac{1}{3}\right), \quad e_{R,i} \sim (1, 1, -1), \quad (4)$$

where $i = 1, 2, 3$. The fields ν'_i are new neutral leptons. For $\beta = \pm\sqrt{3}$, in the minimal case, the triplet includes charged lepton e_i^c instead of ν'_i , and no charged singlets are required. However, as discussed later, the scalar structure of such a model does not suit our purposes.

We choose to assign first quark generation into a triplet and the second and the third into an antitriplet:

$$\begin{aligned} Q_{L,1} &= \begin{pmatrix} u_1 \\ d_1 \\ U \end{pmatrix}_L \sim \left(3, 3, \frac{1}{3}\right), \\ Q_{L,2} &= \begin{pmatrix} d_2 \\ -u_2 \\ D_1 \end{pmatrix}_L, \quad Q_{L,3} = \begin{pmatrix} d_3 \\ -u_3 \\ D_2 \end{pmatrix}_L \sim (3, 3^*, 0), \\ u_{R,i}, \quad U_R &\sim \left(3, 1, \frac{2}{3}\right), \quad d_{R,i}, \\ D_{R,1}, \quad D_{R,2} &\sim \left(3, 1, -\frac{1}{3}\right), \end{aligned} \quad (5)$$

where $i = 1, 2, 3$. We have introduced new quarks D_1 and D_2 with electric charge $-1/3$ and U with electric charge $2/3$.

III. FROGGATT-NIELSEN MECHANISM

Froggatt and Nielsen proposed a new symmetry, the so-called flavor symmetry [e.g., $U(1)$ or Z_N], to be responsible for the huge differences in fermion masses in the SM [6]. The FN charge assignment is such that the SM Yukawa couplings are forbidden, but the following effective operator is allowed,

$$c_{ij}^f (\phi/\Lambda)^{n_{ij}^f} \bar{\psi}_{L,i}^f H f_{R,j} + \text{H.c.}, \quad (6)$$

where c_{ij}^f is a dimensionless order-1 number, Λ is the scale of new physics, ψ_i and f_i are the SM fermions, H is the SM Higgs doublet, and ϕ is a complex scalar field called a flavon. The SM Yukawa couplings are generated as effective couplings when a flavon gets a VEV,

$$y_{ij}^f = c_{ij}^f (\langle \phi \rangle / \Lambda)^{n_{ij}^f}. \quad (7)$$

Our purpose here is to mimic the gauge singlet flavon by a combination of the existing scalars in the model.

IV. FROGGATT-NIELSEN MECHANISM IN THE 331 MODEL

We introduce a global $U(1)_{FN}$ symmetry to our model. Fermions and scalars are charged under it. Two of the three triplets in Eq. (2) carry necessarily the same $U(1)_X$ charge. For our purposes, this is crucial, since the combination $\rho^\dagger \chi$ is a gauge singlet. Therefore, it can act as a flavon field when developing a VEV, if the FN charge of the combination does not vanish. We take the FN charge of scalar triplets as follows:

$$q_\eta = -1, \quad q_\rho = 1, \quad q_\chi = 0. \quad (8)$$

Note that the scalar representations generating tree-level masses for fermions in models with $\beta = \pm\sqrt{3}$ [4,5] cannot form an $SU(3)_L \times U(1)_X$ singlet with a FN charge. Thus, in these models, an enlarged scalar sector is needed for the FN mechanism.

The effective flavon obtains a nonzero vacuum expectation value,

$$\langle \rho^\dagger \chi \rangle = (v_2 u) / 2. \quad (9)$$

The relevant effective operator is now

$$(c_s^f)_{ij} \left(\frac{\rho^\dagger \chi}{\Lambda^2} \right)^{(n_f^s)_{ij}} \bar{\psi}_{L,i}^f S f_{R,j} + \text{H.c.}, \quad (10)$$

where $(c_s^f)_{ij}$ is a dimensionless order-1 number. The fermion (anti)triplets and singlets $\bar{\psi}_{L,i}^f$ and $f_{R,j}$ have FN charges $q(\bar{\psi}_{L,i}^f)$ and $q(f_{R,j})$, respectively. S denotes any of the three scalar triplets η, ρ , or χ with which the combination is a gauge singlet. The $(n_f^s)_{ij}$ is determined by the FN charge assignment,

$$(n_f^s)_{ij} = [q(\bar{\psi}_{L,i}^f) + q(f_{R,j}) + q(S)]. \quad (11)$$

Here, $v_2 \sim u$ is responsible for both the $SU(3)_L \times U(1)_X$ breaking and of the flavor symmetry breaking, while Λ is the scale of new physics. For simplicity, we have chosen to work with charge assignments that keep $(n_f^s)_{ij}$ non-negative. If $(n_f^s)_{ij}$ were negative, we would include $\chi^\dagger \rho$ instead of $\rho^\dagger \chi$.

As one expands (10) around the minimum, one obtains

$$\begin{aligned} &(y_s^f)_{ij} \bar{\psi}_{L,i}^f (S + \langle S \rangle) f_{R,j} + (n_f^s)_{ij} (y_s^f)_{ij} \\ &\times \left[\frac{\rho'^{0*}}{v_2} + \frac{\chi'^0}{u} + \frac{v_1 \chi'^0}{v_2 u} \right] \sqrt{2} \bar{\psi}_{L,i}^f \langle S \rangle f_{R,j} + \text{H.c.} + \dots, \end{aligned} \quad (12)$$

where we have kept only the renormalizable contributions. The first term gives the fermion masses and the usual Yukawa interaction:

$$(y_s^f)_{ij} = (c_s^f)_{ij} \left(\frac{v_2 u}{2\Lambda^2} \right)^{(n_f^s)_{ij}} \equiv (c_s^f)_{ij} e^{(n_f^s)_{ij}}. \quad (13)$$

The other term of (12) is the artifact of the Froggatt-Nielsen mechanism introducing a source of flavor violation into the model. The flavor-violating term is suppressed by the $SU(3)_L$ -breaking scale, u and v_2 , in the case of the SM fermions. Therefore, the flavor violation coming from this additional term is negligible.

V. SYMMETRY BREAKING AND THE BOSONIC SECTOR

The most general scalar potential of our model, consistent with the Froggatt-Nielsen mechanism, is

$$\begin{aligned} V_{\text{FN}} = & \mu_1^2 \eta^\dagger \eta + \mu_2^2 \rho^\dagger \rho + \mu_3^2 \chi^\dagger \chi + \lambda_1 (\eta^\dagger \eta)^2 + \lambda_2 (\rho^\dagger \rho)^2 \\ & + \lambda_3 (\chi^\dagger \chi)^2 + \lambda_{12} (\eta^\dagger \eta) (\rho^\dagger \rho) + \lambda_{13} (\eta^\dagger \eta) (\chi^\dagger \chi) \\ & + \lambda_{23} (\rho^\dagger \rho) (\chi^\dagger \chi) + \tilde{\lambda}_{12} (\eta^\dagger \rho) (\rho^\dagger \eta) + \tilde{\lambda}_{13} (\eta^\dagger \chi) (\chi^\dagger \eta) \\ & + \tilde{\lambda}_{23} (\rho^\dagger \chi) (\chi^\dagger \rho) + \sqrt{2} f (\epsilon_{ijk} \eta^i \rho^j \chi^k + \text{H.c.}). \end{aligned}$$

One scalar and seven pseudoscalar massless degrees of freedom are needed for the missing polarization states of the gauge bosons of the model, namely, the Z , $W^\pm$ of the SM, new heavy charged gauge bosons, a heavy neutral gauge boson, and a non-Hermitian heavy neutral gauge boson. Thus, in the physical spectrum, one has four scalars, two pseudoscalars, and two charged Higgs bosons.

The global $U(1)_{\text{FN}}$ symmetry is spontaneously broken by the scalar field VEVs, and one of the pseudoscalars will be a massless Goldstone boson. To give a mass for the Goldstone boson, we add the following soft FN symmetry-breaking term to the potential:

$$V_{\text{soft}} = b(\rho^\dagger \chi) + \text{H.c.} \quad (14)$$

One of the scalar eigenvalues is given by light VEVs corresponding to the experimentally detected Higgs boson mass. The remaining three scalars as well as the charged Higgses are heavy with masses proportional to large VEVs and decouple. One of the physical pseudoscalars is heavy, and the mass of the pseudo-Goldstone boson, A_2 , is proportional to the soft parameter b .

VI. FERMION MASSES IN THE MODEL

The charged lepton masses are generated by the term

$$\mathcal{L} \supset y_{ij}^e \bar{L}'_{L,i} \eta e'_{R,j} + \text{H.c.}, \quad (15)$$

where the prime denotes a gauge eigenstate. The mass matrix is given by

$$\mathcal{L}_{\ell\text{-mass}} = m_{ij}^e \bar{e}'_{L,i} e'_{R,j} + \text{H.c.} \quad \text{with} \quad m_{ij}^e = (y_{ij}^e)_{ij} \frac{v'}{\sqrt{2}}.$$

The mass matrix is proportional to the Yukawa-coupling matrix. Both will be diagonalized simultaneously. As for

neutrinos, with the fields in (4), one of the neutrinos gets mass only radiatively, at the same time raising degeneracy from the masses of the other two. Such an approach is demonstrated, e.g., in Ref. [2]. This is the approach we adopt here. Tree-level masses can also be produced, if new neutral fermions $N_{R,i}$ are introduced [9].

The up-type quark masses are generated by the following terms:

$$\begin{aligned} \mathcal{L} \supset & \sum_{\gamma=1}^4 (y_\rho^u)_{1\gamma} \bar{Q}'_{L,1} \rho u'_{R,\gamma} + \sum_{\gamma=1}^4 (y_\chi^u)_{1\gamma} \bar{Q}'_{L,1} \chi u'_{R,\gamma} \\ & + \sum_{\alpha=2}^3 \sum_{\gamma=1}^4 (y_{\eta^*}^u)_{\alpha\gamma} \bar{Q}'_{L,\alpha} \eta^* u'_{R,\gamma} + \text{H.c.} \end{aligned} \quad (16)$$

The up-quark mass matrix elements in the gauge eigenbasis are given by

$$\mathcal{L}_{\text{up-mass}} = \bar{u}'_L m^u u'_R + \text{H.c.} \quad \text{with} \quad (17)$$

$$m_{1\gamma}^u = \frac{v_1}{\sqrt{2}} (y_\rho^u)_{1\gamma}, \quad m_{\alpha\gamma}^u = -\frac{v'}{\sqrt{2}} (y_{\eta^*}^u)_{\alpha\gamma}, \quad (18)$$

$$m_{4\gamma}^u = \frac{v_2}{\sqrt{2}} (y_\rho^u)_{1\gamma} + \frac{u}{\sqrt{2}} (y_\chi^u)_{1\gamma}, \quad (19)$$

where $\alpha = 2, 3$ and $\gamma = 1, 2, 3, 4$. The down-quark mass matrix and the terms that generate it are given in the Appendix.

In 331 setting, the hierarchy of the matrix elements is determined by the FN charge assignment and different VEVs, in contrast to the traditional FN mechanism, in which the hierarchy is set solely by the charge assignment. The hierarchy of VEVs, $u, v_2 \gg v_1, v'$, greatly affects the hierarchy in the matrix. We rewrite the elements of the mass matrix to clarify the hierarchy,

$$m_{1\gamma}^u = \frac{v'}{\sqrt{2}} \left[\frac{v_1}{v'} (c_\rho^u)_{1\gamma} \right] \epsilon^{a_1^u + q(u_{R,\gamma})}, \quad (20)$$

$$m_{\alpha\gamma}^u = \frac{v'}{\sqrt{2}} [-(c_{\eta^*}^u)_{\alpha\gamma}] \epsilon^{a_\alpha^u + q(u_{R,\gamma})}, \quad \alpha = 2, 3,$$

$$m_{4\gamma}^u = \frac{v'}{\sqrt{2}} \left[(c_\rho^u)_{1\gamma} \epsilon^{q(\rho) - q(\chi)} + (c_\chi^u)_{1\gamma} \frac{u}{v_2} \right] \epsilon^{a_4^u + q(u_{R,\gamma})}, \quad (21)$$

where the quantities in square brackets are order-1 numbers and therefore the hierarchy is completely set by the powers of ϵ . The a_γ^u are

$$\begin{aligned} a_1^u &= q(\bar{Q}_{L,1}) + q(\rho), \quad a_\alpha^u = q(\bar{Q}_{L,\alpha}) + q(\eta^*), \quad \alpha = 2, 3, \\ a_4^u &= (\log \epsilon)^{-1} \log \left(\frac{v_2}{v'} \right) + q(\bar{Q}_{L,1}) + q(\chi). \end{aligned} \quad (22)$$

The difference between two symmetry-breaking scales manifests itself as effective left-handed charges, analogous to FN charges of the left-handed fermion doublets in the

original FN mechanism. This way, the textures of the diagonalization matrices can be easily obtained.

VII. SUPPRESSION OF FLAVOR-CHANGING NEUTRAL CURRENTS

As explicitly shown in Eq. (16), the quarks couple to multiple scalar triplets, and 331 models generally predict scalar-mediated flavor-changing neutral currents (FCNCs) of quarks at tree level [10], without a natural suppression mechanism. Although this is not the main motivation in our current work, we shortly discuss the suppression of FCNCs in the model and return to this in another work [9]. The gauge sector in our model remains the same as in other 331 models, and the Glashow-Iliopoulos-Maiani (GIM) mechanism [11] has been studied in 331 models previously [12].

Although most of the scalars are heavy, the FCNC mediated by the lightest Higgs, h , or by the pseudo-Goldstone boson A_2 can be large. Here, we assume that b is large, which suppresses the FCNC by A_2 , and concentrate on the Higgs boson.

In our model, the charged lepton generations are assigned to the same representations. The only flavor-violating leptonic couplings to the neutral scalars are due to the FN mechanism, coming from the second term in Eq. (12). However, it is suppressed by the large $SU(3)_L$ -breaking VEVs and is negligible.

The Yukawa interactions of quarks with the lightest Higgs boson can be written as

$$\mathcal{L}_{q,\text{Yukawa}} = \frac{1}{\sqrt{2}} \bar{u}_L U_L^u \Gamma_h^u U_R^{u\dagger} u_R h + \frac{1}{\sqrt{2}} \bar{d}_L U_L^d \Gamma_h^d U_R^{d\dagger} d_R h,$$

where the quark gauge eigenstates have been rotated to the physical states by $U_{L,R}^{u,d}$ matrices.

We ignore here the heavily suppressed FN contributions to Yukawa interactions of the quarks. The physical Yukawa couplings are $\Gamma_h^u = U_L^u \Gamma_h^u U_R^{u\dagger}$ and $\Gamma_h^d = U_L^d \Gamma_h^d U_R^{d\dagger}$.

By using the explicit form for the coupling matrices and approximating the h eigenvector, we can write the coupling matrix in the form

$$(\Gamma_h^u)_{ij} = \sqrt{2} \frac{m_j}{v_{\text{SM}}} \left[\delta_{ij} + \alpha_1 (U_L^u)_{i1} (U_L^{u\dagger})_{1j} - (U_L^u)_{i4} (U_L^{u\dagger})_{4j} + \alpha_2 (U_L^u)_{i1} (U_L^{u\dagger})_{4j} + \alpha_3 (U_L^u)_{i4} (U_L^{u\dagger})_{1j} \right], \quad (23)$$

where $\alpha_i = \mathcal{O}(v_{\text{light}}/v_{\text{heavy}})$, $i = 1, 2, 3$, with $v_{\text{light}} = v'$, v_1 and $v_{\text{heavy}} = u$, v_2 . The corresponding couplings $(\Gamma_h^d)_{ij}$ follow a similar pattern. We assume

$$m_{i,j}^q \leq m_{i+1,j}^q, \quad (24)$$

which can be ensured by a proper choice of FN charges. Rotation matrix elements then satisfy $(U_L^u)_{ij} \sim \epsilon^{|a_i'' - a_j''|}$, which provides additional suppression to FCNCs.

For a concrete example, we set $\epsilon = 0.23$ and consider the following FN charge assignments for the left-handed quark triplets: $q(Q_{L,1}^c) = 2$, $q(Q_{L,2}^c) = 1$, and $q(Q_{L,3}^c) = -1$. These charge assignments will produce the correct texture for the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The FN charges of the right-handed quarks are $q(u_{R,1}) = 5$, $q(u_{R,2}) = 2$, $q(u_{R,3}) = 0$, $q(u_{R,4}) = 0$, $q(d_{R,1}) = 7$, $q(d_{R,2}) = 5$, $q(d_{R,3}) = 4$, $q(d_{R,4}) = 3$, and $q(d_{R,5}) = 2$. The up-type Yukawa-matrix texture becomes

$$\Gamma_h^u \sim \begin{pmatrix} y_u & y_c[\alpha\epsilon^1] & y_t[\alpha\epsilon^x] & \epsilon^2 \\ y_u[\alpha\epsilon^1] & y_c & y_t[\alpha\epsilon^x] & \epsilon^2 \\ y_u[\alpha\epsilon^x] & y_c[\alpha\epsilon^x] & y_t & 1 \\ y_u[\alpha] & y_c[\alpha] & y_t[\epsilon^x] & \epsilon^x \end{pmatrix}, \quad (25)$$

where $x = (\log \epsilon)^{-1} \log(\frac{v'}{v_2}) - 2 \geq 0$ due to Eq. (24) and $\alpha = \mathcal{O}(v_{\text{light}}/v_{\text{heavy}})$. The diagonal couplings of the SM quarks are the SM Yukawa couplings, and the off-diagonal elements are suppressed by the ratio of the two VEV scales and the powers of ϵ . A similar texture applies for the down-type Yukawa matrix. We find that the heavy VEVs $\sim \mathcal{O}(5 \text{ TeV})$ can sufficiently suppress all the meson mixing constraints $M^0 - \bar{M}^0$, $M = K, B_d, B_s, D$, which provide the tightest constraints in our case.

The texture in Eq. (25) produces exotic quark masses that are suppressed compared to the $SU(3)_L$ -breaking scale: $m_U \sim \epsilon^2 v_{\text{heavy}}$, $m_{D_1} \sim \epsilon^3 v_{\text{heavy}}$, and $m_{D_2} \sim \epsilon^0 v_{\text{heavy}}$. The experimental mass limit for exotic quarks is $\mathcal{O}(1 \text{ TeV})$ [13], and thus v_{heavy} should be somewhat larger than required by the suppression of FCNCs. The textures for the SM quark masses are $m_u \sim \epsilon^8 v_{\text{light}}$, $m_c \sim \epsilon^4 v_{\text{light}}$, $m_t \sim \epsilon^0 v_{\text{light}}$, $m_d \sim \epsilon^8 v_{\text{light}}$, $m_s \sim \epsilon^5 v_{\text{light}}$, and $m_b \sim \epsilon^2 v_{\text{light}}$.

The CKM matrix is not a square matrix in this model but a 4×5 matrix:

$$\mathcal{L}_{gCC} = \frac{g_3}{\sqrt{2}} \bar{u}_L \gamma^\mu V_{\text{CKM}}^{331} d_L W_\mu^+ + \text{H.c.} \quad (26)$$

The CKM-matrix texture in our example is

$$V_{\text{CKM}}^{331} \sim \begin{pmatrix} 1 & \epsilon^1 & \epsilon^3 & \epsilon^1 \alpha & \epsilon^3 \alpha \\ \epsilon^1 & 1 & \epsilon^2 & \alpha & \epsilon^2 \alpha \\ \epsilon^3 & \epsilon^2 & 1 & \epsilon^{-2} \alpha & \alpha \\ \alpha & \alpha & \epsilon^{-1} \alpha & \epsilon^{-4} \alpha^2 & \epsilon^{-2} \alpha^2 \end{pmatrix}. \quad (27)$$

The 3×3 block in the upper-left corner corresponds to the CKM matrix of the Standard Model. The W_μ boson couples to the exotic quarks, and they contribute to the neutral meson mixing. We find that the W -mediated meson mixing is always subleading to the Higgs-mediated mixing.

For a numerical example, we set the $SU(3)_L$ -breaking VEVs to be $u = 48 \text{ TeV}$ and $v_2 = 55 \text{ TeV}$, and the $SU(2)_L$ -breaking VEVs are $v_1 = 100 \text{ GeV}$ and

$v' = 237.05$ GeV. The exact quark mass matrices are given in the Appendix, and they produce the experimentally measured quark masses [13]. The exotic quark masses become $m_U = 5$ TeV, $m_{D_1} = 1.295$ TeV, and $m_{D_2} = 50.9$ TeV. The physical up-type Yukawa-coupling matrix is

$$\Gamma_h^u = \begin{pmatrix} 1.7 \times 10^{-5} & 2.9 \times 10^{-9} & -5.0 \times 10^{-5} & -2.5 \times 10^{-2} \\ -7.7 \times 10^{-11} & 7.3 \times 10^{-3} & -6.3 \times 10^{-5} & -3.2 \times 10^{-2} \\ -5.2 \times 10^{-9} & -5.5 \times 10^{-7} & 9.9 \times 10^{-1} & -1.6 \\ -9.2 \times 10^{-8} & -9.7 \times 10^{-6} & -5.7 \times 10^{-2} & 9.3 \times 10^{-2} \end{pmatrix}. \quad (28)$$

A similar Yukawa-coupling matrix can be found for the down-type quarks. The CKM matrix is

$$|V_{\text{CKM}}^{331}| = \begin{pmatrix} 0.974 & 0.226 & 0.00332 & 0.000014 & 0.000048 \\ 0.23 & 0.97 & 0.0434 & 0.00010 & 0.000086 \\ 0.007 & 0.0430 & 0.997 & 0.017 & 0.0001 \\ 0.00073 & 0.0016 & 0.057 & 0.00099 & 6.7 \times 10^{-6} \\ 0.00073 & 0.0016 & 0.057 & 0.00099 & 6.7 \times 10^{-6} \end{pmatrix}. \quad (29)$$

The SM CKM matrix elements are produced correctly at 2σ confidence level. We have checked [9] that the neutral meson mixing bounds [14] are satisfied in this example.

VIII. CONCLUSION

It is interesting that the Froggatt-Nielsen mechanism, with which the hierarchical structure of fermions can be realized using gauge singlet combination of triplets as an effective flavon, can be embedded in a 331 model in which also the number of generations can be understood. Furthermore, the scale of the flavor breaking is the same as the breaking scale of the symmetry of the model. To form an effective flavon, no new scalar triplets beyond those that are needed to generate the tree-level masses for the particles are necessary. We also indicate that the tree-level FCNCs are suppressed in our model.

ACKNOWLEDGMENTS

K. H. acknowledges the H2020-MSCA-RICE-2014 Grant No. 645722 (NonMinimalHiggs). N. K. is supported by the Vilho, Yrjö and Kalle Väisälä Foundation.

APPENDIX: d-QUARK MASS MATRIX AND NUMERICAL PARAMETERS

The down-type quark mass matrix is generated by the following terms in the Lagrangian:

$$\mathcal{L} \supset \sum_{\gamma=1}^5 (y_{\eta}^d)_{1\gamma} \bar{Q}'_{L,1} \eta d'_{R,\gamma} + \sum_{\alpha=2}^3 \sum_{\gamma=1}^5 (y_{\rho^*}^d)_{\alpha\gamma} \bar{Q}'_{L,\alpha} \rho^* d'_{R,\gamma} + \sum_{\alpha=2}^3 \sum_{\gamma=1}^5 (y_{\chi^*}^d)_{\alpha\gamma} \bar{Q}'_{L,\alpha} \chi^* d'_{R,\gamma} + \text{H.c.}$$

The down-type quark mass matrix is

$$m_{1\gamma}^d = \frac{v'}{\sqrt{2}} (c_{\eta}^d)_{1\gamma} \epsilon^{(h^{\eta})_{1\gamma}}, \quad m_{\alpha\gamma}^d = \frac{v_1}{\sqrt{2}} (c_{\rho^*}^d)_{\alpha\gamma} \epsilon^{(h^{\rho^*})_{\alpha\gamma}},$$

$$m_{(2+\alpha)\gamma}^d = \frac{v_2}{v_1} m_{\alpha\gamma}^d + \frac{u}{\sqrt{2}} (c_{\chi^*}^d)_{\alpha\gamma} \epsilon^{(h^{\chi^*})_{\alpha\gamma}},$$

where $\alpha = 2, 3$ and

$$(h^{\eta})_{1\gamma} = q(\bar{Q}_{L,1}) + q(\eta) + q(d_{R,\gamma})$$

$$(h^{\rho^*})_{\alpha\gamma} = q(\bar{Q}_{L,\alpha}) + q(\rho^*) + q(d_{R,\gamma})$$

$$(h^{\chi^*})_{\alpha\gamma} = q(\bar{Q}_{L,\alpha}) + q(\chi^*) + q(d_{R,\gamma}).$$

The order-1 coefficients are restricted to the interval $|c| \in [0.5, 5]$. The order-1 coefficients for down-type quarks are $(c_{\eta}^d)_{11} = 3.1679$, $(c_{\eta}^d)_{12} = 1.0274$, $(c_{\eta}^d)_{13} = 1.3083$, $(c_{\eta}^d)_{14} = 1.0222$, $(c_{\eta}^d)_{15} = -1.0835$, $(c_{\rho^*}^d)_{21} = -1.3342$, $(c_{\rho^*}^d)_{22} = 2.0875$, $(c_{\rho^*}^d)_{23} = -0.8140$, $(c_{\rho^*}^d)_{24} = 1.3463$, $(c_{\rho^*}^d)_{25} = 1.6967$, $(c_{\rho^*}^d)_{31} = -1.1293$, $(c_{\rho^*}^d)_{32} = 1.9257$, $(c_{\rho^*}^d)_{33} = 1.9138$, $(c_{\rho^*}^d)_{34} = 1.8382$, $(c_{\rho^*}^d)_{35} = -0.5945$, $(c_{\chi^*}^d)_{21} = 0.9823$, $(c_{\chi^*}^d)_{22} = -0.9842$, $(c_{\chi^*}^d)_{23} = -1.5314$, $(c_{\chi^*}^d)_{24} = 4.4024$, $(c_{\chi^*}^d)_{25} = -4.7233$, $(c_{\chi^*}^d)_{31} = 1.7299$, $(c_{\chi^*}^d)_{32} = -1.0801$, $(c_{\chi^*}^d)_{33} = -0.5451$, $(c_{\chi^*}^d)_{34} = -4.5226$, and $(c_{\chi^*}^d)_{35} = -3.4529$.

The order-1 coefficients for up-type quarks are $(c_{\rho}^u)_{11} = -3.3709$, $(c_{\rho}^u)_{12} = 2.4799$, $(c_{\rho}^u)_{13} = 1.1381$, $(c_{\rho}^u)_{14} = 1.5495$, $(c_{\eta^*}^u)_{21} = -0.6744$, $(c_{\eta^*}^u)_{22} = -1.6193$, $(c_{\eta^*}^u)_{23} = 1.0648$, $(c_{\eta^*}^u)_{24} = 1.3911$, $(c_{\eta^*}^u)_{31} = 1.3571$, $(c_{\eta^*}^u)_{32} = 1.3324$, $(c_{\eta^*}^u)_{33} = 0.9053$, $(c_{\eta^*}^u)_{34} = 1.7678$, $(c_{\chi^*}^u)_{11} = 1.6150$, $(c_{\chi^*}^u)_{12} = 0.6765$, $(c_{\chi^*}^u)_{13} = 2.0687$, and $(c_{\chi^*}^u)_{14} = 1.0458$.

The scalar potential parameters used are $\lambda_1 = 0.4$, $\lambda_2 = 0.2898$, $\lambda_3 = 0.9$, $\lambda_{12} = 0.5$, $\lambda_{13} = 0.5$, $\lambda_{23} = 0.22$, $\tilde{\lambda}_{12} = 0.8$, $\tilde{\lambda}_{13} = 0.9$, $\tilde{\lambda}_{23} = -0.1$, and $b = -(10 \text{ TeV})^2$. All the scalar masses are positive with these parameters. Except for the SM-like Higgs, all the scalars are heavy with their masses being larger than 10 TeV.

- [1] M. Singer, J. W. F. Valle, and J. Schechter, *Phys. Rev. D* **22**, 738 (1980).
- [2] J. W. F. Valle and M. Singer, *Phys. Rev. D* **28**, 540 (1983).
- [3] R. Foot, H. N. Long, and T. A. Tran, *Phys. Rev. D* **50**, R34 (1994).
- [4] F. Pisano and V. Pleitez, *Phys. Rev. D* **46**, 410 (1992).
- [5] P. H. Frampton, *Phys. Rev. Lett.* **69**, 2889 (1992).
- [6] C. D. Froggatt and H. B. Nielsen, *Nucl. Phys. B* **147**, 277 (1979).
- [7] K. S. Babu and S. Nandi, *Phys. Rev. D* **62**, 033002 (2000); G. F. Giudice and O. Lebedev, *Phys. Lett. B* **665**, 79 (2008); M. Bauer, M. Carena, and K. Gemmler, *J. High Energy Phys.* **11** (2015) 016; *Phys. Rev. D* **94**, 115030 (2016).
- [8] Q. H. Cao and D. M. Zhang, [arXiv:1611.09337](https://arxiv.org/abs/1611.09337).
- [9] K. Huitu and N. Koivunen (to be published).
- [10] S. L. Glashow and S. Weinberg, *Phys. Rev. D* **15**, 1958 (1977); E. A. Paschos, *Phys. Rev. D* **15**, 1966 (1977).
- [11] S. L. Glashow, J. Iliopoulos, and L. Maiani, *Phys. Rev. D* **2**, 1285 (1970).
- [12] H. Georgi and A. Pais, *Phys. Rev. D* **19**, 2746 (1979); J. C. Montero, F. Pisano, and V. Pleitez, *Phys. Rev. D* **47**, 2918 (1993); M. Ozer, *Phys. Rev. D* **54**, 4561 (1996).
- [13] C. Patrignani *et al.* (Particle Data Group), *Chin. Phys. C* **40**, 100001 (2016).
- [14] M. Bona *et al.* (UTfit Collaboration), *J. High Energy Phys.* **03** (2008) 049.