Light-cone distribution amplitudes of vector meson in a large momentum effective theory

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We investigate the leading twist light-cone distribution amplitudes (LCDAs) of vector meson in the framework of large momentum effective theory. We derive the matching equation for the LCDAs and quasidistribution amplitudes. The matching coefficients are determined to one loop accuracy, both in the ultraviolet cutoff and dimensional regularization schemes. This calculation provides the possibility of studying the full x behavior of LCDAs and extracting LCDAs of vector mesons from lattice simulations.

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I. INTRODUCTION

The light-cone distribution amplitudes (LCDAs), defined by the matrix elements of light-cone separated field operators, are essential for studying exclusive processes and hadron structures. They describe the probability amplitudes of finding Fock states in a hadron. The amplitude of an exclusive process can be factorized as the convolution of LCDA and hard kernel, if the collinear factorization theorem is established [1]. The LCDAs of vector mesons are of particular interest since they are necessary in the theoretical analysis of exclusive processes involving vector mesons, for example, the production of vector mesons at high energy colliders, and B meson decays like $B \rightarrow V$, where $V = \rho, K^*, \phi$, etc. Among these processes, the $B \rightarrow K^* \ell^+ \ell^-$ and $B_s \rightarrow \phi \ell^+ \ell^-$ are considered to be sensitive to the new physics. Measurement of the angular distributions and the lepton flavor universality violation may shed light on the evidence of new physics [2–4]. The determination of LCDAs is thus also important for searching new physics beyond the Standard Model. Furthermore, LCDAs are also crucial in light-cone QCD sum rules, perturbative QCD approach, and so on. Because of their nonperturbative nature, LCDAs cannot be calculated with QCD perturbation theory.

The LCDAs of mesons have been studied intensively due to their significance in phenomenology. Among the mesons, the LCDAs for pseudoscalar mesons like pion and kaon are relatively well understood, since their LCDAs are much simpler. The LCDAs for vector mesons, e.g., the ρ meson, are more complex since the vector meson can be either longitudinally or transversely polarized. The LCDAs of vector mesons have been studied in various approaches, e.g., the QCD sum rules [5–7], the lattice QCD (LQCD) calculation, etc. However, at present the LCDAs of the ρ meson are only calculated up to the second moment in the LQCD approach [8,9].

A novel strategy of evaluating light-cone correlators is the large momentum effective theory (LaMET), in which the full x (x is the longitudinal momentum fraction) dependence can be calculated [10,11]. In LaMET, instead of calculating light-cone correlation matrix elements, one can first evaluate the corresponding equal-time correlators, which can be simulated on the lattice. The matrix elements defined by these equal-time correlators are the so-called quasiquantities, e.g., quasiparton distribution functions (quasi-PDFs), quasidistribution amplitudes (quasi-DAs), etc. The quasi- and light-cone quantities have the same infrared (IR) structure, but their ultraviolet (UV) behaviors are different, the difference is involved in the matching coefficient. Under the large P_z limit, the quasiobservables can be factorized as the convolution of perturbatively calculable coefficients and the standard light-cone observables. With such a factorization formula, one can extract light-cone observables from lattice simulation. Some other related proposals, e.g., the lattice cross section approach [12,13] and the pseudo-PDFs [14–16], are also developed. The LaMET has been studied to explore the quark PDFs

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[17], gluon PDFs [18,19], transverse momentum dependent PDFs [20,21], generalized parton distributions [22,23], as well as pion and kaon's distribution amplitudes [24–26] and the LCDAs of heavy quarkonia [27]. The renormalizability of the quasi-PDF has been established recently [28–31], and the nonperturbative renormalization schemes such as regularization invariant momentum subtraction (RI/MOM) have also been employed to study the quasi-PDFs [29,32–35]. LQCD calculations on quasi-PDFs shows the feasibility of evaluating PDFs and LCDAs from the first principle of QCD [24,36–39]. Thus LaMET provides one more approach of accessing LCDAs of vector mesons by lattice simulation. Before the lattice evaluation is performed, it is necessary to determine the matching coefficient between the LCDAs and quasi-DAs, in QCD perturbation theory.

The present paper is devoted to the perturbative matching between the quasi- and light-cone distribution amplitudes of vector mesons in LaMET. We will study the twist-2 LCDAs of the vector meson and the corresponding quasi-DAs. The main aim of this work is to derive the matching equation for quasi- and light-cone distribution amplitudes. To do this, we will calculate the one loop corrections to both the quasi- and light-cone distribution amplitudes, then work out the matching coefficients to one loop accuracy. This work will provide the possibility of extracting LCDAs of the vector meson from future lattice simulations.

The rest of this paper is organized as follows. In Sec. II, we present the definitions of twist-2 LCDAs for the transversely and longitudinally polarized states, and their corresponding quasi-DAs. In Sec. III, we calculate the one loop corrections to the LCDAs and quasi-DAs, in the UV cutoff scheme. In Sec. IV, the LaMET matching equation will be derived. We summarize in Sec. V. The results under dimensional regularization and matching coefficients with a finite UV cutoff Λ will be arranged in the Appendices.

II. DEFINITIONS OF LIGHT-CONE AND QUASIDISTRIBUTION AMPLITUDES

Before introducing the quasi-DAs, we first revisit the LCDAs. We adopt the light-cone coordinate system to discuss the LCDAs. In the light-cone coordinate system, any four-vector a can be expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$. The two unit light-cone vectors are denoted as $n^\mu = (0, 1, \vec{0}_\perp)$ and $l^\mu = (1, 0, \vec{0}_\perp)$. The inner product of four vector a and b then reads $a \cdot b = a^+ b^- + a^- b^+ - \vec{a}_\perp \cdot \vec{b}_\perp$.

In QCD, the LCDAs are defined by the matrix elements of nonlocal gauge invariant quark bilinear operators, in which the two fermion fields are separated in the n direction. At the leading twist, there are two LCDAs $\phi_V^\perp$ and $\phi_V^\parallel$ corresponding to the transversely (denoted by “ $\perp$ ”) and longitudinally (denoted by “ $\parallel$ ”) polarized states of the

vector meson. We first introduce the nonlocal operators in coordinate space

$$\mathcal{O}_V^\Gamma(\xi^-) = \bar{\psi}(\xi^-) \Gamma W(\xi^-, 0) \psi(0), \quad (1)$$

where $\Gamma = \gamma^+ \gamma_\perp^\alpha$ for the transversely polarized vector meson, and $\Gamma = \gamma^+$ for the longitudinally polarized vector meson. $W(\xi^-, 0)$ is the Wilson line with the end points $(0, \xi^-, 0_\perp)$ and $(0, 0, 0_\perp)$. In LCDAs the Wilson line is lightlike

$$W(\xi^-, 0) = P \exp \left[-ig_s \int_0^{\xi^-} n \cdot A(\lambda n) d\lambda \right], \quad (2)$$

where P denotes that the exponential is path ordered. We also need the Fourier transformation of these operators, which are denoted by $\mathcal{O}_V^\Gamma(x)$,

$$\mathcal{O}_V^\Gamma(x) = \int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \mathcal{O}_V^\Gamma(\xi^-), \quad (3)$$

$x \equiv k^+/P^+$ is the longitudinal momentum fraction with k^+ being the momentum of quark. Then, the LCDAs of the transversely and longitudinally polarized vector meson are defined by the matrix elements of $\mathcal{O}_V^\Gamma(x)$, in which $\mathcal{O}_V^\Gamma(x)$ is sandwiched between the meson and vacuum states:

$$f_V^\perp \epsilon_\perp^{*\alpha} \phi_V^\perp(x, \mu) = \langle V, P, \epsilon^* | \mathcal{O}_V^\perp(x) | 0 \rangle, \quad (4a)$$

$$f_V \frac{m_V}{P^+} \epsilon^{*+} \phi_V^\parallel(x, \mu) = \langle V, P, \epsilon^* | \mathcal{O}_V^\parallel(x) | 0 \rangle, \quad (4b)$$

where $f_V^\perp$ and f_V are the decay constants of the vector meson V , P and ϵ^* are the momentum and polarization vector of meson V , respectively. The decay constants are defined by the local operators

$$\begin{aligned} f_V^\perp \epsilon_\perp^{*\alpha} &= \langle V, P, \epsilon^* | \mathcal{O}_V^\perp(0) | 0 \rangle \\ &= \int dx \langle V, P, \epsilon^* | \mathcal{O}_V^\perp(x) | 0 \rangle, \end{aligned} \quad (5a)$$

$$\begin{aligned} f_V \frac{m_V}{P^+} \epsilon^{*+} &= \langle V, P, \epsilon^* | \mathcal{O}_V^\parallel(0) | 0 \rangle \\ &= \int dx \langle V, P, \epsilon^* | \mathcal{O}_V^\parallel(x) | 0 \rangle, \end{aligned} \quad (5b)$$

then the LCDAs can be expressed as the ratio of the nonlocal and local matrix elements,

$$\phi_V^\Gamma(x, \mu) = \frac{\langle V, P, \epsilon^* | \mathcal{O}_V^\Gamma(x) | 0 \rangle}{\langle V, P, \epsilon^* | \mathcal{O}_V^\Gamma(0) | 0 \rangle}. \quad (6)$$

In LaMET, one can define the quasi-DAs similarly. It is more convenient to discuss quasiobservables in the original Cartesian coordinate system. The unit vector of the z

direction is denoted by $n_z^\mu = (0, 0, 0, 1)$. The inner product of n_z and an arbitrary vector a gives $n_z \cdot a = a_z = -a^z$. To define the quasi-DAs, we introduce two nonlocal bilinear operators, in which the fermion fields are separated on the z direction,

$$\tilde{\mathcal{O}}^\Gamma(z) = \bar{\psi}(z)\Gamma W(z, 0)\psi(0), \quad (7)$$

and their Fourier transformation,

$$\tilde{\mathcal{O}}_V^\Gamma(x) = \int \frac{dz}{2\pi} e^{-ixzP_z} \tilde{\mathcal{O}}^\Gamma(z), \quad (8)$$

where $\Gamma = \gamma_z \gamma_\perp^\alpha$ and γ_z for the transverse and longitudinal components, respectively. Here the Wilson line W is along the z direction, with $zn_z^\mu = (0, 0, 0, z)$ and the origin of coordinates $(0, 0, 0, 0)$ as its end points. Then, the quasi-DAs of the transverse and longitudinal components of a vector meson are defined by the matrix elements of the operators as

$$f_V^\perp \epsilon_\perp^{\alpha*} \tilde{\phi}_V^\perp(x, P_z) = \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\perp(x) | 0 \rangle, \quad (9a)$$

$$f_V \epsilon_z^* \frac{m_V}{P_z} \tilde{\phi}_V^\parallel(x, P_z) = \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\parallel(x) | 0 \rangle. \quad (9b)$$

We note that although the light-cone and quasioperators are different, the decay constants in Eqs. (4) and (9) should be the same. The reason is that either the quasi- or light-cone operator is the $\mu = +$ or $\mu = z$ component of the operator $\bar{\psi}\gamma^\mu\gamma_\perp^\alpha\psi$ or $\bar{\psi}\gamma^\mu\psi$. The Lorentz index is only carried by the polarization vector ϵ^* , while the decay constants are scalar quantities, therefore they are independent of the Lorentz indices. The decay constants are related to the matrix elements of local operators by

$$f_V^\perp \epsilon_\perp^{\alpha*} = \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\perp(0) | 0 \rangle = \int dx \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\perp(x) | 0 \rangle, \quad (10a)$$

$$f_V \epsilon_z^* \frac{m_V}{P_z} = \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\parallel(0) | 0 \rangle = \int dx \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\parallel(x) | 0 \rangle. \quad (10b)$$

The quasi-DA can be expressed as the ratio of the nonlocal and the local matrix elements as

$$\tilde{\phi}_V^\Gamma(x, \mu) = \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle}. \quad (11)$$

One can immediately find that the LCDAs and quasi-DAs are normalized to 1, i.e.,

$$\int dx \phi_V^\Gamma(x, \mu) = 1, \quad \int dx \tilde{\phi}_V^\Gamma(x, P_z) = 1 \quad (12)$$

from the definitions.

III. ONE LOOP RESULTS

To examine the factorization and determine the matching coefficients at the one loop level, we first replace the meson state $\langle V, P, \epsilon^* |$ with its lowest Fock state $\langle Q(x_0 P) \bar{Q}((1-x_0)P) |$. P is the total momentum of the quark and antiquark, $x_0 P$ and $(1-x_0)P$ are the momenta of the Q and $\bar{Q}$, respectively, with $0 < x_0 < 1$. Then the matrix elements with the Fock state as their final state can be calculated in perturbation theory. Direct calculation at tree level leads to

$$\phi_V^{\Gamma(0)}(x) = \tilde{\phi}_V^{\Gamma(0)}(x) = \delta(x - x_0). \quad (13)$$

We will perform our calculation under Feynman gauge. The Feynman diagrams at the one loop level are presented by Fig. 1. The distribution amplitudes of the Fock state are calculable in perturbation theory, thus can be expanded in series of α_s . Up to the one loop level, we have

$$\tilde{\phi}_V^\Gamma(x, P_z, \Lambda) = \tilde{\phi}_V^{\Gamma(0)}(x, P_z) + \tilde{\phi}_V^{\Gamma(1)}(x, P_z, \Lambda) + \mathcal{O}(\alpha_s^2). \quad (14)$$

On the other hand, the matrix element of $\tilde{\mathcal{O}}_V^\Gamma(x)$, up to the one loop level, can be expressed by

$$\begin{aligned} \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle &= \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(0)} (1 + \delta Z_F^{(1)}) \\ &\quad + \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(1)} + \mathcal{O}(\alpha_s^2). \end{aligned} \quad (15)$$

Here $\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(0)}$ is the tree level matrix element, and $\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(1)}$ denotes the one loop correction to the matrix element in which the self-energy of the quark has been excluded. The contributions from the quark's self-energy are involved in $\delta Z_F^{(1)}$, where $\delta Z_F^{(1)}$ is the one loop correction of the quark's self-energy. Meanwhile, the local matrix element is also corrected at one loop,

$$\begin{aligned} \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle &= \langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)} \\ &\quad \times (1 + \delta Z_F^{(1)} + \delta Z_V^{\Gamma(1)}) + \mathcal{O}(\alpha_s^2), \end{aligned} \quad (16)$$

where $\delta Z_V^{\Gamma(1)}$ is the one loop vertex correction of the local operator. Since $\mathcal{O}_V^\Gamma$ and $\tilde{\mathcal{O}}_V^\Gamma$ are the $\mu = +$ and $\mu = z$ components of operator $\bar{\psi}\gamma^\mu\gamma_\perp^\alpha\psi$ or $\bar{\psi}\gamma^\mu\psi$, respectively, $\delta Z_V^{\Gamma(1)}$ should be the same for light-cone and quasilocal operators. From Eqs. (5) and (10), one can get that

$$\delta Z_V^{\Gamma(1)} = \int dx \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(1)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}}. \quad (17)$$

FIG. 1. Feynman diagrams for LCDAs and quasi-DAs at one loop level. The double line denotes the Wilson line.

From Eqs. (15) and (16), we immediately have

$$\begin{aligned}\tilde{\phi}_V^\Gamma(x, P_z, \Lambda) &= \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(0)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}} + \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(1)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}} - \delta Z_V^{\Gamma(1)} \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(0)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}} + \mathcal{O}(\alpha_s^2) \\ &= \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(0)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}} + \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(1)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}} - \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(0)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}} \int dy \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(y) | 0 \rangle^{(1)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}} \\ &\quad + \mathcal{O}(\alpha_s^2).\end{aligned}\quad (18)$$

By comparing Eqs. (18) and (14), one can identify that

$$\tilde{\phi}_V^{\Gamma(0)}(x) = \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(0)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}} = \delta(x - x_0), \quad (19)$$

$$\tilde{\phi}_V^{\Gamma(1)}(x) = \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(x) | 0 \rangle^{(1)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}} - \delta(x - x_0) \int dy \frac{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(y) | 0 \rangle^{(1)}}{\langle V, P, \epsilon^* | \tilde{\mathcal{O}}_V^\Gamma(0) | 0 \rangle^{(0)}}. \quad (20)$$

From Eq. (18), one can indicate that since the quark's self-energy cancels between the nonlocal and local matrix elements, it will not contribute to the distribution amplitudes. Therefore we have no need to consider Figs. 1(e) and 1(f). The general discussions of one loop correction above are also applicable to the LCDAs.

In the following calculations, we will introduce a small gluon mass m_g to regularize the collinear divergence. For the UV divergence, we will employ two schemes: one is adding an UV cutoff Λ on the transverse momentum, another one is the dimensional regularization (DR). In this section we devote to the cutoff scheme, the DR results will be arranged in Appendix A.

To express the one loop results of quasi- and light-cone distribution amplitudes, we introduce the generalized plus distribution “+,” which is defined by

$$\int_{-\infty}^{\infty} dx [f(x)]_+ T(x) = \int_{-\infty}^{\infty} dx f(x) (T(x) - T(x_0)), \quad (21)$$

where $T(x)$ is an arbitrary smooth test function. The generalized plus function regularizes the pole of divergent integral at $x = x_0$.

A. Transverse distribution amplitudes

We now list the results of distribution amplitudes for the transversely polarized vector meson diagram by diagram.

In Fig. 1(a), the internal gluon is not connected to the Wilson line. For this diagram, we have

$$\phi_V^{\perp(1)}(x, \Lambda) \Big|_{(a)} = \tilde{\phi}_V^{\perp(1)}(x, P_z, \Lambda) \Big|_{(a)} = 0 \quad (22)$$

for both LCDA and quasi-DA.

In Figs. 1(b) and 1(c), one end of the internal gluon is attached to the Wilson line, thus there is an eikonal propagator, which is proportional to $1/(x - x_0)$. The contributions from Fig. 1(b) read

$$\phi_V^{\perp(1)}(x, \Lambda)|_{(b)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0(x-x_0)} \ln \frac{m_g^2 x}{\Lambda^2 x_0} \right]_+, & 0 < x < x_0 \\ 0, & \text{others} \end{cases} \quad (23)$$

for LCDA, and

$$\tilde{\phi}_V^{\perp(1)}(x, P_z, \Lambda)|_{(b)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0(x-x_0)} \ln \frac{x}{x-x_0} - \frac{1}{2(x-x_0)} \right]_+, & x < 0 \\ \left[\frac{x}{x_0(x-x_0)} \ln \frac{m_g^2}{4P_z^2 x_0(x_0-x)} + \frac{2x-x_0}{2x_0(x-x_0)} \right]_+, & 0 < x < x_0 \\ \left[\frac{x}{x_0(x-x_0)} \ln \frac{x-x_0}{x} + \frac{1}{2(x-x_0)} \right]_+, & x > x_0 \end{cases} \quad (24)$$

for quasi-DA. For Fig. 1(c), we have

$$\phi_V^{\perp(1)}(x, \Lambda)|_{(c)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} -\left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{m_g^2(x-1)}{\Lambda^2(x_0-1)} \right]_+, & x_0 < x < 1 \\ 0, & \text{others} \end{cases} \quad (25)$$

for LCDA, and

$$\tilde{\phi}_V^{\perp(1)}(x, P_z, \Lambda)|_{(c)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{x-1}{x-x_0} - \frac{1}{2(x-x_0)} \right]_+, & x < x_0 \\ \left[-\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{m_g^2}{4P_z^2(1-x_0)(x-x_0)} - \frac{2x-x_0-1}{2(x_0-1)(x-x_0)} \right]_+, & x_0 < x < 1 \\ \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{x-x_0}{x-1} + \frac{1}{2(x-x_0)} \right]_+, & x > 1 \end{cases} \quad (26)$$

for quasi-DA.

Figure 1(d) is the one loop correction to the Wilson line's self-energy, which is proportional to n^2 , n is the direction vector of the Wilson line. This contribution vanishes for LCDA since $n^2 = 0$, but does not vanish for quasi-DA. Then the results read

$$\phi_V^{\perp(1)}(x, \Lambda)|_{(d)} = 0, \quad (27)$$

and

$$\tilde{\phi}_V^{\perp(1)}(x, P_z, \Lambda)|_{(d)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{1}{x-x_0} + \frac{\Lambda}{(x-x_0)^2 P_z} \right]_+, & x < x_0 \\ \left[-\frac{1}{x-x_0} + \frac{\Lambda}{(x-x_0)^2 P_z} \right]_+, & x > x_0. \end{cases} \quad (28)$$

Note that for quasi-DA, this diagram contributes a linear divergence. Perturbative calculation on quasi-PDFs also shows the existence of the powerlike UV divergence [17,18,22]. The power divergences have to be subtracted properly. A renormalization scheme has been proposed to subtract the linear divergence based on the auxiliary field

formalism [19,21,40,41]; another approach is to replace the straight Wilson line with the nondipolar Wilson lines [42].

It has been known that the power divergence in the Wilson line's self-energy can be canceled by introducing a “mass counterterm” of the Wilson line [43]. Since the source of the linear divergence is the Wilson line's self-energy, the improved quasi-PDFs and DAs are proposed by adding such a mass counterterm, to subtract the linear divergence [24,40]. In the same spirit, one can also define the improved quasi-DAs of vector meson. To do this, we replace the operator in Eq. (9) by the “improved” operator

$$\tilde{\mathcal{O}}_V^{\text{Fimp}}(x) = \int \frac{dz}{2\pi} e^{-ixzP_z - \delta m|z|} \tilde{\mathcal{O}}^{\text{F}}(z), \quad (29)$$

where δm is the mass counterterm of the Wilson line. It has been shown that δm can be extracted by using the static quark potential nonperturbatively [44]. Perturbative calculation shows that the contribution from δm cancels the linearly divergent term in Eq. (28). Therefore, one can get the result for improved quasi-DAs just by subtracting the linearly divergent term.

In the above results, the LCDAs are only nonzero in the physical regions $0 < x < x_0$ and $x_0 < x < 1$, while the

quasi-DAs have nonzero support in all of the four regions $x < 0$, $0 < x < x_0$, $x_0 < x < 1$ and $x > 1$. However, the collinear divergence only exists in the physical regions $0 < x < x_0$ and $x_0 < x < 1$. One can also notice that the LCDAs and quasi-DAs are symmetric under variable substitution $x \leftrightarrow 1 - x$, $x_0 \leftrightarrow 1 - x_0$.

B. Longitudinal distribution amplitudes

The one loop results of distribution amplitudes for the longitudinally polarized vector meson are listed below diagram by diagram.

For Fig. 1(a), we have

$$\phi_V^{\parallel(1)}(x, \Lambda)|_{(a)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[-\frac{x}{x_0} \ln \frac{m_g^2 x}{\Lambda^2 x_0} \right]_+, & 0 < x < x_0 \\ \left[-\frac{x-1}{x_0-1} \ln \frac{m_g^2 (x-1)}{\Lambda^2 (x_0-1)} \right]_+, & x_0 < x < 1 \\ 0, & \text{others} \end{cases} \quad (30)$$

for LCDA, and

$$\tilde{\phi}_V^{\parallel(1)}(x, P_z, \Lambda)|_{(a)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x-1}{x_0-1} \ln \frac{x-1}{x-x_0} - \frac{x}{x_0} \ln \frac{x}{x-x_0} \right]_+, & x < 0 \\ \left[-\frac{x}{x_0} \ln \frac{m_g^2}{4P_z^2 x_0 (x_0-x)} - \frac{x-1}{x_0-1} \ln \frac{x-x_0}{x-1} \right]_+, & 0 < x < x_0 \\ \left[-\frac{x-1}{x_0-1} \ln \frac{m_g^2}{4P_z^2 (x_0-1)(x_0-x)} - \frac{x}{x_0} \ln \frac{x-x_0}{x} \right]_+, & x_0 < x < 1 \\ \left[\frac{x}{x_0} \ln \frac{x}{x-x_0} - \frac{x-1}{x_0-1} \ln \frac{x-1}{x-x_0} \right]_+, & x > 1 \end{cases} \quad (31)$$

for quasi-DA. Note that according to Eq. (20), we have subtracted the vertex correction of the local operator which can be expressed as an integral of the nonlocal matrix element. Therefore the contributions above have been reformed to the generalized plus distribution.

For Fig. 1(b), the results are

$$\phi_V^{\parallel(1)}(x, \Lambda)|_{(b)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0(x-x_0)} \ln \frac{m_g^2 x}{\Lambda^2 x_0} \right]_+, & 0 < x < x_0 \\ 0, & \text{others} \end{cases} \quad (32)$$

for LCDA, and

$$\tilde{\phi}_V^{\parallel(1)}(x, P_z, \Lambda)|_{(b)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0(x-x_0)} \ln \frac{x}{x-x_0} - \frac{1}{2(x-x_0)} \right]_+, & x < 0 \\ \left[\frac{x}{x_0(x-x_0)} \ln \frac{m_g^2}{4P_z^2 x_0 (x_0-x)} + \frac{2x-x_0}{2x_0(x-x_0)} \right]_+, & 0 < x < x_0 \\ \left[\frac{x}{x_0(x-x_0)} \ln \frac{x-x_0}{x} + \frac{1}{2(x-x_0)} \right]_+, & x > x_0 \end{cases} \quad (33)$$

for quasi-DA. Similarly, for Fig. 1(c), we have

$$\phi_V^{\parallel(1)}(x, \Lambda)|_{(c)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} -\left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{m_g^2 (x-1)}{\Lambda^2 (x_0-1)} \right]_+, & x_0 < x < 1 \\ 0, & \text{others} \end{cases} \quad (34)$$

for LCDA, and

$$\tilde{\phi}_V^{\parallel(1)}(x, P_z, \Lambda)|_{(c)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{x-1}{x-x_0} - \frac{1}{2(x-x_0)} \right]_+, & x < x_0 \\ \left[-\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{m_g^2}{4P_z^2(1-x_0)(x-x_0)} - \frac{2x-x_0-1}{2(x_0-1)(x-x_0)} \right]_+, & x_0 < x < 1 \\ \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{x-x_0}{x-1} + \frac{1}{2(x-x_0)} \right]_+, & x > 1 \end{cases} \quad (35)$$

for quasi-DA.

Figure 1(d) receives the contribution from the Wilson line's self-energy, which is proportional to n^2 , n is the direction vector of the Wilson line. This contribution vanishes for LCDAs since $n^2 = 0$, but does not vanish for quasi-DAs. The results read

$$\phi_V^{\parallel(1)}(x, \Lambda)|_{(d)} = 0, \quad (36)$$

for LCDA, and

$$\tilde{\phi}_V^{\parallel(1)}(x, P_z, \Lambda)|_{(d)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{1}{x-x_0} + \frac{\Lambda}{(x-x_0)^2 P_z} \right]_+, & x < x_0 \\ \left[-\frac{1}{x-x_0} + \frac{\Lambda}{(x-x_0)^2 P_z} \right]_+, & x > x_0 \end{cases} \quad (37)$$

is the result of quasi-DA. Similar to the transverse quasi-DA, this diagram also contributes a linear divergence to the longitudinal quasi-DA. As we have discussed in the last subsection, the linear divergence can be cured by introducing a mass counterterm of the Wilson line. The improved quasi-DAs have already been defined by Eq. (29). The one loop results under the improved definition can be obtained by subtracting the linearly divergent term in Eq. (37).

At last, since all of the results above are represented by the generalized plus distribution, they are zero under the integration, which is the normalization condition given by Eq. (12).

IV. THE MATCHING EQUATION

In this section, we present the matching equation connecting the LCDAs and quasi-DAs.

In LaMET, if the factorization holds, the quasi-DA $\tilde{\phi}_V^\Gamma$ can be factorized as

$$\tilde{\phi}_V^\Gamma(x, P_z, \Lambda) = \int_0^1 dy Z_\Gamma(x, y, P_z, \Lambda) \phi_V^\Gamma(y, \Lambda) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{P_z^2}, \frac{m_V^2}{P_z^2}\right), \quad (38)$$

where y is constrained by $0 < y < 1$. Here Z_Γ is the perturbatively calculable function, hence can be expanded in the series of α_s as

$$\begin{aligned} Z_\Gamma(x, y, P_z, \Lambda) &= \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n Z_\Gamma^{(n)}(x, y, P_z, \Lambda) \\ &= \delta(x-y) + \frac{\alpha_s}{2\pi} Z_\Gamma^{(1)}(x, y, P_z, \Lambda) + \mathcal{O}(\alpha_s^2). \end{aligned} \quad (39)$$

By recalling the tree level result in Eq. (13), one can find that the one loop correction to the matching coefficient can be attributed to the difference between LCDA and quasi-DA at the one loop level,

$$\frac{\alpha_s}{2\pi} Z_\Gamma^{(1)}(x, x_0, P_z, \Lambda) = \tilde{\phi}_V^{\Gamma(1)}(x, P_z, \Lambda) - \phi_V^{\Gamma(1)}(x, \Lambda). \quad (40)$$

By using Eq. (40), together with the one loop results calculated in Sec. III, one can determine the one loop corrections to the matching coefficients. For $Z_\perp^{(1)}$, we have

$$Z_\perp^{(1)}(x, y, P_z, \Lambda) = C_F \begin{cases} \left[\frac{x}{y(x-y)} \ln \frac{x}{x-y} + \frac{x-1}{(y-1)(x-y)} \ln \frac{x-1}{x-y} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, & x < 0 < y \\ \left[\frac{x}{y(x-y)} \ln \frac{\Lambda^2}{4P_z^2 x(y-x)} + \frac{x-1}{(y-1)(x-y)} \ln \frac{x-1}{x-y} + \frac{x}{y(x-y)} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, & 0 < x < y \\ \left[-\frac{x-1}{(y-1)(x-y)} \ln \frac{\Lambda^2}{4P_z^2 (1-x)(x-y)} + \frac{x}{y(x-y)} \ln \frac{x-y}{x} + \frac{1-x}{(y-1)(x-y)} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, & y < x < 1 \\ \left[\frac{x}{y(x-y)} \ln \frac{x-y}{x} + \frac{x-1}{(y-1)(x-y)} \ln \frac{x-y}{x-1} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, & y < 1 < x \end{cases} \quad (41)$$

and for $Z_\parallel^{(1)}$, the result reads

$$Z_{\parallel}^{(1)}(x, y, P_z, \Lambda) = C_F \begin{cases} \left[\frac{x-1}{y-1} \left(1 + \frac{1}{x-y} \right) \ln \frac{x-1}{x-y} - \frac{x}{y} \left(1 - \frac{1}{x-y} \right) \ln \frac{x}{x-y} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, & x < 0 < y \\ \left[\frac{x}{y} \left(1 - \frac{1}{x-y} \right) \ln \frac{4P_z^2(y-x)x}{\Lambda^2} - \frac{x-1}{y-1} \left(1 + \frac{1}{x-y} \right) \ln \frac{x-y}{x-1} - \frac{x}{y(x-y)} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, & 0 < x < y \\ \left[\frac{x-1}{y-1} \left(1 + \frac{1}{x-y} \right) \ln \frac{4P_z^2(1-x)(x-y)}{\Lambda^2} - \frac{x}{y} \left(1 - \frac{1}{x-y} \right) \ln \frac{x-y}{x} + \frac{1-x}{(y-1)(x-y)} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+, & y < x < 1 \\ \left[\frac{x}{y} \left(1 - \frac{1}{x-y} \right) \ln \frac{x}{x-y} - \frac{x-1}{y-1} \left(1 + \frac{1}{x-y} \right) \ln \frac{x-1}{x-y} + \frac{\Lambda}{(x-y)^2 P_z} \right]_+. & y < 1 < x. \end{cases} \quad (42)$$

In other regions, $Z_{\perp}^{(1)}$ and $Z_{\parallel}^{(1)}$ are zero. One can notice that $Z_{\Gamma}(x, y, P_z, \Lambda) = \bar{Z}_{\Gamma}(1-x, 1-y, P_z, \Lambda)$. We should note that the plus distribution here is to subtract the singularities located at $x = y$, which is a little different from the one defined in Eq. (21). One can immediately find that the collinear divergence, which is regularized by m_{ρ} , canceled out between LCDAs and quasi-DAs, thus the matching coefficients are free of IR divergence. Thus we have proved the LaMET factorization for DAs of vector meson at the one loop level.

There are also UV divergences which are regularized by the cutoff Λ . As we have discussed in Sec. III, the linear divergence will be subtracted by introducing δm , the mass counterterm of the Wilson line. Therefore, the matching coefficients of LCDAs and the improved quasi-DAs are the same to Eqs. (41) and (42) except the linearly divergent terms, hence the improved matching coefficients have only the logarithm UV divergence. The relation between improved matching coefficients and Eqs. (41) and (42) is given by

$$Z_{\Gamma}^{(1), \text{imp.}}(x, y, P_z, \Lambda) = Z_{\Gamma}^{(1)}(x, y, P_z, \Lambda) - C_F \left[\frac{\Lambda}{(x-y)^2 P_z} \right]_+. \quad (43)$$

In Sec. III, we have taken the $\Lambda \rightarrow \infty$ limit, the $\mathcal{O}(P_z/\Lambda)$ contributions have been neglected. However, at present it is difficult to take too large value of P_z in lattice simulations, in fact, Λ and xP_z are of the same order in a practical calculation on the lattice. Therefore, it is valuable to consider the finite Λ corrections to the matching coefficients. The matching coefficients with a finite cutoff have been derived for the quark PDF [17,38] and LCDA of pion [24]. In Appendix B we will list the one loop matching coefficients of the vector meson's distribution amplitudes with a finite cutoff Λ .

Since LCDAs do not depend on P_z , one can take the derivative with $\ln P_z$ on both sides of the factorization formula Eq. (38), and derive the evolution equation of quasi-DAs with P_z :

$$\frac{d\tilde{\phi}_V^{\Gamma, \text{imp.}}(x, P_z)}{d \ln P_z} = \frac{\alpha_s C_F}{\pi} \int dy V_{\Gamma}(x, y) \tilde{\phi}_V^{\Gamma, \text{imp.}}(y, P_z), \quad (44)$$

where $V_{\Gamma}(x, y) = d \ln Z_{\Gamma}^{\text{imp.}}(x, y, P_z, \Lambda) / d \ln P_z$ is the evolution kernel, and the superscript “imp.” denotes that the quasi-DAs are under the improved definition. With the Z_{Γ} calculated in the above, we arrive at

$$V_{\perp}(x, y) = \left[\frac{x}{y(y-x)} \theta(y-x) \theta(x) \right]_+ + \left[\frac{1-x}{(1-y)(x-y)} \theta(x-y) \theta(1-x) \right]_+, \quad (45a)$$

$$V_{\parallel}(x, y) = \left[\frac{x}{y} \left(1 - \frac{1}{x-y} \right) \theta(y-x) \theta(x) \right]_+ + \left[\frac{1-x}{1-y} \left(1 + \frac{1}{x-y} \right) \theta(x-y) \theta(1-x) \right]_+, \quad (45b)$$

where $\theta(x)$ is the Heaviside step function. These functions are the Brodsky-Lepage kernels. It indicates that the evolution of quasi-DAs with P_z shares the same behavior with the scale evolution of LCDAs, which are dominated by the Efremov-Radyushkin-Brodsky-Lepage (ERBL) equation [1,45–47]. This evolution equation can be used to resum the large logarithm of P_z which appears in the perturbative calculations. The P_z evolution behavior for quasi-PDFs has already been reported, see, e.g., Refs. [10,18]. Since the P_z evolution equation of quasi-DAs is equivalent to the ERBL equation of LCDAs, one can expect that when $P_z \rightarrow \infty$, the quasi-DAs converge to the same asymptotic form with LCDAs. Therefore, it seems that the asymptotic form is the UV fixed point for both LCDAs and quasi-DAs.

V. SUMMARY

In the framework of large momentum effective theory, we have performed the one loop calculation on the leading twist light-cone distribution amplitudes as well as the quasisdistribution amplitudes of the vector meson. The distribution amplitudes of both transversely and longitudinally polarized meson have been discussed. Based on the perturbative calculation under UV cutoff and DR schemes, we have examined the LaMET factorization and found that the collinear divergence cancels between light-cone and

quasidistribution amplitudes. The matching coefficients have been determined at one loop accuracy. We also get the meson momentum evolution equation for quasidistribution amplitudes, and find that the evolution kernels are identical with the Brodsky-Lepage kernels of light-cone distribution amplitudes. The results of the present work will be useful to extract light-cone distribution amplitudes of vector mesons from the future lattice simulations.

For practical simulation on the lattice, the renormalization of quasi-DAs is necessary. In the present work the calculation is performed in a naive cutoff scheme and the renormalization is absent. Furthermore, the one loop calculation is not on the discrete but the continuum quasi-PDFs. Therefore, a calculation based on lattice perturbation theory is necessary to fill the gap. Another approach is to renormalize the quasi-DAs in a nonperturbative renormalization scheme, such as the RI/MOM scheme, which has been employed to renormalize quasi-PDFs on the lattice. These issues will be discussed in the future works.

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APPENDIX A: ONE LOOP RESULTS IN DIMENSIONAL REGULARIZATION

In Sec. III, we have introduced a cutoff Λ on the transverse momentum as an UV regulator. A commonly used regularization scheme is the dimensional regularization. In this scheme, the space-time dimensions are modified from 4 to $d = 4 - 2\epsilon$. The UV divergence is expressed by the poles of ϵ . To renormalize the UV divergence one can employ the $\overline{\text{MS}}$ scheme, in which only the terms proportional to $1/\epsilon - \gamma_E + \ln 4\pi$ ($\gamma_E = 0.577\dots$ is the Euler-Mascheroni constant) are subtracted. Since the standard light-cone PDFs and LCDAs are always defined under $\overline{\text{MS}}$, we list here the one loop results under DR and $\overline{\text{MS}}$.

1. Transverse distribution amplitudes

We list here our results of distribution amplitudes for transversely polarized vector meson under dimensional regularization. For Fig. 1(a), we have

$$\phi_V^{\perp(1)}(x, \mu)|_{(a)} = 0, \quad (\text{A1})$$

and

$$\tilde{\phi}_V^{\perp(1)}(x, P_z)|_{(a)} = 0. \quad (\text{A2})$$

For Fig. 1(b), we have

$$\phi_V^{\perp(1)}(x, \mu)|_{(b)} = -\frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0(x-x_0)} \ln \frac{x}{x-x_0} - \frac{1}{2(x-x_0)} \right]_+, & 0 < x < x_0, \\ 0, & \text{others} \end{cases} \quad (\text{A3})$$

$$\tilde{\phi}_V^{\perp(1)}(x, P_z)|_{(b)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0(x-x_0)} \ln \frac{x}{x-x_0} - \frac{1}{2(x-x_0)} \right]_+, & x < 0 \\ \left[\frac{x}{x_0(x-x_0)} \ln \frac{m_g^2}{4P_z^2 x_0(x_0-x)} + \frac{2x-x_0}{2x_0(x-x_0)} \right]_+, & 0 < x < x_0 \\ \left[\frac{x}{x_0(x-x_0)} \ln \frac{x-x_0}{x} + \frac{1}{2(x-x_0)} \right]_+, & x > x_0. \end{cases} \quad (\text{A4})$$

For Fig. 1(c), we have

$$\phi_V^{\perp(1)}(x, \mu)|_{(c)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{\mu^2(1-x_0)}{m_g^2(1-x)} \right]_+, & x_0 < x < 1 \\ 0, & \text{others} \end{cases} \quad (\text{A5})$$

$$\tilde{\phi}_V^{\perp(1)}(x, P_z)|_{(c)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{x-1}{x-x_0} - \frac{1}{2(x-x_0)} \right]_+, & x < x_0 \\ \left[-\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{m_g^2}{4P_z^2(1-x_0)(x-x_0)} - \frac{2x-x_0-1}{2(x_0-1)(x-x_0)} \right]_+, & x_0 < x < 1 \\ \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{x-x_0}{x-1} + \frac{1}{2(x-x_0)} \right]_+, & x > 1. \end{cases} \quad (\text{A6})$$

Figure 1(d) is the self-energy of the Wilson line. For a Wilson line along the light-cone direction, the self-energy is zero. For a spacelike Wilson line, the self-energy is linearly divergent. However, in the DR scheme, one can assign a finite value to the linearly divergent self-energy with analytical continuation. Thus we have

$$\phi_V^{\perp(1)}(x, \mu)|_{(d)} = 0, \quad (\text{A7})$$

$$\tilde{\phi}_V^{\perp(1)}(x, P_z)|_{(d)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{1}{x-x_0} \right]_+, & x < x_0 \\ \left[-\frac{1}{x-x_0} \right]_+, & x > x_0. \end{cases} \quad (\text{A8})$$

In the results of LCDAs, we have performed the $\overline{\text{MS}}$ subtraction. For the quasi-DAs, the results of all the one

loop diagrams are finite. However, one can notice that when $x \rightarrow \pm\infty$, the quasi-DA behaves as $\propto 1/x$, which is logarithmically divergent. One can take the convolution of the quasi-DA and an arbitrary test function $T(x)$, e.g., $T(x) = 1$. The integral is zero since the quasi-DAs are of type $[f(x)]_+$, but it is due to the cancellation of two logarithmically divergent integrals. Thus a renormalization is needed to make the integrals converge. One calculation on the quasi-PDF based on the RI/MOM scheme has been performed in Ref. [35]. The renormalization on quasi-DAs will be discussed in a forthcoming work.

2. Longitudinal distribution amplitudes

Now we list our results for the distribution amplitudes of longitudinally polarized vector meson under dimensional regularization.

For Fig. 1(a), we have

$$\phi_V^{\parallel(1)}(x, \mu)|_{(a)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0} \left(\ln \frac{\mu^2 x_0}{m_g^2 x} - 1 \right) \right]_+, & 0 < x < x_0 \\ \left[\frac{x-1}{x_0-1} \left(\ln \frac{\mu^2 (x_0-1)}{m_g^2 (x-1)} - 1 \right) \right]_+, & x_0 < x < 1 \\ 0, & \text{others} \end{cases} \quad (\text{A9})$$

and

$$\tilde{\phi}_V^{\parallel(1)}(x, P_z)|_{(a)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x-1}{x_0-1} \ln \frac{x-1}{x-x_0} - \frac{x}{x_0} \ln \frac{x}{x-x_0} \right]_+, & x < 0 \\ \left[-\frac{x}{x_0} \ln \frac{m_g^2}{4P_z^2 x_0 (x_0-x)} - \frac{x-1}{x_0-1} \ln \frac{x-x_0}{x-1} \right]_+, & 0 < x < x_0 \\ \left[-\frac{x-1}{x_0-1} \ln \frac{m_g^2}{4P_z^2 (x_0-1)(x_0-x)} - \frac{x}{x_0} \ln \frac{x-x_0}{x} \right]_+, & x_0 < x < 1 \\ \left[\frac{x}{x_0} \ln \frac{x}{x-x_0} - \frac{x-1}{x_0-1} \ln \frac{x-1}{x-x_0} \right]_+, & x > 1. \end{cases} \quad (\text{A10})$$

According to Eq. (18), there is a contribution from the vertex correction of the local operator, which can be expressed as an integral of $\phi_V^\Gamma(x)$ or $\tilde{\phi}_V^\Gamma(x)$. Note that we have added the contribution from $\delta Z_V^{\parallel(1)} \delta(x-x_0)$ here, so the contributions above have been referred to the generalized plus distribution.

For Fig. 1(b), we have

$$\phi_V^{\parallel(1)}(x, \mu)|_{(b)} = -\frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0(x-x_0)} \ln \frac{\mu^2 x_0}{m_g^2 x} \right]_+, & 0 < x_0 < x \\ 0, & \text{others} \end{cases} \quad (\text{A11})$$

$$\tilde{\phi}_V^{\parallel(1)}(x, P_z)|_{(b)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x}{x_0(x-x_0)} \ln \frac{x}{x-x_0} - \frac{1}{2(x-x_0)} \right]_+, & x < 0 \\ \left[\frac{x}{x_0(x-x_0)} \ln \frac{m_g^2}{4P_z^2 x_0 (x_0-x)} + \frac{2x-x_0}{2x_0(x-x_0)} \right]_+, & 0 < x < x_0 \\ \left[\frac{x}{x_0(x-x_0)} \ln \frac{x-x_0}{x} + \frac{1}{2(x-x_0)} \right]_+, & x > x_0 \end{cases} \quad (\text{A12})$$

and for Fig. 1(c), the results are

$$\phi_V^{(1)}(x, \mu)|_{(c)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{\mu^2(1-x_0)}{m_g^2(1-x)} \right]_+, & x_0 < x < 1 \\ 0, & \text{others} \end{cases} \quad (\text{A13})$$

$$\tilde{\phi}_V^{(1)}(x, P_z)|_{(c)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{x-1}{x-x_0} - \frac{1}{2(x-x_0)} \right]_+, & x < x_0 \\ \left[-\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{m_g^2}{4P_z^2(1-x_0)(x-x_0)} - \frac{2x-x_0-1}{2(x_0-1)(x-x_0)} \right]_+, & x_0 < x < 1 \\ \left[\frac{x-1}{(x_0-1)(x-x_0)} \ln \frac{x-x_0}{x-1} + \frac{1}{2(x-x_0)} \right]_+, & x > 1 \end{cases} \quad (\text{A14})$$

For Fig. 1(d), we have

$$\phi_V^{(1)}(x, \mu)|_{(d)} = 0, \quad (\text{A15})$$

$$\tilde{\phi}_V^{(1)}(x, P_z)|_{(d)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{1}{x-x_0} \right]_+, & x < x_0 \\ \left[-\frac{1}{x-x_0} \right]_+, & x > x_0. \end{cases} \quad (\text{A16})$$

3. Matching coefficients and evolution equations

By using Eq. (40), together with the one loop results under DR, one can determine the one loop corrections to the matching coefficients. For $Z_{\perp}^{(1)}$, we have

$$Z_{\perp}^{(1)}(x, y, P_z, \mu) = C_F \begin{cases} \left[\frac{x}{y(x-y)} \ln \frac{x}{x-y} + \frac{x-1}{(y-1)(x-y)} \ln \frac{x-1}{x-y} \right]_+, & x < 0 < y \\ \left[\frac{x}{y(x-y)} \ln \frac{\mu^2}{4P_z^2 x(y-x)} + \frac{x-1}{(y-1)(x-y)} \ln \frac{x-1}{x-y} + \frac{x}{y(x-y)} \right]_+, & 0 < x < y \\ \left[-\frac{x-1}{(y-1)(x-y)} \ln \frac{\mu^2}{4P_z^2(1-x)(x-y)} + \frac{x}{y(x-y)} \ln \frac{x-y}{x} + \frac{1-x}{(y-1)(x-y)} \right]_+, & y < x < 1 \\ \left[\frac{x}{y(x-y)} \ln \frac{x-y}{x} + \frac{x-1}{(y-1)(x-y)} \ln \frac{x-y}{x-1} \right]_+, & y < 1 < x \end{cases} \quad (\text{A17})$$

and for $Z_{\parallel}^{(1)}$, the result reads

$$Z_{\parallel}^{(1)}(x, y, P_z, \mu) = C_F \begin{cases} \left[\frac{x-1}{y-1} \left(1 + \frac{1}{x-y} \right) \ln \frac{x-1}{x-y} - \frac{x}{y} \left(1 - \frac{1}{x-y} \right) \ln \frac{x}{x-y} \right]_+, & x < 0 < y \\ \left[\frac{x}{y} \left(1 - \frac{1}{x-y} \right) \ln \frac{4P_z^2(y-x)x}{\mu^2} - \frac{x-1}{y-1} \left(1 + \frac{1}{x-y} \right) \ln \frac{x-y}{x-1} + \frac{x}{y(x-y)} + \frac{x}{y} \right]_+, & 0 < x < y \\ \left[\frac{x-1}{y-1} \left(1 - \frac{1}{x-y} \right) \ln \frac{4P_z^2(1-x)(x-y)}{\mu^2} - \frac{x}{y} \left(1 - \frac{1}{x-y} \right) \ln \frac{x-y}{x} + \frac{1-x}{(y-1)(x-y)} + \frac{1-x}{1-y} \right]_+, & y < x < 1 \\ \left[\frac{x}{y} \left(1 - \frac{1}{x-y} \right) \ln \frac{x}{x-y} - \frac{x-1}{y-1} \left(1 + \frac{1}{x-y} \right) \ln \frac{x-1}{x-y} \right]_+, & y < 1 < x. \end{cases} \quad (\text{A18})$$

In other regions, $Z_{\perp}^{(1)}$ and $Z_{\parallel}^{(1)}$ are zero.

Based on these matching coefficients one can also derive the P_z evolution equations. Since the $\ln P_z$ dependence is the same in cutoff and DR schemes, the evolution equations are identical.

APPENDIX B: MATCHING COEFFICIENTS WITH A FINITE CUTOFF

In Sec. IV, we have calculated the matching coefficients under the UV cutoff scheme. The cutoff Λ has been taken to be $\Lambda \gg xP_z$. Since it is difficult to achieve the $\Lambda \rightarrow \infty$ limit for lattice simulations at present, Λ and xP_z could be

of the same order. By considering the finite Λ effect, the matching coefficients presented by Eqs. (41) and (42) will be modified to be

$$Z_{\perp}^{(1)}(x, y, P_z, \Lambda) = Z_{\perp}^{(1)}(x, y, P_z, \Lambda)|_{\text{Eq. (41)}} + \delta Z_{\perp}^{(1)}(x, y, P_z, \Lambda), \quad (\text{B1})$$

$$Z_{\parallel}^{(1)}(x, y, P_z, \Lambda) = Z_{\parallel}^{(1)}(x, y, P_z, \Lambda)|_{\text{Eq. (42)}} + \delta Z_{\parallel}^{(1)}(x, y, P_z, \Lambda). \quad (\text{B2})$$

The corrections $\delta Z_{\perp}^{(1)}$ and $\delta Z_{\parallel}^{(1)}$ read

$$\begin{aligned} \delta Z_{\perp}^{(1)}(x, y, P_z, \Lambda) = C_F \left[\frac{x}{y(x-y)} \left(\ln \frac{\Lambda(x) + P_z x}{\Lambda(x-y) + P_z(x-y)} + \frac{\Lambda(x-y) - \Lambda(x)}{2P_z} \right) + \frac{\Lambda(x-y) - \Lambda(0)}{2(x-y)^2 P_z} \right]_+ \\ + (x \rightarrow 1-x, y \rightarrow 1-y), \end{aligned} \quad (\text{B3})$$

$$\begin{aligned} \delta Z_{\parallel}^{(1)}(x, y, P_z, \Lambda) = C_F \left[\frac{x}{y} \ln \frac{\Lambda(x) - P_z x}{\Lambda(x-y) - P_z(x-y)} + \frac{x}{y(x-y)} \left(\ln \frac{\Lambda(x) + P_z x}{\Lambda(x-y) + P_z(x-y)} + \frac{\Lambda(x-y) - \Lambda(x)}{2P_z} \right) \right. \\ \left. + \frac{\Lambda(x-y) - \Lambda(0)}{2(x-y)^2 P_z} \right]_+ + (x \rightarrow 1-x, y \rightarrow 1-y), \end{aligned} \quad (\text{B4})$$

where $\Lambda(x) \equiv \sqrt{\Lambda^2 + x^2 P_z^2}$. One can examine that $\delta Z_{\Gamma}^{(1)} \rightarrow 0$ when $\Lambda \rightarrow \infty$.

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