Production of χ_c pairs with large rapidity separation in k_T factorization

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We calculate the production of pairs of χ_{cJ} mesons with all possible combinations of $J = 0, 1, 2$. The production mechanism which we consider is the crossed-channel gluon exchange in the gluon-gluon fusion reaction, the mechanism relevant for large rapidity separations between χ_c mesons. The salient features of the t and u -channel gluon exchange are the broad distributions in rapidity difference Δy between χ_c mesons. The building blocks are the vertices $g^* g^* \rightarrow \chi_{cJ}$ for off-shell gluons. We stick to the color-singlet model and calculate the gluon fusion vertices in the limit of heavy quarks with nonrelativistic motion in the bound state. These vertices are used to construct the $g^* g^* \rightarrow \chi_{cJ_1} \chi_{cJ_2}$ amplitudes. We then calculate hadron-level cross sections using the k_T -factorization approach. Several differential distributions at the pp center of mass energy $\sqrt{s} = 8$ TeV are shown. We briefly discuss the contribution to $J/\psi J/\psi$ production at large rapidity separation. For reference we show some distributions for production of $\chi_{cJ_1} \chi_{cJ_2}$ pairs in double parton scattering. The discussed single parton scattering mechanisms yield similar shapes of different distributions as the double parton scattering mechanisms.

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I. INTRODUCTION

Recently, cross sections for the production of J/ψ -pairs were measured at the Tevatron [1] and the LHC [2–5]. There remain a number of puzzles, especially with the CMS and ATLAS data. Here the leading order of $\mathcal{O}(\alpha_s^4)$ (see e.g., [6,7]) is clearly not sufficient. The double parton scattering (DPS) contribution was claimed to be large or even dominant in some corners of the phase space, when the rapidity distance Δy between two J/ψ mesons is large. However the effective cross sections σ_{eff} found from empirical analyses are about a factor 2.5 smaller than the usually accepted $\sigma_{\text{eff}} = 15$ mb. It is an open issue at the moment whether this points to a nonuniversality of σ_{eff} or whether there are additional single parton scattering mechanisms which can alleviate the tension.

The production of quarkonium pairs is interesting in a broader context. Here we wish to consider production of

pairs of χ_c mesons. This process is more difficult to measure experimentally but interesting from the theoretical point of view. For example, in the case of χ_c -pair production, the double parton scattering may be somewhat easier to calculate than for other, more involved processes [8]. Furthermore, a feed-down to the double J/ψ channel is interesting in the context of the puzzles mentioned above.

The single-inclusive χ_c meson production recently was a topic of both experimental [9–11] and theoretical studies using collinear factorization [12–14] as well as k_T -factorization [15–19].

The cross section for single χ_c production is rather large. The nonrelativistic perturbative QCD is the standard theoretical approach in this context. In leading order the gluon fusion $g^* g^* \rightarrow \chi_{cJ}$, $J = 0, 1, 2$ is the underlying production mechanism. The k_T -factorization approach provides a reasonable description of the experimental data [17–19]. Recently two of us [18,19] have shown that at not too large transverse momenta we can describe the cross section for $pp \rightarrow \chi_c$ and $pp \rightarrow J/\psi$ with color-singlet contributions only. It is our aim here to include corresponding color-singlet contributions to the $\chi_{cJ_1} \chi_{cJ_2}$ production cross section. Especially in the single-inclusive χ_c production, we do not see much room for color-octet contributions, so that we feel well justified to neglect them in this work.

In this paper we will concentrate only on processes (diagrams) relevant at large rapidity separation between the two χ_c quarkonia. Radiative decays may then generate two

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FIG. 1. A diagrammatic representation of the leading order mechanisms for $pp \rightarrow \chi_{cJ_1} \chi_{cJ_2} \rightarrow (J/\psi + \gamma)(J/\psi + \gamma)$ reaction.

J/ψ mesons with large rapidity separations, where the puzzling empirical results for σ_{eff} were found. This is at the present a topic of active debate [20].

We shall include the production of all combinations of χ_c meson pair production. The cross section will be calculated in k_T -factorization approach using newly derived off-shell matrix elements for the $g^* g^* \rightarrow \chi_{cJ_1} \chi_{cJ_2}$ process.

A first evaluation of the total cross section will be given. We also show some differential distributions. As the title implies, our results are meant to apply for situations where the rapidity distance between χ_c 's is large. We will argue that numerically a rapidity distance $\Delta y \geq 2$ can be considered large in this sense.

II. THE $pp \rightarrow \chi_{cJ_1} \chi_{cJ_2} X$ REACTION, FORMALISM

It was shown in [21,22] that the $\chi_c J/\psi$ pair production is possible only at $O(\alpha_s^5)$, while forbidden at $O(\alpha_s^4)$ due to C parity conservation. In contrast, the production of $\chi_{cJ_1} \chi_{cJ_2}$, see Fig. 1, is possible already at the $O(\alpha_s^4)$ order.

Here, from the many diagrams which can contribute to the χ -pair production, we single out the gauge invariant subset which contains the crossed-channel gluon exchange. The other diagrams (of the “box” type; see, e.g., [23]) always have at least one parton which is off-shell of order $\hat{s} = M_{\chi\chi}^2$. They are therefore strongly suppressed in the limit where the center-of-mass energy of the gluon-gluon collision becomes large. This is not so for the t -channel gluon exchanges, where we have a broad range of $\hat{t}/\hat{s} \ll 1$ or $\hat{u}/\hat{s} \ll 1$ which contributes at high energies. In addition, due to the vector exchange, the Feynman amplitude will be $\propto \hat{s}$, which leads to cross sections which are flat at large $\hat{s}$.

FIG. 2. A diagrammatic representation of the $g^* g^* \rightarrow \chi_c(\lambda)$ vertex being a building block of corresponding $g^* g^* \rightarrow \chi_{cJ_1} \chi_{cJ_2}$ and discussed in this section's amplitudes.

This implies broad distributions in the rapidity distance Δy between the produced χ_c -mesons.

According to our knowledge this contribution was so far only discussed in collinear factorization [23], naturally only using the gluon distributions which were available some twenty years ago. There has been recently also a calculation for $\chi_c \chi_b$ production [24] using collinear gluon distributions. Notice that in the latter case, within the color-singlet model the gluon exchange mechanism is even the exact leading order in this channel.

We consider the gluon-gluon fusion mechanism shown diagrammatically in Fig. 1. There are altogether six possible combinations of pair production of $\chi_{c0}, \chi_{c1}, \chi_{c2}$ quarkonia.

In order to calculate the subprocess amplitudes, we first turn to the $g^* g^* \rightarrow \chi_{cJ}$ vertices.

A. The $g^* g^* \rightarrow \chi_{cJ}$ vertices

The $g^* g^* \rightarrow \chi_c$ vertices with off-shell gluons (see Fig. 2) are building blocks of the elementary $g^* g^* \rightarrow \chi_{cJ_1} \chi_{cJ_2}$ amplitudes.

Here we follow the general rules of Non-Relativistic QCD as explained, e.g., in [25–27]. We restrict ourselves to the color-singlet contribution and can write the amplitude for the production of the χ_{cJ} meson via the fusion of two gluons as:

$$\begin{aligned}
 V_{\mu\nu}^{ab}(J, J_z; q_1, q_2) &= 4\pi\alpha_s \frac{\text{Tr}[t^a t^b]}{\sqrt{N_c}} \sqrt{\frac{2}{M}} \sum_{S_z, L_z} \int \frac{d^4 k}{(2\pi)^3} \delta\left(k^0 - \frac{\vec{k}^2}{M}\right) \Psi_{1, L_z}(\vec{k}) \\
 &\quad \times \langle 1, S_z; 1, L_z | J, J_z \rangle \cdot \text{Tr}[A_{\mu\nu} \Pi_{1, S_z}] \\
 &\equiv -i4\pi\alpha_s \delta^{ab} \frac{2R'(0)}{\sqrt{\pi N_c M^3}} \sqrt{3} \cdot T_{\mu\nu}(J, J_z; q_1, q_2), \quad (2.1)
 \end{aligned}$$

where $R'(0)$ is the derivative of the radial wave function at the (spatial) origin. For more details on the formalism,

see [28–30], where these vertices had been calculated for external Reggeized gluons. Below we will need the amplitudes (2.1) for arbitrary off-shell momenta of gluons, not only the multi-Regge kinematics as in [15,28–30]. There is however no additional difficulty related with this. As we concentrate on the color-singlet mechanism, the three-gluon coupling does not enter and we really deal with

a QED problem. Consequently the amplitudes (2.1) fulfill the QED-like gauge invariance conditions:

$$q_1^\mu V_{\mu\nu}^{ab}(J, J_z; q_1, q_2) = 0, \quad q_2^\nu V_{\mu\nu}^{ab}(J, J_z; q_1, q_2) = 0. \quad (2.2)$$

The explicit expressions for $T_{\mu\nu}(J, J_z; q_1, q_2)$ are:

(i) scalar, $J = 0$:

$$T_{\mu\nu}(0, 0; q_1, q_2) = \frac{1}{\sqrt{3}} \frac{M^2}{(2q_1 \cdot q_2)^2} \left\{ g_{\mu\nu} \left(6(q_1 \cdot q_2) - q_1^2 - q_2^2 + \frac{(q_2^2 - q_1^2)^2}{M^2} \right) + q_{1\mu} q_{2\nu} 2 \left(\frac{q_1^2 + q_2^2}{M^2} - 1 \right) \right. \\ \left. + q_{2\mu} q_{1\nu} 2 \left(\frac{q_1^2 + q_2^2}{M^2} - 3 \right) + q_{1\mu} q_{1\nu} \frac{4q_2^2}{M^2} + q_{2\mu} q_{2\nu} \frac{4q_1^2}{M^2} \right\} \quad (2.3)$$

(ii) axial vector, $J = 1$:

$$T_{\mu\nu}(1, J_z; q_1, q_2) = \frac{i}{\sqrt{2}M} \frac{1}{(q_1 \cdot q_2)} \left\{ (q_1^2 - q_2^2) \epsilon_{\mu\nu\alpha\beta} (q_1 + q_2)^\alpha \epsilon^\beta(J_z) \right. \\ \left. + \frac{q_1^2 + q_2^2}{(q_1 \cdot q_2)} (a_\mu q_{1\nu} - a_\nu q_{2\mu}) + 2(a_\nu q_{1\mu} - a_\mu q_{2\nu}) \right\}, \quad (2.4)$$

with

$$a_\mu = \epsilon_{\mu\rho\alpha\beta} q_1^\rho q_2^\alpha \epsilon^\beta(J_z). \quad (2.5)$$

(iii) tensor, $J = 2$:

$$T_{\mu\nu}(2, J_z; q_1, q_2) = \frac{-M^2}{(2q_1 \cdot q_2)^2} \left\{ -g_{\mu\nu} (q_2 - q_1)^\alpha (q_2 - q_1)^\beta \epsilon_{\alpha\beta}(J_z) \right. \\ \left. + 4(q_1 \cdot q_2) \epsilon_{\mu\nu}(J_z) + 2(q_2 - q_1)^\alpha \epsilon_{\alpha\nu}(J_z) q_{2\mu} \right. \\ \left. - 2(q_2 - q_1)^\alpha \epsilon_{\alpha\mu}(J_z) q_{1\nu} \right\}, \quad (2.6)$$

where $\epsilon_{\mu\nu}(J_z) = \sum_{m_1, m_2} \langle 2, J_z | 1, m_1, 1, m_2 \rangle \epsilon_\mu(m_1) \times \epsilon_\nu(m_2)$ is the polarization tensor of the $J = 2$ state.

Notice that

$$2(q_1 \cdot q_2) = M^2 - q_1^2 - q_2^2, \quad (2.7)$$

and as gluons are always spacelike $q_i^2 < 0$, the denominators of Eqs. (2.3), (2.4), (2.6) are always finite.

Besides the QED-like gauge invariance condition, these amplitudes also fulfill the Bose-symmetry¹

¹The similar expressions found in [24] refer to a situation where one of the gluons is on-shell and can not be used for the purpose of k_T -factorization.

$$T_{\mu\nu}(J, J_z; q_1, q_2) = T_{\nu\mu}(J, J_z; q_2, q_1). \quad (2.8)$$

A comment on the $J = 1$ axial vector is in order. Here the Landau-Yang theorem forbids the decay of the χ_{c1} into $\gamma\gamma$ or gg , and likewise its production through fusion of on-shell photons or gluons. Indeed, in the limit $q_1^2 \rightarrow 0$, $q_2^2 \rightarrow 0$, we have

$$T_{\mu\nu}(1, J_z; q_1, q_2) \propto a_\nu q_{1\mu} - a_\mu q_{2\nu}, \quad (2.9)$$

which vanishes when contracted with the polarization vectors of on-shell photons/gluons

$$\epsilon_1^\mu \epsilon_2^\nu (a_\nu q_{1\mu} - a_\mu q_{2\nu}) = 0, \quad (2.10)$$

as required by the Landau-Yang theorem.

B. The $g^* g^* \rightarrow \chi_{cJ_1} \chi_{cJ_2}$ amplitudes

Now we wish to discuss the elementary $g^* g^* \rightarrow \chi_{cJ_1} \chi_{cJ_2}$ amplitudes, which can be obtained from the building blocks discussed above.

In all cases there are two diagrams [t (left) and u (right) in Fig. 3].

We can write the Feynman amplitudes corresponding to these diagrams as

FIG. 3. A diagrammatic representation of the generic $g^* g^* \rightarrow \chi_{cJ_1} \chi_{cJ_2}$ t -channel (left) and u -channel (right) amplitudes.

$$\begin{aligned}
& \mathcal{M}_{\mu\nu}^{ab}(J_1, J_{1z}, J_2, J_{2z}) \\
&= V_{\mu\alpha}^{ac}(J_1, J_{1z}; q_1, p_1 - q_1) \frac{-g^{\alpha\beta} \delta_{cd}}{\hat{t}} \\
&\quad \times V_{\beta\nu}^{db}(J_2, J_{2z}; p_2 - q_2, q_2) + V_{\mu\alpha}^{ac}(J_2, J_{2z}; q_1, p_2 - q_1) \\
&\quad \times \frac{-g^{\alpha\beta} \delta_{cd}}{\hat{u}} V_{\beta\nu}^{db}(J_1, J_{1z}; p_1 - q_2, q_1), \quad (2.11)
\end{aligned}$$

where $\hat{t} = (p_1 - q_1)^2 = (p_2 - q_2)^2$, $\hat{u} = (p_2 - q_1)^2 = (p_1 - q_2)^2$. These amplitudes are infrared finite and gauge invariant.

To obtain the k_T -factorization amplitude one should contract (2.11) with the polarization vectors of off-shell gluons

$$e_{1\mu} = \frac{q_{1T\mu}}{|\vec{q}_{1T}|}, \quad e_{2\nu} = \frac{q_{2T\nu}}{|\vec{q}_{2T}|}. \quad (2.12)$$

Because of the QED-like Ward identities of the gluon fusion vertices, these polarization vectors are equivalent to the more common Gribov's polarizations n_μ^+ , n_ν^- , for incoming gluons in the high-energy kinematics $q_{1\mu} = q_1^+ n_\mu^+ + q_{1T\mu}$, $q_{2\nu} = q_2^- n_\nu^- + q_{2T\nu}$.

In the nonrelativistic QCD approach the cross section for χ_c pair production is proportional to $|R'(0)|^4$. The result is therefore extremely sensitive to the precise value of the wave function derivative at the origin. In our

opinion the best estimate of the parameter can be obtained from:

$$\Gamma(\chi_c(0^+) \rightarrow \gamma\gamma) = \frac{27e_c^4 \alpha_{\text{em}}}{m_c^4} |R'(0)|^2. \quad (2.13)$$

From the experimental value of the diphoton decay width [31], one obtains for the χ_c P-wave function squared

$$|R'(0)|^2 = 0.042 \text{ GeV}^5. \quad (2.14)$$

In the following, the $\chi_{cJ_1} \chi_{cJ_2}$ cross section is calculated within the k_T -factorization approach including off-shell matrix elements for the $g^* g^* \rightarrow \chi_{cJ_1} \chi_{cJ_2}$ subprocess and modern unintegrated gluon distributions.

It is well known that about 30% of prompt single J/ψ production originates from radiative decays $\chi_c \rightarrow J/\psi + \gamma$ with branching fractions: $\text{Br}(\chi_{c0} \rightarrow J/\psi\gamma) = 1.26 \pm 0.06\%$, $\text{Br}(\chi_{c1} \rightarrow J/\psi\gamma) = 33.9 \pm 1.2\%$, $\text{Br}(\chi_{c2} \rightarrow J/\psi\gamma) = 19.2 \pm 0.7\%$ [31]. Obviously, regarding feed-down into the $J/\psi J/\psi$ channel only the $\chi_{c1} \chi_{c1}$, $\chi_{c1} \chi_{c2}$ and $\chi_{c2} \chi_{c2}$ states could give potentially important contributions. The details depend, however, on corresponding matrix elements and cross sections for the $\chi_{cJ_1} \chi_{cJ_2}$ production.

The cross section for $pp \rightarrow \chi_{cJ_1} \chi_{cJ_2}$ is calculated in the k_T -factorization approach. The corresponding differential cross section for the production of $\chi_{ci} \chi_{cj}$ states, where i and j run through 0,1,2 can be written as:

$$\begin{aligned}
\frac{d\sigma(pp \rightarrow \chi_{ci} \chi_{cj} X)}{dy_1 dy_2 d^2\vec{p}_{1T} d^2\vec{p}_{2T}} &= \frac{1}{16\pi^2 (x_1 x_2 s)^2} \frac{1}{1 + \delta_{ij}} \int \frac{d^2\vec{q}_{1T}}{\pi \vec{q}_{1T}^2} \frac{d^2\vec{q}_{2T}}{\pi \vec{q}_{2T}^2} \overline{|\mathcal{M}_{g^* g^* \rightarrow \chi_{ci} \chi_{cj}}^{\text{off-shell}}|^2} \\
&\quad \times \delta^{(2)}(\vec{q}_{1T} + \vec{q}_{2T} - \vec{p}_{1T} - \vec{p}_{2T}) \mathcal{F}(x_1, \vec{q}_{1T}^2, \mu_F^2) \mathcal{F}(x_2, \vec{q}_{2T}^2, \mu_F^2). \quad (2.15)
\end{aligned}$$

The unintegrated gluon distribution $\mathcal{F}(x_1, \vec{q}_{1T}^2, \mu_F^2)$ is related to the collinear one through

$$xg(x, \mu_F^2) = \int^{\mu_F^2} \frac{d\vec{q}_T^2}{\vec{q}_T^2} \mathcal{F}(x, \vec{q}_{1T}^2, \mu_F^2), \quad (2.16)$$

and the off-shell matrix element is obtained as

$$\begin{aligned}
& \overline{|\mathcal{M}_{g^* g^* \rightarrow \chi_{ci} \chi_{cj}}^{\text{off-shell}}|^2} \\
&= \frac{1}{(N_c^2 - 1)^2} \sum_{a,b,J_{1z},J_{2z}} |e_{1\mu} e_{2\nu} \mathcal{M}_{\mu\nu}^{ab}(J_1, J_{1z}, J_2, J_{2z})|^2. \quad (2.17)
\end{aligned}$$

The longitudinal momentum fractions x_1 and x_2 are calculated from χ_c 's transverse masses $m_{Ti} = \sqrt{M^2 + \vec{p}_{iT}^2}$ and rapidities $y_{1,2}$ of mesons in the pp center-of-mass:

$$\begin{aligned}
x_1 &= \frac{m_{T1}}{\sqrt{s}} e^{y_1} + \frac{m_{T2}}{\sqrt{s}} e^{y_2}, \\
x_2 &= \frac{m_{T1}}{\sqrt{s}} e^{-y_1} + \frac{m_{T2}}{\sqrt{s}} e^{-y_2}. \quad (2.18)
\end{aligned}$$

III. RESULTS FOR $\chi_{cJ_1} \chi_{cJ_2}$ PRODUCTION

A. Integrated cross sections

We start presentation of our results by showing integrated cross sections. As an example, in Table I we show cross sections in a broad range of χ_c rapidities. We used an unintegrated gluon distribution constructed from the

TABLE I. Cross sections in nb for production of different combinations of $\chi_{cJ_1} \chi_{cJ_2}$ dimeson states for $-8 < y_1, y_2 < 8$ at $\sqrt{s} = 8$ TeV. The numbers are obtained in the k_T -factorization approach. We used an unintegrated gluon distribution constructed from the KMR prescription [32] based on the MSTW2008 collinear NLO gluon distribution [33]. In all cases the gauge invariant matrix elements discussed in the present paper were used.

	χ_{c0}	χ_{c1}	χ_{c2}
χ_{c0}	1.32	1.71	4.24
χ_{c1}	...	0.84	2.88
χ_{c2}	...	...	3.45

FIG. 4. Single-particle rapidity distributions of χ_c -quarkonia for different spin combinations. In the upper right panel the distribution of the first meson of the $\chi_c(i)\chi_c(j)$ -pair is shown. The lower panel shows the distributions of the second meson.

Khoze_Martin-Ryskin (KMR) prescription [32] based on the MSTW2008 collinear Next-to-leading order (NLO) gluon distribution [33]. For the renormalization scales μ_{r1}^2 , μ_{r2}^2 of the running coupling and factorization scales μ_{F1}^2 , μ_{F2}^2 entering the unintegrated gluon distribution, we choose

$$\begin{aligned}\mu_{r1}^2 &= \mu_{F1}^2 = \max\{m_c^2, |\vec{q}_{1T}|^2\}, \\ \mu_{r2}^2 &= \mu_{F2}^2 = \max\{m_c^2, |\vec{q}_{2T}|^2\},\end{aligned}\quad (3.1)$$

where these scales refer to the running coupling/gluon distribution coupling to gluon q_1 or q_2 respectively. We refrain from a detailed analysis of dependence on the factorization scale, the distributions shown below simply serve to get an impression of the salient features of the production mechanism. A more detailed analysis, including theoretical errors will be given in a future work [34], where we will address the feed-down into the $J/\psi J/\psi$ channel.

There are six independent cross sections related to the different spin combinations (see Table I). We see that the cross sections for different spin combinations are of the same order of magnitude.

B. Single-particle distributions

Let us now turn to single-particle distributions. In Fig. 4 we show rapidity distributions for χ_c mesons for different pair combinations. In the upper left panel we show: $\chi_{c0}\chi_{c0}$ (solid line), $\chi_{c1}\chi_{c1}$ (dashed line), and $\chi_{c2}\chi_{c2}$ (dotted line). In the other panels we show the rapidity distributions for: $\chi_{c0}\chi_{c1}$ (solid line), $\chi_{c0}\chi_{c2}$ (dashed line), and $\chi_{c1}\chi_{c2}$ (dotted line). In the upper left panel the distribution of the first listed quarkonium is shown, while the distributions of the second listed quarkonium are shown in the lower panel. Evidently for the nonidentical quarkonia the distribution of the first and second meson are not the same.

FIG. 5. Transverse momentum distributions of χ_c -quarkonia for different spin combinations. In the upper right panel the distribution of the first meson of the $\chi_c(i)\chi_c(j)$ -pair is shown. The lower panel shows the distributions of the second meson.

In Fig. 5 we show similar distributions in transverse momentum of a single χ_c . The distributions for χ_{c1} quarkonia are somewhat harder than those for the other spin states. This may have important consequences for large transverse momenta, also for J/ψ pair production (CDF, ATLAS, CMS), but goes beyond the scope of the present paper.

C. χ_c -pair distributions

Let us now turn to distributions/correlations of the $\chi_c\chi_c$ -pair. We start from the rapidity difference Δy between mesons. The exchange of gluons leads to broad distributions in the difference of rapidities Δy of the two quarkonia, as shown in Fig. 6.

A closely related observable is the pair invariant mass shown in Fig. 7, here the effect of the gluon exchange leads to the long tail in invariant mass $M_{\chi_c\chi_c}$, in fact large rapidity

distance corresponds directly to large pair invariant mass (see also the discussion of Fig. 10 below).

We also see that the considered by us gluon-exchange mechanism leads to a deep dip at $\Delta y = 0$ for all final states. Here a brief comment on the possible role of “box diagram” contributions is in order. As mentioned above, these diagrams involve a parton of off-shellness of order $\hat{s} = M_{\chi\chi}^2$. The corresponding parton-parton cross quickly rises close to the threshold and then falls off rapidly $\propto \hat{s}^{-3}$. This behavior gives rise to a rather narrow bump centered around $\Delta y = 0$, which can potentially fill the dip shown in Fig. 6. This behavior is very similar to the direct $gg \rightarrow J/\psi J/\psi$ process; however the $\chi_c\chi_c$ box diagram cross section is expected to be suppressed by the small factor [23]:

$$\lambda^2 = \left(\frac{|R'(0)|^2}{M_\chi^2 |R(0)|^2} \right)^2 \sim 10^{-3}. \quad (3.2)$$

FIG. 6. Distributions in the rapidity separation between χ_c 's for different spin combinations. Notice that the dips at $\Delta y = 0$ will be filled by box diagrams not considered in this paper.

FIG. 7. Distributions in the invariant mass of the χ_c -pair for different spin combinations for the gluon-exchange mechanism.

We will discuss a rough estimate of the box diagram contributions for χ_c and J/ψ pairs to the $J/\psi J/\psi$ final state at the end of this section.

In calculations based on collinear gluon distributions, the two χ_c mesons are produced back-to-back at the lowest order. This is not so in the k_T -factorization approach discussed here. In Fig. 8 we show distributions of the transverse momentum of the meson pair, $p_{T,\text{sum}}$. The distribution for the $\chi_{c1}\chi_{c1}$ extends to large pair transverse momenta, which is related to the corresponding vertex structure.

The χ_c mesons radiatively decay into J/ψ mesons. The double feed-down leads to a new contribution to the $J/\psi J/\psi$ channel. The direct $J/\psi J/\psi$ contribution is more than an order of magnitude larger than the feed-down contribution. However, the $\chi_c\chi_c$ contribution has its own

specificity. In Fig. 9 we show distribution in rapidity difference for all $\chi_c\chi_c$ contributions weighted by branching fractions into J/ψ channel (solid line) compared to the standard direct $J/\psi J/\psi$ contribution (dashed line). At large rapidity difference the feed-down contribution dominates over the contribution of the standard mechanism. Here we assumed that the J/ψ 's from the decay will be collinear to their parent χ_c 's. How important is the feed-down contribution for different experimental situations will be discussed elsewhere [34].

We also show an estimate of the box contribution for χ_c -pairs. Here we simply rescaled the J/ψ -pair box amplitude by the factor λ defined in Eq. (3.2). We calculated the kinematics for the proper masses in the final state and included the relevant branching fractions. We see

FIG. 8. Distributions in the transverse momentum of χ_c -quarkonium pairs for different spin combinations.

that the so-estimated box contribution from χ_c pairs is completely negligible in the $J/\psi J/\psi$ -channel. Notice that it is even much smaller than the contribution from the gluon exchange mechanism. All this is fully consistent with earlier calculations of Baranov in the collinear parton model and in $\gamma\gamma$ collisions [23].

Clearly the box contribution for $\chi_c\chi_c$ is negligible, especially for large rapidity difference. For $|\Delta y| > 2$, even the interference of the contribution with the gluon-exchange mechanisms is negligible as is obvious from standard general rules of quantum mechanics. If one separate contribution is

smaller than the other by more than two orders of magnitude, maximal interference effect can be only a few percent, clearly unimportant effect in the present context.

For completeness in Fig. 10 we show also invariant mass distribution of the $\chi_c\chi_c$ system, compared to the one for $J/\psi J/\psi$ box contribution. Clearly one can expect that the feed-down of $\chi_c\chi_c$ contribution is important in the region of large invariant masses, where the present state-of-art calculations break down. It becomes clear that large

FIG. 9. Distributions in the rapidity difference between two J/ψ mesons. *Short dashed line*: leading-order box mechanism for direct J/ψ -pair production. *Solid line*: feed-down from χ_c produced via the gluon exchange mechanism. Shown is the sum over all $\chi_c\chi_c$ combinations multiplied by combined branching fractions. *Long dashed line*: feed-down from χ_c 's produced via “box diagram” estimated as described in the text.

FIG. 10. Invariant mass distribution of $\chi_c\chi_c$ systems weighted by the corresponding branching fractions compared to the invariant mass distribution of the $J/\psi J/\psi$ system for the dominant box contribution. The dashed line shows the contribution from the box diagrams to direct J/ψ -pair production. The solid line is the χ_c -pair contribution from the gluon exchange mechanism, summed over all states and weighted by respective branching fractions. The dotted line shows the χ_c -pair contribution with the cut $|\Delta y| > 2$ imposed.

FIG. 11. Transverse momentum distributions of χ_c -quarkonia for different spin combinations. Here the cut on Δy was imposed.

invariant masses of the $\chi_c\chi_c$ -system are directly related to a large rapidity separation of the χ_c 's and not to large transverse momenta of χ_c -mesons.

D. Single-particle distributions with the cut $|\Delta y| > 2$

In the calculations presented up to now no additional cuts on χ_c 's were imposed. In Fig. 11 we show the transverse momentum spectra for events where $|\Delta y| > 2$. We see that we lose only less than 20% of the cross sections, and the distributions are very similar to the ones integrated over the full phase space. The same is true for the distributions of the transverse momentum of the meson pair, shown in Fig. 12 with the cut $|\Delta y| > 2$.

E. Double parton scattering

Let us now turn to the DPS contribution to χ_c -pair production. Using the factorized ansatz one writes the DPS cross section as

$$\begin{aligned} & \frac{d\sigma_{\text{DPS}}(pp \rightarrow \chi_{ci}\chi_{cj}X)}{dy_1 dy_2 d^2\vec{p}_{1T} d^2\vec{p}_{2T}} \\ &= \frac{1}{1 + \delta_{ij}} \frac{1}{\sigma_{\text{eff}}} \frac{d\sigma(pp \rightarrow \chi_{ci}X)}{dy_1 d^2\vec{p}_{1T}} \frac{d\sigma(pp \rightarrow \chi_{cj}X)}{dy_2 d^2\vec{p}_{2T}}. \end{aligned} \quad (3.3)$$

In our calculations we take $\sigma_{\text{eff}} = 15$ mb which is a typical value for gluon-gluon initiated processes (see e.g., [35]). The cross section for each step ($pp \rightarrow \chi_c$) is calculated in the k_T -factorization approach as done recently in [18,19], where a rather good agreement with the world experimental data was achieved. Below we show some selected results for the double parton scattering contribution.

In Fig. 13 we show our results for the distribution in transverse momentum of the quarkonium pairs. The cross sections obtained for the DPS contribution are of similar order as those obtained for our single parton scattering mechanism considered in the previous subsection. Also shapes of distributions are rather similar as those for the

FIG. 12. Distributions in the transverse momentum of χ_c -quarkonium pairs for different spin combinations. Here the cut on Δy was imposed.

FIG. 13. χ -pair production in DPS: distributions in the transverse momentum of χ_c -quarkonium pairs for different spin combinations.

FIG. 14. χ -pair production in DPS: distributions in rapidity distance between quarkonia for different spin combinations.

single parton scattering mechanism (compare these results with those shown in Fig. 8).

In Fig. 14 we show distributions in Δy . The reader is asked to compare these distributions with those shown in Fig. 6. Both distributions are very broad. Of course there is no minimum at $\Delta y = 0$ for the DPS distributions.

Summarizing, we observe similar distributions in $p_{T,\text{sum}}$ and Δy for single and double parton scattering production of different quarkonia states. This shows that both contributions must be included in analysis of future data for $\chi_{cJ_i}\chi_{cJ_j}$ production.

IV. CONCLUSIONS

We have made a first exploratory study of χ_c pair production in proton-proton collisions. The $g^*g^* \rightarrow \chi_{cJ_i}\chi_{cJ_j}$ amplitudes for off-shell gluons and different spin combinations $J_i, J_j = 0, 1, 2$ are calculated based on $g^*g^* \rightarrow \chi_{cJ}$ vertices calculated within the color-singlet nonrelativistic perturbative QCD approach. In this approach the vertices are proportional to the derivative of the spatial wave function at the origin $|R'(0)|$. The value of this quantity can be obtained from models of the quarkonia states. Here it has been obtained from the $\chi_{c0} \rightarrow \gamma\gamma$ branching fraction which was measured experimentally.

We have performed calculations within the k_T -factorization approach for the $pp \rightarrow \chi_c\chi_c X$ process at $\sqrt{s} = 8$ TeV using Kimber-Martin-Ryskin [32] type unintegrated gluon distribution based on the MSTW2008 [33] collinear gluons.

In the present paper we have considered only the color-singlet contribution to the $\chi_c\chi_c$ channel. This was in part motivated by a success of the color-singlet approach in k_T -factorization observed in [18,19]. A small color-octet contribution to the $\chi_c\chi_c$ channels is not excluded; however, according to our knowledge there is no reliable method for how to estimate it. First color-octet contribution to single χ_c must be known. The color-octet contribution can be only fitted, provided other mechanisms (such as color-singlet) are well known. In any case, the color-octet contribution would only increase the cross section for $\chi_c\chi_c$ and associated feed-down to the $J/\psi J/\psi$ channel. Probably it would not be a big effect in view that single χ_c production can be roughly understood within color-singlet mechanism [18,19].

We have found that the cross sections for different combinations of χ_c quarkonia are of a similar size. The integrated cross sections for different channels are of the order of a few nb. This is of the same order of magnitude as the cross section for J/ψ pair production. This means that a feed-down from the double χ_c decays $\chi_c \rightarrow J/\psi\gamma$ leads to an extra non-negligible contribution which has to be included in the total prompt production of two J/ψ mesons. Due to specific branching fractions the $\chi_{c1}\chi_{c1}$, $\chi_{c1}\chi_{c2}$ and $\chi_{c2}\chi_{c2}$ channels are the dominant ones. The other three contributions can be safely neglected.

The $\chi_c\chi_c$ contribution to the $J/\psi J/\psi$ final state is interesting but goes beyond the scope of the present analysis and will be studied in detail in future dedicated analyses.

The salient feature of the t and u -channel gluon exchange mechanism are the broad distributions in rapidity difference Δy between χ_c mesons. This is to be contrasted with the narrow Δy distribution of J/ψ pairs at leading order. A feed-down from double χ_c production to the double J/ψ channel is therefore expected to be important at large Δy and/or large invariant mass of the quarkonium pair and may mimic the kinematical behavior of double parton scattering mechanisms.

We have presented also results for double parton scattering mechanisms. The results have been obtained within factorized ansatz with standard σ_{eff} parameter. In general, we have obtained cross sections of similar size for single and double parton scattering mechanisms. Our single parton scattering results are therefore very important ingredients that must be taken into account when extracting σ_{eff} from experimental data for $\chi_{cJ_i}\chi_{cJ_j}$ production. Neglecting single parton scattering contributions is dangerous in the context of similar shapes (e.g., in $p_{T,\text{sum}}$ or Δy) of the DPS and single parton scattering (SPS) contributions as shown in our paper.

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