Standard model effective field theory: Integrating out neutralinos and charginos in the MSSM

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We apply the covariant derivative expansion method to integrate out the neutralinos and charginos in the minimal supersymmetric Standard Model. The results are presented as set of pure bosonic dimension-six operators in the Standard Model effective field theory. Nontrivial chirality dependence in fermionic covariant derivative expansion is discussed carefully. The results are checked by computing the $h\gamma\gamma$ effective coupling and the electroweak oblique parameters using the Standard Model effective field theory with our effective operators and direct loop calculation. In global fitting, the proposed lepton collider constraint projections, special phenomenological emphasis is paid to the gaugino mass unification scenario ($M_2 \simeq 2M_1$) and anomaly mediation scenario ($M_1 \simeq 3.3M_2$). These results show that the precision measurement experiments in future lepton colliders will provide a very useful complementary job in probing the electroweakino sector, in particular, filling the gap of the soft lepton plus the missing E_T channel search left by the traditional collider, where the neutralino as the lightest supersymmetric particle is very degenerated with the next-to-lightest chargino/neutralino.

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I. INTRODUCTION

As an effective tool of connecting the new physics to the experimental observations, the Standard Model effective field theory (SMEFT) is recently being intensively developed and applied (refer to [1,2] for a review). One advantage of the SMEFT is that it provides a model independent way when comparing with experimental results. In the SMEFT, the beyond Standard Model (SM) effects are characterized by a series of high dimensional operators, which are constructed with the SM fields only and satisfy the SM $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry. Regarding the precision of the current and near future experiments, it is usually

sufficient to consider only dimension-six operators, the number of which is finite (59 when demanding baryon number conservation) [3,4]. The experimental observables can be expressed by SM parameters and Wilson coefficients of high dimensional operators. Then, the results of experiments can be transformed into the constraints on these Wilson coefficients. For theorists who have their favorite models and want to test them through experiments, the main work is to calculate those coefficients of the effective operators from the model.

To effectively match a new physics model to the SMEFT, the method of covariant derivative expansion (CDE), initially introduced in [5–7], is being well developed recently [1,8–14]. Comparing to the traditional way of Feynman diagram calculations, which is relatively tedious and not gauge invariant in the process, the CDE method can directly obtain the gauge-invariant operators in a much simpler way. Using this method, the effective operators have been calculated for various new physics models and the corresponding phenomenological implications were discussed [15–20].

A particularly important kind of the beyond SM models is the supersymmetric one, which provides elegant solutions to the naturalness problem, the dark matter

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candidates, and the path to the grand unification. Applying the CDE method, people have already worked out the dimension-six effective operators by integrating out the sfermions in the minimal supersymmetric standard model (MSSM) to one-loop order. In this work we consider another important sector of the MSSM, the neutralinos and charginos (or called the electroweakino sector). The electroweakino sector of the MSSM is of great significance in phenomenology, because the lightest neutralino is often considered as a very competitive dark matter candidate. We calculate dimension-six operators by integrating out the neutralinos and charginos with the CDE method. Since the original form of the electroweakino sector is expressed with two-component Weyl fermions, the procedure is not as straightforward as those in the literature. We show in detail how to transform the Lagrangian to a desired form to apply the CDE and then obtain the effective operators with the universal one-loop effective action [8]. Our results can be used in a lot of phenomenological studies of the effects of the neutralinos and charginos, e.g., cosmology [20–22] and collider indirect searches [23–27]. As an illustration, we apply our results in future lepton colliders, to see how the precise electroweak and Higgs observables constrain the MSSM electroweakino sector.

The paper is organized as follows. Section II presents the key steps to apply the CDE formalism to the electroweakino sector. The analytical cross-check against other calculation methods are displayed in Sec. III, and the constraints on parameter space of MSSM are discussed in Sec. IV. We summarize and conclude in Sec. V. Some discussions about the regularization options and the Wilson coefficients of the effective operators are presented in appendices.

II. THE FORMULISM AND INPUT

Each of the two Higgs superfields has the same degree of freedom with the supersymmetrization of the left chiral SM lepton, and the gaugino is also of Majorana type and has the same degree of freedom with a Weyl fermion, so the natural way to write down the Lagrangian is in the two-component Weyl spinor form. Taking the left chiral form, for example, [28]

$$\begin{aligned} \mathcal{L}_{\text{MSSM}} \supset & i\tilde{B}^\dagger \bar{\sigma}^\mu D_\mu \tilde{B} + i\tilde{W}^\dagger \bar{\sigma}^\mu D_\mu \tilde{W} + i\tilde{H}_u^\dagger \bar{\sigma}^\mu D_\mu \tilde{H}_u \\ & + i\tilde{H}_d^\dagger \bar{\sigma}^\mu D_\mu \tilde{H}_d - \frac{1}{2}(M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}\tilde{W} + \text{c.c.}) \\ & - \mu(\tilde{H}_u^T \epsilon \tilde{H}_d + \text{c.c.}) \\ & + \left[\frac{\sqrt{2}}{2} g(H_u^\dagger \sigma^a \tilde{H}_u + H_d^\dagger \sigma^a \tilde{H}_d) \tilde{W}^a \right. \\ & \left. + \frac{\sqrt{2}}{2} g'(H_u^\dagger \tilde{H}_u - H_d^\dagger \tilde{H}_d) \tilde{B} + \text{c.c.} \right], \end{aligned} \quad (1)$$

where $\tilde{H}_u = (\tilde{H}_u^+, \tilde{H}_u^0)^T$, $\tilde{H}_d = (\tilde{H}_d^{0*}, -\tilde{H}_d^-)^T$ (so that the charge conjugation $\tilde{H}_d^c = \epsilon \tilde{H}_d^*$ has the same phase with $\tilde{H}_u$), $\tilde{W} = (\tilde{W}^1, \tilde{W}^2, \tilde{W}^3)^T$ and $\tilde{B}$ are the Higgsinos and electroweak gauginos, and $()^T$ indicates transpose. Here the $\epsilon = -i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is the two dimensional antisymmetric matrix used to contract the two $SU(2)$ doublets $\tilde{H}_u$ and $\tilde{H}_d$, while other ϵ matrixes used to contract two Weyl spinors are not shown explicitly. And σ^a are the Pauli matrices.

We follow the procedure of the CDE method [1] to integrate out the neutralinos and charginos in the MSSM at one-loop level. Since the CDE directly works in the four-component Dirac spinors, in the following we will show the procedures of organizing Eq. (1) into that form. Because $\xi^\dagger i\bar{\sigma}^\mu \partial_\mu \chi = \chi i\sigma^\mu \partial_\mu \xi^\dagger$ (which follows the identity $\xi^\dagger \bar{\sigma}^\mu \chi = -\chi \sigma^\mu \xi^\dagger$ and integration by parts), both the bino (wino) and the Higgsino kinetic terms can be arranged in the Dirac form with a overall factor of $\frac{1}{2}$. However, it is noted that the CDE formalism requires a unique covariant derivative in the basis where the BSM fields are organized with their SM representation components expanded, but this is not simply true for chiral fermions between different chiralities, if the fermions are in the fundamental representation of some non-Abelian gauge group. The most straightforward way to see it is through the identity $g\psi_i^\dagger \bar{\sigma}^\mu \psi_j A_\mu^a t_{ij}^a = -g\psi_j \sigma^\mu \psi_i^\dagger A_\mu^a t_{ij}^a$ in the two-component spinor basis, which indicates that an extra matrix transpose for the gauge group generator of the fundamental representation is needed.¹

Another way to clarify such an issue with the non-Abelian gauge field is in terms of the raising and lowering operators. For example, such $SU(2)_L$ gauge field on the doublet fundamental representation is usually chosen as $\frac{q}{\sqrt{2}} \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}$, which is consistent with left chiral fermion ordering $(\tilde{H}_u^+, \tilde{H}_u^0, \tilde{H}_d^{0*}, -\tilde{H}_d^-)^T$ if it acts on the first two and last two components in a block diagonal way. For the right chiral fermion ordering to be consistently acted by the same gauge field, apparently the choice is $(\tilde{H}_d^+, \tilde{H}_d^0, \tilde{H}_u^{0*}, -\tilde{H}_u^-)^T$, or the overall charge conjugation of the left chiral one. Combining with the gaugino, the complete Dirac spinor form of the electroweakino is chosen as $\chi = (\chi^B, \chi^1, \chi^2, \chi^3, \chi^+, \chi^0, \chi^{0*}, -\chi^-)^T$ in the SM $SU(2)_L \times U(1)_Y$ component field basis, with each component of Dirac field written in the van der Waerden form

$$\begin{aligned} \chi^B &= \begin{pmatrix} \tilde{B} \\ \tilde{B}^\dagger \end{pmatrix}, & \chi^1 &= \begin{pmatrix} \tilde{W}^1 \\ \tilde{W}^{1\dagger} \end{pmatrix}, \\ \chi^2 &= \begin{pmatrix} \tilde{W}^2 \\ \tilde{W}^{2\dagger} \end{pmatrix}, & \chi^3 &= \begin{pmatrix} \tilde{W}^3 \\ \tilde{W}^{3\dagger} \end{pmatrix}, \end{aligned} \quad (2)$$

¹There is no such problem in the gaugino sector, due to the identity $f^{abc} = -f^{cba}$ for adjoint representation.

$$\begin{aligned}\chi^+ &= \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_d^+ \end{pmatrix}, & \chi^0 &= \begin{pmatrix} \tilde{H}_u^0 \\ \tilde{H}_d^0 \end{pmatrix}, \\ \chi^{0*} &= \begin{pmatrix} \tilde{H}_d^{0*} \\ \tilde{H}_u^{0*} \end{pmatrix}, & \chi^- &= \begin{pmatrix} \tilde{H}_d^- \\ \tilde{H}_u^- \end{pmatrix}.\end{aligned}\quad (3)$$

Here we have suppressed the undotted and dotted spinor index, and for Higgsino the physical electric charge is used instead of the (in)existence of a $\dagger$.

Then we can apply the CDE. The fermionic new physics model path integral is formally written as

$$e^{iS_{\text{eff}}} = \int D\bar{\chi} D\chi \exp \left\{ \frac{1}{2} \bar{\chi} (i\gamma^\mu D_\mu - M - U_L P_L - U_R P_R) \chi \right\}, \quad (4)$$

where the $P_{L,R} = (1 \mp \gamma_5)/2$ are left and right projection operators. We can get the SM field independent M matrix and the dependent U_L, U_R matrix

$$M = \begin{pmatrix} M_1 & & & & \\ & M_2 & & & \\ & & M_2 & & \\ & & & \mu & \\ & & & & \mu \\ & & & & & \mu \\ & & & & & & \mu \end{pmatrix}, \quad (5)$$

$$U_L = \begin{pmatrix} -\frac{g'}{\sqrt{2}} H_d^+ & -\frac{g}{\sqrt{2}} H_d^0 & \frac{ig}{\sqrt{2}} H_d^0 & -\frac{g}{\sqrt{2}} H_d^+ \\ -\frac{g'}{\sqrt{2}} H_d^0 & -\frac{g}{\sqrt{2}} H_d^+ & -\frac{ig}{\sqrt{2}} H_d^+ & \frac{g}{\sqrt{2}} H_d^0 \\ \frac{g'}{\sqrt{2}} H_u^{0*} & \frac{g}{\sqrt{2}} H_u^- & -\frac{ig}{\sqrt{2}} H_u^- & -\frac{g}{\sqrt{2}} H_u^{0*} \\ -\frac{g'}{\sqrt{2}} H_u^- & -\frac{g}{\sqrt{2}} H_u^{0*} & -\frac{ig}{\sqrt{2}} H_u^{0*} & -\frac{g}{\sqrt{2}} H_u^- \end{pmatrix}, \quad (6)$$

$$U_R = U_L^\dagger, \quad (7)$$

where the blank positions in the above matrices are zeroes. Apparently if only the neutral (charged) components are picked out and the two Higgs fields are set to their vacuum expectation value, then up to some column or row switches and minus signs, we will get the standard neutralino (chargino) mass mixing matrix.

When following the technique of adding $\ln(\not{p} + M + U)$ to $\ln(\not{p} - M - U)$ in [1] to convert the above fermionic functional determinant to a bosonic-like one, for $U_L \neq U_R$ a further attention is needed. Note that with the identity $UU^\dagger = U^\dagger U$ under the implicit trace, it is $(\not{p} + U_L P_L + U_L^\dagger P_R)(\not{p} - U_L^\dagger P_L - U_L P_R) = p^2 - (U_L^\dagger U_L)(P_L + P_R) + \dots = p^2 - U_L^\dagger U_L + \dots$ (ignore M which is always the same for both chiralities) that reproduces the canonical kinetic and mass terms, rather than $(\not{p} + U_L P_L + U_L^\dagger P_R) \times (\not{p} - U_L P_L - U_L^\dagger P_R)$. If we rewrite

$$\begin{aligned}U_L P_L + U_R P_R &= U + U_5 \gamma_5 \quad \text{with} \\ U &= \frac{U_L + U_R}{2}, \quad U_5 = \frac{U_R - U_L}{2},\end{aligned}\quad (8)$$

it means to add $\ln(\not{p} + M + U - U_5 \gamma_5)$ to $\ln(\not{p} - M - U - U_5 \gamma_5)$, namely, the $\not{p} \rightarrow -\not{p}$ in the trick in [1] also induces a $\gamma_5 \rightarrow -\gamma_5$. This should bring us no trouble because the trace containing odd numbers of γ_5 will give us CP -odd operators and thus must vanish since we are considering the CP -even Lagrangian in Eq. (1).

Finally, in the CDE, the Lagrangian reads²

$$\mathcal{L}_{\text{eff}} = -\frac{i}{4} \int \frac{d^4 p}{(2\pi)^4} \text{tr} \ln [p^2 - M^2 - \tilde{U}_{\text{ferm}} - \tilde{G}], \quad (9)$$

with

$$\begin{aligned}U_{\text{ferm}} &= \{M, U\} + U^2 - U_5^2 - i[\not{p}, U] \\ &\quad - [M + U - i\not{p}, U_5] \gamma_5 - \frac{g}{2} \sigma^{\mu\nu} F_{\mu\nu}^a t^a\end{aligned}\quad (10)$$

²Note that we have the coefficient $-1/4$ instead of the usual $-1/2$ from the fermionic determinant (and after using the technique in [1]), because we have both the fermions and their conjugates in the χ in Eq. (4).

TABLE I. Dimension-six SMEFT CP -even bosonic operators.

$\mathcal{O}_{GG} = g_s^2 H^\dagger H G_{\mu\nu}^a G^{a,\mu\nu}$	$\mathcal{O}_H = \frac{1}{2}(\partial_\mu H^\dagger H)^2$
$\mathcal{O}_{WW} = g^2 H^\dagger H W_{\mu\nu}^a W^{a,\mu\nu}$	$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2$
$\mathcal{O}_{BB} = g^2 H^\dagger H B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_R = H^\dagger H D_\mu H^\dagger D^\mu H$
$\mathcal{O}_{WB} = 2gg' H^\dagger t^a H W_{\mu\nu}^a B^{\mu\nu}$	$\mathcal{O}_D = D^2 H^\dagger D^2 H$
$\mathcal{O}_W = ig(H^\dagger \overleftrightarrow{D}_\mu t^a H) D_\nu W^{a\mu\nu}$	$\mathcal{O}_6 = (H^\dagger H)^3$
$\mathcal{O}_B = ig'(H^\dagger \overleftrightarrow{D}_\mu H) \partial_\nu B^{\mu\nu}$	$\mathcal{O}_{2G} = -\frac{1}{2}(D^\mu G_{\mu\nu}^a)^2$
$\mathcal{O}_{3G} = \frac{1}{3!} g_s f^{abc} G_p^{a\mu} G_\mu^{b\nu} G_\nu^{c\rho}$	$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W_{\mu\nu}^a)^2$
$\mathcal{O}_{3W} = \frac{1}{3!} g \epsilon^{abc} W_\rho^{a\mu} W_\mu^{b\nu} W_\nu^{c\rho}$	$\mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$
$\mathcal{O}_{HW} = 2ig(D_\mu H)^\dagger t^a (D_\nu H) W^{a\mu\nu}$	
$\mathcal{O}_{HB} = 2ig' Y_H (D_\mu H)^\dagger (D_\nu H) B^{\mu\nu}$	

$$\tilde{U}_{\text{ferm}} = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} D_{\mu_1} \cdots D_{\mu_n} U_{\text{ferm}} \frac{\partial^n}{\partial p_{\mu_1} \cdots \partial p_{\mu_n}}, \quad (11)$$

$$\tilde{F}_\mu = g \sum_{n=0}^{\infty} \frac{n+1}{(n+2)!} (-i)^{n+1} D_{\mu_1} D_{\mu_n} F_{\nu\mu}^a \frac{\partial^{n+1}}{\partial p_{\mu_1} \cdots \partial p_{\mu_n} \partial p_\nu}, \quad (12)$$

$$\tilde{G} = -\{\tilde{F}_\mu, p^\mu\} - \tilde{F}_\mu \tilde{F}^\mu, \quad (13)$$

where $[\cdot, \cdot]$ and $\{\cdot, \cdot\}$ denote commutator and anticommutator, respectively, and $F_{\mu\nu}^a$ is the field strength of gauge field A^a . The terms for the $U_5 = 0$ case have been explicitly expanded in [16]. Note that we adopt different regularization in dealing with the logarithm of Eq. (9) compared with [15]; further relevant discussion is given in Appendix A.

III. ANALYTICAL RESULTS AND CHECK

The target operators $\mathcal{O}_i$ are listed in Table I.³ The analytical Wilson coefficients c_i are shown in Appendix B. Note that the coefficients of $\mathcal{O}_{2W}$, $\mathcal{O}_{3W}$, $\mathcal{O}_{2B}$ are obtained by applying the universal results of [1] to the MSSM. The unshown coefficients of $\mathcal{O}_{GG}$, $\mathcal{O}_{3G}$, $\mathcal{O}_{2G}$ are zeros. The coefficient of $\mathcal{O}_6$ is very tedious, and since it does not contribute to the currently interesting precision measurements we also omit it in this paper.

These results can be checked by inspecting the correction to $h\gamma\gamma$ effective coupling $\bar{c}_{h\gamma\gamma}$, which is defined as

³This is a redundant basis, in the sense that $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ can be switched into $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$, $\mathcal{O}_{WB}$, $\mathcal{O}_W$, and $\mathcal{O}_B$, by using the following relations:

$$\begin{aligned} \mathcal{O}_{HW} &= \mathcal{O}_W - \frac{1}{4}(\mathcal{O}_{WW} + \mathcal{O}_{WB}), \\ \mathcal{O}_{HB} &= \mathcal{O}_B - \frac{1}{4}(\mathcal{O}_{BB} + \mathcal{O}_{WB}). \end{aligned} \quad (14)$$

$$\Delta\mathcal{L}_{h\gamma\gamma} = \bar{c}_{h\gamma\gamma} h A^{\mu\nu} A_{\mu\nu}, \quad (15)$$

where $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength. On the one hand, in the dimension-six SMEFT, the $\bar{c}_{h\gamma\gamma}$ can be expressed as [1]

$$\bar{c}_{h\gamma\gamma} = g^2 \sin^2 \theta_w v (c_{WW} + c_{BB} - c_{WB}), \quad (16)$$

where $v = 246$ GeV is the Higgs vacuum expectation value. Plugging in the corresponding Wilson coefficients in Appendix B, we get

$$\bar{c}_{h\gamma\gamma}^{EFT} = -\frac{g^4 \sin^2 \theta_w v \sin 2\beta}{48\pi^2 \mu M_2}. \quad (17)$$

On the other hand, in the low energy theorem, the correction to the $h\gamma\gamma$ effective coupling from chargino loops can be calculated [29–32]:

$$\bar{c}_{h\gamma\gamma}^{LET} = \frac{g^4 \sin^2 \theta_w v \sin 2\beta}{48\pi^2 (g^2 v^2 \sin 2\beta - \mu M_2)}. \quad (18)$$

From Eqs. (17) and (18) above, one can see that the results derived from the two methods are consistent with each other when the value of μM_2 is much larger than that of m_w^2 .

To further illustrate the validity of the EFT we obtained, we also compare Eq. (17) with that from the direct one-loop Feynman diagram calculation with charginos in the loop. In this loop calculation we use FeynCalc [33,34] to calculate the amplitude with the help of the LoopTools [35] to get the scalar loop integrals numerically. In Fig. 1, blue contours show the deviation of the two methods which is defined as

$\Delta R = \frac{\bar{c}_{h\gamma\gamma}^{EFT}}{\bar{c}_{h\gamma\gamma}^{\text{loop}}} - 1$ [36], and red curves show masses of the lighter chargino $m_{\tilde{\chi}_1^\pm}$ at tree level. One can see that the EFT works well when the charginos are heavier than about 200 GeV, characterized by $\Delta R < 0.1$.

Moreover, we also calculate the oblique parameters S and T analytically to check our results. For simplicity, we assume that $\mu = M_1 = M_2 = M$ and $\tan \beta = 1$, and we find that both the EFT results and the one-loop Feynman diagram induced results are consistent. In this case, the S and T are exactly the same within these two methods, i.e.,

$$S = \frac{v^2(g^2 + 43g^2)}{240\pi M^2}, \quad (19)$$

$$T = \frac{v^2(g^2 + g^2)}{40\pi M^2}. \quad (20)$$

IV. PROJECTED FUTURE CONSTRAINTS

In this section, we mainly discuss the above EFT contributions to the electroweak precision test (EWPT) (the oblique parameters T and S), the triple gauge boson

FIG. 1. Blue contours show the value of ΔR and red contours show the value of $m_{\tilde{\chi}_1^\pm}$. Here the results are independent of M_1 and we choose $\tan\beta = 2$.

coupling (TGC), and Higgs production and decay rates, which will be measured with high precision at the future lepton colliders such as the ILC, the CEPC, and the FCC-ee. The mapping of the Wilson coefficients onto these observables are well summarized in Sec. 4 of Ref. [1]. Note that here we consider only the contribution from the electroweakino sector, rather than the whole MSSM which also includes the sfermions (see [8, 15, 18]) and gluinos, as well as the extra Higgs (see, e.g., [17]). For natural supersymmetry the stop sector usually has the largest contribution, but it is facing more and more severe experimental constraints. In scenarios with heavy sfermions which are usually motivated by the neutralino lightest supersymmetric particle (LSP) dark matter candidate study, e.g., the split supersymmetry scenario, the dominant contribution could come from the electroweakino sector, and the collider search channel based on $\tilde{t} \rightarrow \tilde{\chi}_1^0 + t$ becomes inaccessible. In this paper we assume such a scenario.

The projected sensitivities of the proposed future lepton colliders are listed in Table II. Naively we can see from the EWPT and Higgs data that the FCC-ee would be the best

platform to perform such a precision test. So here we will combine the projected FCC-ee EWPT and Higgs measurements with the projected CEPC TGC measurements as our example of applying our EFT results to constrain model parameters.

The combined constraints from all three kinds of measurements, as the indirect searches for new physics, are presented in Figs. 2 and 3, for the gaugino unification scenario (Fig. 2, $M_1 = M_2/2$) and the anomaly mediation scenario (Fig. 3, $M_1 = 3.3M_2$), respectively. In each scenario we choose a typical small $\tan\beta$ value of 2 (upper panels) and a typical large $\tan\beta$ value of 50 (lower panels), and show two choices of the sign of μ (left and right panels). The 1σ and 2σ allowed regions are green and yellow shaded. We can see that the χ^2 constrained region on the M_2 vs μ plane are almost the same for the two different scenarios, which can be understood because the bino's contribution is not significant. The dependence on $\tan\beta$ is not strong on this plane either. And at large $\tan\beta$ the constraints are more symmetric between different μ choices.

TABLE II. The uncertainties expected at each experiment, for the EWPT experiments (T and S), the TGC experiments (Δg_1^Z , $\Delta\kappa_\gamma$ and λ_γ), and two representative channels of eight Higgs experiment channels (the others refer to the references therein).

Observable	T	S	$10^4 \Delta g_1^Z$	$10^4 \Delta\kappa_\gamma$	$10^4 \lambda_\gamma$	$\frac{\Delta(\sigma_{Zh} \text{Br}_{bb}^{\text{SM}})}{\sigma_{Zh}^{\text{SM}} \text{Br}_{bb}^{\text{SM}}}$	$\frac{\Delta(\sigma_{Zh} \text{Br}_{\tau\tau}^{\text{SM}})}{\sigma_{Zh}^{\text{SM}} \text{Br}_{\tau\tau}^{\text{SM}}}$
ILC (250 GeV)	0.022 [37]	0.017 [37]				1.1% [38]	35% [38]
ILC (500 GeV)	0.022 [37]	0.017 [37]	2.8 [39]	3.1 [39]	4.3 [39]	0.66% [38]	23% [38]
ILC (1 TeV)	0.022 [37]	0.017 [37]	1.8 [39]	1.9 [39]	2.6 [39]	0.47% [38]	8.5% [38]
CEPC	0.009 [40]	0.014 [40]	1.59 [41]	2.30 [41]	1.67 [41]	0.32% [40]	9.1% [40]
FCC-ee	0.004 [42]	0.007 [42]				0.2% [43]	3.0% [43]

FIG. 2. The expected χ^2 constraints from future CEPC and FCC-ee experiments for the gaugino unification scenario ($M_1/M_2 = 0.5$). The upper and lower panels stand for $\tan\beta = 2$ and $\tan\beta = 50$, respectively. The 1σ and 2σ allowed region are green and yellow shaded. The tree level neutralino LSP masses and the tree level mass splitting between the NLSP and LSP are shown as solid and dashed contours, respectively; the $m_{\tilde{\chi}_2^0/\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 8$ GeV region which can also be detected by the soft lepton plus missing E_T search is hatched. Also shown are several individual 1σ constraints (here covariance is not considered), including the FCC-ee EWPT T (blue) and S (cyan), the FCC-ee $h \rightarrow \gamma\gamma$ (magenta), and the CEPC Δg_1^Z (orange), if they are strong enough to be shown.

The tree level neutralino LSP mass contours are shown as solid curves, and the tree level mass splitting between the neutralino LSP and the chargino or neutralino next-to-lightest supersymmetric particle (NLSP) contours are shown as dashed curves. Unlike the shaded χ^2 constrained region and the LSP mass contours, the mass splitting shows the greatest difference in different scenarios or $\tan\beta$ choices. The significant degeneracy between LSP and NLSP only happens for almost pure Higgsino LSP in the gaugino unification scenario, while in the anomaly

mediation scenario the LSP is degenerated with the NLSP in almost full parameter space. Traditional direct collider electroweakino searches using the soft lepton plus missing E_T channel become very inefficient for such a degenerated electroweakino spectrum; e.g., the current CMS search is limited by $m_{\tilde{\chi}_1^\pm \tilde{\chi}_0^0} - m_{\tilde{\chi}_0^0} \gtrsim 8$ GeV [44] (the meshed region in Figs. 2 and 3), and the future high luminosity LHC cannot significantly fill the gap. We can see that the indirect searches can be very useful in probing such a previously unbounded region, especially in the anomaly mediation

FIG. 3. The same as Fig. 2, but for the anomaly mediation scenario ($M_1/M_2 = 3.3$).

scenario. Assuming null signal, the LSP can be constrained to be heavier than about 200 GeV at 2σ level in the anomaly mediation scenario, regardless of the degeneracy of the spectrum.

In Figs. 2 and 3, also shown are the individual 1σ constraints from each best precision measurement in Table II. The ones strong enough to be visible include the FCC-ee EWPT constraints of T (blue curve) and S (cyan curve), the FCC-ee $h \rightarrow \gamma\gamma$ constraint (magenta curve), and the CEPC TGC Δg_1^Z constraint (orange curve). Because the EWPT T and S have some covariance and the $h \rightarrow \gamma\gamma$ branching ratio should be combined with the Higgs production cross section, the total constraint is not necessarily stronger than any single one of them. Since the EWPT T parameter is the only one proportional to the fourth power of

the coupling of new physics to SM Higgs, in many models it gives the strongest constraint. However, here the SM $g \sim 0.65$ is not large enough to give it an edge. In the large $\tan\beta$ case, T is the most sensitive observable in the region $M_2 \simeq \mu$, which dominates the central notch of the χ^2 region. The EWPT S parameter is the most sensitive one in the $\mu > 0$ and small $\tan\beta$ case, leading to the most stringent constraint on the M_2 vs μ plane. The $\tan\beta$ dependence of $h \rightarrow \gamma\gamma$ can be seen from Eq. (17), and at $\beta \sim \pi/4$ ($\sin 2\beta \sim 1$) this channel can be significant, especially if the Higgs total production cross section can be measured independently and precisely. The TGC Δg_1^Z constraint alone looks not very impressive, but actually the covariance of the TGC experiments are in a direction such that the overall χ^2 is still competitive compared with other experiments.

FIG. 4. The projected $1\sigma\chi^2$ constraint of each of the best experiments, including FCC-ee EWPT experiments (blue curves), the CEPC TGC experiments (orange curves), and the FCC-ee Higgs precision measurements (red solid curves). Although the EFT approximation may break down, for illustrative purpose we also show ILC 500 GeV and 1 TeV run Higgs precision measurement constraints (red dashed curves and red dotted curves, respectively). With the expected precisions reported in [38] such sensitivities happen to be close but a little bit weaker than the naive direct production bounds ($m_{\text{LSP}} = \sqrt{s}/2$, grey curves). Here we choose $\tan\beta = 10$ and $M_1/M_2 = 1.3$.

In order to further illustrate our results, in particular, with the effects of covariance of the EWPT and TGC experiments as well as the Higgs production cross section being imposed on the Higgs decay branching ratio measurements, in Fig. 4 we show the 1σ allowed region from the projected FCC-ee EWPT experiments (blue curves), the projected CEPC TGC experiments (orange curves), and the projected FCC-ee Higgs precision measurements (red solid curves). As is shown that $\tan\beta$ and M_1/M_2 are not important for the constraint on the M_2 vs μ plane (except for the NLSP LSP splitting), here we choose general values, $\tan\beta = 10$ and $M_1/M_2 = 1.3$. Rather than the circular FCC-ee which has a fixed $\sqrt{s}$ of 240 or 350 GeV, the linear collider ILC can achieve a much higher $\sqrt{s}$, and in the EFT approximation scheme the Higgs production cross section correction may receive $\sqrt{s}$ boosted contributions (see Table 11 of [1,23]). So we also show our EFT calculation of the projected ILC 500 GeV and 1 TeV run Higgs precision measurements (red dashed curves and red dotted curves respectively) as illustrations. Although apparently the EFT approximation breaks down and the results are not reliable, and a consistent direct Feynman diagram calculation is beyond the scope of the current paper, we can still see the advantage of linear colliders.

V. SUMMARY AND CONCLUSIONS

In this work, neutralinos and charginos in the MSSM are integrated out and the Wilson coefficients for the dimension-six operators are presented analytically. We have

presented a convenient way of dealing two-component Weyl fermions in the covariant derivative expansion method. By adopting our results, we compute the $h\gamma\gamma$ effective coupling and oblique parameters, which are consistent with those in the direct loop calculation method. And we believe that the SMEFT we obtained can be applied in more scenarios in a vast range of parameter space. In the numerical evaluation, we use the projected electroweak oblique parameters T and S , the TGC and Higgs production, and decay rates at the future lepton collider to constrain the electroweakino sector. While implementing the global fitting, we focus on the gaugino unification scenario ($M_2 \simeq 2M_1$) and anomaly mediation scenario ($M_1 \simeq 3.3M_2$), and the allowed parameter space regions are obtained. We also find that the constrained parameter space could stand out with a higher center-of-mass collision energy combined with high experimental statistics. In the parameter space where the neutralino as the lightest supersymmetric particle is very degenerated with the next-to-lightest chargino/neutralino, the traditional collider search of the electroweakino sector is inefficient. However, with the help of the Wilson coefficients of dimension-six operators we obtained, the future lepton collider like CEPC, ILC, FCC-ee could make a difference in precision measurement experiments.

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APPENDIX A: REGULARIZATION OPTIONS

In order to specify an order for the $\frac{\partial}{\partial p}$ [contained in the $(\tilde{U} + \tilde{G})$] to act on the $(p^2 - M^2)$, the CDE formulism is based on the following identity:

$$\begin{aligned} (p^2 - M^2 - \tilde{U} - \tilde{G})^{-1} &= (p^2 - M^2)^{-1} + (p^2 - M^2)^{-1}(\tilde{U} + \tilde{G})(p^2 - M^2)^{-1} \\ &\quad + (p^2 - M^2)^{-1}(\tilde{U} + \tilde{G})(p^2 - M^2)^{-1} \\ &\quad \times (\tilde{U} + \tilde{G})(p^2 - M^2)^{-1} \dots \end{aligned} \quad (\text{A1})$$

However, the tricks used to get the $(p^2 - M^2 - \tilde{U} - \tilde{G})^{-1}$ from the $\ln(p^2 - M^2 - \tilde{U} - \tilde{G})$ have ambiguity. Two convenient options are

$$\begin{aligned} \ln(p^2 - M^2 - \tilde{U} - \tilde{G}) &= \int_1^\infty -d\xi \frac{d}{d\xi} \ln(p^2 - \xi M^2 - \tilde{U} - \tilde{G}) \\ &= \int_1^\infty d\xi M^2 (p^2 - \xi M^2 - \tilde{U} - \tilde{G})^{-1} \end{aligned} \quad (\text{A2})$$

as in [8,45], and

$$\begin{aligned} \ln(p^2 - M^2 - \tilde{U} - \tilde{G}) &= \int_0^\infty -du \frac{d}{du} \ln(p^2 - M^2 - \tilde{U} - \tilde{G} - u) \\ &= \int_0^\infty du (p^2 - M^2 - \tilde{U} - \tilde{G} - u)^{-1} \end{aligned} \quad (\text{A3})$$

as in [15,46]. Both of the boundary terms at infinity ($\xi \rightarrow \infty$ and $u \rightarrow \infty$) are dropped implicitly. Correspondingly, after finishing all the loop integrations,

the implementation of the $\int_1^\infty d\xi$ in the first option and the $\int_0^\infty du$ in the second option, while making no difference in the degenerate case, clearly have a chance to make differences in the intermediate steps. For example, such an integration prototype of the expansion of effective action at the same order in above two options can be different,

$$\begin{aligned} \int_1^\infty d\xi \frac{M_1^2 \text{ or } M_2^2}{\xi^{n+1} (xM_1^2 + (1-x)M_2^2)^{n+1}} &= \frac{1}{n} \frac{M_1^2 \text{ or } M_2^2}{(xM_1^2 + (1-x)M_2^2)^{n+1}}, \\ \int_0^\infty du \frac{1}{(xM_1^2 + (1-x)M_2^2 + u)^{n+1}} &= \frac{1}{n} \frac{1}{(xM_1^2 + (1-x)M_2^2)^n}. \end{aligned}$$

However, it is checked that while the individual integration can differ with different regularization options, the whole result of all the integrations' summation will always agree with each other when including all the contributions correctly.

APPENDIX B: COEFFICIENTS OF OPERATORS

Here we list the Wilson coefficients c_i of operators $\mathcal{O}_i$ in Table. I, except the zero ones and that of c_6 . We introduce the following shorthand notations:

$$\begin{aligned} r_1 &= \frac{M_1}{\mu}, \quad r_2 = \frac{M_2}{\mu}, \\ s_{2\beta} &= \sin(2\beta), \quad c_{4\beta} = \cos(4\beta). \end{aligned} \quad (\text{B1})$$

Then, the Wilson coefficients are

$$c_{2W} = \frac{g^2(r_2^2 + 2)}{120\pi^2\mu^2 r_2^2}, \quad (\text{B2})$$

$$c_{2B} = \frac{g^2}{120\pi^2\mu^2}, \quad (\text{B3})$$

$$c_{3W} = -\frac{g^2(r_2^2 + 2)}{480\pi^2\mu^2 r_2^2}, \quad (\text{B4})$$

$$\begin{aligned} c_{WW} &= g^2 \left\{ -\frac{(2(2r_1^4 + 5r_1^2 - 1)r_1 s_{2\beta} + 13r_1^4 - 2r_1^2 + 1)}{1536\pi^2(r_1^2 - 1)^3\mu^2} + \frac{(2r_1 s_{2\beta} + r_1^2 + 1)r_1^4 \text{Log}[\frac{M_1^2}{\mu^2}]}{256\pi^2(r_1^2 - 1)^4\mu^2} \right\} \\ &\quad + g^2 \left\{ -\frac{2(6r_2^6 - 25r_2^4 + 5r_2^2 - 16)s_{2\beta} - 17r_2^5 + 10r_2^3 - 53r_2}{1536\pi^2(r_2^2 - 1)^3\mu^2 r_2} - \frac{(r_2^4 + 4)(2r_2 s_{2\beta} + r_2^2 + 1) \text{Log}[\frac{M_2^2}{\mu^2}]}{256\pi^2(r_2^2 - 1)^4\mu^2} \right\}, \end{aligned} \quad (\text{B5})$$

$$c_{BB} = g^2 \left\{ -\frac{(2(2r_1^4 + 5r_1^2 - 1)r_1 s_{2\beta} + 13r_1^4 - 2r_1^2 + 1)}{1536\pi^2(r_1^2 - 1)^3\mu^2} + \frac{r_1^4(2r_1 s_{2\beta} + r_1^2 + 1)\text{Log}[\frac{M_1^2}{\mu^2}]}{256\pi^2(r_1^2 - 1)^4\mu^2} \right\} \\ + g^2 \left\{ -\frac{2(2r_2^4 + 5r_2^2 - 1)r_2 s_{2\beta} + 13r_2^4 - 2r_2^2 + 1}{512\pi^2(r_2^2 - 1)^3\mu^2} + \frac{3r_2^4(2r_2 s_{2\beta} + r_2^2 + 1)\text{Log}[\frac{M_2^2}{\mu^2}]}{256\pi^2(r_2^2 - 1)^4\mu^2} \right\}, \quad (\text{B6})$$

$$c_{WB} = g^2 \left\{ -\frac{(2(2r_1^4 + 5r_1^2 - 1)r_1 s_{2\beta} + 13r_1^4 - 2r_1^2 + 1)}{768\pi^2(r_1^2 - 1)^3\mu^2} + \frac{r_1^4(2r_1 s_{2\beta} + r_1^2 + 1)\text{Log}[\frac{M_1^2}{\mu^2}]}{128\pi^2(r_1^2 - 1)^4\mu^2} \right\} \\ + g^2 \left\{ \frac{(4r_2^5 - 38r_2^3 + 46r_2)s_{2\beta} - 11r_2^4 - 2r_2^2 + 25}{768\pi^2(r_2^2 - 1)^3\mu^2} + \frac{(r_2^4 - 2)\text{Log}[\frac{M_2^2}{\mu^2}](2r_2 s_{2\beta} + r_2^2 + 1)}{128\pi^2(r_2^2 - 1)^4\mu^2} \right\}, \quad (\text{B7})$$

$$c_W = g^2 \left\{ \frac{4(7r_1^4 - 11r_1^2 - 2)r_1 s_{2\beta} + 23r_1^6 - 17r_1^4 - 35r_1^2 + 5}{1152\pi^2(r_1^2 - 1)^4\mu^2} - \frac{r_1^3(2(r_1^4 - 3)s_{2\beta} + (r_1^4 + 4r_1^2 - 9)r_1)\text{Log}[\frac{M_1^2}{\mu^2}]}{192\pi^2(r_1^2 - 1)^5\mu^2} \right\} \\ + g^2 \left\{ -\frac{4(5r_2^4 + 11r_2^2 - 10)r_2 s_{2\beta} + r_2^6 + 41r_2^4 + 11r_2^2 - 29}{384\pi^2(r_2^2 - 1)^4\mu^2} \right. \\ \left. + \frac{(2(r_2^6 + 12r_2^4 - 3r_2^2 - 4)r_2 s_{2\beta} + r_2^8 + 4r_2^6 + 27r_2^4 - 16r_2^2 - 4)\text{Log}[\frac{M_2^2}{\mu^2}]}{192\pi^2(r_2^2 - 1)^5\mu^2} \right\}, \quad (\text{B8})$$

$$c_B = g^2 \left\{ \frac{4(7r_1^4 - 11r_1^2 - 2)r_1 s_{2\beta} + 23r_1^6 - 17r_1^4 - 35r_1^2 + 5}{1152\pi^2(r_1^2 - 1)^4\mu^2} - \frac{r_1^3(2(r_1^4 - 3)s_{2\beta} + (r_1^4 + 4r_1^2 - 9)r_1)\text{Log}[\frac{M_1^2}{\mu^2}]}{192\pi^2(r_1^2 - 1)^5\mu^2} \right\} \\ + g^2 \left\{ \frac{4(7r_2^4 - 11r_2^2 - 2)r_2 s_{2\beta} + 23r_2^6 - 17r_2^4 - 35r_2^2 + 5}{384\pi^2(r_2^2 - 1)^4\mu^2} - \frac{r_2^3(2(r_2^4 - 3)s_{2\beta} + (r_2^4 + 4r_2^2 - 9)r_2)\text{Log}[\frac{M_2^2}{\mu^2}]}{64\pi^2(r_2^2 - 1)^5\mu^2} \right\}, \quad (\text{B9})$$

$$c_{HW} = g^2 \left\{ -\frac{(r_1^2 - 1) - 2(r_1^5 + 10r_1^3 + r_1)s_{2\beta} + 11r_1^6 - 35r_1^4}{384\pi^2(r_1^2 - 1)^4\mu^2} + \frac{r_1^3(r_1^2 + 1)(r_1^3 - 3r_1 - 2s_{2\beta})\text{Log}[\frac{M_1^2}{\mu^2}]}{64\pi^2(r_1^2 - 1)^5\mu^2} \right\} \\ + g^2 \left\{ \frac{2(r_2^5 + 10r_2^3 + r_2)s_{2\beta} + 5r_2^6 - 13r_2^4 + 47r_2^2 - 15}{128\pi^2(r_2^2 - 1)^4\mu^2} - \frac{(r_2^2 + 1)(6r_2^3 s_{2\beta} + r_2^6 - 3r_2^4 + 12r_2^2 - 4)\text{Log}[\frac{M_2^2}{\mu^2}]}{64\pi^2(r_2^2 - 1)^5\mu^2} \right\}, \quad (\text{B10})$$

$$c_{HB} = g^2 \left\{ -\frac{(r_1^2 - 1) - 2(r_1^5 + 10r_1^3 + r_1)s_{2\beta} + 11r_1^6 - 35r_1^4}{384\pi^2(r_1^2 - 1)^4\mu^2} + \frac{r_1^3(r_1^2 + 1)(r_1^3 - 3r_1 - 2s_{2\beta})\text{Log}[\frac{M_1^2}{\mu^2}]}{64\pi^2(r_1^2 - 1)^5\mu^2} \right\} \\ + g^2 \left\{ \frac{2(r_2^5 + 10r_2^3 + r_2)s_{2\beta} + 5r_2^6 - 13r_2^4 + 47r_2^2 - 15}{128\pi^2(r_2^2 - 1)^4\mu^2} - \frac{(r_2^2 + 1)(6r_2^3 s_{2\beta} + r_2^6 - 3r_2^4 + 12r_2^2 - 4)\text{Log}[\frac{M_2^2}{\mu^2}]}{64\pi^2(r_2^2 - 1)^5\mu^2} \right\}, \quad (\text{B11})$$

$$c_D = g^2 \left\{ \frac{-(r_1^5 + 10r_1^3 + r_1)s_{2\beta} + r_1^6 - 7r_1^4 - 7r_1^2 + 1}{96\pi^2(r_1^2 - 1)^4\mu^2} + \frac{r_1^3((r_1^2 + 1)s_{2\beta} + 2r_1)\text{Log}[\frac{M_1^2}{\mu^2}]}{16\pi^2(r_1^2 - 1)^5\mu^2} \right\} \\ + g^2 \left\{ \frac{-(r_2^5 + 10r_2^3 + r_2)s_{2\beta} + r_2^6 - 7r_2^4 - 7r_2^2 + 1}{32\pi^2(r_2^2 - 1)^4\mu^2} + \frac{3r_2^3((r_2^2 + 1)s_{2\beta} + 2r_2)\text{Log}[\frac{M_2^2}{\mu^2}]}{16\pi^2(r_2^2 - 1)^5\mu^2} \right\}, \quad (\text{B12})$$

$$\begin{aligned}
c_H = g^4 & \left\{ \frac{1}{1536\pi^2(r_1^2-1)^4\mu^2} [(2r_1^6-45r_1^4-126r_1^2+25)c_{4\beta} + 12(r_1^4+52r_1^2-5)r_1s_{2\beta} - 28r_1^6+309r_1^4+192r_1^2-41] \right. \\
& + \frac{\text{Log}[\frac{M_1^2}{\mu^2}]}{256\pi^2(r_1^2-1)^5\mu^2} [(r_1^6+19r_1^4+6r_1^2-2)c_{4\beta} + 4(r_1^6-15r_1^4-12r_1^2+2)r_1s_{2\beta} + 2r_1^8-15r_1^6-59r_1^4-2r_1^2+2] \\
& + g^2g^2 \left\{ \frac{\text{Log}[\frac{M_1^2}{\mu^2}]r_1^3}{256\pi^2(r_1^2-1)^4(r_1-r_2)(r_1+r_2)\mu^2} [-8(r_1+r_2)(r_1^2-5)r_1s_{2\beta} + (r_1^5+2r_2r_1^4-4r_1^3-2r_2r_1^2-7r_1-6r_2)c_{4\beta} \right. \\
& - 7r_1^5-6r_2r_1^4+20r_1^3+22r_2r_1^2+9r_1+10r_2] \\
& + \frac{\text{Log}[\frac{M_2^2}{\mu^2}]r_2^3}{256\pi^2(r_2^2-1)^4(r_1-r_2)(r_1+r_2)\mu^2} [8(r_1+r_2)(r_2^2-5)r_2s_{2\beta} + (r_1^5+2r_2r_1^4-4r_1^3-2r_2r_1^2-7r_1-6r_2)c_{4\beta} \\
& + 7r_2^5+6r_1r_2^4-20r_2^3-22r_1r_2^2-9r_2-10r_1] \\
& + \frac{1}{768\pi^2(r_1^2-1)^3(r_2^2-1)^3\mu^2} [c_{4\beta}(3(4r_2^4-9r_2^2-1)r_2r_1^5+(-16r_2^4-19r_2^2+5)r_1^4+3(-9r_2^4+14r_2^2+7)r_2r_1^3 \\
& + (-19r_2^4+110r_2^2-31)r_1^2-3r_2(r_2^4-7r_2^2+12)r_1+5r_2^4-31r_2^2-4)-4s_{2\beta}((r_2^4-32r_2^2+7)r_1^5+(r_2^4-32r_2^2+7)r_2r_1^4 \\
& - 4(8r_2^4-25r_2^2+5)r_1^3-4r_2(8r_2^4-25r_2^2+5)r_1^2+(7r_2^4-20r_2^2-11)r_1+(7r_2^4-20r_2^2-11)r_2) \\
& + (-16r_2^5+101r_2^3-7r_2)r_1^5+(28r_2^4+61r_2^2-23)r_1^4+(101r_2^4-262r_2^2+5)r_2r_1^3+(61r_2^4-290r_2^2+97)r_2^2 \\
& + (-7r_2^5+5r_2^3+80r_2)r_1-23r_2^4+97r_2^2-8] \left. \right\} \\
& + g^4 \left\{ \frac{1}{1536\pi^2(r_2^2-1)^4\mu^2} [(30r_2^6-169r_2^4-250r_2^2+53)c_{4\beta} - 4(29r_2^4-412r_2^2+47)r_2s_{2\beta} - 112r_2^6+681r_2^4+564r_2^2-125] \right. \\
& - \frac{\text{Log}[\frac{M_2^2}{\mu^2}]}{256\pi^2(r_2^2-1)^5\mu^2} [(4r_2^8-13r_2^6-35r_2^4-18r_2^2+6)c_{4\beta} - 4(5r_2^6-35r_2^4-32r_2^2+6)r_2s_{2\beta} - 6r_2^8+19r_2^6+155r_2^4+6r_2^2-6] \left. \right\}, \tag{B13}
\end{aligned}$$

$$\begin{aligned}
c_R = g^4 & \left\{ \frac{1}{384\pi^2(r_1^2-1)^4\mu^2} [(r_1^6-10r_1^4-37r_1^2-2)c_{4\beta} + 2(-7r_1^4+92r_1^2+11)r_1s_{2\beta} - 18r_1^6+87r_1^4+78r_1^2-3] \right. \\
& + \frac{\text{Log}[\frac{M_1^2}{\mu^2}]r_1^2}{64\pi^2(r_1^2-1)^5\mu^2} [(5r_1^2+3)c_{4\beta} + 2(r_1^4-7r_1^2-10)r_1s_{2\beta} + r_1^6-2r_1^4-20r_1^2-3] \left. \right\} \\
& + g^2g^2 \left\{ \frac{-r_1^3\text{Log}[\frac{M_1^2}{\mu^2}]}{128\pi^2(r_1^2-1)^4(r_1-r_2)^3(r_1+r_2)\mu^2} [-8s_{2\beta}(r_1^6+r_2r_1^5+3r_1^4-5r_2r_1^3-(5r_2^2+1)r_1^2+(3r_2^3+r_2)r_1+2r_2^2) \right. \\
& + c_{4\beta}(r_1^7-(3r_2^2+2)r_1^5+2(r_2^2+6)r_2r_1^4+(9-6r_2^2)r_1^3-20r_2r_1^2-(3r_2^2+4)r_1+6r_2^3+8r_2)-r_1^7 \\
& - 4r_2r_1^6-5r_2^2r_1^5-18r_1^5+2r_2^3r_1^4+16r_2r_1^4+26r_2^2r_1^3-5r_1^3-12r_2^3r_1^2+20r_2r_1^2-9r_2^2r_1+4r_1-6r_2^3-8r_2] \\
& + \frac{r_2^3\text{Log}[\frac{M_2^2}{\mu^2}]}{128\pi^2(r_2^2-1)^4(r_1-r_2)^3(r_1+r_2)\mu^2} [-8((3r_2r_1^3+(2-5r_2^2)r_1^2+(r_2^5-5r_2^3+r_2)r_1+(r_2^4+3r_2^2-1)r_2^2)s_{2\beta} \\
& + (2(r_2^4+3)r_1^3-3r_2(r_2^2+1)^2r_1^2+4(3r_2^4-5r_2^2+2)r_1+(r_2^6-2r_2^4+9r_2^2-4)r_2)c_{4\beta}-r_2^7-4r_1r_2^6-5r_1^2r_2^5 \\
& - 18r_2^5+2r_1^3r_2^4+16r_1r_2^4+26r_1^2r_2^3-5r_2^3-12r_1^3r_2^2+20r_1r_2^2-9r_1^2r_2+4r_2-6r_1^3-8r_1] \\
& - \frac{1}{384\pi^2(r_1^2-1)^3(r_2^2-1)^3(r_1-r_2)^2\mu^2} [8(r_2^6-11r_2^4+16r_2^2-3)r_1^3r_2+2(4r_2^6+3r_2^4-12r_2^2+5)r_1r_2 \\
& + c_{4\beta}(-(16r_2^6+27r_2^4-12r_2^2+5)r_1^6+8(r_2^5+r_2^3+r_2)r_1^7+(8r_2^7+26r_2^5-88r_2^3+6r_2)r_1^5-5r_2^6+28r_2^4-11r_2^2 \\
& + (-27r_2^6+184r_2^4-101r_2^2+28)r_1^4+(12r_2^6-101r_2^4+40r_2^2-11)r_1^2) \left. \right\}
\end{aligned}$$

$$\begin{aligned}
& -12s_{2\beta}((2r_2^4 + 5r_2^2 - 1)r_1^7 + (-4r_2^5 - 5r_2^3 + 3r_2)r_1^6 + (-4r_2^6 + 8r_2^4 - 31r_2^2 + 9)r_1^5 + (2r_2^6 + 8r_2^4 + 17r_2^2 - 9)r_2r_1^4 \\
& + 3(r_2^2 - 3)r_1r_2^4 + (5r_2^4 - 31r_2^2 + 8)r_1^2r_2^3 - (r_2^4 - 9r_2^2 + 2)r_2^3 + (-5r_2^6 + 17r_2^4 + 8r_2^2 - 2)r_1^3) \\
& + 2(r_2^5 - 26r_2^3 + r_2)r_1^7 + (20r_2^6 + 9r_2^4 + 24r_2^2 - 17)r_1^6 + 2(r_2^6 + r_2^4 + 97r_2^2 - 15)r_2r_1^5 \\
& + (9r_2^6 - 344r_2^4 + 271r_2^2 - 68)r_1^4 - 17r_2^6 - 68r_2^4 + 25r_2^2 - 2(26r_2^6 - 97r_2^4 + 236r_2^2 - 69)r_1^3r_2 \\
& + 2(r_2^6 - 15r_2^4 + 69r_2^2 - 19)r_1r_2 + (24r_2^6 + 271r_2^4 - 164r_2^2 + 25)r_1^2)] \Big\} \\
& + g^4 \left\{ -\frac{1}{384\pi^2(r_2^2 - 1)^4\mu^2} [(5r_2^6 + 76r_2^4 + 103r_2^2 + 8)c_{4\beta} - 2(17r_2^4 + 332r_2^2 + 35)r_2s_{2\beta} + 36r_2^6 - 321r_2^4 - 312r_2^2 + 21] \right. \\
& \left. + \frac{\text{Log}[\frac{M_2^2}{\mu^2}]r_2^2}{64\pi^2(r_2^2 - 1)^5\mu^2} [(6r_2^4 + 17r_2^2 + 9)c_{4\beta} + 2(r_2^4 - 31r_2^2 - 34)r_2s_{2\beta} + r_2^6 - 8r_2^4 - 80r_2^2 - 9] \right\}, \tag{B14}
\end{aligned}$$

$$\begin{aligned}
c_T = & g^4 \left\{ \frac{(17r_1^4 - 7r_1^2 - 4)c_{2\beta}^2}{768\pi^2(r_1^2 - 1)^3\mu^2} - \frac{r_1^2(r_1^4 + 2r_1^2 - 2)c_{2\beta}^2 \text{Log}[\frac{M_1^2}{\mu^2}]}{128\pi^2(r_1^2 - 1)^4\mu^2} \right\} + g^4 \left\{ \frac{(29r_2^4 - 7r_2^2 - 16)c_{2\beta}^2}{768\pi^2(r_2^2 - 1)^3\mu^2} - \frac{r_2^2(r_2^4 + 6r_2^2 - 6)c_{2\beta}^2 \text{Log}[\frac{M_2^2}{\mu^2}]}{128\pi^2(r_2^2 - 1)^4\mu^2} \right\} \\
& + g^2 g^2 \left\{ \frac{r_1^3 \text{Log}[\frac{M_1^2}{\mu^2}]}{128\pi^2(r_1^2 - 1)^4(r_1 - r_2)^3(r_1 + r_2)\mu^2} [-4c_{4\beta}(2(r_2^2 - 1)r_1 + r_1^5 + 3r_2r_1^4 + (2 - 3r_2^2)r_1^3 + (r_2^2 - 8)r_2r_1^2 + 4r_2) \right. \\
& + 2(-(r_2^2 - 1)r_1^4 + (r_2^2 - 5)r_2r_1^3 + 3r_1^6 + r_2r_1^5 - (5r_2^2 + 2)r_1^2 + (r_2^2 + 2)r_2r_1 + 4r_2^2)s_{2\beta} + 3r_1^7 - r_2^2r_1^5 + r_1^5 \\
& + 2r_2^3r_1^4 + r_2r_1^4 - 7r_2^2r_1^3 + r_1^3 - r_2^3r_1^2 - 8r_2r_1^2 + 5r_2^2r_1 - 2r_1 + 2r_2^3 + 4r_2] \\
& - \frac{r_2^3 \text{Log}[\frac{M_2^2}{\mu^2}]}{128\pi^2(r_2^2 - 1)^4(r_1 - r_2)^3(r_1 + r_2)\mu^2} [-c_{4\beta}((r_2^2r_1^3 + (2r_2 - 3r_2^3)r_1^2 + (3r_2^4 - 8r_2^2 + 4)r_1 + (r_2^4 + 2r_2^2 - 2)r_2) \\
& + 2((r_2^3 + r_2)r_1^3 - (r_2^4 + 5r_2^2 - 4)r_1^2 + (r_2^4 - 5r_2^2 + 2)r_2r_1 + (3r_2^4 + r_2^2 - 2)r_2^2)s_{2\beta} + 3r_2^7 - r_1^2r_2^5 + r_2^5 + 2r_1^3r_2^4 + r_1r_2^4 \\
& - 7r_1^2r_2^3 + r_2^3 - r_1^3r_2^2 - 8r_1r_2^2 + 5r_1^2r_2 - 2r_2 + 2r_1^3 + 4r_1] \\
& + \frac{1}{768\pi^2(r_1^2 - 1)^3(r_2^2 - 1)^3(r_1 - r_2)^2\mu^2} [((-2r_2^5 - 5r_2^3 + r_2)r_1^7 + (4r_2^6 + 17r_2^4 - 19r_2^2 + 4)r_1^6 - r_2(2r_2^6 + 12r_2^4 - 57r_2^2 + 25)r_1^5 \\
& + (17r_2^6 - 126r_2^4 + 120r_2^2 - 29)r_1^4 + (-5r_2^7 + 57r_2^5 - 126r_2^3 + 56r_2)r_1^3 + (-19r_2^6 + 120r_2^4 - 102r_2^2 + 19)r_1^2 \\
& + (r_2^6 - 25r_2^4 + 56r_2^2 - 26)r_2r_1 + (4r_2^4 - 29r_2^2 + 19)r_2^2)c_{4\beta} + 2((13r_2^4 - 2r_2^2 + 1)r_1^7 + (-25r_2^5 + 2r_2^3 + 11r_2)r_1^6 \\
& + (-25r_2^6 + 48r_2^4 - 93r_2^2 + 34)r_1^5 + (13r_2^6 + 48r_2^4 + 9r_2^2 - 34)r_2r_1^4 + (2r_2^6 + 9r_2^4 + 48r_2^2 - 23)r_1^3 \\
& + (-2r_2^7 - 93r_2^5 + 48r_2^3 + 11r_2)r_1^2 + (11r_2^4 - 34r_2^2 + 11)r_2^2r_1 + (r_2^4 + 34r_2^2 - 23)r_2^3)s_{2\beta} \\
& + 3(3r_2r_1^5((r_2^2 - 1) + 2r_2^6 - 8r_2^4) + r_2^2((r_2^2 - 5) + 10r_2^4) + 3(2r_2^5 - r_2^3 + r_2)r_1^7 + (-20r_2^6 + 29r_2^4 - 25r_2^2 + 10)r_1^6 \\
& + (29r_2^6 + 6r_2^4 - 18r_2^2 + 1)r_1^4 - 3r_2(r_2^6 - r_2^4 - 10r_2^2 + 4)r_1^3 - (25r_2^6 + 18r_2^4 - 30r_2^2 + 5)r_1^2 + 3(r_2^6 - r_2^4 - 4r_2^2 + 2)r_2r_1)] \Big\}. \tag{B15}
\end{aligned}$$

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