Weak decays of triply heavy baryons

Wei Wang and Ji Xu*

INPAC, Shanghai Key Laboratory for Particle Physics and Cosmology,
MOE Key Laboratory for Particle Physics, Astrophysics and Cosmology,
School of Physics and Astronomy, Shanghai Jiao Tong University,
Shanghai 200240, China

(Received 10 March 2018; published 25 May 2018)

After the experimental establishment of doubly heavy baryons, baryons with three quarks are the last missing pieces of the lowest-lying baryon multiplets in the quark model. In this work, we study semileptonic and nonleptonic weak decays of triply heavy baryons, Ω_{ccc}^{++} , Ω_{ccb}^{+} , Ω_{cbb}^0 , and Ω_{bbb}^{-} . Decay amplitudes for various channels are parametrized in terms of a few SU(3) irreducible amplitudes. We point out that branching fractions for Cabibbo-allowed processes, $\Omega_{ccc}^{++} \rightarrow (\Xi_{cc}^{++} \bar{K}^0, \Xi_{cc}^{++} K^- \pi^+, \Omega_{cc}^{+} \pi^+, \Xi_c^{+} D^+, \Xi_c^{+} D^+, \Lambda_c D^+ \bar{K}^0, \Xi_c^{+} D^0 \pi^+, \Xi_c^{+} D^+ \pi^+)$, may reach a few percent. We suggest our experimental colleagues to perform a search at hadron colliders and the electron and positron collisions in the future, which will presumably lead to discoveries of triply heavy baryons and complete the baryon multiplets. Using the expanded amplitudes, we derive a number of relations for the partial widths that can be examined in the future.

DOI: 10.1103/PhysRevD.97.093007

I. INTRODUCTION

In the past decades, hadron spectroscopy has experienced a rapid renaissance, predominantly propelled by discoveries of a number of hadron resonances that defy the standard quark model interpretations. These resonant states are generically classified as hadron exotics, and for reviews on recent progress, please see Refs. [1–4]. Among all exotic hadrons, $X(3872)$ plays a most important role due to its special properties. Aside from these unexpected discoveries, there has also been gradual progress in the traditional sector of the charmonium and bottomonium spectroscopy. One of the highlights in recent years is the discovery of Ξ_{cc}^{++} by the LHCb Collaboration [5]:

$$m_{\Xi_{cc}^{++}} = (3621.40 \pm 0.72 \pm 0.27 \pm 0.14) \text{ MeV}. \quad (1)$$

This observed state Ξ_{cc}^{++} fills well in the quark model as the lowest-lying ccu baryon [6]. After the tentative establishment of Ξ_{cc}^{++} , it is plausible to fill the baryon family with the last missing members, namely, baryons made of three heavy quarks. Charm and bottom quarks are much heavier than the u , d , and s ; thus, baryons with three heavy quarks will refrain from light-quark contaminations, and the study

of triply heavy baryons can help us better understand the dynamics of strong interactions.

Previous studies of triply heavy baryons concentrated on three facets: spectroscopy, production, and decays. Most theoretical investigations in the literature, especially in recent years, have focused on the masses and magnetic moments [7–20], while less attention has been paid to the production and decay properties. The only available estimate of the production is conducted in Refs. [21,22], where the cross sections at the LHC with $\sqrt{s} = 7$ TeV are found to reach the 0.1 nb level depending on different kinematics cuts. In the $b \rightarrow c$ transitions among triply heavy baryons, Ref. [23] has discussed the implications of heavy-quark spin symmetry. Some decay modes of the Ω_{ccc}^{++} were analyzed for the first time in Ref. [24].

The main focus of this paper is to provide a systematic analysis of weak decays of the lowest-lying triply heavy baryons, $\Omega_{ccc,ccb,cb,bbb}$. The Ω_{ccc} and Ω_{bbb} have spin $3/2$, while the Ω_{ccb} and Ω_{cbb} can be the $J^P = 1/2^+$ or $J^P = 3/2^+$ state. As we will show, various types of weak decays of triply heavy baryons occur, but unfortunately, a universal dynamical (factorization) approach cannot be established yet. This gives a barrier for us to predict their decay widths. Instead, we will use an optional theoretical tool to analyze heavy-quark decays, the flavor SU(3) symmetry [24–55]. One advantage of the SU(3) analysis is that it is independent of the factorization details. In this work, we consider semileptonic decay channels with one or two hadrons in the final state, while for nonleptonic decays, the two-body and three-body modes will be analyzed.

*xuji1991@sjtu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](#). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

The rest of this paper is organized as follows. Section II is devoted to an estimate of the total decay rates or equivalently the lifetimes that will be used in the calculation of branching fractions. In Sec. III, we will collect the representation matrices for various particle multiplets in the SU(3) group. In Sec. IV, semileptonic decay modes with one or two hadrons in the final state are analyzed. In Secs. V, VI, and VII, nonleptonic decays of Ω_{ccc} , Ω_{bbb} , Ω_{ccb} , and Ω_{cbb} will be studied in order. In Sec. VIII, we shall present a collection of golden modes that are most likely to discover the triply heavy baryons. Section IX will briefly discuss the SU(3) symmetry-breaking effects. A short summary is given in the last section.

II. LIFETIMES AND TOTAL DECAY WIDTHS

Lifetimes, the inverse of total decay rates, are among the most fundamental properties of the involved triply heavy baryons. The decay rate of a triply heavy baryon H into an inclusive final state X can be expressed as a phase-space integral over the matrix element square of the effective Hamiltonian sandwiched between the initial and final state X . Summing over all final states with the same quantum numbers, we obtain

$$\Gamma(H \rightarrow X) = \frac{1}{2m_H} \sum_X \int \prod_i \left[\frac{d^3 p_i}{(2\pi)^3 2E_i} \right] \times (2\pi)^4 \delta^{(4)}(p_H - p_X) |\langle X | \mathcal{H}_{\text{eff}}^{ew} | H \rangle|^2, \quad (2)$$

with $p_X = \sum_i p_i$. The $\mathcal{H}_{\text{eff}}^{ew}$ is the effective electroweak Hamiltonian of which the explicit forms can be found in the following sections. With the help of the optical theorem, the total rate is written as

$$\Gamma(H \rightarrow X) = \frac{1}{2m_H} \langle H | \mathcal{T} | H \rangle, \quad (3)$$

with the transition operator

$$\mathcal{T} = \text{Im}i \int d^4x T[\mathcal{H}_{\text{eff}}^{ew}(x), \mathcal{H}_{\text{eff}}^{ew}(0)] \quad (4)$$

consisting of a nonlocal double insertion of the effective Hamiltonian.

A theoretical tool that describes the decay widths of inclusive decays is heavy-quark expansion, making use of operator product expansion in essence. Contracting the off-shell operators in Eq. (4) results in local operators organized by dimensions. Higher-dimensional operators are suppressed by powers of the inverse of heavy-quark mass $1/m_Q$. Up to dimension 6, the transition operator is then expanded as

$$\mathcal{T} = \frac{G_F^2 m_Q^5}{192\pi^3} |V_{\text{CKM}}|^2 \left[c_{3,Q} \bar{Q}Q + \frac{c_{5,Q}}{m_Q^2} \bar{Q}g_s \sigma_{\mu\nu} G^{\mu\nu} Q + 2 \frac{c_{6,Q}}{m_Q^3} (\bar{Q}q)_\Gamma (\bar{q}Q)_\Gamma + \dots \right], \quad (5)$$

where G_F is the Fermi constant and V_{CKM} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The c_i s are the corresponding short-distance coefficients that arise from the semileptonic and nonleptonic heavy-quark decays. These coefficients have been evaluated to a high accuracy in QCD perturbation theory; please see Ref. [56] for a recent review.

The total decay rate is given by matrix elements of these operators:

$$\Gamma(H) = \frac{G_F^2 m_Q^5}{192\pi^3} |V_{\text{CKM}}|^2 \left(c_{3,Q} \frac{\langle H | \bar{c}c | H \rangle}{2m_H} + \frac{c_{5,Q}}{m_Q^2} \frac{\langle H | \bar{Q}g_s \sigma_{\mu\nu} G^{\mu\nu} Q | H \rangle}{2m_H} + \frac{c_{6,Q}}{m_Q^3} \frac{\langle H | (\bar{Q}q)_\Gamma (\bar{q}Q)_\Gamma | H \rangle}{m_H} \right). \quad (6)$$

A. Ω_{ccc}^{++}

The dimension-6 operators arise from the interaction between the decaying heavy quark and the spectator quarks. For the Ω_{ccc}^{++} baryon, such operators do not contribute since two charm quarks cannot scatter at leading order. At the lowest order in $1/m_c$, the $\bar{c}c$ operator gives the charm-quark number in the Ω_{ccc}^{++} baryon:

$$\frac{\langle \Omega_{ccc}^{++} | \bar{c}c | \Omega_{ccc}^{++} \rangle}{2m_{\Omega_{ccc}^{++}}} = 3 + \mathcal{O}(1/m_c). \quad (7)$$

Other matrix elements suppressed by powers of $1/m_c$ request a detailed model calculation in the future and are omitted in this work. The short-distance coefficient $c_{3,c}$ has been calculated as $c_{3,c} = 6.29 \pm 0.72$ at leading order (LO) in QCD and $c_{3,c} = 11.61 \pm 1.55$ at next-to-leading order (NLO) [56]. Keeping only the leading-order contribution, we obtain the total decay rate and lifetime:

$$\Gamma(\Omega_{ccc}^{++}) = \begin{cases} (2.18 \pm 0.25) \times 10^{-12} \text{ GeV}, & \text{LO} \\ (4.03 \pm 0.54) \times 10^{-12} \text{ GeV}, & \text{NLO} \end{cases}, \quad (8)$$

$$\tau(\Omega_{ccc}^{++}) = \begin{cases} (302 \pm 35) \times 10^{-15} \text{ s}, & \text{LO} \\ (164 \pm 22) \times 10^{-15} \text{ s}, & \text{NLO} \end{cases}. \quad (9)$$

In this equation, we have used the charm-quark mass $m_c = 1.4 \text{ GeV}$. This lifetime, especially at NLO, is smaller than the ones for the charmed mesons:

$$\begin{aligned}
\tau(D^\pm) &= (1040 \pm 7) \times 10^{-15} \text{ s}, \\
\tau(D^0) &= (410.1 \pm 1.5) \times 10^{-15} \text{ s}, \\
\tau(D_s^\pm) &= (500 \pm 7) \times 10^{-15} \text{ s}.
\end{aligned} \tag{10}$$

B. Ω_{bbb}^-

The heavy bottom quark is heavier than the charm quark, and thus one would expect the heavy-quark expansion to work better in the Ω_{bbb}^- decays. A perturbative calculation of the short-distance coefficient [57]

$$c_{3,b} = \begin{cases} 5.29 \pm 0.35, & \text{LO} \\ 6.88 \pm 0.74, & \text{NLO} \end{cases} \tag{11}$$

also shows better convergence. Using the above results and $m_b = 4.8 \text{ GeV}$, we obtain the lifetime of the triply bottom baryons:

$$\Gamma(\Omega_{bbb}^-) = \begin{cases} (1.47 \pm 0.01) \times 10^{-12} \text{ GeV}, & \text{LO} \\ (1.92 \pm 0.02) \times 10^{-12} \text{ GeV}, & \text{NLO} \end{cases}, \tag{12}$$

$$\tau(\Omega_{bbb}^-) = \begin{cases} (0.45 \pm 0.03) \times 10^{-12} \text{ s}, & \text{LO} \\ (0.34 \pm 0.04) \times 10^{-12} \text{ s}, & \text{NLO} \end{cases}. \tag{13}$$

As one can see, the lifetime is close to one-third of the ones for the bottomed mesons:

$$\begin{aligned}
\tau(B^-) &= 1.638 \times 10^{-12} \text{ s}, \\
\tau(B^0) &= 1.52 \times 10^{-12} \text{ s}, \\
\tau(B_s^-) &= 1.505 \times 10^{-12} \text{ s}.
\end{aligned} \tag{14}$$

C. Ω_{ccb}^+ and Ω_{cbb}^0

The decay rates of Ω_{ccb}^+ and Ω_{cbb}^0 receive both contributions from the charm-quark and bottom-quark decays. Using the formulas and inputs given in the last two subsections, we obtain

$$\begin{aligned}
\Gamma(\Omega_{ccb}^+) &= \begin{cases} (1.95 \pm 0.20) \times 10^{-12} \text{ GeV}, & \text{LO} \\ (3.32 \pm 0.43) \times 10^{-12} \text{ GeV}, & \text{NLO} \end{cases}, \\
\tau(\Omega_{ccb}^+) &= \begin{cases} (338 \pm 35) \times 10^{-15} \text{ s}, & \text{LO} \\ (198 \pm 25) \times 10^{-15} \text{ s}, & \text{NLO} \end{cases},
\end{aligned} \tag{15}$$

$$\begin{aligned}
\Gamma(\Omega_{cbb}^0) &= \begin{cases} (1.71 \pm 0.15) \times 10^{-12} \text{ GeV}, & \text{LO} \\ (2.62 \pm 0.32) \times 10^{-12} \text{ GeV}, & \text{NLO} \end{cases}, \\
\tau(\Omega_{cbb}^0) &= \begin{cases} (384 \pm 33) \times 10^{-15} \text{ s}, & \text{LO} \\ (251 \pm 30) \times 10^{-15} \text{ s}, & \text{NLO} \end{cases}.
\end{aligned} \tag{16}$$

All these results including the lifetimes for Ω_{ccc}^{++} and Ω_{bbb}^- will be needed in the estimate of branching fractions.

III. PARTICLE MULTIPLETS

In this section, we will collect the representations for hadron multiplets under the flavor SU(3) group. We start with the baryon sector. Light baryons made of three light quarks can group into an SU(3) octet and a decuplet. The light baryon decuplet is symmetric in SU(3) flavor space and is characterized by the following matrix:

$$\begin{aligned}
(T_{10})^{111} &= \Delta^{++}, \quad (T_{10})^{112} = (T_{10})^{121} = (T_{10})^{211} = \frac{1}{\sqrt{3}} \Delta^+, \\
(T_{10})^{222} &= \Delta^-, \quad (T_{10})^{122} = (T_{10})^{212} = (T_{10})^{221} = \frac{1}{\sqrt{3}} \Delta^0, \\
(T_{10})^{113} &= (T_{10})^{131} = (T_{10})^{311} = \frac{1}{\sqrt{3}} \Sigma'^+, \\
(T_{10})^{223} &= (T_{10})^{232} = (T_{10})^{322} = \frac{1}{\sqrt{3}} \Sigma'^-, \\
(T_{10})^{123} &= (T_{10})^{132} = (T_{10})^{213} = (T_{10})^{231} = (T_{10})^{312} \\
&= (T_{10})^{321} = \frac{1}{\sqrt{6}} \Sigma^0, \\
(T_{10})^{133} &= (T_{10})^{313} = (T_{10})^{331} = \frac{1}{\sqrt{3}} \Xi'^0, \\
(T_{10})^{233} &= (T_{10})^{323} = (T_{10})^{332} = \frac{1}{\sqrt{3}} \Xi'^-, \\
(T_{10})^{333} &= \Omega^-.
\end{aligned} \tag{17}$$

The octet has the expression

$$T_8 = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}} \Lambda^0 \end{pmatrix}. \tag{18}$$

The triply heavy baryons form an SU(3) singlet, while doubly heavy baryons are an SU(3) triplet:

$$\begin{aligned}
T_{cc} &= \begin{pmatrix} \Xi_{cc}^{++}(ccu) \\ \Xi_{cc}^+(ccd) \\ \Omega_{cc}^+(ccs) \end{pmatrix}, \quad T_{bc} = \begin{pmatrix} \Xi_{bc}^+(bcu) \\ \Xi_{bc}^0(bcd) \\ \Omega_{bc}^0(bcs) \end{pmatrix}, \\
T_{bb} &= \begin{pmatrix} \Xi_{bb}^0(bbu) \\ \Xi_{bb}^-(bbd) \\ \Omega_{bb}^-(bbs) \end{pmatrix}.
\end{aligned} \tag{19}$$

Singly charmed and bottom baryons with two light quarks can form an antitriplet or sextet. For the antitriplet and sextet, we have the matrix expression

$$T_{c\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix},$$

$$T_{c6} = \begin{pmatrix} \Sigma_c^{++} & \frac{1}{\sqrt{2}}\Sigma_c^+ & \frac{1}{\sqrt{2}}\Xi_c'^+ \\ \frac{1}{\sqrt{2}}\Sigma_c^+ & \Sigma_c^0 & \frac{1}{\sqrt{2}}\Xi_c'^0 \\ \frac{1}{\sqrt{2}}\Xi_c'^+ & \frac{1}{\sqrt{2}}\Xi_c'^0 & \Omega_c^0 \end{pmatrix}. \quad (20)$$

In the meson sector, a light pseudoscalar meson is formed by a light quark and one light antiquark. It forms an octet,

$$M_8 = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \quad (21)$$

while the singlet η_1 is not considered. Our analysis is also applicable to light vector mesons and other light mesons. The charmed meson forms an SU(3) antitriplet,

$$D_i = (D^0, D^+, D_s^+), \quad (22)$$

and the anticharmed meson forms an SU(3) triplet,

$$\bar{D}^i = (\bar{D}^0, D^-, D_s^-). \quad (23)$$

The above two SU(3) triplets are also applicable to the bottom mesons.

In the following, we will construct the hadron-level effective Hamiltonian for various decay modes. It is necessary to point out that a hadron in the final state must be created by its antiparticle field. For instance, to produce a Ξ_{ccu}^{++} , one must use the $\bar{\Xi}_{ccu}^{++}$ in the Hamiltonian, and the doubly heavy baryon antitriplet is abbreviated as $\bar{T}_{cc}$.

IV. SEMILEPTONIC DECAY CHANNELS

A. Semileptonic Ω_{ccc} decays

The $c \rightarrow q\bar{\ell}\nu$ transition is induced by the effective electroweak Hamiltonian

$$\mathcal{H}_{\text{e.w.}} = \frac{G_F}{\sqrt{2}} [V_{cq}^* \bar{q}\gamma^\mu (1 - \gamma_5) c \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell] + \text{H.c.}, \quad (24)$$

where $q = d, s$ and $\ell = e, \mu$. The V_{cd} and V_{cs} are CKM matrix elements. Heavy-to-light operators form an SU(3) triplet, denoted as H_3 with the nonzero components $(H_3)^2 = V_{cd}^*$, $(H_3)^3 = V_{cs}^*$.

At the hadron level, the effective Hamiltonian for three-body and four-body semileptonic Ω_{ccc} decays can be constructed as

FIG. 1. Feynman diagrams for semileptonic decays of Ω_{ccc}^{++} . Panel (a) corresponds to three-body decay modes with a doubly charmed baryon in the final state. In panel (b), one additional light meson is generated. The last panel corresponds to the decays into a charmed baryon and charmed meson.

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & a_1 \Omega_{ccc} (\bar{T}_{cc})_i (H_3)^i \bar{\nu}_\ell \ell \\ & + a_2 \Omega_{ccc} (\bar{T}_{cc})_i (M_8)_j^i (H_3)^j \bar{\nu}_\ell \ell \\ & + a_3 \Omega_{ccc} (\bar{T}_{c\bar{3}})_{[ij]} \bar{D}^i (H_3)^j \bar{\nu}_\ell \ell \\ & + a_4 \Omega_{ccc} (\bar{T}_{c6})_{\{ij\}} \bar{D}^i (H_3)^j \bar{\nu}_\ell \ell, \end{aligned} \quad (25)$$

where the a_i 's are SU(3) irreducible amplitudes. Feynman diagrams for these modes are given in Fig. 1. Decay amplitudes for various channels can be deduced from the Hamiltonian in Eq. (25), and are collected in Table I.

A few remarks are given in order:

- (i) In this table and the following ones, the light pseudoscalar mesons can be replaced by their vector counterparts. For instance, the K^0 can be replaced by a K^{*0} , which is reconstructed by the $K^-\pi^+$ final state. The η meson is difficult to reconstruct at hadron colliders, while the vector ϕ meson can be reconstructed in the K^+K^- final state with high efficiency.
- (ii) The $c \rightarrow s$ transition is proportional to the $V_{cs} \sim 1$, and the $c \rightarrow d$ transition has a smaller CKM matrix

TABLE I. Amplitudes for semileptonic Ω_{ccc} decays derived from Eq. (25). The light meson in the final state can be replaced by its vector counterpart. For instance, the K^0 can be replaced by a K^{*0} decaying into $K^-\pi^+$.

Channel	Amplitude	Channel	Amplitude
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ \ell^+ \nu_\ell$	$a_1 V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^0 \ell^+ \nu_\ell$	$a_3 V_{cd}^*$
$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^+ \ell^+ \nu_\ell$	$a_1 V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^0 \ell^+ \nu_\ell$	$a_3 V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \pi^- \ell^+ \nu_\ell$	$a_2 V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D^+ \ell^+ \nu_\ell$	$a_3 V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^- \ell^+ \nu_\ell$	$a_2 V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D_s^+ \ell^+ \nu_\ell$	$-a_3 V_{cd}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ \pi^0 \ell^+ \nu_\ell$	$-\frac{a_2 V_{cd}^*}{\sqrt{2}}$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^0 \ell^+ \nu_\ell$	$\frac{a_4 V_{cd}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ \bar{K}^0 \ell^+ \nu_\ell$	$a_2 V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^0 D^+ \ell^+ \nu_\ell$	$a_4 V_{cd}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ \eta \ell^+ \nu_\ell$	$\frac{a_2 V_{cd}^*}{\sqrt{6}}$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^0 \ell^+ \nu_\ell$	$\frac{a_4 V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^+ K^0 \ell^+ \nu_\ell$	$a_2 V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D^+ \ell^+ \nu_\ell$	$\frac{a_4 V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^+ \eta \ell^+ \nu_\ell$	$-\sqrt{\frac{2}{3}} a_2 V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D_s^+ \ell^+ \nu_\ell$	$\frac{a_4 V_{cd}^*}{\sqrt{2}}$
		$\Omega_{ccc}^{++} \rightarrow \Omega_c^0 D_s^+ \ell^+ \nu_\ell$	$a_4 V_{cs}^*$

element $V_{cd} \sim 0.2$. Inspired by the experimental data on D -meson decays [6], we can infer that branching fractions for the $c \rightarrow s$ channels are about a few percent, and the ones for the $c \rightarrow d$ transitions are at the order 10^{-3} .

- (iii) A number of relations for decay widths can be easily read off from Table I. For instance, for the $c \rightarrow s$ decays into a doubly charmed baryon, we have

$$\begin{aligned} \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^- \ell^+ \nu_\ell) \\ = \Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0 \ell^+ \nu_\ell) \\ = \frac{3}{2} \Gamma(\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^+ \ell^+ \nu_\ell). \end{aligned} \quad (26)$$

However, it is necessary to point out that the above relations will be modified due to the different masses of the final hadrons. Once the mass of Ω_{ccc} is experimentally measured in the future, kinematical corrections can be included, and these relations can be refined.

FIG. 2. Kinematics for the $\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^+ l^+ \bar{\nu}$ decay. In the Ω_{ccc}^{++} rest frame, the Ω_{cc}^+ moves on the opposite of the z axis. In the lepton pair $l^+ \bar{\nu}$ rest frame, the l^+ flying direction and the z axis intersect with the polar angle θ .

There is only one final hadron in the $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ / \Omega_{cc}^+ \ell^+ \nu_\ell$ decays. Leptonic amplitudes can be calculated in electroweak perturbation theory, while the hadronic matrix element can be parametrized in terms of form factors,

$$\begin{aligned} \langle \Omega_{cc}^+(p') | \bar{s} \gamma^\mu \gamma_5 c | \Omega_{ccc}^{++}(p) \rangle &= \bar{u}(p') \left[\frac{f_1 \gamma^\mu P^\alpha}{m_{\Omega_{ccc}^{++}} - m_{\Omega_{cc}^+}} + \frac{f_2 P^\mu P^\alpha}{m_{\Omega_{ccc}^{++}}^2 - m_{\Omega_{cc}^+}^2} + \frac{i f_3 \sigma^{\mu\nu} q_\nu P^\alpha}{m_{\Omega_{ccc}^{++}}^2} + f_4 g^{\alpha\mu} \right] u^\alpha(p), \\ \langle \Omega_{cc}^+(p') | \bar{s} \gamma^\mu c | \Omega_{ccc}^{++}(p) \rangle &= \bar{u}(p') \left[\frac{g_1 \gamma^\mu P^\alpha}{m_{\Omega_{ccc}^{++}} + m_{\Omega_{cc}^+}} + \frac{g_2 P^\mu P^\alpha}{m_{\Omega_{ccc}^{++}}^2 - m_{\Omega_{cc}^+}^2} + \frac{i g_3 \sigma^{\mu\nu} q_\nu P^\alpha}{m_{\Omega_{ccc}^{++}}^2} + g_4 g^{\alpha\mu} \right] \gamma_5 u^\alpha(p), \end{aligned} \quad (27)$$

with $P = p + p'$ and $q = p - p'$. Here, $u(p)$ is the spinor for the final $1/2^+$ baryon, and $u_\alpha(p')$ is the Rarita-Schwinger vectorial spinor for the initial $3/2^+$ baryon, satisfying

$$\not{p} u^\alpha(p) = m_{\Omega_{ccc}^{++}} u^\alpha(p), \quad p_\alpha u^\alpha(p) = \gamma_\alpha u^\alpha(p) = 0. \quad (28)$$

It can be constructed using the polarization vector and a spinor,

$$u^\alpha(p) = \left(\epsilon^\alpha - \frac{1}{3} \left(\gamma^\alpha + \frac{p^\alpha}{m} \right) \not{p} \right) u(p), \quad (29)$$

which is a product of the Clebsch-Gordon coefficient. The partial decay widths for $\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^+ / \Xi_{cc}^+ l^+ \bar{\nu}$ are then derived as

$$\begin{aligned} \frac{d\Gamma}{dq^2 d\cos\theta} &= \frac{\sqrt{\lambda} G_F^2 q^2}{1024 \pi^3 m_l^3} \left(1 - \frac{m_l^2}{q^2} \right)^2 ((m_l + m_f)^2 - q^2) |V_{CKM}|^2 \left\{ (|H_{1,-1/2}^{V,3/2}|^2 + |H_{1,-1/2}^{A,3/2}|^2 + |H_{1,1/2}^{V,1/2}|^2 + |H_{1,1/2}^{A,1/2}|^2) \right. \\ &\times \left(1 + \cos^2\theta + \frac{m_l^2}{q^2} \sin^2\theta \right) - 4 \cos\theta \text{Re}[H_{1,-1/2}^{V,3/2} (H_{1,-1/2}^{A,3/2})^* + H_{1,1/2}^{V,1/2} (H_{1,1/2}^{A,1/2})^*] + (|H_{0,-1/2}^{V,1/2}|^2 + |H_{0,-1/2}^{A,1/2}|^2) \\ &\times \left(2 \sin^2\theta + \frac{2m_l^2}{q^2} \cos^2\theta \right) + (|H_{t,-1/2}^{V,1/2}|^2 + |H_{t,-1/2}^{A,1/2}|^2) \frac{2m_l^2}{q^2} - 4 \cos\theta \frac{m_l^2}{q^2} (\text{Re}[H_{0,-1/2}^{V,1/2} (H_{t,-1/2}^{V,1/2})^*] \\ &\left. + H_{0,-1/2}^{A,1/2} (H_{t,-1/2}^{A,1/2})^*] \right\}, \end{aligned} \quad (30)$$

where the kinematics is chosen as in Fig. 2. $m_l = m_{\Omega_{ccc}^{++}}$, and $m_f = m_{\Omega_{cc}^+}$ or $m_{\Xi_{cc}^+}$. In the above, the H_i 's are the hadronic helicity amplitudes:

$$H_{1,-1/2}^{V,3/2} = H_{-1,1/2}^{V,-3/2} = f_4, \quad H_{1,-1/2}^{A,3/2} = -H_{-1,-1/2}^{A,-3/2} = \frac{g_4\sqrt{\lambda}}{\hat{f}_+}, \quad (31)$$

$$H_{0,-1/2}^{V,1/2} = H_{0,1/2}^{V,-1/2} = -\frac{f_1(1+m_f)\hat{f}_-}{\sqrt{6}(1-m_f)\sqrt{q^2}} - \frac{\lambda f_2}{\sqrt{6}(1-m_f^2)\sqrt{q^2}} + \frac{f_3\hat{f}_-\sqrt{q^2}}{\sqrt{6}} + \frac{f_4(1+q^2-m_f^2)}{\sqrt{6}\sqrt{q^2}}, \quad (32)$$

$$H_{0,-1/2}^{A,1/2} = -H_{0,1/2}^{A,-1/2} = \frac{g_1\lambda(1-m_f)}{\sqrt{6}(1+m_f)\sqrt{q^2}} - \frac{g_2\sqrt{\lambda}\hat{f}_-}{\sqrt{6}(1-m_f^2)\sqrt{q^2}} + \frac{g_3\sqrt{\lambda}\sqrt{q^2}}{\sqrt{6}} + \frac{g_4\sqrt{\lambda}(1+q^2-m_f^2)}{\sqrt{6}\hat{f}_+\sqrt{q^2}}, \quad (33)$$

$$H_{t,-1/2}^{V,1/2} = H_{t,1/2}^{V,-1/2} = -\frac{(f_1+f_2-f_4)\sqrt{\lambda}}{\sqrt{6}q^2}, \quad (34)$$

$$H_{t,-1/2}^{A,1/2} = -H_{t,1/2}^{A,-1/2} = \frac{(g_1-g_2+g_4)\hat{f}_-}{\sqrt{6}q^2}, \quad (35)$$

$$H_{1,1/2}^{V,1/2} = H_{-1,-1/2}^{V,-1/2} = -\frac{f_1\hat{f}_-}{\sqrt{3}(1-m_f)} + \frac{f_3(1+m_f)\hat{f}_-}{\sqrt{3}} + \frac{f_4}{\sqrt{3}}, \quad (36)$$

$$H_{1,1/2}^{A,1/2} = -H_{-1,-1/2}^{A,-1/2} = -\frac{g_1\sqrt{\lambda}}{\sqrt{3}(1+m_f)} - \frac{g_3\sqrt{\lambda}(1-m_f)}{\sqrt{3}} - \frac{f_4\sqrt{\lambda}}{\sqrt{3}\hat{f}_+}. \quad (37)$$

A factor $i\sqrt{2m_I(E_f+m_f)} = i\sqrt{(m_I+m_f)^2 - q^2}$ has been taken out in the amplitudes. $m_f = m_f/m_I$, and $q^2 = q^2/m_I^2$. The λ function is introduced as

$$\begin{aligned} \lambda &\equiv \lambda(m_I^2, m_f^2, q^2) = (m_I^2 - m_f^2 - q^2)^2 - 4m_f^2q^2, \\ \lambda &\equiv \lambda(1, m_f^2, q^2) = (1 - m_f^2 - q^2)^2 - 4m_f^2q^2, \end{aligned} \quad (38)$$

and the abbreviation has been introduced:

$$\hat{f}_+ = (1+m_f)^2 - q^2, \quad \hat{f}_- = (1-m_f)^2 - q^2. \quad (39)$$

In the above transitions, the two heavy quarks act as spectators and might be viewed as a diquark system. The quark-diquark correspondence implies that the above transition might be related to the $B_c \rightarrow B$ transition, in which the bottom quark acts a spectator. Using the $B_c \rightarrow B$ form factors as a *very* rough approximation [58], we obtain an order-of-magnitude estimate of partial decay widths:

$$\Gamma(\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^+ l^+ \nu) = 1.41 \times 10^{-13} \text{ GeV}, \quad (40)$$

$$\Gamma(\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ l^+ \nu) = 1.29 \times 10^{-14} \text{ GeV}. \quad (41)$$

Masses for triply and doubly heavy baryons are taken from the QCD sum rules calculations [13,59]:

$$\begin{aligned} m_{\Omega_{ccc}^{++}} &= 4.99 \text{ GeV}, & m_{\Xi_{cc}^+} &= 3.57 \text{ GeV}, \\ m_{\Omega_{cc}^+} &= 3.71 \text{ GeV}. \end{aligned} \quad (42)$$

Notice the mass for Ξ_{cc}^+ is slightly below the experimental result for $m_{\Xi_{cc}^{++}}$ shown in Eq. (1), but this difference will not cause sizable modifications to decay widths. Using the NLO lifetime given in Eq. (13), we can derive the branching fractions:

$$\mathcal{B}(\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^+ l^+ \nu) = 3.6\%, \quad (43)$$

$$\mathcal{B}(\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ l^+ \nu) = 0.33\%. \quad (44)$$

It is worth noting that the CKM matrix element of $\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^+ l^+ \nu$ is $|V_{cs}|^2$ and matrix element of $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ l^+ \nu$ is $|V_{cd}|^2$. Numerically, $|V_{cs}|^2/|V_{cd}|^2 \sim 20$, but due to the larger phase-space region of the latter process, the ratio of these two branching fractions is about 10, as one can see in the equations above.

B. Semileptonic Ω_{bbb} decays

The b -quark decay is controlled by the electroweak Hamiltonian

$$\mathcal{H}_{\text{e.e.}} = \frac{G_F}{\sqrt{2}} [V_{q'b} \bar{q}' \gamma^\mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell] + \text{H.c.}, \quad (45)$$

with $q' = u, c$, and here $\ell = e, \mu, \tau$. The $b \rightarrow c$ transition is an SU(3) singlet, and thus the transition is simply a singlet. The $b \rightarrow u$ transition forms an SU(3) triplet H'_3 with $(H'_3)^1 = 1$ and $(H'_3)^{2,3} = 0$. The hadron-level Hamiltonian is given as

$$\begin{aligned}
\mathcal{H}_{\text{eff}} = & b_1 \Omega_{bbb} \bar{\Omega}_{cbb} \bar{\ell} \nu_\ell + b_2 \Omega_{bbb} (\bar{T}_{bb})_i \bar{D}^i \bar{\ell} \nu_\ell \\
& + b_3 \Omega_{bbb} (\bar{T}_{bc})_i \bar{B}^i \bar{\ell} \nu_\ell + b_4 \Omega_{bbb} (\bar{T}_{bb})_i (H'_3)^i \bar{\ell} \nu_\ell \\
& + b_5 \Omega_{bbb} (\bar{T}_{bb})_i (M_8)_j^i (H'_3)^j \bar{\ell} \nu_\ell \\
& + b_6 \Omega_{bbb} (\bar{T}_{b\bar{3}})_{[ij]} \bar{B}^i (H'_3)^j \bar{\ell} \nu_\ell \\
& + b_7 \Omega_{bbb} (\bar{T}_{b6})_{\{ij\}} \bar{B}^i (H'_3)^j \bar{\ell} \nu_\ell.
\end{aligned} \quad (46)$$

The b_i 's are the SU(3)-independent amplitudes like the a_i 's in Eq. (25). Feynman diagrams are shown in Fig. 3. Decay amplitudes can be deduced from this Hamiltonian, and the results are given in Table II.

A few remarks are given in order:

- (i) The Ω_{cbb} in the final state can be $1/2^+$ or $3/2^+$. It is similar for T_{bb} , T_{bc} , and others.
- (ii) Decay modes induced by the $b \rightarrow c$ transition still contain a triply heavy baryon, which must be detected through its subsequent weak decays.

C. Semileptonic Ω_{cbb} decays

Both the charm quark and bottom quark in Ω_{cbb} can weakly decay. Thus, the hadron-level Hamiltonian for semileptonic Ω_{cbb} decays is given as

$$\begin{aligned}
\mathcal{H}_{\text{eff}} = & c_1 \Omega_{cbb} (\bar{T}_{bc})_i (H_3)^i \bar{\nu}_\ell \ell + c_2 \Omega_{cbb} (\bar{T}_{bc})_i (M_8)_j^i (H_3)^j \bar{\nu}_\ell \ell + c_3 \Omega_{cbb} (\bar{T}_{b\bar{3}})_{[ij]} \bar{D}^i (H_3)^j \bar{\nu}_\ell \ell + c_4 \Omega_{cbb} (\bar{T}_{c\bar{3}})_{[ij]} \bar{B}^i (H_3)^j \bar{\nu}_\ell \ell \\
& + c_5 \Omega_{cbb} (\bar{T}_{b6})_{\{ij\}} \bar{D}^i (H_3)^j \bar{\nu}_\ell \ell + c_6 \Omega_{cbb} (\bar{T}_{c6})_{\{ij\}} \bar{B}^i (H_3)^j \bar{\nu}_\ell \ell + c_7 \Omega_{cbb} \bar{\Omega}_{ccc} \bar{\ell} \nu_\ell + c_8 \Omega_{cbb} (\bar{T}_{cc})_i \bar{D}^i \bar{\ell} \nu_\ell \\
& + c_9 \Omega_{cbb} (\bar{T}_{cc})_i (H'_3)^i \bar{\ell} \nu_\ell + c_{10} \Omega_{cbb} (\bar{T}_{cc})_i (M_8)_j^i (H'_3)^j \bar{\ell} \nu_\ell + c_{11} \Omega_{cbb} (\bar{T}_{c\bar{3}})_{[ij]} \bar{D}^i (H'_3)^j \bar{\ell} \nu_\ell \\
& + c_{12} \Omega_{cbb} (\bar{T}_{c6})_{\{ij\}} \bar{D}^i (H'_3)^j \bar{\ell} \nu_\ell.
\end{aligned} \quad (47)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table III.

D. Semileptonic Ω_{cbb} decays

Similarly, the hadron-level Hamiltonian for semileptonic Ω_{cbb} decay is given as

$$\begin{aligned}
\mathcal{H}_{\text{eff}} = & d_1 \Omega_{cbb} \Omega_{cbb} \bar{\ell} \nu_\ell + d_2 \Omega_{cbb} (\bar{T}_{cc})_i \bar{B}^i \bar{\ell} \nu_\ell + d_3 \Omega_{cbb} (\bar{T}_{bc})_i \bar{D}^i \bar{\ell} \nu_\ell + d_4 \Omega_{cbb} (T_{bc})_i (H'_3)^i \bar{\ell} \nu_\ell + d_5 \Omega_{cbb} (T_{bc})_i M_j^i (H'_3)^j \bar{\ell} \nu_\ell \\
& + d_6 \Omega_{cbb} (\bar{T}_{b\bar{3}})_{[ij]} \bar{D}^i (H'_3)^j \bar{\ell} \nu_\ell + d_7 \Omega_{cbb} (\bar{T}_{c\bar{3}})_{[ij]} \bar{B}^i (H'_3)^j \bar{\ell} \nu_\ell + d_8 \Omega_{cbb} (\bar{T}_{b6})_{\{ij\}} \bar{D}^i (H'_3)^j \bar{\ell} \nu_\ell \\
& + d_9 \Omega_{cbb} (\bar{T}_{c6})_{\{ij\}} \bar{B}^i (H'_3)^j \bar{\ell} \nu_\ell + d_{10} \Omega_{cbb} (T_{bb})_i (H_3)^i \bar{\nu}_\ell \ell + d_{11} \Omega_{cbb} (T_{bb})_i M_j^i (H_3)^j \bar{\nu}_\ell \ell \\
& + d_{12} \Omega_{cbb} (\bar{T}_{b\bar{3}})_{[ij]} \bar{B}^i (H_3)^j \bar{\nu}_\ell \ell + d_{13} \Omega_{cbb} (\bar{T}_{b6})_{\{ij\}} \bar{B}^i (H_3)^j \bar{\nu}_\ell \ell.
\end{aligned} \quad (48)$$

TABLE II. Decay amplitudes for Ω_{bbb} semileptonic decays with one or two hadrons in the final state.

Channel	Amplitude	Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 \ell^- \bar{\nu}_\ell$	$b_1 V_{cb}$	$\Omega_{bbb}^- \rightarrow \Xi_{cbb}^0 \ell^- \bar{\nu}_\ell$	$b_4 V_{ub}$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D_s^+ \ell^- \bar{\nu}_\ell$	$b_2 V_{cb}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D^0 \ell^- \bar{\nu}_\ell$	$b_2 V_{cb}$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^+ \ell^- \bar{\nu}_\ell$	$b_2 V_{cb}$	$\Omega_{bbb}^- \rightarrow \Xi_{bc}^+ B^- \ell^- \bar{\nu}_\ell$	$b_3 V_{cb}$
$\Omega_{bbb}^- \rightarrow \Xi_{bc}^0 \bar{B}^0 \ell^- \bar{\nu}_\ell$	$b_3 V_{cb}$	$\Omega_{bbb}^- \rightarrow \Omega_{bc}^0 \bar{B}_s^0 \ell^- \bar{\nu}_\ell$	$b_3 V_{cb}$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \pi^+ \ell^- \bar{\nu}_\ell$	$b_5 V_{ub}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \pi^0 \ell^- \bar{\nu}_\ell$	$\frac{b_5 V_{ub}}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \eta \ell^- \bar{\nu}_\ell$	$\frac{b_5 V_{ub}}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 \ell^- \bar{\nu}_\ell$	$-b_6 V_{ub}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- K^+ \ell^- \bar{\nu}_\ell$	$b_5 V_{ub}$	$\Omega_{bbb}^- \rightarrow \Lambda_b^0 \bar{B}^0 \ell^- \bar{\nu}_\ell$	$-b_6 V_{ub}$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 \ell^- \bar{\nu}_\ell$	$-b_6 V_{ub}$
$\Omega_{bbb}^- \rightarrow \Sigma_b^+ B^- \ell^- \bar{\nu}_\ell$	$b_7 V_{ub}$	$\Omega_{bbb}^- \rightarrow \Sigma_b^0 \bar{B}^0 \ell^- \bar{\nu}_\ell$	$\frac{b_7 V_{ub}}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 \ell^- \bar{\nu}_\ell$	$\frac{b_7 V_{ub}}{\sqrt{2}}$

FIG. 3. Feynman diagrams for semileptonic decays of Ω_{bbb}^- . Panel (a) corresponds to three-body decay modes with a doubly charmed baryon in the final state. In panel (b), one additional light meson is generated. The last panel corresponds to the decays into a charmed baryon and charmed meson.

TABLE III. Decay amplitudes for Ω_{ccb}^+ semileptonic decays.

Channel	Amplitude	Channel	Amplitude
$\Omega_{ccb}^+ \rightarrow \Omega_{ccc}^+ \ell^- \bar{\nu}_\ell$	$c_7 V_{cb}$	$\Omega_{ccb}^+ \rightarrow \Sigma_b^0 D^0 \ell^+ \nu_\ell$	$\frac{c_5 V_{cd}^*}{\sqrt{2}}$
$\Omega_{ccb}^+ \rightarrow \Xi_{cc}^+ \ell^- \bar{\nu}_\ell$	$c_9 V_{ub}$	$\Omega_{ccb}^+ \rightarrow \Sigma_b^- D^+ \ell^+ \nu_\ell$	$c_5 V_{cd}^*$
$\Omega_{ccb}^+ \rightarrow \Xi_{bc}^0 \ell^+ \nu_\ell$	$c_1 V_{cd}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_b^0 D^0 \ell^+ \nu_\ell$	$\frac{c_5 V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccb}^+ \rightarrow \Omega_{bc}^0 \ell^+ \nu_\ell$	$c_1 V_{cs}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_b^+ D^+ \ell^+ \nu_\ell$	$\frac{c_5 V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccb}^+ \rightarrow \Xi_{bc}^+ \pi^- \ell^+ \nu_\ell$	$c_2 V_{cd}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_b^+ D_s^+ \ell^+ \nu_\ell$	$\frac{c_5 V_{cd}^*}{\sqrt{2}}$
$\Omega_{ccb}^+ \rightarrow \Xi_{bc}^+ K^- \ell^+ \nu_\ell$	$c_2 V_{cs}^*$	$\Omega_{ccb}^+ \rightarrow \Omega_b^- D_s^+ \ell^+ \nu_\ell$	$c_5 V_{cs}^*$
$\Omega_{ccb}^+ \rightarrow \Xi_{bc}^0 \pi^0 \ell^+ \nu_\ell$	$-\frac{c_2 V_{cd}^*}{\sqrt{2}}$	$\Omega_{ccb}^+ \rightarrow \Sigma_c^+ B^- \ell^+ \nu_\ell$	$\frac{c_6 V_{cd}^*}{\sqrt{2}}$
$\Omega_{ccb}^+ \rightarrow \Xi_{bc}^0 \bar{K}^0 \ell^+ \nu_\ell$	$c_2 V_{cs}^*$	$\Omega_{ccb}^+ \rightarrow \Sigma_c^0 \bar{B}^0 \ell^+ \nu_\ell$	$c_6 V_{cd}^*$
$\Omega_{ccb}^+ \rightarrow \Xi_{bc}^0 \eta \ell^+ \nu_\ell$	$\frac{c_2 V_{cd}^*}{\sqrt{6}}$	$\Omega_{ccb}^+ \rightarrow \Xi_c^+ B^- \ell^+ \nu_\ell$	$\frac{c_6 V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccb}^+ \rightarrow \Omega_{bc}^0 K^0 \ell^+ \nu_\ell$	$c_2 V_{cd}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_c^0 \bar{B}^0 \ell^+ \nu_\ell$	$\frac{c_6 V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccb}^+ \rightarrow \Omega_{bc}^0 \eta \ell^+ \nu_\ell$	$-\sqrt{\frac{2}{3}} c_2 V_{cs}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_c^0 \bar{B}_s^0 \ell^+ \nu_\ell$	$\frac{c_6 V_{cd}^*}{\sqrt{2}}$
$\Omega_{ccb}^+ \rightarrow \Lambda_b^0 D^0 \ell^+ \nu_\ell$	$c_3 V_{cd}^*$	$\Omega_{ccb}^+ \rightarrow \Omega_c^0 \bar{B}_s^0 \ell^+ \nu_\ell$	$c_6 V_{cs}^*$
$\Omega_{ccb}^+ \rightarrow \Xi_b^0 D^0 \ell^+ \nu_\ell$	$c_3 V_{cs}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_{cc}^+ D^0 \ell^- \bar{\nu}_\ell$	$c_8 V_{cb}$
$\Omega_{ccb}^+ \rightarrow \Xi_b^- D^+ \ell^+ \nu_\ell$	$c_3 V_{cs}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_{cc}^+ D^+ \ell^- \bar{\nu}_\ell$	$c_8 V_{cb}$
$\Omega_{ccb}^+ \rightarrow \Xi_b^- D_s^+ \ell^+ \nu_\ell$	$-c_3 V_{cd}^*$	$\Omega_{ccb}^+ \rightarrow \Omega_{cc}^+ D_s^+ \ell^- \bar{\nu}_\ell$	$c_8 V_{cb}$
$\Omega_{ccb}^+ \rightarrow \Lambda_c^+ B^- \ell^+ \nu_\ell$	$c_4 V_{cd}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_{cc}^+ \pi^0 \ell^- \bar{\nu}_\ell$	$\frac{c_{10} V_{ub}}{\sqrt{2}}$
$\Omega_{ccb}^+ \rightarrow \Xi_c^+ B^- \ell^+ \nu_\ell$	$c_4 V_{cs}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_{cc}^+ \eta \ell^- \bar{\nu}_\ell$	$\frac{c_{10} V_{ub}}{\sqrt{6}}$
$\Omega_{ccb}^+ \rightarrow \Xi_c^0 \bar{B}^0 \ell^+ \nu_\ell$	$c_4 V_{cs}^*$	$\Omega_{ccb}^+ \rightarrow \Xi_{cc}^+ \pi^+ \ell^- \bar{\nu}_\ell$	$c_{10} V_{ub}$
$\Omega_{ccb}^+ \rightarrow \Xi_c^0 \bar{B}_s^0 \ell^+ \nu_\ell$	$-c_4 V_{cd}^*$	$\Omega_{ccb}^+ \rightarrow \Omega_{cc}^+ K^+ \ell^- \bar{\nu}_\ell$	$c_{10} V_{ub}$
$\Omega_{ccb}^+ \rightarrow \Sigma_c^{++} D^0 \ell^- \bar{\nu}_\ell$	$c_{12} V_{ub}$	$\Omega_{ccb}^+ \rightarrow \Lambda_c^+ D^+ \ell^- \bar{\nu}_\ell$	$-c_{11} V_{ub}$
$\Omega_{ccb}^+ \rightarrow \Sigma_c^+ D^+ \ell^- \bar{\nu}_\ell$	$\frac{c_{12} V_{ub}}{\sqrt{2}}$	$\Omega_{ccb}^+ \rightarrow \Xi_c^+ D_s^+ \ell^- \bar{\nu}_\ell$	$-c_{11} V_{ub}$
$\Omega_{ccb}^+ \rightarrow \Xi_c^+ D_s^+ \ell^- \bar{\nu}_\ell$	$\frac{c_{12} V_{ub}}{\sqrt{2}}$		

TABLE IV. Amplitudes for Ω_{cbb} semileptonic decay.

Channel	Amplitude	Channel	Amplitude
$\Omega_{cbb}^0 \rightarrow \Omega_{ccb}^+ \ell^- \bar{\nu}_\ell$	$d_1 V_{cb}$	$\Omega_{cbb}^0 \rightarrow \Xi_{cc}^{++} B^- \ell^- \bar{\nu}_\ell$	$d_2 V_{cb}$
$\Omega_{cbb}^0 \rightarrow \Xi_{bc}^+ \ell^- \bar{\nu}_\ell$	$d_4 V_{ub}$	$\Omega_{cbb}^0 \rightarrow \Xi_{cc}^+ \bar{B}^0 \ell^- \bar{\nu}_\ell$	$d_2 V_{cb}$
$\Omega_{cbb}^0 \rightarrow \Xi_{bb}^- \ell^+ \nu_\ell$	$d_{10} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Omega_{cc}^+ \bar{B}_s^0 \ell^- \bar{\nu}_\ell$	$d_2 V_{cb}$
$\Omega_{cbb}^0 \rightarrow \Omega_{bb}^- \ell^+ \nu_\ell$	$d_{10} V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_{bc}^+ D^0 \ell^- \bar{\nu}_\ell$	$d_3 V_{cb}$
$\Omega_{cbb}^0 \rightarrow \Xi_{bb}^0 \pi^- \ell^+ \nu_\ell$	$d_{11} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_{bc}^0 D^+ \ell^- \bar{\nu}_\ell$	$d_3 V_{cb}$
$\Omega_{cbb}^0 \rightarrow \Xi_{bb}^0 K^- \ell^+ \nu_\ell$	$d_{11} V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Omega_{bc}^0 D_s^+ \ell^- \bar{\nu}_\ell$	$d_3 V_{cb}$
$\Omega_{cbb}^0 \rightarrow \Xi_{bb}^- \pi^0 \ell^+ \nu_\ell$	$-\frac{d_{11} V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Xi_{bc}^0 \pi^0 \ell^- \bar{\nu}_\ell$	$\frac{d_5 V_{ub}}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Xi_{bb}^- \bar{K}^0 \ell^+ \nu_\ell$	$d_{11} V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_{bc}^+ \eta \ell^- \bar{\nu}_\ell$	$\frac{d_5 V_{ub}}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow \Xi_{bb}^- \eta \ell^+ \nu_\ell$	$\frac{d_{11} V_{cd}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \Xi_{bc}^0 \pi^+ \ell^- \bar{\nu}_\ell$	$d_5 V_{ub}$
$\Omega_{cbb}^0 \rightarrow \Omega_{bb}^- K^0 \ell^+ \nu_\ell$	$d_{11} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Omega_{bc}^0 K^+ \ell^- \bar{\nu}_\ell$	$d_5 V_{ub}$
$\Omega_{cbb}^0 \rightarrow \Omega_{bb}^- \eta \ell^+ \nu_\ell$	$-\sqrt{\frac{2}{3}} d_{11} V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 D^+ \ell^- \bar{\nu}_\ell$	$-d_6 V_{ub}$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 B^- \ell^+ \nu_\ell$	$d_{12} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 D_s^+ \ell^- \bar{\nu}_\ell$	$-d_6 V_{ub}$
$\Omega_{cbb}^0 \rightarrow \Xi_b^0 B^- \ell^+ \nu_\ell$	$d_{12} V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Lambda_c^+ \bar{B}^0 \ell^- \bar{\nu}_\ell$	$-d_7 V_{ub}$
$\Omega_{cbb}^0 \rightarrow \Xi_b^- \bar{B}^0 \ell^+ \nu_\ell$	$d_{12} V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_c^+ \bar{B}_s^0 \ell^- \bar{\nu}_\ell$	$-d_7 V_{ub}$
$\Omega_{cbb}^0 \rightarrow \Xi_b^- \bar{B}_s^0 \ell^+ \nu_\ell$	$-d_{12} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Sigma_b^+ D^0 \ell^- \bar{\nu}_\ell$	$d_8 V_{ub}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 B^- \ell^+ \nu_\ell$	$\frac{d_{13} V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 D^+ \ell^- \bar{\nu}_\ell$	$\frac{d_8 V_{ub}}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^- \bar{B}^0 \ell^+ \nu_\ell$	$d_{13} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 D_s^+ \ell^- \bar{\nu}_\ell$	$\frac{d_8 V_{ub}}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Xi_b^0 B^- \ell^+ \nu_\ell$	$\frac{d_{13} V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Sigma_c^{++} B^- \ell^- \bar{\nu}_\ell$	$d_9 V_{ub}$
$\Omega_{cbb}^0 \rightarrow \Xi_b^- \bar{B}^0 \ell^+ \nu_\ell$	$\frac{d_{13} V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Sigma_c^+ \bar{B}^0 \ell^- \bar{\nu}_\ell$	$\frac{d_9 V_{ub}}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Xi_b^- \bar{B}_s^0 \ell^+ \nu_\ell$	$\frac{d_{13} V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Xi_c^+ \bar{B}_s^0 \ell^- \bar{\nu}_\ell$	$\frac{d_9 V_{ub}}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Omega_b^- \bar{B}_s^0 \ell^+ \nu_\ell$	$d_{13} V_{cs}^*$		

Expanding the above equations, we will obtain the decay amplitudes given in Table IV.

V. NONLEPTONIC Ω_{ccc} DECAYS

Nonleptonic charm-quark decays into light quarks are classified into three groups:

FIG. 4. Feynman diagrams for two-body decay modes induced by Cabibbo-allowed transitions. In the first two panels, the final state contains a doubly charmed baryon and a light meson, described by Eq. (53). The last panel corresponds to decays into a charmed baryon and a charmed meson. The first panel is color allowed, while the last two panels are suppressed by $1/N_c$.

$$c \rightarrow \bar{s}du, \quad c \rightarrow u\bar{d}d/\bar{s}s, \quad c \rightarrow d\bar{s}u. \quad (49)$$

Feynman diagrams for two-body decays induced by the $c \rightarrow \bar{s}du$ are given in Fig. 4. In the first two panels, the final state contains a doubly charmed baryon and a light meson, while the last panel corresponds to decays into a charmed baryon and a charmed meson. The first panel is color allowed, while the last two panels are suppressed by $1/N_c$.

TABLE V. Amplitudes for Ω_{ccc} decays into a doubly charmed baryon and a light meson. Cabibbo-allowed, singly Cabibbo-suppressed, and doubly Cabibbo-suppressed decay channels are included in this table and the following ones. The amplitudes a_i 's are different than the ones in Table I.

Cabibbo-allowed	Amplitude	Singly Cabibbo-suppressed	Amplitude
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0$	$(a_2 - a_1)V_{ud}V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \pi^0$	$\frac{(a_2 - a_1)V_{us}V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{++} \pi^+$	$(a_1 + a_2)V_{ud}V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \eta$	$\sqrt{\frac{3}{2}}(a_1 - a_2)V_{us}V_{cs}^*$
Doubly Cabibbo-suppressed	Amplitude	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{+} \pi^+$	$(a_1 + a_2)(-V_{us}V_{cs}^*)$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^0$	$(a_1 - a_2)(-V_{us}V_{cd}^*)$	$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{+} K^+$	$(a_1 + a_2)V_{us}V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{+} K^+$	$(a_1 + a_2)V_{us}V_{cd}^*$		

Penguin contributions in charm-quark decays are highly suppressed and thus are neglected in our analysis. Tree operators transform under the flavor SU(3) symmetry as $\mathbf{3} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} = \mathbf{3} \oplus \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}$. For charm-quark decays, the vector representation H_3 will vanish as an approximation. For the $c \rightarrow su\bar{d}$ transition, we have

$$\begin{aligned} (H_{\bar{6}})_2^{31} &= -(H_{\bar{6}})_2^{13} = V_{ud}V_{cs}^*, \\ (H_{15})_2^{31} &= (H_{15})_2^{13} = V_{ud}V_{cs}^*, \end{aligned} \quad (50)$$

while for the doubly Cabibbo-suppressed $c \rightarrow du\bar{s}$ transition, we have

$$\begin{aligned} (H_{\bar{6}})_3^{21} &= -(H_{\bar{6}})_3^{12} = V_{us}V_{cd}^*, \\ (H_{15})_3^{21} &= (H_{15})_3^{12} = V_{us}V_{cd}^*. \end{aligned} \quad (51)$$

CKM matrix elements for $c \rightarrow u\bar{d}d$ and $c \rightarrow u\bar{s}s$ transitions are approximately equal in magnitude but different in sign. Here, we take $V_{ud}V_{cd}^* = -V_{us}V_{cs}^*$, with both contributions, and one has the nonzero components:

$$\begin{aligned} (H_{\bar{6}})_3^{31} &= -(H_{\bar{6}})_3^{13} = (H_{\bar{6}})_2^{12} = -(H_{\bar{6}})_2^{21} = V_{us}V_{cs}^*, \\ (H_{15})_3^{31} &= (H_{15})_3^{13} = -(H_{15})_2^{12} = -(H_{15})_2^{21} = V_{us}V_{cs}^*. \end{aligned} \quad (52)$$

A. Decays into a doubly charmed baryon and one (two) light meson(s)

For decays into a doubly charmed baryon and a light meson, one may derive the effective Hamiltonian

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= a_1 \Omega_{ccc} (\bar{T}_{cc})_i (M_8)_j^k (H_{\bar{6}})_k^{ij} \\ &+ a_2 \Omega_{ccc} (\bar{T}_{cc})_i (M_8)_j^k (H_{15})_k^{ij}, \end{aligned} \quad (53)$$

the Feynman diagrams of which are given in Fig. 4. It is necessary to stress that the above SU(3) independent amplitudes a_i 's are different than the ones in Eq. (25).

Expanding the above equations, we will obtain the decay amplitudes given in Table V. From Eq. (53) and Table V, one can see that there are two SU(3)-independent amplitudes, and thus there exist a few relations for the decay widths. These relations can be directly read off from Table V and can be examined by future experiments.

For the color-allowed decay channel, one may use the factorization approach to predict its decay widths. Using the form factors, we have the decay width

$$\begin{aligned} \Gamma(\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{+} \pi^+) &= \frac{\sqrt{\lambda} G_F^2}{64\pi m_{\Omega_{ccc}^{++}}^3} |V_{cs}V_{ud}|^2 f_\pi^2 [|H_{t,-1/2}^{V,1/2}|^2 + |H_{t,-1/2}^{A,1/2}|^2], \end{aligned} \quad (54)$$

where the H_i 's are helicity amplitudes:

$$\begin{aligned} H_{t,-1/2}^{V,1/2} &= -\frac{(f_1 + f_2 - f_4)\sqrt{\lambda}}{\sqrt{6}m_\pi^2}, \\ H_{t,-1/2}^{A,1/2} &= \frac{(g_1 - g_2 + g_4)\hat{f}_-}{\sqrt{6}m_\pi^2}. \end{aligned} \quad (55)$$

In the above equation, we used $m_{\Omega_{cc}^+} = m_{\Omega_{cc}^+}/m_{\Omega_{ccc}^{++}}$, $m_\pi = m_\pi/m_{\Omega_{ccc}^{++}}$, and the abbreviations

$$\begin{aligned} \lambda &\equiv \lambda(m_{\Omega_{ccc}^{++}}^2, m_{\Omega_{cc}^+}^2, m_\pi^2) = (m_{\Omega_{ccc}^{++}}^2 - m_{\Omega_{cc}^+}^2 - m_\pi^2)^2 \\ &- 4m_{\Omega_{cc}^+}^2 m_\pi^2, \\ \lambda &\equiv \lambda(1, m_{\Omega_{cc}^+}^2, m_\pi^2) = (1 - m_{\Omega_{cc}^+}^2 - m_\pi^2)^2 - 4m_{\Omega_{cc}^+}^2 m_\pi^2, \\ \hat{f}_- &= (1 - m_{\Omega_{cc}^+}^2)^2 - m_\pi^2. \end{aligned} \quad (56)$$

Employing the same numerical inputs with the previous section, we obtain a very rough estimate of partial decay widths:

$$\Gamma(\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{+} \pi^+) = 6.20 \times 10^{-14} \text{ GeV}. \quad (57)$$

Here, $f_\pi = 0.13 \text{ GeV}$ is used [6]. With the NLO lifetime given in Eq. (13), we can derive the branching fractions:

TABLE VI. Amplitudes for Ω_{ccc} decays into a doubly charmed baryon and two light mesons. The amplitude b_2 defined in Eq. (59) is not shown since it always accompanies b_1 in the form $b_1 - b_2$.

Cabibbo-allowed	Amplitude	Singly Cabibbo-suppressed	Amplitude
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0 \pi^0$	$\frac{b_1 - b_3 + b_4}{\sqrt{2}} V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \pi^0 \pi^0$	$(b_1 - b_3 + b_4) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^- \pi^+$	$(-b_1 + b_3 + b_4) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \pi^- \pi^+$	$(b_1 - b_3 - b_4) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \eta \bar{K}^0$	$\frac{b_1 - b_3 + b_4}{\sqrt{6}} V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^- K^+$	$(-b_1 + b_3 + b_4) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \pi^+ \bar{K}^0$	$2b_4 V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \eta \pi^0$	$-\frac{(b_1 - b_3 + b_4) V_{us} V_{cs}^*}{\sqrt{3}}$
$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{++} \pi^+ \eta$	$\sqrt{\frac{2}{3}}(b_1 + b_3 - b_4) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \eta \eta$	$-(b_1 - b_3 + b_4) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{++} K^+ \bar{K}^0$	$(b_1 + b_3 + b_4) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \pi^0 \pi^+$	$\sqrt{2}b_4 V_{us} V_{cs}^*$
Doubly Cabibbo-suppressed	Amplitude		
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^+ \pi^-$	$(b_1 - b_3 - b_4)(-V_{us} V_{cd}^*)$	$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{++} K^0 \pi^+$	$(b_1 + b_3 - b_4) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^0 \pi^0$	$\frac{(b_1 - b_3 + b_4) V_{us} V_{cd}^*}{\sqrt{2}}$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0 K^+$	$(b_1 + b_3 - b_4)(-V_{us} V_{cs}^*)$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \eta K^0$	$\frac{(b_1 - b_3 + b_4) V_{us} V_{cd}^*}{\sqrt{6}}$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \eta \pi^+$	$-\sqrt{\frac{2}{3}}(b_1 + b_3 + 2b_4) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^+ \pi^0$	$\frac{(b_1 + b_3 - b_4) V_{us} V_{cd}^*}{\sqrt{2}}$	$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{++} \pi^+ K^0$	$(b_1 + b_3 - b_4) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^+ \eta$	$-\frac{(b_1 + b_3 - b_4) V_{us} V_{cd}^*}{\sqrt{6}}$	$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{++} K^+ \pi^0$	$\frac{(b_1 + b_3 + b_4) V_{us} V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^0 \pi^+$	$(b_1 + b_3 + b_4) V_{us} V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{++} K^+ \eta$	$-\frac{(b_1 + b_3 + 5b_4) V_{us} V_{cs}^*}{\sqrt{6}}$
$\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{++} K^+ K^0$	$2b_4 V_{us} V_{cd}^*$		

$$\mathcal{B}(\Omega_{ccc}^{++} \rightarrow \Omega_{cc}^{++} \pi^+) = 1.5\%. \quad (58)$$

Such a large decay branching fraction indicates that this channel is very promising in future experimental searches.

For the reactions with one additional light meson in the final state, one has the Hamiltonian

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & b_1 \Omega_{ccc} (\bar{T}_{cc})_i (M_8)_j^k (M_8)_l^j (H_6)_k^{il} \\ & + b_2 \Omega_{ccc} (\bar{T}_{cc})_i (M_8)_j^i (M_8)_l^k (H_6)_k^{jl} \\ & + b_3 \Omega_{ccc} (\bar{T}_{cc})_i (M_8)_j^k (M_8)_l^j (H_{15})_k^{il} \\ & + b_4 \Omega_{ccc} (\bar{T}_{cc})_i (M_8)_j^i (M_8)_l^k (H_{15})_k^{jl}. \end{aligned} \quad (59)$$

Expanding the above equation, we will obtain the decay amplitudes given in Table VI. The following remarks are in order:

- (i) From the expanded Hamiltonian, one can find that the amplitudes b_1 and b_2 always appear in the combination $b_1 - b_2$. Thus, we have removed the amplitude b_2 in Table VI.
- (ii) For channels with two identical particles, there is a factor 1/2 in the decay width.

B. Decays into a charmed baryon and a charmed meson

For the two-body decays into a charmed baryon and a charmed meson, the effective Hamiltonian for the decays of Ω_{ccc} into a singly charmed baryon and a charmed meson is given as

$$\mathcal{H}_{\text{eff}} = c_1 \Omega_{ccc} (\bar{T}_{c\bar{3}})_{[ij]} \bar{D}^k (H_{\bar{6}})_k^{ij} + c_2 \Omega_{ccc} (\bar{T}_{c6})_{[ij]} \bar{D}^k (H_{15})_k^{ij}. \quad (60)$$

TABLE VII. Amplitudes for Ω_{ccc} decays into a singly charmed baryon and a charmed meson.

Cabibbo-allowed	Amplitude	Singly Cabibbo-suppressed	Amplitude
$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+$	$-2c_1 V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^+$	$2c_1 V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^+$	$\sqrt{2}c_2 V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D_s^+$	$-2c_1 V_{us} V_{cs}^*$
Doubly Cabibbo-suppressed	Amplitude		
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D_s^+$	$\sqrt{2}c_2 V_{us} V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D_s^+$	$\sqrt{2}c_2 V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D_s^+$	$-2c_1 V_{us} V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^+$	$-\sqrt{2}c_2 V_{us} V_{cs}^*$

TABLE VIII. Amplitudes for three-body Ω_{ccc} decays into a singly charmed baryon (antitriplet), D meson, and a light meson.

Cabibbo-allowed	Amplitude	Singly Cabibbo-suppressed	Amplitude
$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^+ \bar{K}^0$	$(-d_2 + d_3 + d_4 - d_5) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^0 \pi^+$	$(2d_1 + d_2 + d_4) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^0 \pi^+$	$(-2d_1 - d_2 - d_4) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^+ \pi^0$	$-\frac{(2d_1 + d_2 - d_4 + 2d_5) V_{us} V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ \pi^0$	$\frac{2d_1 + d_3 + d_5}{\sqrt{2}} V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^+ \eta$	$\frac{(2d_1 + 3d_2 - 2d_3 - 3d_4) V_{us} V_{cs}^*}{\sqrt{6}}$
$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ \eta$	$-\frac{2d_1 + d_3 - 3d_5}{\sqrt{6}} V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D_s^+ \bar{K}^0$	$(2d_1 + d_3 - d_5) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D_s^+ \bar{K}^0$	$(-2d_1 - d_2 + d_4) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^0 K^+$	$(2d_1 + d_2 + d_4)(-V_{us} V_{cs}^*)$
$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D^+ \pi^+$	$(-d_2 + d_3 - d_4 + d_5) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ K^0$	$(2d_1 + d_3 - d_5)(-V_{us} V_{cs}^*)$
Doubly Cabibbo-suppressed	Amplitude	$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D_s^+ \pi^+$	$(d_2 - d_3 + d_4 - d_5)(-V_{us} V_{cs}^*)$
$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^0 K^+$	$(2d_1 + d_2 + d_4)(-V_{us} V_{cd}^*)$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D_s^+ \pi^0$	$\frac{(-d_2 + d_3 + d_4 + d_5) V_{us} V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^+ K^0$	$(2d_1 + d_2 - d_4)(-V_{us} V_{cd}^*)$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D_s^+ \eta$	$\frac{(4d_1 + 3d_2 - d_3 - 3d_4 + 3d_5) V_{us} V_{cs}^*}{\sqrt{6}}$
$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D_s^+ \pi^0$	$\sqrt{2} d_5 V_{us} V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D^+ K^+$	$(d_2 - d_3 + d_4 - d_5)(-V_{us} V_{cs}^*)$
$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D_s^+ \eta$	$\sqrt{\frac{2}{3}}(2d_1 + d_3) V_{us} V_{cd}^*$		
$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D_s^+ K^0$	$(d_2 - d_3 - d_4 + d_5)(-V_{us} V_{cd}^*)$		
$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D_s^+ K^+$	$(d_2 - d_3 + d_4 - d_5) V_{us} V_{cd}^*$		

TABLE IX. Amplitudes for three-body Ω_{ccc} decays into a singly charmed baryon (sextet), D meson, and a light meson.

Cabibbo-allowed	Amplitude	Singly Cabibbo-suppressed	Amplitude
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^0 \bar{K}^0$	$(e_2 - e_4) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^0 \pi^0$	$\frac{(e_2 - e_4) V_{us} V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^+ K^-$	$(e_3 - e_5) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^0 \eta$	$-\sqrt{\frac{3}{2}}(e_2 - e_4) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^+ \bar{K}^0$	$\frac{e_2 + e_3 - e_4 - e_5}{\sqrt{2}} V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^+ \pi^-$	$(e_5 - e_3) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^0 \pi^+$	$\frac{2e_1 + e_2 + e_4}{\sqrt{2}} V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D_s^+ K^-$	$(e_3 - e_5) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^+ \pi^0$	$\frac{1}{2}(-2e_1 + e_3 + e_5) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^0 \pi^+$	$-\frac{(2e_1 + e_2 + e_4) V_{us} V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^+ \eta$	$\frac{2e_1 - e_3 + 3e_5}{2\sqrt{3}} V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^+ \pi^0$	$\frac{1}{2}(2e_1 + e_2 - e_4 - 2e_5) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D_s^+ \bar{K}^0$	$\frac{2e_1 + e_2 - e_4}{\sqrt{2}} V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^+ \eta$	$-\frac{(2e_1 + 3e_2 + 2e_3 - 3e_4) V_{us} V_{cs}^*}{2\sqrt{3}}$
$\Omega_{ccc}^{++} \rightarrow \Xi_c'^0 D^+ \pi^+$	$\frac{e_2 + e_3 + e_4 + e_5}{\sqrt{2}} V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D_s^+ \bar{K}^0$	$-\frac{(2e_1 - e_3 + e_5) V_{us} V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Omega_c^0 D^+ K^+$	$(e_3 + e_5) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Sigma_c^0 D^+ \pi^+$	$(e_2 + e_3 + e_4 + e_5)(-V_{us} V_{cs}^*)$
$\Omega_{ccc}^{++} \rightarrow \Omega_c^0 D_s^+ \pi^+$	$(e_2 + e_4) V_{ud} V_{cs}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^0 K^+$	$\frac{(2e_1 + e_2 + e_4) V_{us} V_{cs}^*}{\sqrt{2}}$
Doubly Cabibbo-suppressed	Amplitude	$\Omega_{ccc}^{++} \rightarrow \Omega_c^0 D_s^+ K^+$	$(e_2 + e_3 + e_4 + e_5) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^0 K^0$	$(e_2 - e_4) V_{us} V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^+ K^0$	$\frac{(2e_1 - e_3 + e_5) V_{us} V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D_s^+ \pi^-$	$(e_3 - e_5) V_{us} V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D_s^+ \pi^0$	$\frac{1}{2}(e_2 + e_3 - e_4 + e_5) V_{us} V_{cs}^*$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^0 K^+$	$\frac{(2e_1 + e_2 + e_4) V_{us} V_{cd}^*}{\sqrt{2}}$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D_s^+ \eta$	$-\frac{(4e_1 + 3e_2 + e_3 - 3e_4 - 3e_5) V_{us} V_{cs}^*}{2\sqrt{3}}$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^+ K^0$	$\frac{(2e_1 + e_2 - e_4) V_{us} V_{cd}^*}{\sqrt{2}}$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^0 D^+ K^+$	$\frac{(e_2 - e_3 + e_4 - e_5) V_{us} V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D_s^+ \pi^0$	$e_5 V_{us} V_{cd}^*$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^0 D_s^+ \pi^+$	$-\frac{(e_2 - e_3 + e_4 - e_5) V_{us} V_{cs}^*}{\sqrt{2}}$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D_s^+ \eta$	$\frac{(e_3 - 2e_1) V_{us} V_{cd}^*}{\sqrt{3}}$		
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^0 D^+ K^+$	$(e_2 + e_4) V_{us} V_{cd}^*$		
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^0 D_s^+ \pi^+$	$(e_3 + e_5) V_{us} V_{cd}^*$		
$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D_s^+ K^0$	$\frac{(e_2 + e_3 - e_4 - e_5) V_{us} V_{cd}^*}{\sqrt{2}}$		
$\Omega_{ccc}^{++} \rightarrow \Xi_c'^0 D_s^+ K^+$	$\frac{(e_2 + e_3 + e_4 + e_5) V_{us} V_{cd}^*}{\sqrt{2}}$		

The Feynman diagram is shown in the last panel of Fig. 4, and decay amplitudes are collected in Table VII.

The three-body decays of Ω_{ccc} can involve an additional light meson in the final state. For the modes with an antitriplet baryon, we have

$$\begin{aligned}\mathcal{H}_{\text{eff}} = & d_1 \Omega_{ccc}(\bar{T}_{\mathbf{c}\bar{3}})_{[ij]} \bar{D}^k(M_8)_k^l (H_{\bar{6}})_l^{ij} \\ & + d_2 \Omega_{ccc}(\bar{T}_{\mathbf{c}\bar{3}})_{[ij]} \bar{D}^j(M_8)_k^l (H_{\bar{6}})_l^{ik} \\ & + d_3 \Omega_{ccc}(\bar{T}_{\mathbf{c}\bar{3}})_{[ij]} \bar{D}^l(M_8)_k^i (H_{\bar{6}})_l^{jk} \\ & + d_4 \Omega_{ccc}(\bar{T}_{\mathbf{c}\bar{3}})_{[ij]} \bar{D}^j(M_8)_k^l (H_{15})_l^{ik} \\ & + d_5 \Omega_{ccc}(\bar{T}_{\mathbf{c}\bar{3}})_{[ij]} \bar{D}^l(M_8)_k^i (H_{15})_l^{jk},\end{aligned}\quad (61)$$

while the effective Hamiltonian for a sextet baryon is constructed as

$$\begin{aligned}\mathcal{H}_{\text{eff}} = & e_1 \Omega_{ccc}(\bar{T}_{\mathbf{c}\mathbf{6}})_{\{ij\}} \bar{D}^k(M_8)_k^l (H_{15})_l^{ij} \\ & + e_2 \Omega_{ccc}(\bar{T}_{\mathbf{c}\mathbf{6}})_{\{ij\}} \bar{D}^j(M_8)_k^l (H_{15})_l^{ik} \\ & + e_3 \Omega_{ccc}(\bar{T}_{\mathbf{c}\mathbf{6}})_{\{ij\}} \bar{D}^l(M_8)_k^i (H_{15})_l^{jk} \\ & + e_4 \Omega_{ccc}(\bar{T}_{\mathbf{c}\mathbf{6}})_{\{ij\}} \bar{D}^j(M_8)_k^l (H_{\bar{6}})_l^{ik} \\ & + e_5 \Omega_{ccc}(\bar{T}_{\mathbf{c}\mathbf{6}})_{\{ij\}} \bar{D}^l(M_8)_k^i (H_{\bar{6}})_l^{jk}.\end{aligned}\quad (62)$$

Expanding the above equations, we will obtain the decay amplitudes given in Table VIII for antitriplet baryon and in Table IX for the sextet baryon.

VI. NONLEPTONIC Ω_{bbb}^- DECAYS

For the bottom-quark decay, there are four kinds of quark-level transitions,

$$\begin{aligned}b & \rightarrow c\bar{c}d/s, & b & \rightarrow c\bar{u}d/s, \\ b & \rightarrow u\bar{c}d/s, & b & \rightarrow q\bar{q}q,\end{aligned}\quad (63)$$

which will be studied in order.

A. $b \rightarrow c\bar{c}d/s$

1. Decays into a charmonium J/ψ

Such decays will have the same topology with the $b \rightarrow s\ell^+\ell^-$ decays. The transition operator $b \rightarrow c\bar{c}d/s$ can form an SU(3) triplet,

$$\begin{aligned}\mathcal{H}_{\text{eff}} = & f_1 \Omega_{bbb}(\bar{T}_{bb})_i (H_3)^i J/\psi \\ & + f_2 \Omega_{bbb}(\bar{T}_{bb})_i (M_8)_j^i (H_3)^j J/\psi \\ & + f_3 \Omega_{bbb}(\bar{T}_{b\bar{3}})_{[ij]} \bar{B}^i (H_3)^j J/\psi \\ & + f_4 \Omega_{bbb}(\bar{T}_{b6})_{\{ij\}} \bar{B}^i (H_3)^j J/\psi,\end{aligned}\quad (64)$$

with $(H_3)_2 = V_{cd}^*$ and $(H_3)_3 = V_{cs}^*$. Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table X.

TABLE X. Amplitudes for Ω_{bbb} decays into a J/ψ and a doubly bottom baryon or two bottom hadrons.

Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- J/\psi$	$f_1 V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Lambda_b^0 B^- J/\psi$	$f_3 V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- J/\psi$	$f_1 V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^- J/\psi$	$f_3 V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \pi^- J/\psi$	$f_2 V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}^0 J/\psi$	$f_3 V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 K^- J/\psi$	$f_2 V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}_s^0 J/\psi$	$-f_3 V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \pi^0 J/\psi$	$-\frac{f_2 V_{cd}^*}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Sigma_b^- B^- J/\psi$	$\frac{f_4 V_{cd}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{K}^0 J/\psi$	$f_2 V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Sigma_b^- \bar{B}^0 J/\psi$	$f_4 V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \eta J/\psi$	$\frac{f_2 V_{cd}^*}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^- J/\psi$	$\frac{f_4 V_{cs}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- K^0 J/\psi$	$f_2 V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}^0 J/\psi$	$\frac{f_4 V_{cs}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \eta J/\psi$	$-\sqrt{\frac{2}{3}} f_2 V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}_s^0 J/\psi$	$\frac{f_4 V_{cd}^*}{\sqrt{2}}$
		$\Omega_{bbb}^- \rightarrow \Omega_b^- \bar{B}_s^0 J/\psi$	$f_4 V_{cs}^*$

2. Decays into a triply heavy baryon cbb plus an anticharmed meson

The $b \rightarrow c\bar{c}d/s$ transition can lead to another type of effective Hamiltonian,

$$\mathcal{H}_{\text{eff}} = g_1 \Omega_{bbb} \bar{\Omega}_{cbb} D_i (H_3)^i + g_2 \Omega_{bbb} \bar{\Omega}_{cbb} D_j (M_8)_i^j (H_3)^i, \quad (65)$$

which corresponds to the decays into triply heavy baryon Ω_{cbb} plus an anticharmed meson. Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XI.

B. $b \rightarrow c\bar{u}d/s$ transition

1. Decays into a triply heavy baryon cbb plus light mesons

The operator to produce a charm quark from the b -quark decay, $\bar{c}b\bar{q}u$, is given by

$$\mathcal{H}_{\text{e.w.}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{uq}^* [C_1 O_1^{\bar{c}u} + C_2 O_2^{\bar{c}u}] + \text{H.c.}, \quad (66)$$

TABLE XI. Amplitudes for Ω_{bbb} decays into Ω_{cbb} and an anticharmed meson.

Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 D^-$	$g_1 V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 D_s^-$	$g_1 V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 \bar{D}^0 \pi^-$	$g_2 V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 \bar{D}^0 K^-$	$g_2 V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 D^- \pi^0$	$-\frac{g_2 V_{cd}^*}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 D^- \bar{K}^0$	$g_2 V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 D^- \eta$	$\frac{g_2 V_{cd}^*}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 D_s^- K^0$	$g_2 V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{cbb}^0 D_s^- \eta$	$-\sqrt{\frac{2}{3}} g_2 V_{cs}^*$		

with

$$\begin{aligned} O_1^{\bar{c}u} &= \bar{q}^i \gamma^\mu (1 - \gamma_5) u^j \bar{c}^j \gamma_\mu (1 - \gamma_5) b^i, \\ O_2^{\bar{c}u} &= \bar{q} \gamma^\mu (1 - \gamma_5) u \bar{c} \gamma_\mu (1 - \gamma_5) b. \end{aligned} \quad (67)$$

The light quarks in this effective Hamiltonian form an octet with the nonzero entry

$$(H_8)_1^2 = V_{cb} V_{ud}^* \quad (68)$$

for the $b \rightarrow \bar{c}ud$ transition and $(H_8)_1^3 = V_{cb} V_{us}^*$ for the $b \rightarrow \bar{c}us$ transition,

$$\mathcal{H}_{\text{eff}} = h_1 \Omega_{bbb} \bar{\Omega}_{cbb} M_i^j (H_8)_j^i + h_2 \Omega_{bbb} \bar{\Omega}_{cbb} M_k^j M_i^k (H_8)_j^i. \quad (69)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XII.

TABLE XII. Amplitudes for Ω_{bbb} decays into Ω_{cbb} and light meson(s).

Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 \pi^-$	$h_1 V_{cb} V_{ud}^*$	$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 K^-$	$h_1 V_{cb} V_{us}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 \pi^0 K^-$	$\frac{h_2 V_{cb} V_{us}^*}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 \pi^- \eta$	$\sqrt{\frac{2}{3}} h_2 V_{cb} V_{ud}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 \pi^- \bar{K}^0$	$h_2 V_{cb} V_{us}^*$	$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 K^0 K^-$	$h_2 V_{cb} V_{ud}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 K^- \eta$	$-\frac{h_2 V_{cb} V_{us}^*}{\sqrt{6}}$		

2. Decays into a charmed meson plus a doubly bottom baryon or two bottom hadrons

If the $c\bar{c}$ are separated, then the final state could be a doubly bottom baryon and a charmed meson. The three-body modes can also include decays into a bottom baryon, a bottom meson, and a charmed meson. Thus, one can have the effective Hamiltonian

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= j_1 \Omega_{bbb} (\bar{T}_{bb})_i \bar{D}^j (H_8)_j^i + j_2 \Omega_{bbb} (\bar{T}_{bb})_k \bar{D}^k (M_8)_i^j (H_8)_j^i + j_3 \Omega_{bbb} (\bar{T}_{bb})_i \bar{D}^k (M_8)_k^j (H_8)_j^i + j_4 \Omega_{bbb} (\bar{T}_{bb})_i \bar{D}^k (M_8)_j^i (H_8)_k^j \\ &+ j_5 \Omega_{bbb} (\bar{T}_{b\bar{3}})_{[ik]} \bar{B}^j \bar{D}^k (H_8)_j^i + j_6 \Omega_{bbb} (\bar{T}_{b\bar{3}})_{[ik]} \bar{B}^k \bar{D}^j (H_8)_j^i + j_7 \Omega_{bbb} (\bar{T}_{b6})_{\{ik\}} \bar{B}^j \bar{D}^k (H_8)_j^i + j_8 \Omega_{bbb} (\bar{T}_{b6})_{\{ik\}} \bar{B}^k \bar{D}^j (H_8)_j^i. \end{aligned} \quad (70)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XIII.

TABLE XIII. Amplitudes for nonleptonic Ω_{bbb}^- decays into a charmed meson plus a doubly bottom baryon or two bottom hadrons.

Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^0$	$j_1 V_{cb} V_{ud}^*$	$\Omega_{bbb}^- \rightarrow \Lambda_b^0 B^- D^0$	$-(j_5 + j_6) V_{cb} V_{ud}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D^0$	$j_1 V_{cb} V_{us}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^- D^0$	$-(j_5 + j_6) V_{cb} V_{us}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D^0 \pi^-$	$(j_2 + j_4) V_{cb} V_{ud}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^- B^- D^+$	$-j_5 V_{cb} V_{us}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D^0 K^-$	$(j_2 + j_4) V_{cb} V_{us}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^- B^- D_s^+$	$j_5 V_{cb} V_{ud}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^0 \pi^0$	$\frac{(j_3 - j_4) V_{cb} V_{ud}^*}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}^0 D^0$	$-j_6 V_{cb} V_{us}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^0 \bar{K}^0$	$j_4 V_{cb} V_{us}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}_s^0 D^0$	$j_6 V_{cb} V_{ud}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^0 \eta$	$\frac{(j_3 + j_4) V_{cb} V_{ud}^*}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Sigma_b^0 B^- D^0$	$\frac{(j_7 + j_8) V_{cb} V_{ud}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^+ \pi^-$	$(j_2 + j_3) V_{cb} V_{ud}^*$	$\Omega_{bbb}^- \rightarrow \Sigma_b^- B^- D^+$	$j_7 V_{cb} V_{ud}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^+ K^-$	$j_2 V_{cb} V_{us}^*$	$\Omega_{bbb}^- \rightarrow \Sigma_b^- \bar{B}^0 D^0$	$j_8 V_{cb} V_{ud}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D_s^+ K^-$	$j_3 V_{cb} V_{ud}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^- D^0$	$\frac{(j_7 + j_8) V_{cb} V_{us}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D^0 \pi^0$	$\frac{j_3 V_{cb} V_{us}^*}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b'^- B^- D^+$	$\frac{j_7 V_{cb} V_{us}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D^0 K^0$	$j_4 V_{cb} V_{ud}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b'^- B^- D_s^+$	$\frac{j_7 V_{cb} V_{ud}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D^0 \eta$	$\frac{(j_3 - 2j_4) V_{cb} V_{us}^*}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Xi_b'^- \bar{B}^0 D^0$	$\frac{j_8 V_{cb} V_{us}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D^+ \pi^-$	$j_3 V_{cb} V_{us}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b'^- \bar{B}_s^0 D^0$	$\frac{j_8 V_{cb} V_{ud}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D_s^+ \pi^-$	$j_2 V_{cb} V_{ud}^*$	$\Omega_{bbb}^- \rightarrow \Omega_b^- B^- D_s^+$	$j_7 V_{cb} V_{us}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D_s^+ K^-$	$(j_2 + j_3) V_{cb} V_{us}^*$	$\Omega_{bbb}^- \rightarrow \Omega_b^- \bar{B}_s^0 D^0$	$j_8 V_{cb} V_{us}^*$

C. CKM-suppressed $b \rightarrow u\bar{c}d/s$ transition

For the anticharm production, the operator having the quark contents $(\bar{u}b)(\bar{q}c)$ is given by

$$\mathcal{H}_{\text{e.w.}} = \frac{G_F}{\sqrt{2}} V_{ub} V_{cq}^* [C_1 O_1^{\bar{u}c} + C_2 O_2^{\bar{u}c}] + \text{H.c.}, \quad (71)$$

with

$$\begin{aligned} O_1^{\bar{u}c} &= \bar{q}^i \gamma^\mu (1 - \gamma_5) c^j \bar{u}^j \gamma_\mu (1 - \gamma_5) b^i, \\ O_2^{\bar{u}c} &= \bar{q} \gamma^\mu (1 - \gamma_5) c \bar{u} \gamma_\mu (1 - \gamma_5) b. \end{aligned} \quad (72)$$

The two light antiquarks form the $\bar{\mathbf{3}}$ and $\mathbf{6}$ representations. The antisymmetric tensor H_3'' and the symmetric tensor H_6 have nonzero components

$$\begin{aligned} (H_3'')^{13} &= -(H_3'')^{31} = V_{ub} V_{cs}^*, \\ (H_6)^{13} &= (H_6)^{31} = V_{ub} V_{cs}^*, \end{aligned} \quad (73)$$

for the $b \rightarrow u\bar{c}s$ transition. For the transition $b \rightarrow u\bar{c}d$, one requests the interchange of $2 \leftrightarrow 3$ in the subscripts, and V_{cs} is replaced by V_{cd} .

The effective Hamiltonian is derived as

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= k_1 \Omega_{bbb}(\bar{T}_{bb})_i D_j (H_3)^{ij} + k_2 \Omega_{bbb}(\bar{T}_{bb})_i D_j (H_6)^{ij} + k_3 \Omega_{bbb}(\bar{T}_{bb})_i D_j (M_8)_k^i (H_3)^{jk} + k_4 \Omega_{bbb}(\bar{T}_{bb})_i D_j (M_8)_k^j (H_3)^{ik} \\ &+ k_5 \Omega_{bbb}(\bar{T}_{bb})_i D_j (M_8)_k^i (H_6)^{jk} + k_6 \Omega_{bbb}(\bar{T}_{bb})_i D_j (M_8)_k^j (H_6)^{ik} + k_7 \Omega_{bbb}(\bar{T}_{b\bar{3}})_{[ij]} \bar{B}^k D_k (H_3)^{ij} \\ &+ k_8 \Omega_{bbb}(\bar{T}_{b\bar{3}})_{[ik]} \bar{B}^k D_j (H_3)^{ij} + k_9 \Omega_{bbb}(\bar{T}_{b\bar{3}})_{[ik]} \bar{B}^k D_j (H_6)^{ij} + k_{10} \Omega_{bbb}(\bar{T}_{b6})_{\{ik\}} \bar{B}^k D_j (H_3)^{ij} \\ &+ k_{11} \Omega_{bbb}(\bar{T}_{b6})_{\{ij\}} \bar{B}^k D_k (H_6)^{ij} + k_{12} \Omega_{bbb}(\bar{T}_{b6})_{\{ik\}} \bar{B}^k D_j (H_6)^{ij}. \end{aligned} \quad (74)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XIV.

D. Charmless $b \rightarrow q_1 \bar{q}_2 q_3$ decays

1. Decays into a doubly bottom baryon bbq and a light meson

The charmless $b \rightarrow q$ ($q = d, s$) transition is controlled by the weak Hamiltonian $\mathcal{H}_{\text{eff}}$,

$$\mathcal{H}_{\text{e.w.}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{uq}^* [C_1 O_1^{\bar{u}u} + C_2 O_2^{\bar{u}u}] - V_{tb} V_{tq}^* \left[\sum_{i=3}^{10} C_i O_i \right] \right\} + \text{H.c.}, \quad (75)$$

where O_i is a four-quark operator or a moment-type operator. The four-quark operators O_i are given as follows:

$$\begin{aligned} O_1^{\bar{u}u} &= (\bar{q}^i u^j)_{V-A} (\bar{u}^j b^i)_{V-A}, \quad O_2^{\bar{u}u} = (\bar{q}u)_{V-A} (\bar{u}b)_{V-A}, \quad O_3 = (\bar{q}b)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A}, \quad O_4 = (\bar{q}^i b^j)_{V-A} \sum_{q'} (\bar{q}'^j q'^i)_{V-A}, \\ O_5 &= (\bar{q}b)_{V-A} \sum_{q'} (\bar{q}' q')_{V+A}, \quad O_6 = (\bar{q}^i b^j)_{V-A} \sum_{q'} (\bar{q}'^j q'^i)_{V+A}, \quad O_7 = \frac{3}{2} (\bar{q}b)_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V+A}, \\ O_8 &= \frac{3}{2} (\bar{q}^i b^j)_{V-A} \sum_{q'} e_{q'} (\bar{q}'^j q'^i)_{V+A}, \quad O_9 = \frac{3}{2} (\bar{q}b)_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V-A}, \quad O_{10} = \frac{3}{2} (\bar{q}^i b^j)_{V-A} \sum_{q'} e_{q'} (\bar{q}'^j q'^i)_{V-A}. \end{aligned} \quad (76)$$

In the above, q denotes a d quark for the $b \rightarrow d$ transition or an s quark for the $b \rightarrow s$ transition, while $q' = u, d, s$. V and A denote the vector and axial-vector currents. In the SU(3) group, penguin operators behave as the $\mathbf{3}$ representation, while tree operators can be decomposed in terms of a vector H_3 , a traceless tensor antisymmetric in the upper indices H_6 , and a traceless tensor symmetric in the upper indices H_{15} .

For the $\Delta S = 0(b \rightarrow d)$ decays, the nonzero components of the effective Hamiltonian are

$$\begin{aligned} (H_3)^2 &= 1, \quad (H_6)_1^{12} = -(H_6)_1^{21} = (H_6)_3^{23} = -(H_6)_3^{32} = 1, \\ 2(H_{15})_1^{12} &= 2(H_{15})_1^{21} = -3(H_{15})_2^{22} = -6(H_{15})_3^{23} = -6(H_{15})_3^{32} = 6, \end{aligned} \quad (77)$$

and all other remaining entries are zero. For the $\Delta S = 1(b \rightarrow s)$ decays, the nonzero entries in the H_3 , H_6 , and H_{15} are obtained from Eq. (77) with the exchange $2 \leftrightarrow 3$.

TABLE XIV. Amplitudes for the CKM-suppressed Ω_{bbb} decays into an anticharmed meson.

Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D^-$	$(k_1 + k_2)V_{ub}V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Lambda_b^0 B^- \bar{D}^0$	$(2k_7 + k_8 - k_9)V_{ub}V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D_s^-$	$(k_1 + k_2)V_{ub}V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Lambda_b^0 \bar{B}^0 D^-$	$(2k_7 + k_8 + k_9)V_{ub}V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{D}^0$	$(k_2 - k_1)V_{ub}V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Lambda_b^0 \bar{B}^0 D_s^-$	$(k_8 + k_9)V_{ub}V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \bar{D}^0$	$(k_2 - k_1)V_{ub}V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Lambda_b^0 \bar{B}_s^0 D_s^-$	$2k_7 V_{ub}V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \bar{D}^0 \pi^-$	$(k_3 + k_4 + k_5 + k_6)V_{ub}V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^- \bar{D}^0$	$(2k_7 + k_8 - k_9)V_{ub}V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \bar{D}^0 K^-$	$(k_3 + k_4 + k_5 + k_6)V_{ub}V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}^0 D^-$	$2k_7 V_{ub}V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D^- \pi^0$	$-\frac{(k_3 + k_4 - k_5 + k_6)V_{ub}V_{cd}^*}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 D^-$	$(k_8 + k_9)V_{ub}V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D^- \bar{K}^0$	$(k_4 + k_6)V_{ub}V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 D_s^-$	$(2k_7 + k_8 + k_9)V_{ub}V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D^- \eta$	$\frac{(-k_3 + k_4 + k_5 + k_6)V_{ub}V_{cd}^*}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}^0 \bar{D}^0$	$(k_8 - k_9)V_{ub}V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D_s^- \pi^0$	$\frac{(k_5 - k_3)V_{ub}V_{cs}^*}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 \bar{D}^0$	$(k_9 - k_8)V_{ub}V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D_s^- K^0$	$(k_4 + k_6)V_{ub}V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Sigma_b^+ B^- D^-$	$(k_{10} + k_{12})V_{ub}V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D_s^- \eta$	$-\frac{(k_3 + 2k_4 - k_5 + 2k_6)V_{ub}V_{cs}^*}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Sigma_b^+ B^- D_s^-$	$(k_{10} + k_{12})V_{ub}V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{D}^0 \pi^0$	$-\frac{(k_3 + k_4 + k_5 - k_6)V_{ub}V_{cd}^*}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Sigma_b^0 B^- \bar{D}^0$	$\frac{(-k_{10} + 2k_{11} + k_{12})V_{ub}V_{cs}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{D}^0 \bar{K}^0$	$(k_3 + k_5)V_{ub}V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Sigma_b^0 \bar{B}^0 D^-$	$\frac{(k_{10} + 2k_{11} + k_{12})V_{ub}V_{cd}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{D}^0 \eta$	$\frac{(k_3 - k_4 + k_5 + k_6)V_{ub}V_{cd}^*}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Sigma_b^0 \bar{B}^0 D_s^-$	$\frac{(k_{10} + k_{12})V_{ub}V_{cs}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^- \pi^+$	$(-k_3 - k_4 + k_5 + k_6)V_{ub}V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Sigma_b^0 \bar{B}_s^0 D_s^-$	$\sqrt{2}k_{11}V_{ub}V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D_s^- \pi^+$	$(k_5 - k_3)V_{ub}V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Sigma_b^- \bar{B}^0 \bar{D}^0$	$(k_{12} - k_{10})V_{ub}V_{cd}^*$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D_s^- K^+$	$(k_6 - k_4)V_{ub}V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^- \bar{D}^0$	$\frac{(-k_{10} + 2k_{11} + k_{12})V_{ub}V_{cs}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \bar{D}^0 \pi^0$	$\frac{(k_6 - k_4)V_{ub}V_{cs}^*}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}^0 D^-$	$\sqrt{2}k_{11}V_{ub}V_{cs}^*$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \bar{D}^0 K^0$	$(k_3 + k_5)V_{ub}V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 D^-$	$\frac{(k_{10} + k_{12})V_{ub}V_{cd}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \bar{D}^0 \eta$	$-\frac{(2k_3 + k_4 + 2k_5 - k_6)V_{ub}V_{cs}^*}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 D_s^-$	$\frac{(k_{10} + 2k_{11} + k_{12})V_{ub}V_{cs}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D^- \pi^+$	$(k_6 - k_4)V_{ub}V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}^0 \bar{D}^0$	$\frac{(k_{12} - k_{10})V_{ub}V_{cs}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D^- K^+$	$(k_5 - k_3)V_{ub}V_{cd}^*$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 \bar{D}^0$	$\frac{(k_{12} - k_{10})V_{ub}V_{cd}^*}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D_s^- K^+$	$(-k_3 - k_4 + k_5 + k_6)V_{ub}V_{cs}^*$	$\Omega_{bbb}^- \rightarrow \Omega_b^- \bar{B}_s^0 \bar{D}^0$	$(k_{12} - k_{10})V_{ub}V_{cs}^*$

If the final state contains one light meson, the effective Hamiltonian is given as

$$\mathcal{H}_{\text{eff}} = l_1 \Omega_{bbb} (\bar{T}_{bb})_j (M_8)_i^j (H_3)^i + l_2 \Omega_{bbb} (\bar{T}_{bb})_i (M_8)_j^k (H_6)_{jk}^{ij} + l_3 \Omega_{bbb} (\bar{T}_{bb})_i (M_8)_j^k (H_{15})_{jk}^{ij}. \quad (78)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XV.

With one additional light meson, we have

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & l_4 \Omega_{bbb} (\bar{T}_{bb})_i (M_8)_j^i (M_8)_k^j (H_3)^k + l_5 \Omega_{bbb} (\bar{T}_{bb})_i (M_8)_k^j (M_8)_j^k (H_3)^i + l_6 \Omega_{bbb} (\bar{T}_{bb})_i (M_8)_j^k (M_8)_j^l (H_6)_{lk}^{ij} \\ & + l_7 \Omega_{bbb} (\bar{T}_{bb})_i (M_8)_j^i (M_8)_k^l (H_6)_{lk}^{jk} + l_8 \Omega_{bbb} (\bar{T}_{bb})_i (M_8)_l^k (M_8)_j^l (H_{15})_{jk}^{ij} + l_9 \Omega_{bbb} (\bar{T}_{bb})_i (M_8)_j^i (M_8)_k^l (H_{15})_{lk}^{jk}. \end{aligned} \quad (79)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XVI.

A few remarks are given in order:

TABLE XV. Amplitudes for Ω_{bbb} decays into a doubly bottom baryon and a light meson. The $b \rightarrow d$ transitions are given in the left columns, and the $b \rightarrow s$ ones are shown in the right columns.

Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \pi^-$	$l_1 + l_2 + 3l_3$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 K^-$	$l'_1 + l'_2 + 3l'_3$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \pi^0$	$-\frac{l_1 + l_2 - 5l_3}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{K}^0$	$l'_1 - l'_2 - l'_3$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \eta$	$\frac{l_1 - 3l_2 + 3l_3}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \pi^0$	$-\sqrt{2}(l'_2 - 2l'_3)$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- K^0$	$l_1 - l_2 - l_3$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \eta$	$-\sqrt{\frac{2}{3}}(l'_1 - 3l'_3)$

- (i) In Tables XV and XVI, both the $b \rightarrow d$ and $b \rightarrow s$ channels are included. Since the CKM matrix elements are different, the SU(3) irreducible amplitudes for the $b \rightarrow s$ transition are primed.
- (ii) Expanding Eq. (79), one can find the amplitudes l_6 and l_7 are not independent and they always appear in the product $l_6 - l_7$. So, in the two tables, we did not show the l_7 .
- (iii) Inspired from the B -meson decay data, we can infer that the typical branching fractions are at the order 10^{-6} . Thus, these channels are rare decays and can be studied with a large amount of data. However, the direct CP asymmetries in these channels are typically sizable.
- (iv) For the $b \rightarrow q_1 \bar{q}_2 q_3$ decays, there are two amplitudes with different CKM factors. One can consider the U-spin connected decays with the decay amplitudes for $\bar{B}$ decays,

$$A(\Delta S = 0, \bar{B}) = r(V_{ub}V_{ud}^*A^T + V_{tb}V_{td}^*A^P),$$

$$A(\Delta S = 1, \bar{B}) = V_{ub}V_{us}^*A^T + V_{tb}V_{ts}^*A^P, \quad (80)$$

where r is a constant factor and A^T and A^P are the amplitudes without the CKM factors. For the B -meson decays, we have the amplitudes

$$A(\Delta S = 0, B) = r(V_{ub}^*V_{ud}A^T + V_{tb}^*V_{td}A^P),$$

$$A(\Delta S = 1, B) = V_{ub}^*V_{us}A^T + V_{tb}^*V_{ts}A^P, \quad (81)$$

where the weak phases have been reversed. The CP -violating quantity $\Delta = \Gamma(\bar{B}) - \Gamma(B)$ is then derived as

$$\Delta(\Delta S = 0) = 2r^2(\text{Re}[V_{ub}V_{ud}^*V_{tb}^*V_{td}A^TA^{P*}] - \text{Re}[V_{ub}^*V_{ud}V_{tb}V_{td}^*A^TA^{P*}])$$

$$= -4r^2\text{Im}[V_{ub}V_{ud}^*V_{tb}^*V_{td}]\text{Im}[A^TA^{P*}], \quad (82)$$

$$\Delta(\Delta S = 1) = 2(\text{Re}[V_{ub}V_{us}^*V_{tb}^*V_{ts}A^TA^{P*}] - \text{Re}[V_{ub}^*V_{us}V_{tb}V_{ts}^*A^TA^{P*}])$$

$$= -4\text{Im}[V_{ub}V_{us}^*V_{tb}^*V_{ts}]\text{Im}[A^TA^{P*}]. \quad (83)$$

Because of the property of the CKM matrix element $\text{Im}[V_{ub}V_{ud}^*V_{tb}^*V_{td}] = -\text{Im}[V_{ub}V_{us}^*V_{tb}^*V_{ts}]$, one has

$$\Delta(\Delta S = 0) = -r^2\Delta(\Delta S = 1). \quad (84)$$

TABLE XVI. Amplitudes for Ω_{bbb} decays into a doubly bottom baryon and two light mesons. The $b \rightarrow d$ transitions are given in the left columns, and the $b \rightarrow s$ ones are shown in the right columns. The amplitude l_7 is neglected since it correlates with l_6 .

Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \pi^- \pi^0$	$4\sqrt{2}l_9$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \bar{K}^0 \pi^-$	$l'_4 + l'_6 + 3l'_8 - l'_9$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 K^- K^0$	$l_4 + l_6 + 3l_8 - l_9$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 K^- \pi^0$	$\frac{l'_4 + l'_6 + 3l'_8 + 7l'_9}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \eta \pi^-$	$\sqrt{\frac{2}{3}}(l_4 + l_6 + 3l_8 + 3l_9)$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 K^- \eta$	$-\frac{l'_4 + l'_6 + 3l'_8 - 9l'_9}{\sqrt{6}}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \pi^0 \pi^0$	$l_4 + 2l_5 - l_6 + l_8 - 5l_9$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \pi^+ K^-$	$l'_4 - l'_6 - l'_8 + 3l'_9$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \pi^- \pi^+$	$l_4 + 2l_5 - l_6 + l_8 + 3l_9$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{K}^0 \pi^0$	$\frac{-l'_4 + l'_6 + l'_8 + 5l'_9}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{K}^0 K^0$	$l_4 + 2l_5 + l_6 - 3l_8 - l_9$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{K}^0 \eta$	$\frac{-l'_4 + l'_6 + l'_8 + 5l'_9}{\sqrt{6}}$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- K^- K^+$	$2(l_5 + l_8)$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \pi^+ \pi^-$	$2(l'_5 + l'_8)$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \eta \pi^0$	$\frac{-l_4 - l_6 + 5l_8 + l_9}{\sqrt{3}}$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \pi^0 \pi^0$	$2l'_5 + 2l'_8$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \eta \eta$	$\frac{1}{3}(l_4 + 3(2l_5 + l_6 - l_8 + l_9))$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \pi^0 \eta$	$-\frac{2(l'_6 - 2l'_8 + 2l'_9)}{\sqrt{3}}$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \pi^0 K^0$	$\frac{-l_4 + l_6 + l_8 + 5l_9}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- K^+ K^-$	$l'_4 + 2l'_5 - l'_6 + l'_8 + 3l'_9$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- K^+ \pi^-$	$l_4 - l_6 - l_8 + 3l_9$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \bar{K}^0 K^0$	$l'_4 + 2l'_5 + l'_6 - 3l'_8 - l'_9$
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \eta K^0$	$\frac{-l_4 + l_6 + l_8 + 5l_9}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- \eta \eta$	$2(\frac{2l'_4}{3} + l'_5 - l'_8 - 2l'_9)$

TABLE XVII. Amplitudes for Ω_{bbb} decays into a bottom baryon (antitriplet).

Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Lambda_b^0 B^-$	$m_1 + 2m_2$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^-$	$m'_1 + 2m'_2$
$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}_s^0$	$2m_2 - m_1$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}^0$	$m'_1 - 2m'_2$
$\Omega_{bbb}^- \rightarrow \Lambda_b^0 B^- \pi^0$	$\frac{m_3 - m_4 + 2m_5 - m_6 - 5m_8 + 6m_9}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Lambda_b^0 B^- \bar{K}^0$	$m'_4 - m'_6 - m'_7 + m'_8 - 3m'_9$
$\Omega_{bbb}^- \rightarrow \Lambda_b^0 B^- \eta$	$\frac{m_3 + m_4 + 2m_5 - 3m_6 - 2m_7 - 3m_8}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Lambda_b^0 \bar{B}^0 K^-$	$-m'_4 - m'_6 - m'_7 + 3m'_8 - m'_9$
$\Omega_{bbb}^- \rightarrow \Lambda_b^0 \bar{B}^0 \pi^-$	$m_3 - m_4 + 2m_5 - m_6 + 3m_8 - 2m_9$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^- \pi^0$	$\frac{m'_3 + 2m'_5 - 2m'_6 - m'_7 - 4m'_8 + 3m'_9}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Lambda_b^0 \bar{B}_s^0 K^-$	$m_3 + 2m_5 + m_7 - m_9$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^- \eta$	$\frac{m'_3 - 2m'_4 + 2m'_5 + m'_7 - 6m'_8 + 9m'_9}{\sqrt{6}}$
$\Omega_{bbb}^- \rightarrow \Xi_b^0 B^- K^0$	$m_4 - m_6 - m_7 + m_8 - 3m_9$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}^0 \pi^-$	$m'_3 + 2m'_5 + m'_7 - m'_9$
$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 \pi^-$	$-m_4 - m_6 - m_7 + 3m_8 - m_9$	$\Omega_{bbb}^- \rightarrow \Xi_b^0 \bar{B}_s^0 K^-$	$m'_3 - m'_4 + 2m'_5 - m'_6 + 3m'_8 - 2m'_9$
$\Omega_{bbb}^- \rightarrow \Xi_b^- B^- K^+$	$-m_3 + 2m_5 + m_7 - 3m_9$	$\Omega_{bbb}^- \rightarrow \Xi_b^- B^- \pi^+$	$m'_3 - 2m'_5 - m'_7 + 3m'_9$
$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}^0 K^0$	$-m_3 + m_4 + 2m_5 - m_6 + m_8 + 2m_9$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}^0 \pi^0$	$-\frac{m'_3 - 2m'_5 + 2m'_6 + m'_7 + 4m'_8 - m'_9}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}_s^0 \pi^0$	$\frac{m_4 + m_6 + m_7 + 5m_8 + m_9}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}^0 \eta$	$\frac{m'_3 - 2m'_4 - 2m'_5 - m'_7 - 6m'_8 - 3m'_9}{\sqrt{6}}$
$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}_s^0 \eta$	$\frac{2m_3 - m_4 - 4m_5 + 3m_6 + m_7 + 3m_8 - 3m_9}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}_s^0 \bar{K}^0$	$m'_3 - m'_4 - 2m'_5 + m'_6 - m'_8 - 2m'_9$

TABLE XVIII. Amplitudes for Ω_{bbb} decays into a bottom baryon (sextet).

Channel	Amplitude	Channel	Amplitude
$\Omega_{bbb}^- \rightarrow \Sigma_b^0 B^-$	$\frac{n_1 + 6n_2}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime 0} B^-$	$\frac{n'_1 + 6n'_2}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Sigma_b^- \bar{B}^0$	$n_1 - 2n_2$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} \bar{B}^0$	$\frac{n'_1 - 2n'_2}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} \bar{B}_s^0$	$\frac{n_1 - 2n_2}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Omega_b^- \bar{B}_s^0$	$n'_1 - 2n'_2$
$\Omega_{bbb}^- \rightarrow \Sigma_b^+ B^- \pi^-$	$n_4 + 3n_6 + 3n_7 + n_8 + n_9$	$\Omega_{bbb}^- \rightarrow \Sigma_b^+ B^- K^-$	$n'_4 + 3n'_6 + 3n'_7 + n'_8 + n'_9$
$\Omega_{bbb}^- \rightarrow \Sigma_b^0 B^- \pi^0$	$\frac{1}{2}(n_3 - n_4 + 6n_5 + 5n_6 - n_8 - 2n_9)$	$\Omega_{bbb}^- \rightarrow \Sigma_b^0 B^- \bar{K}^0$	$\frac{n'_4 - n'_6 + 3n'_7 - n'_8 + n'_9}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Sigma_b^0 B^- \eta$	$\frac{n_3 + n_4 + 6n_5 + 3n_6 + 6n_7 - 3n_8}{2\sqrt{3}}$	$\Omega_{bbb}^- \rightarrow \Sigma_b^0 \bar{B}^0 K^-$	$\frac{n'_4 + 3n'_6 - n'_7 + n'_8 - n'_9}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Sigma_b^0 \bar{B}^0 \pi^-$	$\frac{n_3 + n_4 + 6n_5 + 3n_6 - 2n_7 + n_8}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Sigma_b^- \bar{B}^0 \bar{K}^0$	$n'_4 - n'_6 - n'_7 - n'_8 - n'_9$
$\Omega_{bbb}^- \rightarrow \Sigma_b^0 \bar{B}_s^0 K^-$	$\frac{n_3 + 6n_5 - n_7 + n_9}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime 0} B^- \pi^0$	$\frac{1}{2}(n'_3 + 6n'_5 + 4n'_6 + 3n'_7 - 2n'_8 - n'_9)$
$\Omega_{bbb}^- \rightarrow \Sigma_b^- B^- \pi^+$	$n_3 - 2n_5 + 3n_7 - n_9$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime 0} B^- \eta$	$\frac{n'_3 - 2n'_4 + 6n'_5 + 6n'_6 - 3n'_7 - 3n'_9}{2\sqrt{3}}$
$\Omega_{bbb}^- \rightarrow \Sigma_b^- \bar{B}^0 \pi^0$	$-\frac{n_3 + n_4 - 2n_5 - 5n_6 - 2n_7 + n_8}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime 0} \bar{B}^0 \pi^-$	$\frac{n'_3 + 6n'_5 - n'_7 + n'_9}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Sigma_b^- \bar{B}^0 \eta$	$\frac{n_3 + n_4 - 2n_5 + 3n_6 - 2n_7 - 3n_8}{\sqrt{6}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime 0} \bar{B}_s^0 K^-$	$\frac{n'_3 + n'_4 + 6n'_5 + 3n'_6 - 2n'_7 + n'_8}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Sigma_b^- \bar{B}_s^0 \bar{K}^0$	$n_3 - 2n_5 - n_7 + n_9$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} B^- \pi^+$	$\frac{n'_3 - 2n'_5 + 3n'_7 - n'_9}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime 0} B^- K^0$	$\frac{n_4 - n_6 + 3n_7 - n_8 + n_9}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} \bar{B}^0 \pi^0$	$\frac{1}{2}(-n'_3 + 2n'_5 + 4n'_6 + n'_7 - 2n'_8 - n'_9)$
$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime 0} \bar{B}_s^0 \pi^-$	$\frac{n_4 + 3n_6 - n_7 + n_8 - n_9}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} \bar{B}^0 \eta$	$\frac{n'_3 - 2n'_4 - 2n'_5 + 6n'_6 + n'_7 + 3n'_9}{2\sqrt{3}}$
$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} B^- K^+$	$\frac{n_3 - 2n_5 + 3n_7 - n_9}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} \bar{B}_s^0 \bar{K}^0$	$\frac{n'_3 + n'_4 - 2n'_5 - n'_6 - 2n'_7 - n'_8}{\sqrt{2}}$
$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} \bar{B}^0 K^0$	$\frac{n_3 + n_4 - 2n_5 - n_6 - 2n_7 - n_8}{\sqrt{2}}$	$\Omega_{bbb}^- \rightarrow \Omega_b^- B^- K^+$	$n'_3 - 2n'_5 + 3n'_7 - n'_9$
$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} \bar{B}_s^0 \pi^0$	$\frac{1}{2}(-n_4 + 5n_6 + n_7 - n_8 + n_9)$	$\Omega_{bbb}^- \rightarrow \Omega_b^- \bar{B}^0 K^0$	$n'_3 - 2n'_5 - n'_7 + n'_9$
$\Omega_{bbb}^- \rightarrow \Xi_b^{\prime -} \bar{B}_s^0 \eta$	$\frac{-2n_3 + n_4 + 4n_5 + 3n_6 + n_7 - 3n_8 - 3n_9}{2\sqrt{3}}$	$\Omega_{bbb}^- \rightarrow \Omega_b^- \bar{B}_s^0 \pi^0$	$\sqrt{2}(2n'_6 - n'_8)$
$\Omega_{bbb}^- \rightarrow \Omega_b^- \bar{B}_s^0 K^0$	$n_4 - n_6 - n_7 - n_8 - n_9$	$\Omega_{bbb}^- \rightarrow \Omega_b^- \bar{B}_s^0 \eta$	$-\sqrt{\frac{2}{3}}(n'_3 + n'_4 - 2n'_5 - 3n'_6 - 2n'_7)$

FIG. 5. Feynman diagrams for W exchange. The spectator is a bottom or a charm quark. If the final u quark is replaced by a charm quark, the W-exchange contribution is an SU(3) triplet, and this triplet contribution is been incorporated in the $b \rightarrow q_1 \bar{q}_2 q_3$.

Since the CP -violating quantity is related to the CP asymmetry $\Delta = A_{CP}\Gamma$, there exists a relation between the CP asymmetry and decay widths [29,32,60]:

$$\frac{A_{CP}(\Delta S = 0)}{A_{CP}(\Delta S = 1)} = -r^2 \frac{\Gamma(\Delta S = 1)}{\Gamma(\Delta S = 0)}. \quad (85)$$

Such channel pairs include $[\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 \pi^-, \Omega_{bbb}^- \rightarrow \Xi_{bb}^0 K^-]$, $[\Omega_{bbb}^- \rightarrow \Omega_{bb}^- K^0, \Omega_{bbb}^- \rightarrow \Omega_{bb}^- \bar{K}^0]$, etc. The future experimental data will be valuable to test flavor SU(3) symmetry and the CKM mechanism for CP violation.

2. Decays into a bottom meson and a bottom baryon bqq

If the bottom baryon is an antitriplet, we have the effective Hamiltonian for two-body and three-body decays:

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & m_1 \Omega_{bbb}(\bar{T}_{\mathbf{b}\bar{3}})_{[ij]} \bar{B}^i(H_3)^j + m_2 \Omega_{bbb}(\bar{T}_{\mathbf{b}\bar{3}})_{[ij]} \bar{B}^k(H_6)^{ij} + m_3 \Omega_{bbb}(\bar{T}_{\mathbf{b}\bar{3}})_{[ij]} \bar{B}^k(M_8)_k^i(H_3)^j \\ & + m_4 \Omega_{bbb}(\bar{T}_{\mathbf{b}\bar{3}})_{[ij]} \bar{B}^i(M_8)_k^j(H_3)^k + m_5 \Omega_{bbb}(\bar{T}_{\mathbf{b}\bar{3}})_{[ij]} \bar{B}^l(M_8)_l^k(H_6)_k^{ij} + m_6 \Omega_{bbb}(\bar{T}_{\mathbf{b}\bar{3}})_{[ij]} \bar{B}^i(M_8)_l^k(H_6)_k^{jl} \\ & + m_7 \Omega_{bbb}(\bar{T}_{\mathbf{b}\bar{3}})_{[ij]} \bar{B}^k(M_8)_l^i(H_6)_k^{jl} + m_8 \Omega_{bbb}(\bar{T}_{\mathbf{b}\bar{3}})_{[ij]} \bar{B}^j(M_8)_k^l(H_{15})_l^{ik} + m_9 \Omega_{bbb}(\bar{T}_{\mathbf{b}\bar{3}})_{[ij]} \bar{B}^l(M_8)_k^i(H_{15})_l^{jk}. \end{aligned} \quad (86)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XVII.

In the case of a sextet, we have

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & n_1 \Omega_{bbb}(\bar{T}_{\mathbf{b6}})_{\{ij\}} \bar{B}^i(H_3)^j + n_2 \Omega_{bbb}(\bar{T}_{\mathbf{b6}})_{\{ij\}} \bar{B}^k(H_{15})_k^{ij} + n_3 \Omega_{bbb}(\bar{T}_{\mathbf{b6}})_{\{ij\}} \bar{B}^k(M_8)_k^i(H_3)^j \\ & + n_4 \Omega_{bbb}(\bar{T}_{\mathbf{b6}})_{\{ij\}} \bar{B}^i(M_8)_k^j(H_3)^k + n_5 \Omega_{bbb}(\bar{T}_{\mathbf{b6}})_{\{ij\}} \bar{B}^l(M_8)_l^k(H_{15})_k^{ij} + n_6 \Omega_{bbb}(\bar{T}_{\mathbf{b6}})_{\{ij\}} \bar{B}^i(M_8)_l^k(H_{15})_k^{jl} \\ & + n_7 \Omega_{bbb}(\bar{T}_{\mathbf{b6}})_{\{ij\}} \bar{B}^k(M_8)_l^i(H_{15})_k^{jl} + n_8 \Omega_{bbb}(\bar{T}_{\mathbf{b6}})_{\{ij\}} \bar{B}^j(M_8)_k^l(H_6)_l^{ik} + n_9 \Omega_{bbb}(\bar{T}_{\mathbf{b6}})_{\{ij\}} \bar{B}^l(M_8)_k^i(H_6)_l^{jk}. \end{aligned} \quad (87)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XVIII.

VII. NONLEPTONIC Ω_{ccb} AND Ω_{cbb} DECAYS

For the mixed triply heavy baryons, Ω_{ccb} and Ω_{cbb} , most of their weak decays can be obtained from the Ω_{ccc} and Ω_{bbb} decay channels with some replacements. For instance, decays of Ω_{ccb} induced by the charm quark can be obtained from the ones of Ω_{ccc} by replacing one charmed meson by the corresponding bottom meson, a charmed baryon by the corresponding bottom baryon, or a doubly charmed baryon H_{cc} by its counterpart H_{bc} .

In addition, there are new decay modes, which are induced by W-exchange transition shown in Fig. 5,¹ $bc \rightarrow ud$ or $bc \rightarrow us$, with two heavy quarks annihilating into two light quarks. The spectator quark is a bottom or charm quark. These diagrams are dynamically suppressed by factors of $1/m_{b,c}$. The electroweak Hamiltonian is similar to Eq. (73). Taking Ω_{cbb} as the example, one should notice that the final state contains only one heavy bottom quark. Thus, at the hadron level, the final state can be a bottom baryon (antitriplet) that has

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & a_1 \Omega_{cbb}(\bar{T}_{b\bar{3}})_{[ik]} (M_8)_j^k (H_3'')^{[ij]} + a_2 \Omega_{cbb}(\bar{T}_{b\bar{3}})_{[ij]} (M_8)_l^k (M_8)_k^l (H_3'')^{[ij]} + a_3 \Omega_{cbb}(\bar{T}_{b\bar{3}})_{[ik]} (M_8)_l^k (M_8)_j^l (H_3'')^{[ij]} \\ & + a_4 \Omega_{cbb}(\bar{T}_{b\bar{3}})_{[kl]} (M_8)_i^k (M_8)_j^l (H_3'')^{[ij]} + a_5 \Omega_{cbb}(\bar{T}_{b\bar{3}})_{[ik]} (M_8)_j^k (H_6'')^{ij} + a_6 \Omega_{cbb}(\bar{T}_{b\bar{3}})_{[ik]} (M_8)_l^k (M_8)_j^l (H_6'')^{ij}. \end{aligned} \quad (88)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XIX.

¹If the final u quark is replaced by a charm quark, the W-exchange contribution is an SU(3) triplet, and this triplet contribution is incorporated in the $b \rightarrow q_1 \bar{q}_2 q_3$. In this case, the CKM matrix element is $V_{cb} V_{cd/cs}^*$.

TABLE XIX. Amplitudes for Ω_{cbb} decays into a singly bottom baryon (triplet).

Channel	Amplitude	Channel	Amplitude
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 \pi^0$	$-\sqrt{2}a_5 V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 K^0$	$(a_1 + a_5)V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 \bar{K}^0$	$(a_1 + a_5)V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 \eta$	$-\frac{(a_1+3a_5)V_{cs}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 \eta$	$\sqrt{\frac{2}{3}}a_1 V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^- \pi^+$	$(a_1 - a_5)V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Xi_b^0 \pi^0$	$\frac{(a_1-a_5)V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^- K^+$	$(a_5 - a_1)V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 \pi^0 \pi^0$	$2(2a_2 + a_3 - a_4)V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 K^+ \pi^-$	$(a_3 - 2a_4 + a_6)V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 \pi^0 \bar{K}^0$	$-\frac{(a_3-2a_4+a_6)V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 K^0 \pi^0$	$-\frac{(a_3-2a_4+a_6)V_{cd}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 \pi^- \pi^+$	$2(2a_2 + a_3 - a_4)V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 \bar{K}^0 K^0$	$(4a_2 + a_3 + a_6)V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 K^+ K^-$	$(4a_2 + a_3 - a_6)V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 K^- K^+$	$2(2a_2 + a_3 - a_4)V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 K^0 \bar{K}^0$	$(4a_2 + a_3 + a_6)V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 \eta K^0$	$-\frac{(a_3-2a_4+a_6)V_{cd}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 \bar{K}^0 \eta$	$-\frac{(a_3-2a_4+a_6)V_{cs}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^0 \eta \eta$	$\frac{1}{3}(12a_2 + 5a_3 - 4a_4 + 3a_6)V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 K^- \pi^+$	$(a_3 - 2a_4 + a_6)V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^- \pi^+ K^0$	$(-a_3 + 2a_4 + a_6)V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 \eta \pi^0$	$-\frac{2a_6 V_{cd}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^- \pi^+ \eta$	$\sqrt{\frac{2}{3}}(a_3 - 2a_4 - a_6)V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Lambda_b^0 \eta \eta$	$\frac{2}{3}(6a_2 + a_3 + a_4)V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^- K^+ \pi^0$	$\frac{(-a_3+2a_4+a_6)V_{cd}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Xi_b^0 \pi^+ \pi^-$	$(4a_2 + a_3 - a_6)V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^- K^+ \eta$	$\frac{(a_3-2a_4-a_6)V_{cd}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow \Xi_b^0 \pi^0 \pi^0$	$(4a_2 + a_3 - a_6)V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^- \bar{K}^0 K^+$	$(a_3 - 2a_4 - a_6)V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Xi_b^0 \pi^0 \eta$	$\frac{(a_3-2a_4-a_6)V_{cs}^*}{\sqrt{3}}$		

The final state can be a bottom baryon (sextet) that has

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & b_1 \Omega_{cbb}(\bar{T}_{b6})_{\{ik\}}(M_8)_j^k(H_3'')^{[ij]} + b_2 \Omega_{cbb}(\bar{T}_{b6})_{\{ik\}}(M_8)_l^k(M_8)_j^l(H_3'')^{[ij]} + b_3 \Omega_{cbb}(\bar{T}_{b6})_{\{ik\}}(M_8)_j^k(H_6'')^{\{ij\}} \\ & + b_5 \Omega_{cbb}(\bar{T}_{b6})_{\{ij\}}(M_8)_l^k(M_8)_k^l(H_6'')^{\{ij\}} + b_6 \Omega_{cbb}(\bar{T}_{b6})_{\{ik\}}(M_8)_l^k(M_8)_j^l(H_6'')^{\{ij\}} + b_7 \Omega_{cbb}(\bar{T}_{b6})_{\{kl\}}(M_8)_i^k M_j^l(H_6'')^{\{ij\}}. \end{aligned} \quad (89)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XX.

If the final state contains a bottom meson and a light baryon (octet), we have

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & c_1 \Omega_{cbb} \bar{B}^k \epsilon_{ijl}(\bar{T}_8)_k^l(H_3'')^{[ij]} + c_2 \Omega_{cbb} \bar{B}^i \epsilon_{ijl}(\bar{T}_8)_k^l(H_3'')^{[jk]} + c_3 \Omega_{cbb} \bar{B}^m \epsilon_{ijl}(\bar{T}_8)_k^l M_m^k(H_3'')^{[ij]} \\ & + c_4 \Omega_{cbb} \bar{B}^j \epsilon_{ijl}(\bar{T}_8)_k^l M_m^k(H_3'')^{[im]} + c_5 \Omega_{cbb} \bar{B}^k \epsilon_{ijl}(\bar{T}_8)_k^l M_m^j(H_3'')^{[im]} + c_6 \Omega_{cbb} \bar{B}^m \epsilon_{ijl}(\bar{T}_8)_k^l M_m^j(H_3'')^{[ik]} \\ & + c_7 \Omega_{cbb} \bar{B}^i \epsilon_{ijl}(\bar{T}_8)_k^l M_m^j(H_3'')^{[km]} + c_8 \Omega_{cbb} \bar{B}^i \epsilon_{ijl}(\bar{T}_8)_k^l(H_6'')^{\{jk\}} + c_9 \Omega_{cbb} \bar{B}^j \epsilon_{ijl}(\bar{T}_8)_k^l M_m^k(H_6'')^{\{im\}} \\ & + c_{10} \Omega_{cbb} \bar{B}^k \epsilon_{ijl}(\bar{T}_8)_k^l M_m^j(H_6'')^{\{im\}} + c_{11} \Omega_{cbb} \bar{B}^m \epsilon_{ijl}(\bar{T}_8)_k^l M_m^j(H_6'')^{\{ik\}} + c_{12} \Omega_{cbb} \bar{B}^i \epsilon_{ijl}(\bar{T}_8)_k^l M_m^j(H_6'')^{\{km\}}. \end{aligned} \quad (90)$$

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XXI. One can find the amplitudes c_1 and c_2 are not independent; they always appear in the product $2c_1 - c_2$. So, in Table XXI, we do not show c_2 .

If the final state contains a bottom meson and a light baryon (decuplet), we have

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & d_1 \Omega_{cbb} \bar{B}^j(\bar{T}_{10})_{ijk} M_m^k(H_3'')^{[im]} + d_2 \Omega_{cbb} \bar{B}^k(\bar{T}_{10})_{ijk} M_m^j(H_3'')^{[im]} + d_3 \Omega_{cbb} \bar{B}^i(\bar{T}_{10})_{ijk} M_m^j(H_3'')^{[km]} \\ & + d_4 \Omega_{cbb} \bar{B}^k(\bar{T}_{10})_{ijk}(H_6'')^{\{ij\}} + d_5 \Omega_{cbb} \bar{B}^i(\bar{T}_{10})_{ijk}(H_6'')^{\{jk\}} + d_6 \Omega_{cbb} \bar{B}^m(\bar{T}_{10})_{ijk} M_m^k(H_6'')^{\{ij\}} \\ & + d_7 \Omega_{cbb} \bar{B}^j(\bar{T}_{10})_{ijk} M_m^k(H_6'')^{\{im\}} + d_8 \Omega_{cbb} \bar{B}^k(\bar{T}_{10})_{ijk} M_m^j(H_6'')^{\{im\}} \\ & + d_9 \Omega_{cbb} \bar{B}^m(\bar{T}_{10})_{ijk} M_m^j(H_6'')^{\{ik\}} + d_{10} \Omega_{cbb} \bar{B}^i(\bar{T}_{10})_{ijk} M_m^j(H_6'')^{\{km\}}. \end{aligned} \quad (91)$$

TABLE XX. Amplitudes for Ω_{cbb} decays into a singly bottom baryon (sextet).

Channel	Amplitude	Channel	Amplitude
$\Omega_{cbb}^0 \rightarrow \Sigma_b^+ \pi^-$	$(b_1 + b_4) V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} \pi^0$	$\frac{1}{2} (b_4 - b_1) V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^+ K^-$	$(b_1 + b_4) V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} K^0$	$\frac{(b_1 + b_4) V_{cd}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \pi^0$	$-b_1 V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} \eta$	$-\frac{(3b_1 + b_4) V_{cs}^*}{2\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \bar{K}^0$	$\frac{(b_1 + b_4) V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime -} \pi^+$	$\frac{(b_4 - b_1) V_{cs}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \eta$	$\frac{b_4 V_{cd}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime -} K^+$	$\frac{(b_4 - b_1) V_{cd}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^- \pi^+$	$(b_4 - b_1) V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Omega_b^- K^+$	$(b_4 - b_1) V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^- \pi^+ \bar{K}^0$	$2b_7 V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Sigma_b^- \bar{K}^0 K^+$	$(b_6 - b_2) V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^+ \pi^0 K^-$	$\frac{(b_2 + b_6 + 2b_7) V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Sigma_b^- \eta \pi^+$	$\sqrt{\frac{2}{3}} (-b_2 + b_6 + b_7) V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^+ \pi^- \pi^0$	$\sqrt{2} b_7 V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} \pi^+ \pi^-$	$\frac{(-b_2 + 4b_5 + b_6) V_{cs}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^+ \pi^- \bar{K}^0$	$(b_2 + b_6) V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} \pi^0 \pi^0$	$\frac{(-b_2 + 4b_5 + b_6) V_{cs}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^+ \pi^- \eta$	$\sqrt{\frac{2}{3}} (b_2 + b_6 + b_7) V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} \pi^0 K^0$	$-\frac{1}{2} (b_2 + b_6 - 2b_7) V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^+ K^0 K^-$	$(b_2 + b_6) V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} \pi^0 \eta$	$\frac{(-b_2 + b_6 - 2b_7) V_{cs}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \eta \eta$	$\frac{2(6b_5 + b_6 + b_7) V_{cd}^*}{3\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} K^+ \pi^-$	$\frac{(b_2 + b_6 + 2b_7) V_{cd}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^+ K^- \eta$	$-\frac{(b_2 + b_6 - 2b_7) V_{cs}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} K^+ K^-$	$\sqrt{2} (2b_5 + b_6 + b_7) V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Omega_b^- K^+ \eta$	$\frac{(b_2 - b_6 - 4b_7) V_{cs}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} K^0 \bar{K}^0$	$\frac{(b_2 + 4b_5 + b_6) V_{cs}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \pi^+ \pi^-$	$\sqrt{2} (2b_5 + b_6 + b_7) V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} K^0 \eta$	$-\frac{(b_2 + b_6 - 2b_7) V_{cd}^*}{2\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \pi^+ K^-$	$\frac{(b_2 + b_6 + 2b_7) V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Omega_b^- K^+ \pi^0$	$\frac{(b_6 - b_2) V_{cs}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \pi^0 \pi^0$	$\frac{2(2b_5 + b_6 - b_7) V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Omega_b^- \pi^+ K^0$	$(b_6 - b_2) V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \pi^0 \bar{K}^0$	$-\frac{1}{2} (b_2 + b_6 - 2b_7) V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime -} K^+ \eta$	$\frac{(b_2 - b_6 + 2b_7) V_{cd}^*}{2\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \pi^0 \eta$	$-\sqrt{\frac{2}{3}} b_2 V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime -} K^+ \bar{K}^0$	$\frac{(-b_2 + b_6 + 2b_7) V_{cs}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Omega_b^- K^+ K^0$	$2b_7 V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime 0} \eta \eta$	$\frac{(3b_2 + 12b_5 + 5b_6 - 4b_7) V_{cs}^*}{3\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 K^+ K^-$	$\frac{(-b_2 + 4b_5 + b_6) V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime -} \pi^+ K^0$	$\frac{(-b_2 + b_6 + 2b_7) V_{cd}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 K^0 \bar{K}^0$	$\frac{(b_2 + 4b_5 + b_6) V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime -} \pi^+ \eta$	$\frac{(-b_2 + b_6 - 2b_7) V_{cs}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^- \pi^+ \pi^0$	$-\sqrt{2} b_7 V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \Xi_b^{\prime -} K^+ \pi^0$	$-\frac{1}{2} (b_2 - b_6 + 2b_7) V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \Sigma_b^0 \bar{K}^0 \eta$	$-\frac{(b_2 + b_6 - 2b_7) V_{cs}^*}{2\sqrt{3}}$		

Decay amplitudes for different channels are obtained by expanding the above Hamiltonian and are collected in Table XXII. It is interesting to notice that the above amplitudes d_i are not all independent. The combinations of d_i s, $d_1 + d_2 + d_3$, $d_4 + d_5$, $d_6 + d_9$, and $d_7 + d_8 + d_9$ always appear in the amplitudes; thus, we have removed the d_2 , d_3 , d_5 , d_9 , d_8 and d_{10} in Table XXII.

VIII. GOLDEN CHANNELS

Based on the above analysis, we first give a collection of the CKM-allowed decay channels for the Ω_{ccc}^{++} in

Table XXIII and for the Ω_{bbb} in Table XXIV. The ones for Ω_{ccb} and Ω_{cbb} can be obtained by the replacements as discussed in the above section:

- (i) The light pseudoscalar meson in these two tables can be replaced by its vector counterpart. For instance, a $\bar{K}^0$ can be replaced by a $\bar{K}^{*0}$ decaying into $K^- \pi^+$.
- (ii) Branching fractions for semileptonic Ω_{ccc} decay channels in Table XXIII can reach a few percent, but there is a neutrino in the final state, reducing somewhat the experimental efficiency.
- (iii) Nonleptonic Ω_{ccc} such as $\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^- \pi^+$ might be used to search for Ω_{ccc} especially at the LHC,

TABLE XXI. Amplitudes for Ω_{cbb} decays into a bottom meson and a light baryon (octet).

Channel	Amplitude	Channel	Amplitude
$\Omega_{cbb}^0 \rightarrow B^-\Sigma^+$	$(-2c_1 - c_8)V_{ub}V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Lambda^0$	$\sqrt{\frac{2}{3}}(-2c_1)V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Lambda^0$	$\frac{(-2c_1+3c_8)V_{ub}V_{cs}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Sigma^0$	$-\sqrt{2}c_8V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Sigma^0$	$\frac{(2c_1+c_8)V_{ub}V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Xi^0$	$(-2c_1 + c_8)V_{ub}V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0n$	$(2c_1 - c_8)V_{ub}V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow B^-p$	$(2c_1 + c_8)V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow B^-\Lambda^0\pi^+$	$-\frac{(2c_3+c_4-c_5+c_6-2c_7-c_9+c_{10}+3c_{11}+2c_{12})V_{ub}V_{cs}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0n\bar{K}^0$	$(c_4 + c_5 + c_9 + c_{10})V_{ub}V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow B^-\Lambda^0K^+$	$-\frac{(4c_3+2c_4+c_5+2c_6-c_7-2c_9-c_{10}+c_{11})V_{ub}V_{cd}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0n\eta$	$\frac{(2c_3+c_4+2c_5+c_6+c_7+c_9+c_{11}-c_{12})V_{ub}V_{cd}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow B^-\Sigma^+\pi^0$	$\frac{(-2c_3-c_4-c_5-c_6+c_9+c_{10}+c_{11})V_{ub}V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Xi^-K^+$	$(c_4 - c_7 - c_9 + c_{12})V_{ub}V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow B^-\Sigma^+K^0$	$-(c_5 + c_7 + c_{10} + c_{12})V_{ub}V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Xi^0K^0$	$-(2c_3 + c_6 + c_{11})V_{ub}V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow B^-\Sigma^+\eta$	$\frac{(-2c_3-c_4+c_5-c_6+2c_7+c_9+3c_{10}+c_{11}+2c_{12})V_{ub}V_{cs}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Lambda^0\pi^0$	$\frac{(c_9+2c_{10}+c_{12})V_{ub}V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^-\Sigma^0\pi^+$	$\frac{(2c_3+c_4+c_5+c_6-c_9-c_{10}-c_{11})V_{ub}V_{cs}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Lambda^0\bar{K}^0$	$-\frac{(2c_3+c_4+2c_5+c_6+c_7+c_9+2c_{10}+3c_{11}+c_{12})V_{ub}V_{cs}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow B^-\Sigma^0K^+$	$\frac{(-c_5-c_7+c_{10}+2c_{11}+c_{12})V_{ub}V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Lambda^0\eta$	$\frac{1}{3}(4c_3 - c_4 - 2c_5 + 2c_6 - c_7)V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow B^-n\pi^+$	$(2c_3 + c_4 + c_6 - c_7 - c_9 + c_{11} + c_{12})V_{ub}V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Sigma^+\pi^-$	$(-c_4 + c_7 - c_9 + c_{12})V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow B^-\Xi^0K^+$	$-(2c_3 + c_4 + c_6 - c_7 - c_9 + c_{11} + c_{12})V_{ub}V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Sigma^+K^-$	$(-2c_3 - c_4 - c_6 + c_7 - c_9 + c_{11} + c_{12})V_{ub}V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Lambda^0\pi^0$	$\frac{(2c_3+c_4-c_5+c_6-2c_7-c_9+c_{10}+3c_{11}+2c_{12})V_{ub}V_{cs}^*}{2\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Sigma^0\pi^0$	$(c_7 - c_4)V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Lambda^0K^0$	$-\frac{(4c_3+2c_4+c_5+2c_6-c_7+2c_9+c_{10}-c_{12})V_{ub}V_{cd}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Sigma^0\bar{K}^0$	$\frac{(2c_3+c_4+c_6-c_7+c_9-c_{11}-c_{12})V_{ub}V_{cs}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Lambda^0\eta$	$\frac{1}{6}(-2c_3 + 5c_4 + c_5 - c_6 - 4c_7 + 3c_9 + 3c_{10} - 3c_{11})V_{ub}V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Sigma^0\eta$	$\frac{(c_9-2c_{11}-c_{12})V_{ub}V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Sigma^+\pi^-$	$(-2c_3 - c_6 + c_{11})V_{ub}V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Sigma^-\pi^+$	$(-c_4 + c_7 + c_9 - c_{12})V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Sigma^0\pi^0$	$-\frac{1}{2}(2c_3 - c_4 - c_5 + c_6 + c_9 + c_{10} - c_{11})V_{ub}V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0n\bar{K}^0$	$(2c_3 + c_6 + c_{11})V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Sigma^0K^0$	$\frac{(c_5+c_7+c_{10}+2c_{11}+c_{12})V_{ub}V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Xi^-\pi^+$	$(c_5 + c_7 - c_{10} - c_{12})V_{ub}V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Sigma^0\eta$	$\frac{(2c_3+c_4-c_5+c_6-2c_7-c_9-3c_{10}-c_{11}-2c_{12})V_{ub}V_{cs}^*}{2\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Xi^-K^+$	$(-c_4 - c_5 + c_9 + c_{10})V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Sigma^-\pi^+$	$(c_4 + c_5 - c_9 - c_{10})V_{ub}V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Xi^0\pi^0$	$\frac{(-c_5-c_7+c_{10}+c_{12})V_{ub}V_{cs}^*}{\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0\Sigma^-K^+$	$(-c_5 - c_7 + c_{10} + c_{12})V_{ub}V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Xi^0K^0$	$-(c_4 + c_5 + c_9 + c_{10})V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0n\pi^0$	$-\frac{(2c_3+c_4+c_6-c_7+c_9+2c_{10}+c_{11}+c_{12})V_{ub}V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0\Xi^0\eta$	$\frac{(4c_3+2c_4+c_5+2c_6-c_7+2c_9+3c_{10}+2c_{11}+c_{12})V_{ub}V_{cs}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow B^-p\pi^0$	$\frac{(2c_3+c_4+c_6-c_7-c_9-2c_{10}-c_{11}-c_{12})V_{ub}V_{cd}^*}{\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow B^-p\bar{K}^0$	$(c_5 + c_7 + c_{10} + c_{12})V_{ub}V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow B^-p\eta$	$\frac{(2c_3+c_4+2c_5+c_6+c_7-c_9-c_{11}+c_{12})V_{ub}V_{cd}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0p\pi^-$	$(2c_3 + c_4 + c_6 - c_7 + c_9 - c_{11} - c_{12})V_{ub}V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0pK^-$	$(c_4 - c_7 + c_9 - c_{12})V_{ub}V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0pK^-$	$(2c_3 + c_6 - c_{11})V_{ub}V_{cd}^*$

since their branching fractions are sizable, and the final state can be easy to identify. This will make use of the doubly heavy baryon Ξ_{cc}^{++} , which has just been discovered by LHCb.

- (iv) For nonleptonic decays of Ω_{bbb}^- , the largest branching fraction might reach 10^{-3} . Taking into account its daughter decays, we expect that the branching fraction for Ω_{bbb}^- decaying into charmless final state is at most 10^{-9} . Thus, the triply bottom baryon can be only observed with a large amount of data in the future, such as the high luminosity LHC.

IX. DISCUSSIONS ON SU(3) SYMMETRY-BREAKING EFFECTS

Our analysis of weak decays of triply heavy baryons heavily relies on the flavor SU(3) symmetry for light u , d , and s quarks. At this stage, the SU(3) symmetry is helpful in figuring out the most promising decay channels toward the discovery. After the establishment of these states on the experimental side, it is valuable to use future experimental data to explore the flavor symmetry and the symmetry-breaking effects.

TABLE XXII. Amplitudes for Ω_{cbb} decays into a bottom meson and a light baryon (decuplet).

Channel	Amplitude	Channel	Amplitude
$\Omega_{cbb}^0 \rightarrow B^- \Delta^+$	$\frac{2d_4 V_{ub} V_{cd}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Sigma'^0$	$\sqrt{\frac{2}{3}} d_4 V_{ub} V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow B^- \Sigma'^+$	$\frac{2d_4 V_{ub} V_{cs}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Sigma'^0$	$\sqrt{\frac{2}{3}} d_4 V_{ub} V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Delta^0$	$\frac{2d_4 V_{ub} V_{cd}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Xi'^0$	$\frac{2d_4 V_{ub} V_{cs}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Delta^{++} \pi^-$	$(d_1 + d_7) V_{ub} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Sigma'^0 K^0$	$\frac{(d_1 + 2d_6 + d_7) V_{ub} V_{cd}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow B^- \Delta^{++} K^-$	$(d_1 + d_7) V_{ub} V_{cs}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Sigma'^0 \eta$	$-\frac{1}{6} (3d_1 - 2d_6 + d_7) V_{ub} V_{cs}^*$
$\Omega_{cbb}^0 \rightarrow B^- \Delta^+ \pi^0$	$-\sqrt{\frac{2}{3}} (d_1 - d_6) V_{ub} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Sigma'^- \pi^+$	$\frac{(-d_1 + d_7) V_{ub} V_{cs}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Delta^+ \bar{K}^0$	$\frac{(d_1 + d_7) V_{ub} V_{cs}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Sigma'^- K^+$	$\frac{(-d_1 + d_7) V_{ub} V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Delta^+ \eta$	$\frac{1}{3} \sqrt{2} (d_6 + d_7) V_{ub} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Xi'^0 K^0$	$\frac{2d_6 V_{ub} V_{cs}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Delta^0 \pi^+$	$\frac{(-d_1 + 2d_6 + d_7) V_{ub} V_{cd}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Xi'^- K^+$	$\frac{(-d_1 + d_7) V_{ub} V_{cs}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Sigma'^+ \pi^0$	$\frac{(-d_1 + 2d_6 + d_7) V_{ub} V_{cs}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Delta^+ K^-$	$\frac{2d_6 V_{ub} V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Sigma'^+ K^0$	$\frac{(d_1 + d_7) V_{ub} V_{cd}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Delta^0 \bar{K}^0$	$\frac{2d_6 V_{ub} V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Sigma'^+ \eta$	$-\frac{(3d_1 - 2d_6 + d_7) V_{ub} V_{cs}^*}{3\sqrt{2}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Sigma'^+ \pi^-$	$\frac{(d_1 + d_7) V_{ub} V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Sigma'^0 \pi^+$	$\frac{(-d_1 + 2d_6 + d_7) V_{ub} V_{cs}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Sigma'^+ K^-$	$\frac{(d_1 + 2d_6 + d_7) V_{ub} V_{cs}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Sigma'^0 K^+$	$\frac{(-d_1 + 2d_6 + d_7) V_{ub} V_{cd}^*}{\sqrt{6}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Sigma'^0 \pi^0$	$-\frac{d_1 V_{ub} V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow B^- \Xi'^0 K^+$	$\frac{(-d_1 + 2d_6 + d_7) V_{ub} V_{cs}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Sigma'^0 \bar{K}^0$	$\frac{(d_1 + 2d_6 + d_7) V_{ub} V_{cs}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Delta^+ \pi^-$	$\frac{(d_1 + 2d_6 + d_7) V_{ub} V_{cd}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Sigma'^0 \eta$	$\frac{1}{3} (-2d_6 + d_7) V_{ub} V_{cd}^*$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Delta^+ K^-$	$\frac{(d_1 + d_7) V_{ub} V_{cs}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Sigma'^- \pi^+$	$\frac{(-d_1 + d_7) V_{ub} V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Delta^0 \pi^0$	$-\sqrt{\frac{2}{3}} (d_1 + d_6) V_{ub} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Xi'^0 \pi^0$	$\frac{(-d_1 + d_7) V_{ub} V_{cs}^*}{\sqrt{6}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Delta^0 \bar{K}^0$	$\frac{(d_1 + d_7) V_{ub} V_{cs}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Xi'^0 K^0$	$\frac{(d_1 + d_7) V_{ub} V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Delta^0 \eta$	$\frac{1}{3} \sqrt{2} (d_6 + d_7) V_{ub} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Xi'^0 \eta$	$-\frac{(3d_1 + 4d_6 + d_7) V_{ub} V_{cs}^*}{3\sqrt{2}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Delta^- \pi^+$	$(-d_1 + d_7) V_{ub} V_{cd}^*$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Xi'^- \pi^+$	$\frac{(-d_1 + d_7) V_{ub} V_{cs}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Sigma'^+ \pi^-$	$\frac{2d_6 V_{ub} V_{cs}^*}{\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Xi'^- K^+$	$\frac{(-d_1 + d_7) V_{ub} V_{cd}^*}{\sqrt{3}}$
$\Omega_{cbb}^0 \rightarrow \bar{B}^0 \Sigma'^0 \pi^0$	$-\frac{(d_1 + 2d_6 - d_7) V_{ub} V_{cs}^*}{2\sqrt{3}}$	$\Omega_{cbb}^0 \rightarrow \bar{B}_s^0 \Omega^- K^+$	$(-d_1 + d_7) V_{ub} V_{cs}^*$

TABLE XXIII. Cabibbo-allowed decays of Ω_{ccc} with typical branching fractions at a few percent level. A light meson $\bar{K}^0$ can be replaced by a $\bar{K}^{*0}$.

Channel	Channel	Channel	Channel
$\Omega_{ccc}^{++} \rightarrow \Omega_c^+ \ell^+ \nu_\ell$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^0 \ell^+ \nu_\ell$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D^+ \ell^+ \nu_\ell$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ \bar{K}^0 \ell^+ \nu_\ell$
$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^0 \ell^+ \nu_\ell$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^0 D^+ \ell^+ \nu_\ell$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^- \ell^+ \nu_\ell$	$\Omega_{ccc}^{++} \rightarrow \Omega_c^+ D_s^+ \ell^+ \nu_\ell$
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} \bar{K}^0$	$\Omega_{ccc}^{++} \rightarrow \Omega_c^+ \pi^+$		
$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ \bar{K}^0 \pi^0$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^+ \pi^+ \bar{K}^0$	$\Omega_{ccc}^{++} \rightarrow \Xi_{cc}^{++} K^- \pi^+$	$\Omega_{ccc}^{++} \rightarrow \Omega_c^+ K^+ \bar{K}^0$
$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^+$		
$\Omega_{ccc}^{++} \rightarrow \Lambda_c^+ D^+ \bar{K}^0$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D_s^+ \bar{K}^0$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^+ \pi^0$	$\Omega_{ccc}^{++} \rightarrow \Xi_c^0 D^+ \pi^+$
$\Omega_{ccc}^{++} \rightarrow \Xi_c^+ D^0 \pi^+$			
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^0 \bar{K}^0$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^0 D^+ \pi^+$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^0 \pi^+$	$\Omega_{ccc}^{++} \rightarrow \Omega_c^0 D^+ K^+$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^{++} D^+ K^-$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D_s^+ \bar{K}^0$	$\Omega_{ccc}^{++} \rightarrow \Xi_c'^+ D^+ \pi^0$	$\Omega_{ccc}^{++} \rightarrow \Omega_c^0 D_s^+ \pi^+$
$\Omega_{ccc}^{++} \rightarrow \Sigma_c^+ D^+ \bar{K}^0$			

TABLE XXIV. CKM-allowed decay channels of Ω_{bbb}^- .

Channel	Channel	Channel	Channel
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D^0 \ell^- \bar{\nu}_\ell$	$\Omega_{bbb}^- \rightarrow \Xi_{bc}^+ B^- \ell^- \bar{\nu}_\ell$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D_s^+ \ell^- \bar{\nu}_\ell$	$\Omega_{bbb}^- \rightarrow \Omega_{bc}^0 \bar{B}_s^0 \ell^- \bar{\nu}_\ell$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^+ \ell^- \bar{\nu}_\ell$	$\Omega_{bbb}^- \rightarrow \Xi_{bc}^0 \bar{B}^0 \ell^- \bar{\nu}_\ell$		
$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- J/\psi$	$\Omega_{bbb}^- \rightarrow \Xi_b^- B^- J/\psi$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 K^- J/\psi$	$\Omega_{bbb}^- \rightarrow \Omega_b^- \bar{B}_s^0 J/\psi$
$\Omega_{bbb}^- \rightarrow \Xi_b^- B^- J/\psi$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}^0 J/\psi$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- \bar{K}^0 J/\psi$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}^0 J/\psi$
$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 D_s^-$	$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 D^- \bar{K}^0$	$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 \bar{D}^0 K^-$	
$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 \pi^-$	$\Omega_{bbb}^- \rightarrow \Omega_{bbc}^0 K^0 K^-$		
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^0$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^+ \pi^-$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D^0 \pi^0$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D^0 K^0$
$\Omega_{bbb}^- \rightarrow \Lambda_b^0 B^- D^0$	$\Omega_{bbb}^- \rightarrow \Sigma_b^- B^- D^+$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}_s^0 D^0$	$\Omega_{bbb}^- \rightarrow \Xi_b^- B^- D_s^+$
$\Omega_{bbb}^- \rightarrow \Xi_{bb}^0 D^0 \pi^-$	$\Omega_{bbb}^- \rightarrow \Sigma_b^- \bar{B}^0 D^0$	$\Omega_{bbb}^- \rightarrow \Omega_{bb}^- D_s^+ \pi^-$	$\Omega_{bbb}^- \rightarrow \Xi_b^- \bar{B}_s^0 D^0$
$\Omega_{bbb}^- \rightarrow \Xi_b^- B^- D_s^+$	$\Omega_{bbb}^- \rightarrow \Xi_{bb}^- D_s^+ K^-$	$\Omega_{bbb}^- \rightarrow \Sigma_b^- B^- D^0$	

In the Standard Model, symmetry-breaking effects arise from the unequal quark masses, $m_s \gg m_u \sim m_d$. In heavy bottom-quark decays, we expect the heavy-quark expansion might fairly work, and thus the symmetry-breaking effects emerging as power corrections $m_{u,d,s}/m_b$ might not be severe. Current studies on B decays indicate the flavor symmetry-breaking effects are typically smaller than 20%.

It is widely believed that the symmetry breaking in charm-quark decays could be sizable. In a recent analysis [61], the authors explored the breaking effects of the SU(3) flavor symmetry in singly Cabibbo-suppressed decays of charmed baryons, namely, Λ_c and Ξ_c . We quote their results for the three amplitudes due to the (H_6) in the SU(3) limit,

$$\begin{aligned}
|a_1| &= (0.252 \pm 0.005) \text{ GeV}^3, \\
|a_2| &= (0.127 \pm 0.009) \text{ GeV}^3, \\
|a_3| &= (0.091 \pm 0.015) \text{ GeV}^3,
\end{aligned} \tag{92}$$

while the H_{15} contributions have been neglected. Instead, three amplitudes due to the symmetry breaking are incorporated and have been fitted as [61]

$$\begin{aligned}
|v_1| &= (0.090 \pm 0.032) \text{ GeV}^3, \\
|v_2| &= (0.037 \pm 0.013) \text{ GeV}^3, \\
|v_3| &= (0.025 \pm 0.012) \text{ GeV}^3.
\end{aligned} \tag{93}$$

Compared to the three SU(3) amplitudes in Eq. (92), these symmetry-breaking terms are sizable but still smaller in magnitude. Thus, we expect that the relations derived in our analysis will not be completely ruined, and many of them can be precisely examined on the experimental side in the future.

X. CONCLUSIONS

Up to date, the quark model is a most successful theoretical tool to describe the hadron spectrum, especially the lowest-lying hadrons. Since the charm and bottom quarks are much heavier than the lighter ones, hadrons with a different number of heavy quarks will have distinct dynamics. On the experimental side, light hadrons with no heavy quark, singly heavy baryons, and doubly heavy baryons have been established, but triply heavy baryons are still missing. Thus, it deserves more theoretical and experimental effort to study various properties of triply heavy baryons from both the theoretical and experimental sides.

In this work, we have systematically analyzed weak decays of triply heavy baryons for the first time in the literature. Decay amplitudes for various transitions have been parametrized in terms of the SU(3)-independent amplitudes. Using these results, we find a number of relations for the partial decay widths. We also give a list of decay channels with sizable branching fractions. We suggest our experimental colleagues perform a search at hadron colliders and the electron and positron collisions in the future.

ACKNOWLEDGMENTS

The authors are grateful to Professor Ji-Bo He for useful discussions and valuable comments. We thank Zhen-Xing Zhao for checking the formulas of decay widths. W.W. is thankful for the hospitality from Professor C. Q. Geng and Professor Xiao-Gang He at National Center for Theoretical Sciences, Hsinchu, where this work was finalized. This work is supported in part by National Natural Science Foundation of China under Grants No. 11575110, No. 11655002, and No. 11735010 and Natural Science Foundation of Shanghai under Grant No. 15DZ2272100.

- [1] H. X. Chen, W. Chen, X. Liu, and S. L. Zhu, *Phys. Rep.* **639**, 1 (2016).
- [2] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, *Rep. Prog. Phys.* **80**, 076201 (2017).
- [3] A. Ali, J. S. Lange, and S. Stone, *Prog. Part. Nucl. Phys.* **97**, 123 (2017).
- [4] F. K. Guo, C. Hanhart, U. G. Meiner, Q. Wang, Q. Zhao, and B. S. Zou, *Rev. Mod. Phys.* **90**, 015004 (2018).
- [5] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **119**, 112001 (2017).
- [6] C. Patrignani *et al.* (Particle Data Group Collaboration), *Chin. Phys. C* **40**, 100001 (2016).
- [7] Y. Jia, *J. High Energy Phys.* **10** (2006) 073.
- [8] A. P. Martynenko, *Phys. Lett. B* **663**, 317 (2008).
- [9] B. Patel, A. Majethiya, and P. C. Vinodkumar, *Pramana* **72**, 679 (2009).
- [10] J. R. Zhang and M. Q. Huang, *Phys. Lett. B* **674**, 28 (2009).
- [11] S. Meinel, *Phys. Rev. D* **82**, 114514 (2010).
- [12] F. J. Llanes-Estrada, O. I. Pavlova, and R. Williams, *Eur. Phys. J. C* **72**, 2019 (2012).
- [13] Z. G. Wang, *Commun. Theor. Phys.* **58**, 723 (2012).
- [14] S. Meinel, *Phys. Rev. D* **85**, 114510 (2012).
- [15] T. M. Aliev, K. Azizi, and M. Savci, *J. High Energy Phys.* **04** (2013) 042.
- [16] M. Padmanath, R. G. Edwards, N. Mathur, and M. Peardon, *Phys. Rev. D* **90**, 074504 (2014).
- [17] Z. S. Brown, W. Detmold, S. Meinel, and K. Orginos, *Phys. Rev. D* **90**, 094507 (2014).
- [18] K. W. Wei, B. Chen, and X. H. Guo, *Phys. Rev. D* **92**, 076008 (2015).
- [19] K. W. Wei, B. Chen, N. Liu, Q. Q. Wang, and X. H. Guo, *Phys. Rev. D* **95**, 116005 (2017).
- [20] K. Thakkar, A. Majethiya, and P. C. Vinodkumar, *Eur. Phys. J. Plus* **131**, 339 (2016).
- [21] Y. Q. Chen and S. Z. Wu, *J. High Energy Phys.* **08** (2011) 144; **09** (2011) 89.
- [22] M. A. Gomshi Nobary and R. Sepahvand, *Phys. Rev. D* **71**, 034024 (2005).
- [23] J. M. Flynn, E. Hernandez, and J. Nieves, *Phys. Rev. D* **85**, 014012 (2012).
- [24] C. Q. Geng, Y. K. Hsiao, C. W. Liu, and T. H. Tsai, *J. High Energy Phys.* **11** (2017) 147.
- [25] D. Zeppenfeld, *Z. Phys. C* **8**, 77 (1981).
- [26] M. J. Savage and M. B. Wise, *Phys. Rev. D* **39**, 3346 (1989); **40**, 3127(E) (1989).
- [27] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, *Phys. Rev. D* **50**, 4529 (1994).
- [28] B. Grinstein and R. F. Lebed, *Phys. Rev. D* **53**, 6344 (1996).
- [29] X. G. He, *Eur. Phys. J. C* **9**, 443 (1999).
- [30] N. G. Deshpande, X. G. He, and J. Q. Shi, *Phys. Rev. D* **62**, 034018 (2000).
- [31] X. G. He, J. Y. Leou, and C. Y. Wu, *Phys. Rev. D* **62**, 114015 (2000).
- [32] N. G. Deshpande and X. G. He, *Phys. Rev. Lett.* **75**, 1703 (1995).
- [33] X. G. He, Y. K. Hsiao, J. Q. Shi, Y. L. Wu, and Y. F. Zhou, *Phys. Rev. D* **64**, 034002 (2001).
- [34] H. K. Fu, X. G. He, Y. K. Hsiao, and J. Q. Shi, *Chin. J. Phys. (Taipei)* **41**, 601 (2003).
- [35] C. W. Chiang, M. Gronau, Z. Luo, J. L. Rosner, and D. A. Suprun, *Phys. Rev. D* **69**, 034001 (2004).
- [36] C. W. Chiang, M. Gronau, J. L. Rosner, and D. A. Suprun, *Phys. Rev. D* **70**, 034020 (2004).
- [37] C. W. Chiang and Y. F. Zhou, *J. High Energy Phys.* **12** (2006) 027.
- [38] Y. Li, C. D. Lu, and W. Wang, *Phys. Rev. D* **77**, 054001 (2008).
- [39] W. Wang, Y. M. Wang, D. S. Yang, and C. D. Lu, *Phys. Rev. D* **78**, 034011 (2008).
- [40] C. W. Chiang and Y. F. Zhou, *J. High Energy Phys.* **03** (2009) 055.
- [41] H. Y. Cheng, C. W. Chiang, and A. L. Kuo, *Phys. Rev. D* **91**, 014011 (2015).
- [42] X. G. He, G. N. Li, and D. Xu, *Phys. Rev. D* **91**, 014029 (2015).
- [43] X. G. He and G. N. Li, *Phys. Lett. B* **750**, 82 (2015).
- [44] Y. K. Hsiao, C. F. Chang, and X. G. He, *Phys. Rev. D* **93**, 114002 (2016).
- [45] X. G. He, W. Wang, and R. L. Zhu, *J. Phys. G* **44**, 014003 (2017).
- [46] X. G. He, S. F. Li, B. Ren, and X. B. Yuan, *Phys. Rev. D* **96**, 053004 (2017).
- [47] C.-D. Lü, W. Wang, and F. S. Yu, *Phys. Rev. D* **93**, 056008 (2016).
- [48] H. Y. Cheng, C. W. Chiang, and A. L. Kuo, *Phys. Rev. D* **93**, 114010 (2016).
- [49] H. Y. Cheng and C. W. Chiang, *Phys. Rev. D* **86**, 014014 (2012).
- [50] H. Y. Cheng and S. Oh, *J. High Energy Phys.* **09** (2011) 024.
- [51] W. Wang, Z. P. Xing, and J. Xu, *Eur. Phys. J. C* **77**, 800 (2017).
- [52] Y. J. Shi, W. Wang, Y. Xing, and J. Xu, *Eur. Phys. J. C* **78**, 56 (2018).
- [53] C. Q. Geng, Y. K. Hsiao, Y. H. Lin, and L. L. Liu, *Phys. Lett. B* **776**, 265 (2018).
- [54] C. Q. Geng, Y. K. Hsiao, C. W. Liu, and T. H. Tsai, *arXiv:1801.03276*.
- [55] X. G. He and W. Wang, *arXiv:1803.04227*.
- [56] A. Lenz, *Int. J. Mod. Phys. A* **30**, 1543005 (2015).
- [57] F. Krinner, A. Lenz, and T. Rauh, *Nucl. Phys. B* **876**, 31 (2013).
- [58] W. Wang, Y. L. Shen, and C. D. Lu, *Phys. Rev. D* **79**, 054012 (2009).
- [59] Z. G. Wang, *Eur. Phys. J. A* **45**, 267 (2010).
- [60] M. Gronau, *Phys. Lett. B* **492**, 297 (2000).
- [61] C. Q. Geng, Y. K. Hsiao, C. W. Liu, and T. H. Tsai, *arXiv:1804.01666*.