24 + 24 real scalar multiplet in four dimensional $N = 2$ conformal supergravity

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(Received 13 February 2018; published 29 March 2018)

Starting from the $48 + 48$ component multiplet of supercurrents for a rigid $N = 2$ tensor multiplet in four spacetime dimensions, we obtain the transformation of the linearized supergravity multiplet which couples to this supercurrent multiplet. At the linearized level, this $48 + 48$ component supergravity multiplet decouples into the $24 + 24$ component linearized standard Weyl multiplet and a $24 + 24$ component irreducible matter multiplet containing a real scalar field. By a consistent application of the supersymmetry algebra with field-dependent structure constants appropriate to $N = 2$ conformal supergravity, we find the full transformation law for this multiplet in a conformal supergravity background. By performing a suitable field redefinition, we find that the multiplet is a generalization of the flat space multiplet, obtained by Howe *et al.* in *Nucl. Phys.* **B214**, 519 (1983), to a conformal supergravity background. We also present a set of constraints which can be consistently imposed on this multiplet to obtain a restricted minimal $8 + 8$ off-shell matter multiplet. We also show, as an example, the precise embedding of the tensor multiplet inside this multiplet.

DOI: [10.1103/PhysRevD.97.066026](https://doi.org/10.1103/PhysRevD.97.066026)

I. INTRODUCTION

The use of conformal symmetries plays a crucial role in the construction of theories of supergravity. While the physical Poincaré theory can be obtained via a simple gauge-fixing procedure, the higher degree of symmetry in the conformal theories allows for a rigorous rearrangement of the off-shell degrees of freedom (d.o.f.) within multiplets shorter than the one that contains Poincaré supergravity d.o.f. Furthermore, techniques generally referred to as “multiplet calculus” have been developed over the years to simplify the often massive task of constructing invariant couplings of supergravity, especially higher-derivative ones.

Off-shell conformal supergravities exist for dimensions $D \leq 6$. The higher-dimensional theories are typically related to the lower dimensional theory via dimensional reduction. For a connection between the $N = (1, 0)$ chiral theory in six dimensions as well as $N = 2$ theory in four

dimensions with the $N = 1$ theory in five dimensions,¹ via dimensional reduction (uplift) see [4,5], respectively. Hence, one can expect that given a result in higher-dimensional theories, a similar result would exist in the lower dimensional theory from dimensional reduction arguments. A relevant example of such connection, the existence of the two versions of Weyl multiplets—the standard and dilaton Weyl, which were constructed in six dimensions [6] and later on in five dimensions [1,2]—also exists in the four-dimensional case [7–10]. While the precise reduction procedure and identification of off-shell d.o.f. is being currently worked out, it is easy to realize that the reduction of the $32 + 32$ off-shell five-dimensional dilaton Weyl multiplet would give the $24 + 24$ four-dimensional dilaton-Weyl multiplet plus a short $8 + 8$ vector matter multiplet in four dimensions.

It is worth pointing out that the dilaton-Weyl multiplet could be obtained in two different yet equivalent ways in five dimensions. The first procedure involves analyzing the coupling of a standard Weyl multiplet with an on-shell vector multiplet [1]. The vanishing of the vector multiplet equations of motion furnishes a set of constraints, which can be used to fix the auxiliary fields of the standard Weyl

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¹We refer to the minimal supergravity in five dimensions formulated in terms of eight supercharges as $N = 1$. However, sometimes, for example in [1–3], it is referred to as $N = 2$.

multiplet in terms of the auxiliary fields of the dilaton Weyl multiplet. The same procedure is used in [10] to obtain the $N = 2$ dilaton-Weyl multiplet in four dimensions.

The second method in five dimensions was based on the construction of the current multiplet for a nonconformal vector multiplet. The standard action for a nonconformal vector multiplet in five dimensions would not be scale invariant and hence the trace of the energy-momentum tensor $\sigma = \theta^\mu_\mu$ would be a nontrivial component of the current multiplet. When coupling the current multiplet to linearized gravity (Weyl) multiplet, the scalar current σ will couple to a scalar field of dimension 1: the dilaton. From the linearized gravity multiplet it is, in principle, a simple, yet computationally challenging, task to derive the full nonlinear transformations of the dilaton version of the Weyl multiplet. It is worth emphasizing that, in this method, the nonconformal nature of the matter multiplet chosen was crucial to obtain the Weyl multiplet with a dilaton.

One naïvely expects the same method to produce the dilaton Weyl multiplet in four dimensions, the only caveat being that the vector multiplet in four dimensions is conformally invariant. Hence, a different off-shell nonconformal multiplet needs to be chosen for the construction of the current multiplet. Such nonconformal current multiplets have been discussed in [11]. We use the current multiplet associated with a rigid tensor multiplet in four spacetime dimensions and show that the naïve expectation of getting a dilaton Weyl multiplet from this nonconformal current multiplet turns out to be incorrect.

Specifically, we find that the linearized multiplet of gravity that couples to the current multiplet of the rigid tensor multiplet has $48 + 48$ components, which can be arranged into the standard Weyl multiplet and an irreducible and unconstrained off-shell $24 + 24$ matter multiplet containing a real scalar field. We will, henceforth, refer to it as the “real scalar multiplet.” After a suitable field redefinition, we show that our result is a generalization of the flat space results of [12] to a conformal supergravity background. We also find that this multiplet can be consistently restricted to an $8 + 8$ component multiplet and show the precise identifications encoding the embedding of the tensor multiplet inside this multiplet.

The paper is organized as follows. In Sec. II, we will describe the features of four-dimensional conformal supergravity relevant to our paper (see [13] for all the details and [14] for an extensive review on the subject). In Secs. III and IV, we will present the details of the construction of the linearized real scalar multiplet starting from the current multiplet of a rigid tensor multiplet. In Sec. V, we will present the real scalar multiplet along with its complete supersymmetry transformations in a conformal supergravity background. In Sec. VI, we will introduce field redefinitions which will simplify the supersymmetry transformations presented in Sec. V. This is one of the main results of our paper which generalizes the flat space results of [12] to a

conformal supergravity background. In Sec. VII, we will find a set of consistent constraints that restricts the multiplet to $8 + 8$ components. Finally, in Sec. VIII, we will find the embedding of the $8 + 8$ tensor multiplet within the $24 + 24$ real scalar multiplet that satisfies the constraints obtained in the previous section. The important results obtained in Secs. VII and VIII have potential applications that we will discuss in the concluding section.

II. $N = 2$ CONFORMAL SUPERGRAVITY IN FOUR SPACETIME DIMENSIONS

Conformal supergravity with N -extended supersymmetry is a gauge theory based on $SU(2, 2|N)$ superconformal algebra, the supersymmetric generalization of the conformal algebra. It is invariant under two different types of supersymmetry generators, the Q-supersymmetry and the special S-supersymmetry. The multiplet of fields that contains the gauge fields of the superconformal algebra is known as Weyl multiplet. For $N = 2$, the Weyl multiplet has $24 + 24$ (bosonic + fermionic) off-shell d.o.f. In order to describe the multiplet, we fix our notations as follows. We denote the spacetime indices by greek letters $\mu, \nu, \dots$, the local Lorentz indices by latin letters $a, b, \dots$, $SU(2)$ indices by $i, j, \dots$. As we mentioned before, two different versions of the Weyl multiplet exists in four dimensions: here we will focus on the standard formulation. The field content of the standard Weyl multiplet contains the gauge fields of the $SU(2, 2|2)$ algebra: e_μ^a (vielbein), an $SU(2)$ doublet of Majorana spinors whose positive and negative chiral projections are denoted by ψ_μ^i and $\bar{\psi}_{\mu i}$, respectively, (gravitinos), an $SU(2)$ triplet of gauge fields $V_\mu^i{}_j$ corresponding to $SU(2)$ R-symmetry, gauge field A_μ corresponding to $U(1)$ R-symmetry, gauge field b_μ corresponding to the dilatation symmetry. Apart from the above-mentioned gauge fields, the Weyl multiplet also contains several auxiliary fields that are required to balance bosonic and fermionic d.o.f. They include an $SU(2)$ doublet of Majorana spinors whose positive and negative chiral projections are denoted by χ^i and $\bar{\chi}_i$, respectively, a real tensor T_{ab} antisymmetric in its Lorentz indices and a real scalar field D . The above-mentioned field content of the

TABLE I. Weights and chiralities of the standard Weyl multiplet.

Field	$SU(2)$ Irreps	Weyl weight (w)	Chiral weight (c)	Chirality fermions
e_μ^a	1	-1	0	...
$V_\mu^i{}_j$	3	0	0	...
A_μ	1	0	0	...
b_μ	1	0	0	...
T_{ab}^-	1	1	-1	...
D	1	2	0	...
ψ_μ^i	2	-1/2	-1/2	+1
$\bar{\chi}^i$	2	3/2	-1/2	+1

Weyl multiplet can be tabulated in Table I along with their chiral and Weyl weights. Here $T_{ab}^\mp$ refers to the (anti)self-dual components of T_{ab} . Complex conjugation is used to raise or lower SU(2) indices and change the duality properties of a tensor. It hence flips the chiral weight as well as chirality (for fermions) but not the Weyl weight. The gauge fields ω_μ^{ab} corresponding to local Lorentz transformations, f_μ^a corresponding to special conformal transformation and $\phi_{\mu i}$ corresponding to S-supersymmetry are dependent and are determined by the following set of conventional constraints:

$$\begin{aligned} R(P)_{\mu\nu}{}^a &= 0, \\ \gamma^\mu \left(R(Q)_{\mu\nu}{}^i + \frac{1}{2} \gamma_{\mu\nu} \chi^i \right) &= 0, \\ e^\nu{}_b R(M)_{\mu\nu}{}^b - i\tilde{R}(A)_{\mu a} + \frac{1}{4} T^+{}_{ab} T^-{}_\mu{}^b - \frac{3}{2} D e_{\mu a} &= 0. \end{aligned} \quad (1)$$

The supercovariant curvature $R(P)$, $R(Q)$, and $R(M)$ appearing above are associated with local translation, Q-supersymmetry and local Lorentz transformations, respectively, and $\tilde{R}(A)$ is the dual of the curvature associated with the U(1) R-symmetry. The supersymmetry commutators, in particular $[\delta_Q, \delta_Q]$, gets modified by field dependent structure constants as mentioned below:

$$\begin{aligned} [\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] &= \delta^{(\text{cov})}(\xi) + \delta_M(\epsilon) + \delta_K(\Lambda_K) + \delta_S(\eta) \\ &\quad + \delta_{\text{gauge}}, \\ [\delta_S(\eta), \delta_Q(\epsilon)] &= \delta_M(\tilde{\eta}_i \gamma^{ab} \epsilon^i + \text{H.c.}) + \delta_D(\tilde{\eta}^i \epsilon_i + \text{H.c.}) \\ &\quad + \delta_A(i\tilde{\eta}_i \epsilon^i + \text{H.c.}) \\ &\quad + \delta_V(-2\tilde{\eta}^i \epsilon_j - (\text{H.c.; traceless})), \\ [\delta_S(\eta_1), \delta_S(\eta_2)] &= \delta_K(\tilde{\eta}_{2i} \gamma^a \eta_1^i + \text{H.c.}). \end{aligned} \quad (2)$$

The infinitesimal transformations, δ_Q , δ_S , δ_M , δ_K , δ_D , δ_A and δ_V , correspond to Q-supersymmetry, S-supersymmetry, local Lorentz, special conformal, dilatation, U(1) and SU(2) R-symmetry, respectively. The infinitesimal covariant transformation, $\delta^{(\text{cov})}$ is defined as

$$\delta^{(\text{cov})}(\xi) = \delta_{\text{gct}}(\xi) + \sum_T \delta^T(-\xi^\mu h_\mu(T)), \quad (3)$$

where δ_{gct} is general coordinate transformation. The sum is over all superconformal transformation (and any other gauge transformation present in the multiplet) except local translation and the field $h_\mu(T)$ is the corresponding gauge field, except for Q and S-supersymmetry, where $h_\mu(T)$ is $\frac{1}{2}$ times the corresponding gauge fields and for SU(2) R-symmetry where $h_\mu(T)$ is $-\frac{1}{2}$ times the corresponding gauge field. Finally the effect of any other gauge transformation (for example in vector multiplet) is encoded in

δ_{gauge} that appears on the RHS of $[\delta_Q, \delta_Q]$ algebra in (2). The parameters that appear above in (2) read:

$$\begin{aligned} \xi^\mu &= 2\tilde{\epsilon}_2^i \gamma^\mu \epsilon_{1i} + \text{H.c.}, \\ \epsilon_{ab} &= \epsilon^{ij} \tilde{\epsilon}_{1i} \epsilon_{2j} T_{ab}^- + \text{H.c.}, \\ \Lambda_K^a &= \epsilon^{ij} \tilde{\epsilon}_{1i} \epsilon_{2j} D_b T^{-ba} - \frac{3}{2} \tilde{\epsilon}_2^i \gamma^a \epsilon_{1i} D + \text{H.c.}, \\ \eta^i &= 6\tilde{\epsilon}_1^i \epsilon_2^j \chi_j. \end{aligned} \quad (4)$$

Apart from the Weyl multiplet(s), there exists a large variety of $N = 2$ superconformal matter multiplets in four dimensions [15–21]. These multiplets all contain $8 + 8$ off-shell d.o.f., with the exception of the hypermultiplet which is an on-shell multiplet.²

In Sec. V, we will present a large $N = 2$ off-shell matter multiplet, which contains instead $24 + 24$ d.o.f. and forms an irreducible representation of the SU(2, 2|2) superconformal algebra.

III. TENSOR MULTIPLY AND THE MULTIPLY OF SUPERCURRENTS

The current multiplet for a rigid $N = 2$ tensor multiplet in four dimensions has been given in [11] as a reducible multiplet containing two parts: a conformal supercurrent V and a trace supermultiplet L_{ijkl} satisfying the superspace constraints.³

$$D^{ij}V = -\frac{4}{5} D_{kl} L^{ijkl}, \quad (5)$$

where the superspace derivatives D_{ij} and D^{ij} appearing above are defined as:

$$\begin{aligned} D_{ij} &\equiv \epsilon^{\alpha\beta} D_{\alpha i} D_{\beta j}, \\ D^{ij} &\equiv \epsilon^{ik} \epsilon^{jl} D_{kl}. \end{aligned} \quad (6)$$

Here, $D_{\alpha i}$ is the superspace covariant derivative, given in terms of the superspace coordinates $(x^\mu, \theta^{\alpha i}, \bar{\theta}_{\dot{\alpha} i})$, as shown below⁴:

$$D_{\alpha i} = \frac{\partial}{\partial \theta^{\alpha i}} + i\sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}_{\dot{\alpha} i} \frac{\partial}{\partial x^\mu}. \quad (7)$$

The supersymmetry transformation of the components of this current multiplet can be read off from the above superspace constraints (5). The components of this current

²The existence of off-shell hypermultiplet is subtle. One can formulate it in the presence of an off-shell central charge. Alternatively one can also formulate it in harmonic superspace [22], without the need for a central charge. The latter formulation requires infinite d.o.f.

³See also [23] for a complete classification of all $N = 2$ supercurrent multiplets in four spacetime dimensions.

⁴Here, we have used the two component notation of writing four dimensional Weyl spinors in terms of dotted and undotted spinor index.

multiplet and its supersymmetry transformations are central to our construction of the real scalar multiplet in four-dimensional $N = 2$ conformal supergravity. Hence, for clarity of exposition, we will discuss in this section the details of this current multiplet construction, in component notation.

An off-shell rigid $N = 2$ tensor multiplet in four space-time dimensions [21] encompasses $8 + 8$ d.o.f., organized in an $SU(2)$ triplet of pseudo-real scalars L_{ij} satisfying the reality constraint $(L_{ij})^* \equiv L^{ij} = \epsilon^{ik}\epsilon^{jl}L_{kl}$; an $SU(2)$ doublet of Majorana spinors, whose positive components are denoted by ϕ^i ; a real tensor gauge field $E_{\mu\nu}$ with vector gauge transformation $\delta E_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]}$. To close the algebra off-shell, an auxiliary complex scalar field G is required. When G is set to zero through its algebraic equations of motion, the supersymmetry transformations of the tensor multiplet components are given by

$$\begin{aligned}\delta E_{\mu\nu} &= i\bar{\epsilon}^i\gamma_{\mu\nu}\phi^j\epsilon_{ij} + \text{H.c.}, \\ \delta\phi^i &= \partial L^{ij}\epsilon_j + \epsilon^{ij}\not{H}\epsilon_j, \\ \delta L^{ij} &= 2\bar{\epsilon}^i(\phi^j) + 2\epsilon^{ik}\epsilon^{jl}\bar{\epsilon}_{(k}\phi_{l)}.\end{aligned}\quad (8)$$

The action that is invariant under the above SUSY transformation reads

$$S = \int d^4x \left[H_\mu H^\mu - \bar{\phi}^i \not{\partial} \phi_i - \frac{1}{2} \partial_\mu L^{ij} \partial^\mu L_{ij} \right], \quad (9)$$

where we defined

$$H^\mu = \frac{i}{6} \epsilon^{\mu\nu\rho\sigma} H_{\nu\rho\sigma}. \quad (10)$$

The three form $H_{\mu\nu\rho}$ is the gauge-invariant field strength of the tensor gauge field $E_{\mu\nu}$ defined as $H_{\mu\nu\rho} \equiv 3\partial_{[\mu}E_{\nu\rho]}$. The equations of motion for the fields are given by:

$$\not{\partial}\phi^i = 0, \quad \partial^2 L_{ij} = 0, \quad \partial_{[\mu}H_{\nu]} = 0. \quad (11)$$

The tensor gauge field also satisfies the Bianchi identity $\partial^\mu H_\mu = 0$. Apart from the above mentioned supersymmetry transformation (8), the action (9) is also invariant under an $SU(2) \times U(1)$ R-symmetry.

The current multiplet associated to the tensor multiplet action contains the following components: the energy-momentum tensor $\theta_{\mu\nu}$ arising from translation invariance, a supersymmetry current $J_{\mu i}$ arising from the supersymmetry invariance, an $SU(2)$ current $v_\mu{}^i{}_j$ and a $U(1)$ current a_μ arising from the $SU(2) \times U(1)$ R-symmetry. Since the multiplet is nonconformal, the trace σ of the energy-momentum tensor and the gamma-trace λ_i of the

supersymmetry current will also be a part of the current multiplet. The above-mentioned components of the current multiplet in terms of the tensor multiplet fields are given as below:

$$\begin{aligned}\theta_{\mu\nu} &= H_\mu H_\nu - \frac{1}{2} \eta_{\mu\nu} H^2 + \frac{1}{2} \bar{\phi}^i \gamma_{(\mu} \overleftrightarrow{\partial}_{\nu)} \phi_i + \frac{1}{2} \partial_\mu L^{ij} \partial_\nu L_{ij} \\ &\quad - \frac{1}{4} \eta_{\mu\nu} \partial_\rho L^{ij} \partial^\rho L_{ij}, \\ \sigma \equiv \theta^\mu{}_\mu &= -H^2 - \frac{1}{2} \partial^\mu L^{ij} \partial_\mu L_{ij}, \\ v_\mu{}^i{}_j &= \frac{1}{8} L^{ik} \overleftrightarrow{\partial}_\mu L_{jk} - \frac{1}{4} \left(\bar{\phi}^i \gamma_\mu \phi_j - \frac{1}{2} \delta_j^i \bar{\phi}^k \gamma_\mu \phi_k \right), \\ a_\mu &= -\frac{i}{8} \bar{\phi}^i \gamma_\mu \phi_i, \\ J_{\mu i} &= \frac{1}{2} \gamma_\mu \not{H} \phi^j \epsilon_{ij} - H_\mu \phi^j \epsilon_{ij} + \phi^j \partial_\mu L_{ij} - \frac{1}{2} \gamma_\mu \not{\partial} L_{ij} \phi^j, \\ \lambda_i &\equiv \gamma^\mu J_{\mu i} = \not{H} \phi^j \epsilon_{ij} - \not{\partial} L_{ij} \phi^j.\end{aligned}\quad (12)$$

The other components of the current multiplet are obtained by supersymmetry and read

$$\begin{aligned}t_\mu{}^i{}_j &= \frac{1}{2} H_\mu L^{ik} \epsilon_{kj} + \frac{1}{4} \left(\bar{\phi}^i \gamma_\mu \phi_j - \frac{1}{2} \delta_j^i \bar{\phi}^k \gamma_\mu \phi_k \right), \\ e^{ij} &= \bar{\phi}^i \phi^j, \quad \tilde{a}_\mu = \epsilon_{ij} \bar{\phi}^i \partial_\mu \phi^j, \quad d = \frac{1}{2} L^{ij} L_{ij}, \\ b_{\mu\nu}^- &= \frac{1}{2} \bar{\phi}^i \gamma_{\mu\nu} \phi^j \epsilon_{ij}, \quad c^{ijkl} = L^{(ij} L^{kl)}, \\ \Sigma^{ijk} &= L^{(ij} \phi^k), \quad \xi_j = L_{jk} \phi^k.\end{aligned}\quad (13)$$

Together, Eqs. (12) and (13) form the $48 + 48$ component current multiplet of the rigid $N = 2$ tensor multiplet in four spacetime dimensions. They satisfy the following conservation, reality and trace relations.

$$\begin{aligned}\partial^\mu \theta_{\mu\nu} &= 0, \quad \sigma = \theta^\mu{}_\mu, \quad \partial^\mu J_{\mu i} = 0, \quad \lambda_i = \gamma^\mu J_{\mu i}, \\ (v_\mu{}^i{}_j)^* &= -v_\mu{}^j{}_i, \quad v_\mu{}^i{}_i = 0, \quad \partial_\mu v^\mu{}^i{}_j = 0, \\ (t_\mu{}^i{}_j)^* &= -t_\mu{}^j{}_i, \quad t_\mu{}^i{}_i = 0, \\ (c_{ijkl})^* &\equiv c^{ijkl} = \epsilon^{im} \epsilon^{jn} \epsilon^{kp} \epsilon^{lq} c_{mnpq}, \\ \partial_\mu a^\mu &= 0, \quad (a_\mu)^* = a_\mu, \quad \partial_\mu \tilde{a}^\mu = 0, \\ \partial^\mu b_{\mu\nu}^- &= \tilde{a}_\nu, \quad \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} b^{-\rho\sigma} = -b_{\mu\nu}^-, \end{aligned}\quad (14)$$

and transform under supersymmetry as:

$$\begin{aligned}
\delta\theta_{\mu\nu} &= \bar{\epsilon}^i \gamma_{\lambda(\mu} \partial^\lambda J_{\nu)i} + \text{H.c.}, \\
\delta\sigma &= \bar{\epsilon}^i \not{\partial} \lambda_i + \text{H.c.}, \\
\delta t_{\mu}^i{}^j &= \frac{1}{2} \bar{\epsilon}^i J_{\mu j} - \frac{1}{2} \bar{\epsilon}^i \gamma_{\mu} \lambda_j + \frac{1}{4} \epsilon_{lm} \epsilon_{jk} \bar{\epsilon}^l \not{\partial} (\gamma_{\mu} \Sigma^{imk}) - \frac{1}{3} \bar{\epsilon}^i \gamma_{\mu} \not{\partial} \xi_j - \frac{1}{3} \bar{\epsilon}^i \partial_{\mu} \xi_j - (\text{H.c.}; \text{traceless}), \\
\delta v_{\mu}^i{}^j &= \frac{1}{2} \bar{\epsilon}^i J_{\mu j} - \frac{1}{4} \epsilon_{kj} \epsilon_{lm} \bar{\epsilon}^l \gamma_{\mu} \not{\partial} \Sigma^{ikm} - \frac{1}{3} \bar{\epsilon}^i \partial_{\mu} \xi_j + \frac{1}{4} \epsilon_{kj} \epsilon_{lm} \bar{\epsilon}^l \partial_{\mu} \Sigma^{ikm} + \frac{1}{3} \bar{\epsilon}^i \gamma_{\mu} \not{\partial} \xi_j - (\text{H.c.}; \text{traceless}), \\
\delta a_{\mu} &= \frac{i}{4} \bar{\epsilon}_k J_{\mu}^k + \text{H.c.}, \\
\delta b_{\mu\nu}^- &= -\epsilon^{ik} \bar{\epsilon}_i \gamma^{\alpha} \gamma_{\mu\nu} J_{\alpha k}, \\
\delta \tilde{a}_{\mu} &= 2\epsilon^{ik} \bar{\epsilon}_i \not{\partial} J_{\mu k} + \epsilon^{ik} \bar{\epsilon}_i \gamma_{\mu} \not{\partial} \lambda_k - \epsilon^{ik} \bar{\epsilon}_i \partial_{\mu} \lambda_k, \\
\delta e^{ij} &= -2\bar{\epsilon}_k \not{\partial} \Sigma^{ijk} - 2\epsilon^{k(i} \epsilon^{j)l} \bar{\epsilon}_k \lambda_l - \frac{8}{3} \epsilon^{k(i} \epsilon^{j)l} \bar{\epsilon}_k \not{\partial} \xi_l, \\
\delta d &= 2\bar{\epsilon}_i \xi^i + \text{H.c.}, \\
\delta c^{ijkl} &= 4\bar{\epsilon}^{(i} \Sigma^{jkl)} + 4\epsilon^{im} \epsilon^{jn} \epsilon^{kp} \epsilon^{lq} \bar{\epsilon}_{(m} \Sigma_{npq)}, \\
\delta J_{\mu i} &= \theta_{\mu\nu} \gamma^{\nu} \epsilon_i + (\gamma_{\rho} \gamma_{\mu\sigma} + \gamma_{\mu\sigma} \gamma_{\rho}) \partial^{\sigma} v^{\rho k} \epsilon_k + (\gamma_{\rho} \gamma_{\mu\sigma} + \gamma_{\mu\sigma} \gamma_{\rho}) \partial^{\sigma} t^{\rho k} \epsilon_k - \frac{1}{4} \partial_{\mu} b_{\nu\rho}^- \gamma^{\nu\rho} \epsilon^j \epsilon_{ij} + \gamma_{\mu\rho} \tilde{a}^{\rho} \epsilon^j \epsilon_{ij} \\
&\quad + i(\gamma_{\rho} \gamma_{\mu\sigma} - 3\gamma_{\mu\sigma} \gamma_{\rho}) \partial^{\sigma} a^{\rho} \epsilon_i, \\
\delta \lambda_i &= \sigma \epsilon_i + 4\bar{\not{p}}_i^k \epsilon_k + 4\bar{\not{t}}_i^k \epsilon_k - 4\partial_{\mu} t_i^{\mu k} \epsilon_k - 8i\bar{\not{d}} \epsilon_i + 3\bar{\not{d}} \epsilon^j \epsilon_{ij} - \frac{1}{4} \not{\partial} (\gamma \cdot b^-) \epsilon^j \epsilon_{ij}, \\
\delta \xi_j &= -2\bar{t}^l{}_j \epsilon_l - 4\bar{p}^l{}_j \epsilon_l + 6i\bar{\not{d}} \epsilon_j - \frac{3}{8} b^- \cdot \gamma \epsilon^m \epsilon_{jm} - \frac{1}{2} \epsilon_{jl} \epsilon_{km} e^{lk} \epsilon^m + \frac{1}{2} \not{\partial} d \epsilon_j, \\
\delta \Sigma^{ijk} &= -2\epsilon^{l(i} \bar{\not{p}}^{jk)} \epsilon_l + 2\epsilon^{l(i} \bar{\not{p}}^{jk)} \epsilon_l + \frac{1}{2} \not{\partial} c^{ijkl} \epsilon_l - e^{(ij} \epsilon^{k)}. \tag{15}
\end{aligned}$$

IV. LINEARIZED SUPERGRAVITY MULTIPLY

The components of the current multiplet will couple to the linearized components of a supergravity multiplet as shown below in (16). This will fix the basis and normalization of the fields belonging to our linearized gravity multiplet.⁵ The invariant action reads

$$\begin{aligned}
S &= \int d^4x \left[\frac{1}{2} \theta^{\mu\nu} h_{\mu\nu} + \sigma \varphi + d\mathcal{D} + \frac{1}{24} c^{ijkl} C_{ijkl} + \frac{1}{4} b_{\mu\nu}^- T^{-\mu\nu} - 2v_{\mu}^i{}_j V^{\mu j}{}_i + e^{ij} E_{ij} + 4a_{\mu} A^{\mu} + (t_{\mu}^i{}_j - v_{\mu}^i{}_j) S^{\mu j}{}_i \right. \\
&\quad \left. + 2\bar{J}_{\mu i} \psi^{\mu i} + \bar{\lambda}_i \Lambda^i + \bar{\xi}_i \zeta^i + \frac{1}{3} \bar{\Sigma}_{ijk} \Xi^{ijk} + \bar{a}_{\mu} \tilde{A}^{\mu} + \text{H.c.} \right]. \tag{16}
\end{aligned}$$

Supersymmetry invariance of the above action (16) gives us the linearized transformation of the fields. Due to the constraints (14) satisfied by the currents, the fields coupling to them will have gauge symmetries. For instance, the field $V_{\mu}^i{}_j$ will have an SU(2) gauge symmetry $\delta V_{\mu}^i{}_j \sim \partial_{\mu} \Lambda^i{}_j$ because it couples to a conserved current (second line of (14)). As a consequence, we can neglect some pure gauge terms $\sim \bar{\epsilon}^i \partial_{\mu} \Lambda_j$ that will appear in the supersymmetry variation of $V_{\mu}^i{}_j$. Alternatively one can add terms proportional to $\partial^{\mu} v_{\mu}^j{}_i \epsilon_j$ (which is zero due to the relations (14) in the supersymmetry variation of λ_i in (15), which cancels against any pure gauge-like terms of the form shown above in the supersymmetry variation of $V_{\mu}^i{}_j$).

⁵If we choose different numerical factors in the coupling or take different linear combination of fields coupling to the currents, we will have a different choice of basis and normalization for the fields which will be related to our choice via field redefinition.

Analogously, due to the last line of (14), the fields $T_{\mu\nu}^-$ and $\tilde{A}_\mu$ that couple to the current $b_{\mu\nu}^-$ and $\tilde{a}_\mu$ have a complex vector gauge transformation as shown below.

$$\begin{aligned}\delta T_{\mu\nu}^- &= \partial_{[\mu}\Lambda_{\nu]} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\rho\Lambda^\sigma, \\ \delta\tilde{A}_\mu &= \Lambda_\mu.\end{aligned}\quad (17)$$

From the above equation, it is obvious that the field $\tilde{A}_\mu$ is a pure gauge under this complex vector gauge transformation and can be gauged away by using the gauge fixing condition $\tilde{A}_\mu = 0$. However, we need to compensate the supersymmetry transformation of $\tilde{A}_\mu$ by a field-dependent complex vector gauge transformation. This will only affect the supersymmetry transformation of $T_{\mu\nu}^-$. After doing the above set of exercises, we obtain the linearized transformation of the fields belonging to the linearized multiplet:

$$\begin{aligned}\delta C^{ijkl} &= -2\bar{\epsilon}^{(l}\partial\Xi^{ijk)} - 2\epsilon^{im}\epsilon^{jn}\epsilon^{kp}\epsilon^{lq}\bar{\epsilon}_{(m}\partial\Xi_{npq)}, \\ \delta\mathcal{D} &= -\frac{1}{4}\bar{\epsilon}_i\partial\chi^i + \text{H.c.}, \\ \delta h^{\mu\nu} &= 2\bar{\epsilon}_i\gamma^{(\mu}\psi^{\nu)i} + \text{H.c.}, \\ \delta\varphi &= -\frac{1}{2}\bar{\epsilon}_i\Lambda^i + \text{H.c.}, \\ \delta E_{ij} &= \frac{1}{3}\bar{\epsilon}^k\Xi_{ijk} - \frac{1}{2}\bar{\epsilon}^{(l}\zeta^k)\epsilon_{ik}\epsilon_{jl}, \\ \delta V_{\mu}{}^j{}_i &= -\bar{\epsilon}_i(\gamma_{\rho\sigma}\gamma_\mu + \gamma_\mu\gamma_{\rho\sigma})\partial^\sigma\psi^{\rho j} - 2\bar{\epsilon}_i\gamma_\mu\partial\Lambda^j \\ &\quad + \frac{3}{2}\bar{\epsilon}_i\gamma_\mu\zeta^j - (\text{H.c.}; \text{traceless}), \\ \delta S_{\mu}{}^j{}_i &= \bar{\epsilon}_i(\gamma_{\rho\sigma}\gamma_\mu + \gamma_\mu\gamma_{\rho\sigma})\partial^\sigma\psi^{\rho j} + 2\bar{\epsilon}_i\gamma_\mu\partial\Lambda^j - 2\bar{\epsilon}_i\partial_\mu\Lambda^j \\ &\quad - \bar{\epsilon}_i\gamma_\mu\zeta^j - \frac{1}{3}\epsilon^{lj}\epsilon^{nk}\bar{\epsilon}_n\gamma^\mu\Xi_{ilk} - (\text{H.c.}; \text{traceless}), \\ \delta T_{\mu\nu}^- &= \frac{3}{2}\bar{\epsilon}^i\gamma_{\mu\nu}\zeta^j\epsilon_{ij} - 2\bar{\epsilon}^i\gamma_{\mu\nu}\partial\Lambda^j\epsilon_{ij} + 8\epsilon_{ij}\bar{\epsilon}^i(\partial_{[\mu}\psi_{\nu]}^j)_{|A.S.D} \\ &\quad + 2\bar{\epsilon}^i\gamma_{\mu\nu}\gamma_{\rho\sigma}\partial^\rho\psi^{\sigma j}\epsilon_{ij}, \\ \delta A_\mu &= +\frac{i}{4}\bar{\epsilon}_i(\gamma_{\rho\sigma}\gamma_\mu + 3\gamma_\mu\gamma_{\rho\sigma})\partial^\sigma\psi^{\rho i} + \frac{3i}{4}\bar{\epsilon}_j\gamma_\mu\zeta^j \\ &\quad - i\bar{\epsilon}_j\gamma_\mu\partial\Lambda^j + \text{H.c.}, \\ \delta\psi^{\mu i} &= -\frac{1}{2}\gamma_{\lambda\nu}\epsilon^i\partial^\lambda h^{\mu\nu} + V^{\mu i}{}_j\epsilon^j + iA^\mu\epsilon^i \\ &\quad - \frac{1}{8}\epsilon^{ij}(\gamma\cdot T^-)\gamma^\mu\epsilon_j, \\ \delta\Lambda^i &= -2\partial\varphi\epsilon^i - \delta^i{}_j\epsilon^j - 2\epsilon^{ik}\epsilon^{jl}\epsilon_j E_{lk}, \\ \delta\zeta^i &= -\frac{4}{3}\partial\delta^i{}_j\epsilon^j - \frac{4}{3}\gamma^{\mu\nu}\partial_\mu V_{\nu}{}^i{}_j\epsilon^j + \frac{8}{3}\epsilon^{ik}\epsilon^{lj}\partial E_{lk}\epsilon_j - 4\mathcal{D}\epsilon^i, \\ \delta\Xi_{ijk} &= \frac{3}{2}\epsilon_{mn}\epsilon_{lp}[\partial_\mu S^{\mu l}{}_{(i}\delta_j^p\delta_k^p - 2\gamma^{\mu\nu}\partial_\mu S_{\nu}{}^l{}_{(i}\delta_j^p\delta_k^p \\ &\quad - 2\gamma^{\mu\nu}\partial_\mu V_{\nu}{}^l{}_{(i}\delta_j^p\delta_k^p)]\epsilon^m + 6\partial E_{(ij}\epsilon_{k)} - C_{ijkl}\epsilon^l.\end{aligned}\quad (18)$$

In the above variations, we observe that the derivative of the gravitino ψ_μ^i appears in antisymmetric form $\sim\partial_{[\mu}\psi_{\nu]}^i$, in the supersymmetry variation of some of the fields like $V_{\mu}{}^i{}_j$, $S_{\mu}{}^i{}_j$,

$T_{\mu\nu}^-$ and A_μ . This is important because derivatives of gauge fields should be traded off for the corresponding curvature $R(Q)$ in this case) and it can only happen if the derivatives appear in an antisymmetric form. But before trading off $\partial\psi$ in terms of $R(Q)$ in the above variations, we recall that the curvature $R(Q)$ will contain a term proportional to the S-gauge field ϕ_μ^i at the linearized level. This happens because the gravitino ψ_μ^i transforms under S-supersymmetry which is known to be $\delta_S\psi_\mu^i = -\gamma_\mu\eta^i$ from the superconformal algebra. Hence, the linearized form of the curvature $R(Q)_{\mu\nu}^i$, is given as

$$R(Q)_{\mu\nu}^i = 2\partial_{[\mu}\psi_{\nu]}^i - \gamma_{[\mu}\phi_{\nu]}^i, \quad (19)$$

where ϕ_μ^i is the gauge field corresponding to S-supersymmetry. Using (19) one can trade off the antisymmetric derivative on the gravitino for the curvature $R(Q)$ in the supersymmetry variation of the above-mentioned fields. However, we also need to impose the curvature constraint on $R(Q)$. The conventional constraint for $R(Q)$ (1) involves the standard Weyl multiplet field χ^i . However, at this point, since we have not yet identified the field χ^i , which would be some linear combination of the fields appearing in (18), it is difficult to implement the conventional constraint for the curvature $R(Q)$ from (1). We will instead impose the following unconventional constraint on $R(Q)$,⁶

$$\gamma^\mu R(Q)_{\mu\nu}^i = 0, \quad (20)$$

which, at the linearized level, gives

$$\gamma^{\mu\nu}\partial_\mu\psi_\nu^i = -\frac{3}{2}\gamma^\nu\phi_\nu^i. \quad (21)$$

We can see that, when we try to rewrite $\partial_{[\mu}\psi_{\nu]}^i$ in terms of $R(Q)$ and use the above constraint in the linearized transformation (18), the bare S-gauge field ϕ_μ^i will pop out as shown below.

$$\begin{aligned}\delta V_{\mu}{}^j{}_i &= 2\bar{\epsilon}_j\phi_\mu^i - 2\bar{\epsilon}_j\gamma_\mu\gamma\cdot\phi^i - 2\bar{\epsilon}_j\partial\Lambda^i + \frac{3}{2}\bar{\epsilon}_j\gamma_\mu\zeta^i \\ &\quad - (\text{H.c.}; \text{traceless}), \\ \delta A_\mu &= \frac{i}{2}\bar{\epsilon}_i\phi_\mu^i - i\bar{\epsilon}_j\gamma_\mu\gamma\cdot\phi^j + \frac{3i}{4}\bar{\epsilon}_j\gamma_\mu\zeta^j - i\bar{\epsilon}_j\gamma_\mu\partial\Lambda^j + \text{H.c.}, \\ \delta T_{\mu\nu}^- &= 4\epsilon_{ij}\bar{\epsilon}^i R(Q)_{\mu\nu}^j - 2\bar{\epsilon}^i\gamma_{\mu\nu}\partial\Lambda^j\epsilon_{ij} - 2\bar{\epsilon}^i\gamma_{\mu\nu}\gamma\cdot\phi^j\epsilon_{ij} \\ &\quad + \frac{3}{2}\bar{\epsilon}^i\gamma_{\mu\nu}\zeta^j\epsilon_{ij}, \\ \delta S_{\mu}{}^j{}_i &= 2\bar{\epsilon}_j\gamma_\mu\partial\Lambda^j - 2\bar{\epsilon}_i\partial_\mu\Lambda^j + 2\bar{\epsilon}_i\gamma_\mu\gamma\cdot\phi^j - 2\bar{\epsilon}_i\gamma_\mu\phi^j \\ &\quad - \bar{\epsilon}_i\gamma_\mu\zeta^j - \frac{1}{3}\epsilon^{lj}\epsilon^{nk}\bar{\epsilon}_n\gamma^\mu\Xi_{ilk} - (\text{H.c.}; \text{traceless}).\end{aligned}\quad (22)$$

⁶The conventional and the unconventional constraints are related by a shift in the S-gauge field by a term proportional to the auxiliary fermion χ^i . Once we correctly identify the χ^i , we can go back to the conventional constraint by making an appropriate shift in the S-gauge field.

Among the fields that appear above in (22), the fields like S_{μ}^i and $T_{\mu\nu}$ do not couple to any conserved current, and as a result they will not correspond to any gauge fields of the multiplet. Rather they will be the auxiliary fields. Such fields should not have any bare S-gauge field in their transformation rules, which should be either be inside a covariant derivative or a curvature. Hence, terms involving bare S-gauge field that appears in (22) for $T_{\mu\nu}$ and S_{μ}^i , should be absorbed into the covariant derivative of Λ^i . This can be done, if we let Λ^i to transform under S-supersymmetry as

$$\delta_S \Lambda^i = -2\eta^i. \quad (23)$$

As a consequence, the covariant derivative of Λ^i at the linearized level is as shown below:

$$D_{\mu} \Lambda^i = \partial_{\mu} \Lambda^i + \phi_{\mu}^i. \quad (24)$$

In terms of the covariant derivative on Λ^i , the supersymmetry variation (22) takes the following form:

$$\begin{aligned} \delta V_{\mu}^i &= 2\bar{\epsilon}_j \phi_{\mu}^i - 2\bar{\epsilon}_j \not{D} \Lambda^i + \frac{3}{2} \bar{\epsilon}_j \gamma_{\mu} \zeta^i - (\text{H.c.}; \text{traceless}), \\ \delta A_{\mu} &= \frac{i}{2} \bar{\epsilon}_j \phi_{\mu}^i + \frac{3i}{4} \bar{\epsilon}_j \gamma_{\mu} \zeta^i - i\bar{\epsilon}_j \gamma_{\mu} \not{D} \Lambda^i + \text{H.c.}, \\ \delta T_{\mu\nu} &= 4\epsilon_{ij} \bar{\epsilon}^i R(Q)^j_{\mu\nu} - 2\bar{\epsilon}^i \gamma_{\mu\nu} \not{D} \Lambda^j \epsilon_{ij} + \frac{3}{2} \bar{\epsilon}^i \gamma_{\mu\nu} \zeta^j \epsilon_{ij}, \\ \delta S_{\mu}^i &= 2\bar{\epsilon}_i \gamma_{\mu} \not{D} \Lambda^i - 2\bar{\epsilon}_i D_{\mu} \Lambda^i - \bar{\epsilon}_i \gamma_{\mu} \zeta^i \\ &\quad - \frac{1}{3} \epsilon^{lj} \epsilon^{nk} \bar{\epsilon}_n \gamma^{\mu} \Xi_{ilk} - (\text{H.c.}; \text{traceless}). \end{aligned} \quad (25)$$

By looking at the transformation of the R-symmetry gauge fields V_{μ}^i and A_{μ} , we find that the standard Weyl multiplet χ^i can be identified with the following linear combination at the linearized level.⁷

$$\chi^i = -\frac{3}{4} \zeta^i + \not{D} \Lambda^i. \quad (26)$$

By making a supersymmetry transformation on the above, we find that the following linear combination is identified with the standard-Weyl multiplet field “D” at the linearized level.

$$D = 3\mathcal{D} - 2\partial^2 \varphi. \quad (27)$$

Finally, we also redefine the scalar field φ as⁸

$$\phi = e^{\varphi}. \quad (28)$$

Having identified the standard-Weyl multiplet field χ^i , we would prefer going back to the conventional constraint by making a field redefinition on the S-gauge field:

⁷Here, we need to keep in mind that we are working with the unconventional constraint and hence the variation of V_{μ}^i and A_{μ} should be compared with the results known in the literature after shifting the S-gauge field so that we can correctly identify the χ field (see for instance [10]).

⁸The redefinitions are redundant at the linearized level, but important at the nonlinear level.

TABLE II. Weights and chiralities of the real scalar multiplet.

Field	SU(2) Irreps	Weyl weight (w)	Chiral weight (c)	Chirality (Fermions)
ϕ	1	1	0	...
S_a^i	3	1	0	...
E_{ij}	3	1	-1	...
C_{ijkl}	5	2	0	...
Λ_i	2	1/2	1/2	+1
Ξ_{ijk}	4	3/2	-1/2	+1

$\phi_{\mu}^i \rightarrow \phi_{\mu}^i - \frac{1}{2} \gamma_{\mu} \chi^i$. In terms of the above redefined fields [(26)–(28) as well as the redefined S-gauge field], we see that the linearized components $h_{\mu\nu}$, V_{μ}^i , A_{μ} , $T_{\mu\nu}$, D , ψ_{μ}^i , χ^i decouple from the rest and they transform, under Q-supersymmetry, exactly like the linearization of the standard Weyl multiplet known in the literature in the conventional constraints. The full supersymmetry transformation of the above fields including the S-supersymmetry transformation follows from the superconformal algebra (2) and is already known. The remaining 24 + 24 components ϕ , E_{ij} , S_{μ}^i , C_{ijkl} , Λ^i , Ξ_{ijk} transform as given below.

$$\begin{aligned} \delta \phi &= -\frac{\phi}{2} \bar{\epsilon}^i \Lambda_i + \text{H.c.}, \\ \delta \Lambda^i &= -2\bar{P} \epsilon^i - (\delta^i_j \epsilon^j + 2\epsilon^{ik} \epsilon^{jl} \epsilon_j E_{lk}) - 2\eta^i, \\ \delta S_a^i &= \bar{\epsilon}_j \gamma_a \chi^i + \frac{2}{3} \bar{\epsilon}_j \gamma_a \not{D} \Lambda^i - 2\bar{\epsilon}_j D_a \Lambda^i \\ &\quad - \frac{1}{3} \epsilon^{li} \epsilon^{nk} \bar{\epsilon}_n \gamma_a \Xi_{ljk} - (\text{H.c.}; \text{traceless}), \\ \delta E_{ij} &= 2\bar{\epsilon}^{(l} \chi^{k)} \epsilon_{ik} \epsilon_{jl} - \frac{2}{3} \bar{\epsilon}^{(l} \not{D} \Lambda^{k)} \epsilon_{ik} \epsilon_{jl} + \frac{1}{3} \bar{\epsilon}^k \Xi_{ijk}, \\ \delta \Xi_{ijk} &= \frac{3}{2} \epsilon_{mn} \epsilon_{lp} [\partial_{\mu} S^{\mu l} (\delta_j^n \delta_k^p - 2\gamma^{\mu\nu} \partial_{\mu} S_{\nu}^l (\delta_j^n \delta_k^p \\ &\quad - \gamma \cdot R(V)^l_{(i} \delta_j^n \delta_k^p) \epsilon^m + 6\partial E_{(ij} \epsilon_{k)} - C_{ijkl} \epsilon^l, \\ \delta C^{ijkl} &= -2\bar{\epsilon}^{(l} \partial \Xi^{ijk)} - 2\epsilon^{im} \epsilon^{jn} \epsilon^{kp} \epsilon^{lq} \bar{\epsilon}_{(m} \partial \Xi_{npq)}. \end{aligned} \quad (29)$$

Here, we have defined $P_a = \phi^{-1} \partial_a \phi$ and $R(V)_{\mu\nu}^i = 2\partial_{[\mu} V_{\nu]}^i$. The above result (29) is a linearization of our final result (31).⁹ The above linearized transformation gives us the Weyl and chiral weights of the field (See Table II). Because of its S-SUSY transformation, Λ^i will have the same chiral and Weyl weight as η^i . Further, we obtain the weights of E_{ij} , S_a^i , C_{ijkl} and Ξ_{ijk} by looking at the above SUSY transformation (29) and the knowledge of the weights of the Q-SUSY parameter ϵ^i . The chiral weight of ϕ has to be zero because it is a real scalar field while its Weyl weight is obtained by acting the commutator $[\delta_S, \delta_Q]$ on it. We present a schematic Table II with the full field content of the multiplet, along with their SU(2) irreps, Weyl and chiral weights as well as chirality for fermions. The chirality and the SU(2) irreps of the fields are

⁹We have partial nonlinear results (see $\delta \phi$ and first term of $\delta \Lambda^i$) because we worked with the field ϕ instead of φ which are related by an exponentiation as given in (28).

obvious from the currents they couple to. In addition, the fields $S_a^i{}_j$ and C_{ijkl} satisfy the following reality conditions which again follow from the reality conditions satisfied by the corresponding currents.

$$(S_a^i{}_j)^* \equiv S_{a i}^j = -S_a^j{}_i, \\ (C_{ijkl})^* \equiv C^{ijkl} = \varepsilon^{im}\varepsilon^{jn}\varepsilon^{kp}\varepsilon^{lq}C_{mnpq}. \quad (30)$$

In the next section, we will show the fully nonlinear result (31), obtained by applying the supersymmetry algebra (2) on the linearized result (29). We will also assume, without loss of generality, that the S-transformation of $S_a^i{}_j$, E_{ij} , C_{ijkl} and

Ξ_{ijk} is zero. This is because, given its S-SUSY transformation, Λ^i can be straightforwardly used as a compensator to give S-SUSY invariant fields.

V. THE REAL SCALAR MULTIPLY

Completing the linearized analysis of the previous section gives us the full supersymmetry transformation of the real scalar multiplet. All the field components are invariant under special conformal transformation,¹⁰ and they transform under Q- and S-supersymmetry as shown below.¹¹

$$\begin{aligned} \delta\phi &= -\frac{\phi}{2}\bar{\epsilon}^i\Lambda_i + \text{H.c.}, \\ \delta\Lambda^i &= -2\bar{P}\epsilon^i - (\delta^i{}_j\epsilon^j + 2\varepsilon^{ik}\varepsilon^{jl}\epsilon_j E_{lk}) - \frac{1}{2}\bar{\Lambda}^i\Lambda^j\epsilon_j - \frac{1}{4}\bar{\Lambda}^j\gamma_a\Lambda_j\gamma^a\epsilon^i + \frac{1}{8}\bar{\Lambda}^i\gamma_{ab}\Lambda^j\gamma^{ab}\epsilon_j - 2\eta^i, \\ \delta S_a^i{}_j &= \bar{\epsilon}_j\gamma_a\chi^i + \frac{2}{3}\bar{\epsilon}_j\gamma_a\bar{D}\Lambda^i - 2\bar{\epsilon}_j D_a\Lambda^i - \frac{1}{3}\varepsilon^{li}\varepsilon^{nk}\bar{\epsilon}_n\gamma_a\Xi_{ljk} + \frac{1}{24}\bar{\epsilon}_j\gamma_a\gamma.T^-\Lambda_k\varepsilon^{ik} - \frac{1}{3}\bar{\epsilon}_j\gamma_a\Lambda_k E_{lm}\varepsilon^{il}\varepsilon^{km} - \frac{2}{3}\bar{\epsilon}^i\gamma_a\delta^k{}_j\Lambda_k \\ &\quad - \frac{1}{2}\bar{\epsilon}^i\delta^k{}_j\gamma_a\Lambda_k + \frac{1}{2}\bar{\epsilon}^k\gamma_a\delta^i{}_j\Lambda_k - \frac{2}{3}\bar{\epsilon}^i\gamma_a\bar{P}\Lambda_j - \bar{\epsilon}^i\bar{P}\gamma_a\Lambda_j - \frac{1}{24}\bar{\Lambda}^i\Lambda^k\bar{\epsilon}_j\gamma_a\Lambda_k - \frac{1}{32}\bar{\Lambda}^i\gamma_{bc}\Lambda^k\bar{\epsilon}_j\gamma^{bc}\gamma_a\Lambda_k - (\text{H.c.}; \text{traceless}), \\ \delta E_{ij} &= 2\bar{\epsilon}^{(l}\chi^{k)}\varepsilon_{ik}\varepsilon_{jl} - \frac{2}{3}\bar{\epsilon}^{(l}\bar{D}\Lambda^{k)}\varepsilon_{ik}\varepsilon_{jl} + \frac{1}{3}\bar{\epsilon}^k\Xi_{ijk} - \frac{1}{12}\bar{\epsilon}^k\gamma.T^-\Lambda_{(i}\varepsilon_{j)k} + \frac{2}{3}\bar{\epsilon}^k\Lambda_{(i}E_{j)k} - 2\bar{\epsilon}_{(i}\Lambda^k E_{j)k} - \frac{2}{3}\bar{\epsilon}^k\Lambda_k E_{ij} \\ &\quad + \bar{\epsilon}_k\Lambda^k E_{ij} - \frac{1}{3}\bar{\epsilon}^{(l}\delta^{m)}{}_k\Lambda^k\varepsilon_{il}\varepsilon_{jm} - \frac{2}{3}\bar{\epsilon}^{(k}\bar{P}\Lambda^{l)}\varepsilon_{ik}\varepsilon_{jl} - \frac{1}{12}\bar{\epsilon}^{(l}\gamma_a\Lambda^{k)}\bar{\Lambda}^m\gamma^a\Lambda_m\varepsilon_{il}\varepsilon_{jk}, \\ \delta\Xi_{ijk} &= \frac{3}{2}\epsilon_{mn}\epsilon_{lp}[D_a S^{al}{}_{(i}\delta_j^n\delta_k^p - 2\gamma^{ab}D_a S_b{}^l{}_{(i}\delta_j^n\delta_k^p) - \gamma.R(V)^l{}_{(i}\delta_j^n\delta_k^p)]\epsilon^m + 6\bar{D}E_{(ij}\epsilon_{k)} - C_{ijkl}\epsilon^l - 6E^{ln}E_{m(i}\varepsilon_{j|l}\varepsilon_{k)n}\epsilon^m \\ &\quad - 6\bar{P}E_{(ij}\epsilon_{k)} + 3\delta^m{}_{(i}E_{j)k}\epsilon_m - 6\delta^m{}_{(i}E_{j|m}\epsilon_{k)} + 3S^{am}{}_{(i}S^{bn}{}_{j}\epsilon_{k)m}\epsilon_{ln}\gamma_{ab}\epsilon^l + 3P^a S_a{}^l{}_{(i}\varepsilon_{j|l}\varepsilon_{k)m}\epsilon^m \\ &\quad - \frac{3}{4}\gamma.T^-E^{lm}\epsilon^n\varepsilon_{(i|l}\varepsilon_{j|m}\varepsilon_{k)n} + \bar{\Lambda}^l\gamma_a\bar{D}\Lambda^m\gamma^a\epsilon_{(i}\varepsilon_{j|l}\varepsilon_{k)m} + \frac{1}{4}\bar{\Lambda}^l\bar{D}\Lambda_{(i}\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} - \frac{3}{8}\bar{\Lambda}^l\bar{D}\gamma_{ab}\Lambda_{(i}\gamma^{ab}\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} \\ &\quad - \frac{1}{4}\bar{\Lambda}_{(i}\bar{D}\Lambda^{lm}\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} + \frac{1}{8}\bar{\Lambda}_{(i}\gamma_{ab}\bar{D}\Lambda^{ab}\gamma^m\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} - \frac{3}{2}\bar{\Lambda}_{(i}R(Q)_{ab}\gamma^{ab}\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} + \frac{1}{2}\bar{\Lambda}^p\Xi^{lmn}\epsilon^q\varepsilon_{il}\varepsilon_{jm}\varepsilon_{kn}\epsilon_{pq} \\ &\quad - \frac{1}{2}\bar{\Lambda}^{(m}\Xi^{np)l}\epsilon^q\varepsilon_{im}\varepsilon_{jn}\varepsilon_{kp}\varepsilon_{lq} - \frac{1}{8}\bar{\Lambda}_l\gamma_{ab}\Xi_{ijk}\gamma^{ab}\epsilon^l - \bar{\Lambda}_{(i}\Xi_{jk)l}\epsilon^l + \frac{1}{8}\bar{\Lambda}_{(i}\gamma_{ab}\Xi_{jk)l}\gamma^{ab}\epsilon^l - \frac{1}{2}\bar{\Lambda}^l\gamma_a\Xi_{ijk}\gamma^a\epsilon_l + \bar{\Lambda}^l\gamma_a\Xi_{l(ij}\gamma^a\epsilon_{k)} \\ &\quad - \frac{3}{2}\bar{\Lambda}_{(i}\chi^{lm}\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} - \frac{3}{2}\bar{\Lambda}_{(i}\gamma_{ab}\chi^{lm}\gamma^{ab}\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} + \frac{3}{2}\bar{\Lambda}^l\chi_{(i}\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} - 3\bar{\Lambda}^l\gamma_a\chi^m\gamma^a\epsilon_{(i}\varepsilon_{j|l}\varepsilon_{k)m} + \frac{1}{4}\varepsilon_{l(k}\bar{\Lambda}_{i)}\Lambda_{j)}\gamma.T^-\epsilon^l \\ &\quad - \frac{1}{8}\varepsilon_{l(k}\bar{\Lambda}_{i)}\gamma_a\Lambda_{j)}\gamma.T^-\gamma^a\epsilon_j + \frac{1}{2}\bar{\Lambda}^{(l}\Lambda^m E^{n)p}\epsilon^q\varepsilon_{il}\varepsilon_{jm}\varepsilon_{kn}\epsilon_{pq} - \frac{1}{2}\bar{\Lambda}^p\Lambda^{(l}E^{mn)}\epsilon^q\varepsilon_{il}\varepsilon_{jm}\varepsilon_{kn}\epsilon_{pq} + \frac{1}{2}\bar{\Lambda}_l\Lambda_{(i}E_{j)k}\epsilon^l \\ &\quad - \frac{1}{2}\bar{\Lambda}_{(i}\Lambda_{j)}E_{k)l}\epsilon^l + \frac{1}{4}\bar{\Lambda}_l\gamma_{ab}\Lambda_{(i}E_{j)k}\gamma^{ab}\epsilon^l + \frac{1}{4}\bar{\Lambda}^l\gamma_a\Lambda_l\gamma^a\epsilon_{(i}E_{j)k} - \bar{\Lambda}^l\gamma_a\Lambda_{(i}\gamma^a\epsilon_{j)k} - \frac{1}{2}\bar{\Lambda}^l\delta^n{}_l\Lambda_{(i}\epsilon^m\varepsilon_{j|n}\varepsilon_{k)m} \\ &\quad + \frac{1}{4}\varepsilon_{(i|n}\varepsilon_{j|m}\bar{\Lambda}^l\delta^n{}_k\Lambda_l\epsilon^m - \frac{1}{8}\bar{\Lambda}^l\delta^n{}_l\gamma_{ab}\Lambda_{(i}\gamma^{ab}\epsilon^m\varepsilon_{j|n}\varepsilon_{k)m} + \frac{3}{16}\varepsilon_{(i|n}\varepsilon_{j|m}\bar{\Lambda}^l\delta^n{}_k\gamma^{ab}\Lambda_l\gamma_{ab}\epsilon^m \\ &\quad + \frac{1}{2}\bar{\Lambda}^n\Lambda^m\delta^l{}_{(i}\varepsilon_{j)k}\epsilon_{nl} + \frac{1}{2}\bar{\Lambda}^l\bar{P}\Lambda_{(i}\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} + \frac{1}{16}\bar{\Lambda}^l\gamma_{ab}\Lambda^m\delta^n{}_{(i}\gamma^{ab}\epsilon_{j)k}\epsilon_{lm} + \frac{1}{2}\bar{\Lambda}^l\bar{P}\gamma_{ab}\Lambda_{(i}\gamma^{ab}\epsilon^m\varepsilon_{j|l}\varepsilon_{k)m} \\ &\quad + \bar{\Lambda}^l\Lambda^m\bar{P}\epsilon_{(i}\varepsilon_{j|l}\varepsilon_{k)m} + \frac{1}{8}\bar{\Lambda}^m\Lambda^n\bar{\Lambda}^l\gamma_a\Lambda_l\gamma^a\epsilon_{(i}\varepsilon_{j|m}\varepsilon_{k)n} - \frac{1}{16}\bar{\Lambda}^l\Lambda^m\bar{\Lambda}_n\gamma_{ab}\Lambda_{(i}\gamma^{ab}\epsilon^n\varepsilon_{j|l}\varepsilon_{k)m}, \\ \delta C_{ijkl} &= \bar{\epsilon}_{(i}\Gamma_{jkl)} + \varepsilon_{ip}\varepsilon_{jq}\varepsilon_{kr}\varepsilon_{ls}\bar{\epsilon}^{(p}\Gamma^{qrs)} + (\bar{\epsilon}_m\Lambda^m + \bar{\epsilon}^m\Lambda_m)C_{ijkl}. \end{aligned} \quad (31)$$

¹⁰This follows in a straightforward way from the $[\delta_S, \delta_S]$ commutators (2).

¹¹We use the usual conventions for symmetrized nonconsecutive indices, i.e. the indices that are not symmetrized are surrounded by vertical lines. For instance, we will write a multi-indices tensor product $A_i B_{jkl}$, symmetrized in (ijl) as $A_{(i} B_{j|k|l)}$.

In the above, the composite field $P_a \equiv \phi^{-1} D_a \phi$ and Γ_{ijk} is defined as

$$\begin{aligned}
\Gamma_{ijk} = & -2\bar{D}\Xi_{ijk} - 3D_a S^{an} (i\Lambda^m \epsilon_{j|n|} \epsilon_{k|m}) + 6\bar{D}E_{(ij}\Lambda_{k)} - 2C_{ijkl}\Lambda^l + 2\bar{D}\Lambda_{(i}E_{jk)} + 2D_a \Lambda^l S^{an} (i\epsilon_{j|n|} \epsilon_{k|l}) - 2\gamma^{ab} D_a \Lambda^l S_b^n (i\epsilon_{j|n|} \epsilon_{k|l}) \\
& - 4\Xi^{lmn} E_{l(i} \epsilon_{j|m|} \epsilon_{k)n} + 2\mathcal{S}^l (i\Xi_{jk})_l + 12\chi_{(i} E_{jk)} - 6\mathcal{S}^l (i\chi^m \epsilon_{j|l|} \epsilon_{k|m}) - 4\bar{P}E_{(ij}\Lambda_{k)} - 4P_a S^{an} (i\epsilon_{j|n|} \epsilon_{k|l}) \Lambda^l - 2P_a \gamma^{ab} S_b^n (i\epsilon_{j|n|} \epsilon_{k|l}) \Lambda^l \\
& - 2\mathcal{S}^l (iE_{jk}) \Lambda_l - 2\mathcal{S}^l (iE_{j|l|} \Lambda_k) - 4E_{(ij} \epsilon_{k)m} \epsilon_{ln} E^{mn} \Lambda^l + 12\epsilon_{(i|m|} \epsilon_{j|n|} E_{k)l} E^{mn} \Lambda^l + S_a^m (iS^{an} \epsilon_{j|m|} \epsilon_{k|n}) \epsilon_{ln} \Lambda^l \\
& + \gamma_{ab} S^{am} (iS^{bn} \epsilon_{k|m} \epsilon_{ln} \Lambda^l + \frac{1}{2} \gamma \cdot T^+ E_{(ij} \epsilon_{k)l} \Lambda^l + \frac{1}{4} \mathcal{S}^l (i\gamma \cdot T^- \Lambda_j \epsilon_{k|l}) + \frac{1}{2} \bar{\Lambda}^l \Lambda^m \bar{D} \Lambda_{(i} \epsilon_{j|l|} \epsilon_{k|m}) + \frac{5}{4} \bar{\Lambda}^l \gamma^a \Lambda_{(i} \epsilon_{j|l|} \epsilon_{k|m}) D_a \Lambda^m \\
& + \frac{5}{16} \bar{\Lambda}^l \gamma^{bc} \gamma^a \Lambda_{(i} \epsilon_{j|l|} \epsilon_{k|m}) \gamma_{bc} D_a \Lambda^m + \frac{3}{2} \bar{\Lambda}^l \Lambda^m \chi_{(i} \epsilon_{j|l|} \epsilon_{k|m}) - \frac{3}{2} \bar{\Lambda}^l \gamma^a \Lambda_{(i} \epsilon_{j|l|} \epsilon_{k|m}) \gamma_a \chi^m + \frac{1}{2} \bar{\Lambda}^q \Lambda^{(p} \Xi^{m)l} \epsilon_{ip} \epsilon_{jm} \epsilon_{kl} \epsilon_{qn} \\
& + \frac{1}{2} \bar{\Lambda}^l \gamma_a \Lambda_{(i} \gamma^a \Xi_{jk)l} + \frac{3}{2} \bar{\Lambda}^l \Lambda^m \bar{P} \Lambda_{(i} \epsilon_{j|l|} \epsilon_{k|m}) - \frac{1}{4} \bar{\Lambda}^l \Lambda^m \mathcal{S}^n (i\epsilon_{j|n|} \epsilon_{k|m}) \Lambda_l - \frac{1}{32} \epsilon_{mn} \bar{\Lambda}^m \gamma^{ab} \Lambda^n \gamma_{ab} \mathcal{S}^l (i\Lambda_j \epsilon_{k|l}) \\
& - \frac{1}{2} \bar{\Lambda}^l \Lambda^{(m} E^{pq)} \Lambda^n \epsilon_{ip} \epsilon_{jq} \epsilon_{km} \epsilon_{ln} - \bar{\Lambda}_{(i} \Lambda_j E_{k)l} \Lambda^l - \bar{\Lambda}_l \Lambda_{(i} E_{jk)} \Lambda^l + \frac{1}{8} \bar{\Lambda}^l \Lambda^m \bar{\Lambda}^n \gamma_a \Lambda_n \gamma^a \Lambda_{(i} \epsilon_{j|l|} \epsilon_{k|m}). \tag{32}
\end{aligned}$$

VI. FIELD REDEFINITIONS AND SIMPLIFYING THE SUPERSYMMETRY TRANSFORMATION LAWS

The supersymmetry transformation of the multiplet presented in the previous section has the advantage of the fields (with the exception of Λ_i) being S-invariant, which can sometimes be useful. However, the Q-supersymmetry transformations turn out to be extremely complicated and nonlinear. In this section, we will present a field redefinition in which the supersymmetry transformation becomes simpler and linear with the only nonlinear terms arising from the background Weyl multiplet. The field redefinitions that we find are

$$\begin{aligned}
V &= \phi^{-2}, \\
\psi^i &= \phi^{-2} \Lambda^i, \\
K^{ij} &= \phi^{-2} \bar{E}^{ij} + \frac{1}{2} \phi^{-2} \bar{\Lambda}^i \Lambda^j, \\
A_a^i{}_j &= \phi^{-2} S_a^i{}_j - \frac{1}{2} \phi^{-2} \left[\bar{\Lambda}^i \gamma_a \Lambda_j - \frac{1}{2} \delta_j^i \bar{\Lambda}^m \gamma_a \Lambda_m \right], \\
\xi_{ijk} &= \phi^{-2} \Xi_{ijk} + 3\phi^{-2} \Lambda_{(i} E_{jk)} + 3\phi^{-2} \mathcal{S}^m (i\Lambda^n \epsilon_{j|m|} \epsilon_{k|n}) - \frac{3}{4} \phi^{-2} \bar{\Lambda}^m \gamma^a \Lambda_{(i} \gamma_a \Lambda^n \epsilon_{j|m|} \epsilon_{k|n}), \\
C_{ijkl} &= \phi^{-2} C_{ijkl} + 12\phi^{-2} E_{(ij} \bar{E}_{kl)} + 2\phi^{-2} \bar{\Lambda}_{(i} \Xi_{jkl)} + 3\phi^{-2} S^n_{(i} \cdot S^m_{j} \epsilon_{k|m|} \epsilon_{l)n} + 3\phi^{-2} \bar{\Lambda}_{(i} \Lambda_j E_{kl)} + 3\phi^{-2} \epsilon_{im} \epsilon_{jn} \epsilon_{kp} \epsilon_{lq} \bar{\Lambda}^{(m} \Lambda^n E^{pq)} \\
& + 2\phi^{-2} \epsilon_{im} \epsilon_{jn} \epsilon_{kp} \epsilon_{lq} \bar{\Lambda}^{(m} \Xi^{npq)} - 6\phi^{-2} \bar{\Lambda}^n \mathcal{S}^p (i\Lambda_j \epsilon_{k|n|} \epsilon_{l)p} + \frac{3}{2} \bar{\Lambda}_{(i} \Lambda_j \bar{\Lambda}^n \Lambda^p \epsilon_{k|n|} \epsilon_{l)p}. \tag{33}
\end{aligned}$$

Here, we have defined:

$$\bar{E}^{ij} = \epsilon^{ik} \epsilon^{jl} E_{kl}. \tag{34}$$

The above redefined fields have the same reality properties, SU(2) irreps, chiral weight and chirality as the corresponding old fields. But the Weyl weight is shifted to a negative value as shown in Table III because of the multiplication by an overall ϕ^{-2} . The Q and S-supersymmetry transformation of the multiplet in terms of the redefined fields take the following simple form:

TABLE III. Weights and chiralities of the field-redefined real scalar multiplet.

Field	SU(2) Irreps	Weyl weight (w)	Chiral weight (c)	Chirality (Fermions)
V	1	-2	0	...
$A_a^i{}_j$	3	-1	0	...
K^{ij}	3	-1	-1	...
C_{ijkl}	5	0	0	...
Λ_i	2	-3/2	1/2	+1
ξ_{ijk}	4	-1/2	-1/2	+1

$$\begin{aligned}
\delta V &= \bar{\epsilon}_k \psi^k + \text{H.c.}, \\
\delta \psi^i &= \bar{D} V \epsilon^i - \mathcal{A}^i_j \epsilon^j - 2K^{ij} \epsilon_j - 2V \eta^i, \\
\delta K^{ij} &= 2V \bar{\epsilon}^{(i} \chi^{j)} - \frac{2}{3} \bar{\epsilon}^{(i} \bar{D} \psi^{j)} + \frac{1}{3} \bar{\epsilon}^k \xi_{lmk} \epsilon^{il} \epsilon^{jm} + \frac{1}{12} \bar{\epsilon}^{(i} \gamma \cdot T^- \psi_l \epsilon^{j)l} - 2\bar{\eta}^{(i} \psi^{j)}, \\
\delta A_a^i{}_j &= V \bar{\epsilon}_j \gamma_a \chi^i + \frac{2}{3} \bar{\epsilon}_j \gamma_a \bar{D} \psi^i - 2\bar{\epsilon}_j D_a \psi^i - \frac{1}{3} \epsilon^{li} \epsilon^{nk} \bar{\epsilon}_n \gamma_a \xi_{ljk} + \frac{1}{24} \bar{\epsilon}_j \gamma_a \gamma \cdot T^- \psi_k \epsilon^{ik} - \bar{\eta}_j \gamma_a \psi^i - (\text{H.c.}; \text{traceless}), \\
\delta \xi_{ijk} &= \frac{3}{2} D_a A^{al}{}_{(i} \epsilon_{j|m} \epsilon_{k)l} \epsilon^m - 3D_a A_b{}^l{}_{(i} \epsilon_{j|m} \epsilon_{k)l} \gamma^{ab} \epsilon^m - \frac{3}{2} V \gamma \cdot R(V)^l{}_{(i} \epsilon_{j|m} \epsilon_{k)l} \epsilon^m + 6\bar{D} K^{lm} \epsilon_{(i} \epsilon_{j|l} \epsilon_{k)m} - C_{ijkl} \epsilon^l \\
&\quad - \frac{3}{4} \gamma \cdot T^- \epsilon^n K_{(ij} \epsilon_{k)n} - \frac{3}{2} \bar{R}(Q)^l{}_{ab} \psi_{(i} \gamma^{ab} \epsilon^m \epsilon_{j|l} \epsilon_{k)m} - \frac{3}{2} \bar{\chi}^l \psi_{(i} \epsilon^m \epsilon_{j|l} \epsilon_{k)m} + \frac{3}{2} \bar{\chi}^l \gamma_{ab} \psi_{(i} \gamma^{ab} \epsilon^m \epsilon_{j|l} \epsilon_{k)m} \\
&\quad + \frac{3}{2} \bar{\chi}_{(i} \psi^l \epsilon^m \epsilon_{j|l} \epsilon_{k)m} + 3\bar{\chi}^m \gamma_a \psi^l \gamma^a \epsilon_{(i} \epsilon_{j|l} \epsilon_{k)m} - 6K^{nm} \eta_{(i} \epsilon_{j|n} \epsilon_{k)m} - 6\mathcal{A}^m{}_{(i} \eta^n \epsilon_{j|m} \epsilon_{k)n}, \\
\delta C_{ijkl} &= \bar{\epsilon}_{(i} \tilde{\Gamma}_{jkl)} + \epsilon_{im} \epsilon_{jn} \epsilon_{kp} \epsilon_{lq} \bar{\epsilon}^{(m} \tilde{\Gamma}^{npq)} - 4\bar{\eta}_{(i} \xi_{jkl)},
\end{aligned} \tag{35}$$

where

$$\tilde{\Gamma}_{ijk} = -2\bar{D} \xi_{ijk} + 12\chi_{(i} K^{lm} \epsilon_{j|l} \epsilon_{k)m}. \tag{36}$$

We can see that upon switching off the background Weyl multiplet fields, we recover the supersymmetry transformation of the multiplet presented in [12], which was introduced in flat superspace as a real scalar superfield V satisfying the superspace constraints:

$$D_{\alpha\beta} V = [D_{\alpha}^i, \bar{D}_{\dot{\alpha}i}] V = 0. \tag{37}$$

Thus, the multiplet that we have found is a generalization of the flat-space multiplet of [12], to conformal supergravity background.

VII. CONSTRAINING THE MULTIPLT

In this section, we will show how to consistently constrain the real scalar multiplet to obtain a restricted multiplet with $8 + 8$ components. For this purpose, we use the fields of Table II. This discussion is in line with the one on $N = 2$ chiral multiplet in the literature [24]. The chiral multiplet of $N = 2$ supergravity is a large multiplet with $16 + 16$ components. It is known that a chiral multiplet of Weyl weight $w = 1$ for the lowest component can be consistently reduced by imposing a reality constraint on an $SU(2)$ triplet of fields B_{ij} belonging to the chiral multiplet. Analogously, the $24 + 24$ component real scalar multiplet presented in this paper can be reduced to an $8 + 8$ matter multiplet, by imposing a reality constraint on the field E_{ij} . However, since E_{ij} has a nontrivial chiral weight $c = -1$ (as opposed to B_{ij} of chiral multiplet which has $c = 0$ if $w = 1$ for the lowest component), we can only constrain it to a “real” field up to an overall phase factor, which carries the chiral weight, i.e we can constrain E_{ij} to take the following form:

$$E_{ij} = e^{-i\sigma/2} \mathcal{L}_{ij}, \tag{38}$$

where $\mathcal{L}_{ij}$ satisfies the reality constraint:

$$\mathcal{L}^{ij} = \epsilon^{ik} \epsilon^{jl} \mathcal{L}_{kl}. \tag{39}$$

Trivially, under chiral transformation, the phase σ transforms as:

$$\delta_{U(1)}(\Lambda) \sigma = 2\Lambda. \tag{40}$$

As a consequence of (38), E_{ij} and its complex conjugate $\bar{E}_{ij} \equiv \epsilon_{ik} \epsilon_{jl} E^{kl}$ are related by the constraint:

$$\mathcal{R}_{ij} := \bar{E}_{ij} - e^{i\sigma} E_{ij} = 0. \tag{41}$$

The above constraint can be taken as the starting point of a procedure to obtain the full set of constraints by applying supersymmetry. For the supersymmetry transformation of $\mathcal{R}_{ij}$, we get:

$$\begin{aligned}
\delta \mathcal{R}_{ij} &= \bar{\epsilon}_{(i} \Theta_{j)} + \bar{\epsilon}_k \Upsilon^{lmk} \epsilon_{il} \epsilon_{jm} \\
&\quad - e^{-i\sigma} \epsilon_{ik} \epsilon_{jl} (\bar{\epsilon}^{(k} \Theta^{l)} + \bar{\epsilon}^m \Upsilon_{mnp} \epsilon^{kn} \epsilon^{lp}),
\end{aligned} \tag{42}$$

with

$$\begin{aligned}
\Theta_j &= 2\chi_j - \frac{2}{3} \bar{D} \Lambda_j + \frac{1}{12} \gamma \cdot T^+ \Lambda^l \epsilon_{jl} + \frac{2}{3} \Lambda^l \bar{E}_{jl} + \frac{1}{3} \mathcal{G}^k{}_j \Lambda_k \\
&\quad - \frac{2}{3} \mathcal{P} \Lambda_j + \frac{1}{12} \bar{\Lambda}^m \gamma_a \Lambda_m \gamma^a \Lambda_j - \frac{2i}{3} \zeta^l \bar{E}_{jl}, \\
\Upsilon^{lmk} &= \frac{1}{3} \Xi^{lmk} - \Lambda^{(l} E^{mk)} - i\zeta^{(l} E^{mk)}.
\end{aligned} \tag{43}$$

Here, we have defined ζ_l as the supersymmetry variation of the phase σ :

$$\delta \sigma = \bar{\epsilon}_k \zeta^k + \text{H.c.} \tag{44}$$

Since $\mathcal{R}_{ij} = 0$ is a supersymmetric invariant constraint, we need to set its supersymmetry variation to zero, i.e. $\Theta_i = 0 = \Upsilon_{ijk}$, to get the other set of constraints. From the first identity, we derive the relation between ζ^k and the components of the dilaton matter multiplet. Specifically,

$$i\zeta^k = \Lambda^k + 3E^{-2} \alpha_j \bar{E}^{jk}, \tag{45}$$

with

$$\begin{aligned}\alpha_j &\equiv 2\chi_j - \frac{2}{3}D\Lambda_j + \frac{1}{12}\gamma.T^+\Lambda^l\epsilon_{jl} + \frac{1}{3}g^k{}_j\Lambda_k - \frac{2}{3}P\Lambda_j \\ &\quad + \frac{1}{12}\bar{\Lambda}^m\gamma_a\Lambda_m\gamma^a\Lambda_j, \\ E^2 &\equiv E^{ij}E_{ij}.\end{aligned}\quad (46)$$

The constraint $\Upsilon^{lmk} = 0$, together with (45), gives an expression for Ξ^{ijk} completely determined in terms of the lower weight fields:

$$\Xi^{ijk} = 9E^{-2}\alpha_i E^{(ij}\bar{E}^{kl)} + 6\Lambda^{(i}E^{jk)}. \quad (47)$$

At this point, one can check that $\delta_S\Xi_{ijk} = 0$ by using the S-transformation of α_j :

$$\delta_S\alpha_j = \frac{8}{3}\bar{E}_{jk}\eta^k.$$

Also from (45), one finds that $\delta_S\zeta^i = -2i\eta^i$, and hence the $[\delta_Q, \delta_S]$ commutator acts on σ as:

$$[\delta_S(\eta), \delta_Q(\epsilon)]\sigma = -2i\bar{\epsilon}_k\eta^k + 2i\bar{\epsilon}^k\eta_k. \quad (48)$$

This is consistent with the U(1) transformation of σ given in (40).

Now, since Ξ_{ijk} is completely determined in terms of the lower weight fields, the number of independent fermionic components of the multiplet is reduced to 8 (Λ_i).

We obtain further constraints on the bosonic fields by setting the supersymmetry variation of Υ_{ijk} to zero. For simplicity, we present only the bosonic terms:

$$\begin{aligned}\delta\Upsilon_{ijk} &= A_{ijkl}\epsilon^l + B_{(ij}\epsilon_k)l\epsilon^l + D^{ab}{}_{ijkl}\gamma_{ab}\epsilon^l \\ &\quad + F^{ab}{}_{(ij}\epsilon_k)l\gamma_{ab}\epsilon^l + G_{(ij}\epsilon_k)\epsilon_m\epsilon^{lm},\end{aligned}\quad (49)$$

with

$$\begin{aligned}A_{ijkl} &= \frac{1}{3}[-C_{ijkl} - 12E^{-2}D_a P^a E_{(ij}\bar{E}_{kl)} - 18E^{-2}DE_{(ij}\bar{E}_{kl)} \\ &\quad - 6E^{-2}D_a S^{am}{}_{(i}E_{jk}\bar{E}_{l)m} - 12E^{-2}P_a P^a E_{(ij}\bar{E}_{kl)} \\ &\quad + 6E_{(ij}\bar{E}_{kl)} - 12E^{-2}P_a S^{am}{}_{(i}E_{jk}\bar{E}_{l)m} \\ &\quad - \frac{3}{2}E^{-2}S^n{}_m.S^m{}_n E_{(ij}\bar{E}_{kl)}] + \text{fermions}, \\ B_{ij} &= \mathcal{L}^{-2}\mathcal{L}_{ij}\mathcal{L}_{kl}\mathcal{G}^{kl} + \text{fermions}, \\ F^{ab}{}_{ij} &= \mathcal{H}_{abij} - \mathcal{L}^{-2}\mathcal{L}_{ij}\mathcal{L}^{kl}\mathcal{H}_{abkl} + \text{fermions}, \\ D^{ab}{}_{ijkl} &= \frac{4}{3}\mathcal{L}^{-2}\mathcal{H}^{ab}{}_{m(i}\mathcal{L}_{jk}\mathcal{L}_{l)n}\epsilon^{mn} + \text{fermions}, \\ J^a{}_{ijkl} &= \frac{4}{3}\mathcal{L}^{-2}G^a{}_{n(i}\mathcal{L}_{jk}\mathcal{L}_{l)m}\epsilon^{mn} + \text{fermions}, \\ G^a{}_{ij} &= 3e^{-i\sigma/2}(D^a\mathcal{L}_{ij} - (\mathcal{L}^{-1}D^a\mathcal{L})\mathcal{L}_{ij} \\ &\quad - S^{ak}{}_{(i}\mathcal{L}_{j)k}) + \text{fermions}.\end{aligned}\quad (50)$$

In the above expressions, we have defined¹²

$$\begin{aligned}\mathcal{G}^{ij} &= \frac{3}{2}[D_a S^{aij} + 2P_a S^{aij}], \\ \mathcal{H}_{abij} &= -\frac{3}{2}\left[D_{[a}S_{b]ij} + \frac{1}{2}S_{am(i}S_{b}{}^m{}_{j)} + \frac{1}{2}R(V)_{abij}\right].\end{aligned}\quad (51)$$

By setting (49) to zero, we obtain the following set of constraints:

$$\begin{aligned}A_{ijkl} &= 0, \\ \mathcal{G}^{ij}\mathcal{L}_{ij} &= 0, \\ \mathcal{H}_{abij} &= \mathcal{L}^{-2}\mathcal{L}_{ij}\mathcal{L}^{mn}\mathcal{H}_{abmn}, \\ G^a{}_{ij} &= 0.\end{aligned}\quad (52)$$

The first constraint fixes C_{ijkl} in terms of the lower weight field. The third constraint in (52) is not an independent one, since it is equivalent to $\mathcal{L}^{lm}\epsilon_{jm}D_{[b}G_{a]il} = 0$. Hence, we are left with a scalar (1) and a vector-tensor (12) constraints, which read explicitly

$$\begin{aligned}(D_a S^{aij} + 2P_a S^{aij})\mathcal{L}_{ij} &= 0, \\ (D^a\mathcal{L}_{ij} - (\mathcal{L}^{-1}D^a\mathcal{L})\mathcal{L}_{ij} - S^{ak}{}_{(i}\mathcal{L}_{j)k}) &= 0.\end{aligned}\quad (53)$$

At this point, it is trivial to check that the second constraint above encompasses only $12 - 4 = 8$ independent components, since a contraction with $\mathcal{L}^{ij}$ gives zero identically. Together, (53) constitute a set of 9 independent constraints on $S_a{}^i{}_j$. Hence, $S_a{}^i{}_j$ has only 3 independent components. Furthermore, the second identity in (53) can be solved for $S_a{}^i{}_j$ in terms of a 4-vector $\mathcal{H}_a$. To see this, consider the set of equalities:

$$\begin{aligned}S_a{}^k{}_{(i}\mathcal{L}_{j)k} &= S_a{}^k{}_i\mathcal{L}_{jk} - S_a{}^k{}_{[i}\mathcal{L}_{j]k} \\ &= S_a{}^k{}_i\mathcal{L}_{jk} - \frac{1}{2}S_a{}^{km}\mathcal{L}_{mk}\epsilon_{ij} \\ &= S_a{}^k{}_i\mathcal{L}_{jk} - \frac{1}{2}\phi^{-2}\mathcal{L}\mathcal{H}_a\epsilon_{ij},\end{aligned}\quad (54)$$

where we defined $S_a{}^{mk}\mathcal{L}_{mk} = \phi^{-2}\mathcal{L}\mathcal{H}_a$. The pre-factor $\phi^{-2}\mathcal{L}$ is chosen conveniently so that $\mathcal{H}_a$ has Weyl weight +3 and it satisfies a simple Bianchi identity as we will see below. Substituting (54) into the second identity in (53) and contracting with $\mathcal{L}^{ij}$, we can solve explicitly for $S_a{}^i{}_j$:

$$S_a{}^i{}_j = \mathcal{L}^{-2}[\mathcal{L}^{im}D_a\mathcal{L}_{jm} - \mathcal{L}_{jm}D_a\mathcal{L}^{im}] + \phi^{-2}\mathcal{L}^{-1}\mathcal{H}_a\mathcal{L}^{ik}\epsilon_{kj}.\quad (55)$$

Plugging this solution in the first identity in (53), we obtain a Bianchi identity on $\mathcal{H}_a$, i.e.:

¹²The indices of $S_a{}^i{}_j$ and $R(V)_{ab}{}^i{}_j$ are raised or lowered by using the rule $A_{ij} = \epsilon_{ik}A^k{}_j$ and $A^{ij} = A^i{}_k\epsilon^{kj}$.

$$D_a \mathcal{H}^a = 0. \quad (56)$$

Thus, $\mathcal{H}_a$ can be interpreted as the dual of a three-form field strength corresponding to a two-form gauge field, up to possible fermionic terms.

To sum up, we have seen that, S_a^i has three independent components which can be traded off for the dual of a three-form field strength $\mathcal{H}_a$ satisfying a Bianchi identity. Together with the real scalar field ϕ (1), the phase σ (1) and $\mathcal{L}_{ij}$ (3), they make up the total 8 bosonic components expected for a minimal matter multiplet of $N = 2$ supergravity.

VIII. THE EMBEDDING OF THE TENSOR MULTIPLY IN THE REAL SCALAR MULTIPLY

In this section, we want to show the precise embedding of the $N = 2$ tensor multiplet in the real scalar multiplet, which satisfies the constraint discussed in the previous section. This will show the equivalence between the tensor multiplet and the restricted real scalar multiplet. For completeness we write the full supersymmetry transformation of the tensor multiplet in a conformal supergravity background [21,25]:

$$\begin{aligned} \delta L_{ij} &= 2\bar{\epsilon}_{(i}\varphi_{j)} + 2\epsilon_{ik}\epsilon_{jl}\bar{\epsilon}^{(k}\varphi^{l)}, \\ \delta\varphi^i &= \bar{D}L^{ij}\epsilon_j + \not{H}\epsilon^{ij}\epsilon_j - G\epsilon^i + 2L^{ij}\eta_j, \\ \delta G &= -2\bar{\epsilon}_i\bar{D}\varphi^i - 6\bar{\epsilon}_i\chi_j L^{ij} + \frac{1}{4}\epsilon^{ij}\bar{\epsilon}_i\gamma \cdot T^+ \varphi_j + 2\bar{\eta}_i\varphi^i, \\ \delta E_{\mu\nu} &= i\bar{\epsilon}^i\gamma_{\mu\nu}\varphi^j\epsilon_{ij} + 2iL_{ij}\epsilon^{jk}\bar{\epsilon}^i\gamma_{[\mu}\psi_{\nu]k} + \text{H.c.} \end{aligned} \quad (57)$$

Here we have defined H_a as the dual of the three-form field strength H_{abc} as shown below:

$$H_a = \frac{i}{6}\epsilon_{abcd}H^{bcd}, \quad (58)$$

where $H_{\mu\nu\rho}$ is the super-covariant field strength associated with tensor gauge field $E_{\mu\nu}$ given as:

$$\begin{aligned} H_{\mu\nu\rho} &= 3\partial_{[\mu}E_{\nu\rho]} - \frac{3i}{2}\bar{\psi}_{[\mu}^i\gamma_{\nu\rho]}\varphi^j\epsilon_{ij} + \frac{3i}{2}\bar{\psi}_{[\mu}^i\gamma_{\nu\rho]}\varphi_j\epsilon^{ij} \\ &\quad - \frac{3i}{2}L_{ij}\epsilon^{jk}\bar{\psi}_{[\mu}^i\gamma_{\nu\rho]k}. \end{aligned} \quad (59)$$

Now, it is possible to verify that the following combinations of the tensor multiplet fields,

$$\begin{aligned} \phi^4 &= L^2, \\ \Lambda^i &= -2L^{-2}L^{ij}\varphi_j, \\ E_{ij} &= L^{-4}L_{ij}L^{kl}\bar{\varphi}_k\varphi_l - L^{-2}\bar{G}L_{ij}, \\ S_a^i &= 2L^{-2}H_a L^{ik}\epsilon_{kj} + 4L^{-4}L^{ik}L_{jm}\bar{\varphi}^m\gamma_a\varphi_k - L^{-2}\bar{\varphi}^i\gamma_a\varphi_j - \frac{1}{2}L^{-2}\delta_j^i\bar{\varphi}^m\gamma_a\varphi_m + L^{-2}(L^{ik}D_a L_{jk} - L_{jk}D_a L^{ik}), \\ \Xi_{ijk} &= -24L^{-6}L^{mn}\bar{\varphi}_m\varphi_n L_{(ij}L_{k)l}\varphi^l + 6L^{-4}\bar{\varphi}_l\varphi_{(i}L_{jk)}\varphi^l - 6L^{-4}L^{lm}\bar{D}L_{l(i}L_{jk)}\varphi_m + 6L^{-4}L^{mn}\not{H}\varphi_n L_{(ij}\epsilon_{k)m} \\ &\quad + 12L^{-4}\bar{G}L_{(ij}L_{k)l}\varphi^l + 6L^{-2}\bar{D}\varphi_{(i}L_{jk)} + 18L^{-2}L_{(ij}L_{k)l}\chi^l - \frac{3}{4}L^{-2}\gamma \cdot T^-\varphi^l L_{(ij}\epsilon_{k)l}, \\ C_{ijkl} &= -18L^{-2}DL_{(ij}L_{kl)} + 6L^{-4}\bar{G}\bar{G}L_{(ij}L_{kl)} + 6L^{-4}H^a H_a L_{(ij}L_{kl)} - 12L^{-4}H^a D_a L^{mn}\epsilon_{m(i}L_{jk}L_{l)n} \\ &\quad - 6L^{-2}D_a D^a L_{(ij}L_{kl)} + 6L^{-4}L^{mn}D_a L_{mn}D^a L_{(ij}L_{kl)} - 3L^{-2}D^a L^{mn}D_a L_{mn}L_{(ij}L_{kl)} \\ &\quad - 9L^{-2}\bar{\chi}^m\varphi^n L_{(ij}\epsilon_{k|m}\epsilon_{l)n} - 36L^{-4}\bar{\chi}^m\varphi^n L_{(ij}L_{k|m}L_{l)n} - 36L^{-4}L^{mn}\bar{\chi}_m\varphi_n L_{(ij}L_{kl)} + 9L^{-2}\bar{\chi}_{(i}\varphi_{j}L_{kl)} \\ &\quad + 6L^{-4}\bar{G}\bar{G}\varphi_{(i}\varphi_{j}L_{kl)} - 18L^{-6}GL^{mn}\bar{\varphi}_m\varphi_n L_{(ij}L_{kl)} + 6L^{-6}\bar{G}L_{mn}\bar{\varphi}^m\varphi^n L_{(ij}L_{kl)} - 6L^{-4}\bar{G}\bar{\varphi}^m\varphi^n L_{(ij}\epsilon_{k|m}\epsilon_{l)n} \\ &\quad - 24L^{-6}\bar{G}\bar{\varphi}^m\varphi^n L_{(ij}L_{k|m}L_{l)n} - 6L^{-4}\bar{\varphi}^m\gamma^a\varphi_{(i}L_{jk}D_a L_{l)m} + 36L^{-6}\bar{\varphi}^m\gamma^a\varphi_n L^{ns}D_a L_{s(i}L_{jk}L_{l)m} \\ &\quad - 6L^{-4}\bar{\varphi}^m\gamma^a\varphi_m D_a L_{(ij}L_{kl)} + 6L^{-4}\bar{\varphi}^m\not{H}\varphi_{(i}L_{jk}\epsilon_{l)m} - 36L^{-6}\bar{\varphi}^m\not{H}\varphi_n L^{ns}L_{(ij}L_{k|m}\epsilon_{l)s} \\ &\quad - 12L^{-4}L^{st}\varphi_s\bar{D}\varphi^m L_{(ij}\epsilon_{k|t}\epsilon_{l)m} - 12L^{-4}\bar{\varphi}^m\bar{D}\varphi_{(i}L_{jk}L_{l)m} + \frac{3}{4}L^{-4}\epsilon_{mn}\bar{\varphi}^m\gamma \cdot T^-\varphi^n L_{(ij}L_{kl)} \\ &\quad + \frac{3}{4}L^{-4}\epsilon^{mn}\bar{\varphi}_m\gamma \cdot T^+\varphi_n L_{(ij}L_{kl)} - 36L^{-6}\bar{\varphi}^m\varphi^n\bar{\varphi}_m\varphi_n L_{(ij}L_{kl)} - 36L^{-6}L_{mn}\bar{\varphi}^m\varphi^n\bar{\varphi}_{(i}\varphi_{j}L_{kl)} \\ &\quad + 36L^{-6}\bar{\varphi}^m\varphi^n\bar{\varphi}_m\varphi_{(i}L_{jk}L_{l)n} + 90L^{-8}L^{mn}\bar{\varphi}_m\varphi_n L_{st}\bar{\varphi}^s\varphi^t L_{(ij}L_{kl)}, \end{aligned} \quad (60)$$

transform as the components of the real scalar multiplet (31). One can check that the above combinations satisfy the constraints presented in the previous section. One can also check that the combinations appearing as S_a^i and C_{ijkl} above also satisfy the reality constraints (30). The relation between the tensor multiplet and the independent components of the restricted real scalar multiplet can be obtained from the identification (60) and are given as:

$$\begin{aligned}
\phi^4 &= L^2, \\
\Lambda^i &= -2L^{-2}L^{ij}\varphi_j, \\
e^{-i\sigma/2} &= \left(\frac{\mathcal{Z}}{\bar{\mathcal{Z}}}\right)^{1/2}, \\
\mathcal{L}_{ij} &= |\mathcal{Z}|L_{ij}, \\
\mathcal{H}_a &= H_a + \text{fermions},
\end{aligned} \tag{61}$$

Here $\mathcal{Z}$ is defined as the following combination of tensor multiplet fields:

$$\mathcal{Z} = L^{-4}L^{kl}\bar{\varphi}_k\varphi_l - L^{-2}\bar{G}. \tag{62}$$

Thus, we see that the off-shell $8 + 8N = 2$ tensor multiplet can be embedded within the $24 + 24$ real scalar multiplet that satisfies the constraints (41), (47), (52), mentioned in the previous section.¹³

IX. CONCLUSIONS AND FUTURE DIRECTIONS

Superconformal couplings of the matter multiplets and the gauge multiplet are crucial in our understanding of the structure of supergravity that can arise from them. Apart from the structure of the higher-derivative terms that one can obtain, they also tell us about the geometric structure of the matter couplings. For example, it is well-known that the scalar manifold corresponding to vector multiplet coupled to $N = 2$ conformal supergravity in four dimensions, upon gauge fixing and eliminating the auxiliary fields, corresponds to a special Kähler manifold (see [26–28] and [14] for a review). Similar geometric structure exists for other matter couplings in four-dimensional as well as five-dimensional supergravity. Hence, the discovery of new matter multiplets opens up the possibilities of gaining new insights into supergravity structures arising from them.

In this paper, we have presented one such multiplet, the real scalar multiplet, which contains $24 + 24$ off-shell d.o.f. and generalizes the flat space result of [12] to a conformal supergravity background.¹⁴ We have also shown that one can impose a consistent set of constraints to reduce the

multiplet to $8 + 8$ components and as an example, we have shown the embedding of the tensor multiplet within the real scalar multiplet that satisfies these constraints.

As a potential application of this result, one can construct a density formula in terms of the real scalar multiplet, which by virtue of the tensor multiplet embedding (60), will allow us to look for new couplings of tensor multiplet to conformal supergravity. This would generalize the improved tensor multiplet action obtained in [25] using an invariant density formula that involved the chiral multiplet. We leave this study for a future work. It would also be interesting to see if results analogous to the ones presented in Sec. VIII can be obtained for vector and Weyl multiplets, and the possibility of new formulations of conformal and Poincaré supergravity.

A complete knowledge of the structure of supergravity arising from various matter and gauge couplings, in an off-shell formulation, is crucial to our understanding of black holes, in particular, the effective quantum corrections to their entropy. In fact, using the standard Weyl multiplet and other matter multiplets known so far, several higher-derivative invariants have been constructed in four space-time dimensions [30–33] and their effect on black hole entropy have been studied [32,34–36]. It has also been shown that the above results are able to correctly reproduce the microscopic entropy of BPS black holes [32,34,36] as well as some class of non-BPS black holes [35]. However, the result presented in this paper practically allows for the existence of new, unknown couplings in supergravity, and their connection to higher-dimensional theories and string theory in general. As an inevitable consequence, the need will arise to revisit and put under scrutiny every result mentioned above (and more), based on the analysis of invariant couplings of supergravity.

ACKNOWLEDGMENTS

The authors would like to thank Daniel Butter and Bernard de Wit for their helpful comments on this work. We would like to thank Daniel Butter and Sergei Kuzenko for bringing to our attention Refs. [11,12,23,29]. We would also like to thank the organizers of the Indian National Strings Meeting (NSM) (IISER Mohali) for giving us a platform for illuminating discussions which led to the idea behind this work.

¹³We thank Daniel Butter for pointing out this possibility.

¹⁴Such multiplets have also recently been constructed in six dimensions [29].

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