Temperature effect on shear and bulk viscosities of QCD matter

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With developing a scheme to determine the thermal width of a pion in a medium, we investigate the shear and bulk viscosities and their ratios to entropy density. We calculate the temperature (T)-dependence of the pion mass and decay constant via the Dyson-Schwinger equations of QCD and get the pion thermal width with the QCD approach. We obtain then the T -dependence of the viscosities, especially a novel feature of the bulk viscosity, via the continuum QCD approach. The novel concavo-convex behavior of the bulk viscosity to entropy density ratio against the temperature is a demonstration of the QCD phase transitions.

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I. INTRODUCTION

It has been well known that the shear viscosity (η) and its ratio to the entropy density (s) are excellent signatures to identify the phase transitions of various kinds of matters (e.g., Refs. [1,2]). Investigations relating the η/s to the observables in relativistic heavy ion collisions (RHIC) (e.g., Refs. [3–7]) yield $1 < 4\pi\eta/s < 3.75$, which is quite close to its lower bound $1/4\pi$ (KSS bound) [8] and indicates that the matter created by the RHIC is in the strongly coupled quark gluon plasma (sQGP) state. Due to the sensitivity of the observables to the temperature (T)- and chemical potential (μ)-dependence of the η/s (e.g., Refs. [1,4,9]), the ratio may also be a signature of the critical end point (CEP) in the QCD phase diagram [4,10]. However, the T -dependence of the η/s is still controversial. Some of them manifest the “U”-like feature [1,10–18], and some others give only the decreasing behavior [19–28] or the increasing feature [29–33]. Even whether there exists the KSS bound is also in debate (e.g., Refs. [32–34]).

It is also well known that there exists another kind of viscosity, namely, the bulk viscosity (ζ), which is generally recognized to be very small (see, e.g., Refs. [26,35–39]). However, studies relating the ζ with the trace anomaly suggest [40,41] and explaining recent experiments requires [32,42] a large value. Nevertheless, the relation connecting the ζ with the trace anomaly requires particular ansatz for

the spectral function [43] and a sophisticated technique to rule out the nonthermal contributions [35,44]. Moreover, presently existing results of the T -dependence of the ζ/s are in serious contradiction with each other, e.g., lattice QCD simulations and holographic correspondence methods give concave functions with different features in particular [40,41,45,46], and chiral perturbation theory gives double peak or convex relations [36] or decreasing function at low temperature [47]. Some hadron resonance models and Nambu–Jona-Lasinio model calculations yield convex or concave function [28,31]. Some others give decreasing features [18,26,39,48] or increasing behavior [22]. The quasiparticle model produces a decreasing or convex function depending on the σ -meson mass [12] and recent quasiparticle anisotropic hydrodynamics also gives a convex function [49]. The linear sigma model yields convex or concave or more complicated behavior relying on also the masses of the pion and σ -meson [37]. Controversial results also exist for its $(1/3 - c_s^2)$ (c_s^2 , the sound speed squared) dependence (see, e.g., Refs. [35,46,50–53]). One may then doubt the validity for the η/s and ζ/s to label a phase transition, even the power of statistical physics principle to describe properties of strong interaction matters. Looking over the schemes taken in previous investigations more carefully, one can recognize that restudying the problems via sophisticated QCD approaches is imperative.

It has been known that the Dyson-Schwinger equations (DSEs) (e.g., Ref. [54]) are almost uniquely a continuum QCD approach including both the confinement and the dynamical chiral symmetry breaking (DCSB) simultaneously [55] and are successful in describing QCD phase transitions and hadron properties (see, e.g., Refs. [54,56–59]). We then, in this article, take the DSE approach of QCD to shed light on the issues.

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II. FRAMEWORK

Strong interaction matter (QCD matter) can be considered as a system consisting of quarks and antiquarks (most of them are confined to form pions, the lowest mass hadrons, at low temperature). The viscosities of the system can be obtained in the kinetic theory approach [12,27,36,39] as

$$\eta = \frac{1}{10\pi^2 T} \int_0^\infty \frac{d|\vec{k}| |\vec{k}|^6}{(\omega_{|\vec{k}|}^\pi)^2 \Gamma_\pi^{\text{TW}}(|\vec{k}|)} n_{|\vec{k}|}(\omega_{|\vec{k}|}^\pi) \{1 + n_{|\vec{k}|}(\omega_{|\vec{k}|}^\pi)\},$$

$$\zeta = \frac{3}{T} \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{1}{(\omega_{|\vec{k}|}^\pi)^2 \Gamma_\pi^{\text{TW}}(|\vec{k}|)} n_{|\vec{k}|}(\omega_{|\vec{k}|}^\pi) \{1 + n_{|\vec{k}|}(\omega_{|\vec{k}|}^\pi)\}$$

$$\times \left\{ \left(\frac{1}{3} - c_s^2 \right) |\vec{k}|^2 - c_s^2 m_\pi^2 \right\}, \quad (1)$$

where $n_{|\vec{k}|}(\omega_{|\vec{k}|}^\pi) = 1/(e^{\omega_{|\vec{k}|}^\pi/T} - 1)$ is the Bose-Einstein distribution with $\omega_{|\vec{k}|}^\pi = \sqrt{|\vec{k}|^2 + m_\pi^2}$, m_π is the pion mass, and $\Gamma_\pi^{\text{TW}}(|\vec{k}|)$ the pion thermal width. Here, c_s^2 is the sound speed squared defined as $c_s^2 = \frac{\partial P}{\partial \varepsilon}$ with P the pressure and ε the energy density.

III. PION PROPERTY

It is apparent that to investigate the property of the viscosities one needs precise thermal property of pion. The property of the pion in vacuum has been attentively investigated in the DSE approach of QCD. Within the bare vertex truncation for quark DSE, it is accurate for light-flavor ground-state vector- and pseudoscalar-mesons [54,60,61]. It has also been proved that in a chiral limit, the massless pion can be connected with the DCSB with $f_\pi E(p) = B_0(p)$, where f_π is the pion decay constant, $E(p)$ the dress function for the component γ_5 of pion's Bethe-Salpeter amplitude $\Gamma_\pi(p)$, and $B_0(p)$ the scalar part of the inverse of quark propagator in chiral limit [62]. In the case of finite temperature and beyond the chiral limit, one may take the similar scheme to describe the meson properties; that is, $\Gamma_\pi(p) \propto i\gamma_5 B(p)$ with $B(p)$ the counterpart at the current quark mass of the $B_0(p)$ and $p = (\vec{p}, \omega_n)$. After considering the right ultraviolet feature, we get the explicit relation as

$$\Gamma_\pi(p) = i\gamma_5 B_d(p)/f_\pi,$$

with $B_d = B(p) - m \frac{\partial B(p)}{\partial m}$, (2)

where m is the current mass of the light quark. The pion mass and decay constant can be formulated as similar as those in Ref. [63],

$$f_\pi N_\pi = 4N_c \int_p B_d \left\{ \sigma_A \sigma_B + \frac{2}{3} |\vec{p}|^2 (\sigma'_A \sigma_B - \sigma_A \sigma'_B) \right\}, \quad (3)$$

$$m_\pi^2 f_\pi^2 = 8N_c \int_p \frac{m B_d}{\omega_n^2 C^2 + |\vec{p}|^2 A^2 + B^2}, \quad (4)$$

$$N_\pi^2 = 2N_c \int_p B_d \left\{ \sigma_A^2 - 2(\omega_n^2 \sigma_C \sigma'_C + |\vec{p}|^2 \sigma_A \sigma'_A + \sigma_B \sigma'_B) \right. \\ \left. - \frac{4}{3} |\vec{p}|^2 [\omega_n^2 (\sigma_C \sigma''_C - (\sigma'_C)^2) + |\vec{p}|^2 (\sigma_A \sigma''_A - (\sigma'_A)^2) \right. \\ \left. + \sigma_B \sigma''_B - (\sigma'_B)^2] \right\}, \quad (5)$$

where N_π is the normalization factor of the Bethe-Salpeter amplitudes. Here, σ_F and F ($F = A, B, C$) are defined in the quark propagator as

$$S(p) = (i\vec{\gamma} \cdot \vec{p} A + \gamma_4 \omega_n \cdot C + B)^{-1} \\ = i\vec{\gamma} \cdot \vec{p} \sigma_A + \gamma_4 \omega_n \cdot \sigma_C + \sigma_B,$$

and $\sigma'_F = \partial \sigma_F / (\partial |\vec{p}|^2)$, which can be fixed with the DSE,

$$S(\vec{p}, i\omega_n)^{-1} = Z_2^A i\vec{\gamma} \cdot \vec{p} + Z_2 i\gamma_4 \omega_n + Z_4 m \\ + T \sum_l \frac{4}{3} \int \frac{d^3 q}{(2\pi)^3} g^2 D_{\mu\nu}(\vec{p} - \vec{q}, \Omega_{nl}) \\ \times (Z_1^A \gamma_\mu^T + Z_1 \gamma_\mu^L) S(\vec{q}, i\omega_l) \Gamma_\nu, \quad (6)$$

where $\Gamma_\nu = \Gamma_\nu(\vec{q}, \omega_l, \vec{p}, \omega_n)$ is the quark-gluon interaction vertex, Z_2^A , Z_2 , Z_4 , Z_1^A , and Z_1 is the respective renormalization factor, $\Omega_{nl} = \omega_n - \omega_l$ with $\omega_n = (2n+1)\pi T$ being the quark's Matsubara frequency, and $\gamma_\mu^{T,L} = P_{\mu\nu}^{T,L} \gamma_\nu$ with $P_{\mu\nu}^{T,L}$ the transverse and longitudinal projection operators, respectively. Here, $g^2 D_{\mu\nu}(\vec{k}, \Omega_{nl})$ is the interaction kernel together with the dressed-gluon propagator, which has generally the form

$$g^2 D_{\mu\nu}(\vec{k}, \Omega_{nl}) = P_{\mu\nu}^T D_T(\vec{k}^2, \Omega_{nl}^2, m_g^2) \\ + P_{\mu\nu}^L D_L(\vec{k}^2, \Omega_{nl}^2, m_g^2). \quad (7)$$

Extending the infrared constant model proposed in Ref. [64] with which one can describe many mesons' properties at $T = 0$ very well, we can have

$$\mathcal{D}_{L,T}(\vec{k}^2, \Omega_{nl}^2, m_g^2) = 8\pi^2 \mathcal{D} \frac{1}{\omega_{L,T}^4} e^{-s_\Omega/\omega_{L,T}^2} \\ + \frac{8\pi^2 \gamma_m \mathcal{F}(s_\Omega)}{\ln[\tau + (1 + s_\Omega/\Lambda_{\text{QCD}}^2)^2]}, \quad (8)$$

with $\mathcal{F}(s_\Omega) = (1 - \exp(-s_\Omega/4m_t^2))/s_\Omega$, $s_\Omega = \Omega^2 + \vec{k}^2 + m_g^2$ with m_g the thermal mass of gluon, $\gamma_m = 12/25$, $\tau = e^2 - 1$,

FIG. 1. Calculated temperature dependence of the pion's mass (solid) and decay constant (dashed).

$m_t = 0.5$ GeV, and $\Lambda_{\text{QCD}} = 0.234$ GeV. In the original model [64], the ω and $\mathcal{D}$ are parameters with relation $\mathcal{D}\omega \approx (0.8 \text{ GeV})^3$ independent of T , but could not describe the difference between the T -dependence of the longitudinal and the transverse parts of the gluon propagator given in recent lattice QCD simulations (e.g., Ref. [65]) and other modern DSE calculations (e.g., Ref. [59]). We then correct the constant ω with T -dependence as

$$\omega_L = \omega_0 - T, \quad \omega_T = \omega_0 + 1.6T. \quad (9)$$

To include the T -screening effect at high temperature, we take a damping factor for the $\mathcal{D}$ in the way as in Ref. [56] and $m_g^2 = 0, 16T^2/5$ for the $\mathcal{D}_T, \mathcal{D}_L$, respectively, according to perturbative QCD (pQCD) calculations [66,67].

Then, in our calculations, we take the bare vertex approximation $\Gamma_\nu = \gamma_\nu$ which has been well known to be appropriate to describe pion properties. The obtained T -dependence of the pion mass and decay constant with the Bethe-Salpeter equation and Eqs. (2)–(5) (with parameters $m = 3.4$ MeV, $D = 1.024 \text{ GeV}^2$ and $\omega_0 = 0.5 \text{ GeV}$) are shown in Fig. 1. It is evident that the $m_\pi = 143 \text{ MeV}$ and $f_\pi = 87 \text{ MeV}$ at $T = 0$ agree with empirical data very well. The decreasing behavior of the $f_\pi(T)$ and the increasing feature of the $m_\pi(T)$ are also consistent with previous results (e.g., Ref. [56]).

To evaluate the pion width, we resort to the properties of the resonant states in π - π scattering. It has been well known that σ - and ρ -mesons are, respectively, the S - and P -wave resonant states of the scattering. The width Γ_π^{TW} can then be formulated as [27]

$$\Gamma_\pi^{\text{TW}} = \sum_{r=\rho,\sigma} \frac{1}{N_r} \int_{(m_r^-)^2}^{(m_r^+)^2} dM^2 \rho_r(M) \Gamma_{\pi\pi,r}, \quad (10)$$

with

$$\Gamma_{\pi\pi,r} = \frac{1}{16\pi m_\pi |\vec{k}|} \int_{\omega_+}^{\omega_-} d\omega L_r(\omega) [n(\omega) - n(\omega_{|\vec{k}|}^\pi + \omega)],$$

$$N_r = \int_{(m_r^-)^2}^{(m_r^+)^2} dM^2 \rho_r(M),$$

where $m_r^\pm = M_r \pm \Gamma_r$ with M_r, Γ_r being the mass, the width of the resonance r , respectively. $\omega^\pm = R^2(-\omega_{|\vec{k}|}^\pi \pm |\vec{k}|(1 - 1/R^4)^{1/2})$ with $R^2 = 1 - \frac{M_\pi^2}{2m_\pi^2}$. In narrow-width approximation, the resonance width Γ_r can be fixed by the Breit-Wigner formula,

$$\rho_r(M) = \frac{1}{\pi} \text{Im} \left[\frac{-1}{M^2 - M_r^2 + i\Gamma_r M_r} \right]. \quad (11)$$

In one-loop calculation, $L_\sigma(\omega) = \frac{-g_\sigma^2 M_\sigma^2}{4}$ and $L_\rho(\omega) = \frac{-g_\rho^2}{M_\rho^2} [2m_\pi^2(m_\pi^2 - M_\rho^2) - 2(-\omega_{|\vec{k}|}^\pi \omega M_\rho^2 + m_\pi^4)]$, with parameters $g_\rho = 6$ and $g_\sigma = 6.85$.

It is very hard to compute the mass and width of the ρ - and σ -resonances in this scenario directly in the DSE scheme (in fact, it is impossible for the width of a meson at the present stage). We then appeal to the Roy equations (REs) [68,69], which are a set of coupled integral equations of various resonance channels. In the scheme of REs, the partial wave amplitude $t_l^I(s)$ connects with the scattering lengths. The amplitudes contain mass poles of the resonance on the second sheet of the s -plane, if and only if the S matrix, $S(s) = 1 - 2\sqrt{4m_\pi^2 s - 1}t(s)$, has a zero on the physical sheet [70]. The position of the zero is $s = M_r + \frac{i}{2}\Gamma_r$ with $r = \sigma$ for $t_0^0(s)$ and $r = \rho$ for $t_1^1(s)$, respectively. Solving the REs including the relevant channels t_0^0, t_0^2 , and t_1^1 (for their explicit expresses, see Ref. [71]) with the scattering lengths $a_0^0 = \frac{7m_\pi^2}{32\pi f_\pi^2}$ and $a_0^2 = -\frac{m_\pi^2}{16\pi f_\pi^2}$ (Weinberg's formulas [72]), we get the T -dependence of the mass and the width of the σ - and ρ -resonances with the above-obtained pion mass and decay constant as input. The obtained results are illustrated in Fig. 2. It is apparent that the masses and their widths of the resonances at zero temperature, $m_{\sigma,0} = 427 \text{ MeV}$, $\Gamma_{\sigma,0} = 624 \text{ MeV}$, $m_{\rho,0} = 770 \text{ MeV}$, and $\Gamma_{\rho,0} = 134 \text{ MeV}$, agree with experimental data [73] very well. Meanwhile as temperature increases, the masses of the mesons decrease slightly at low temperature and turn to increase after reaching their respective minimum. At $T \approx 183 \text{ MeV}$, $M_\sigma = M_\rho = 2M_\pi = 4M_q$, with M_q the constituent quark mass fixed by $B(T, \vec{p} = 0)/C(T, \vec{p} = 0)$ [$B(T, \vec{p})$ and $C(T, \vec{p})$, the functions in the quark propagator]. It manifests that the mesons get melted to quarks; i.e., deconfinement phase transition takes place at $T \approx 183 \text{ MeV}$ (marked as T_c^d hereafter). In addition, such a straightforward computation obtained widths of the ρ - and σ -mesons decrease with increasing temperature, as the dashed line and dotted-dashed line in the lower panel of Fig. 2 display.

Recalling the calculation scheme, we know that since the REs used are still those in vacuum even though the T -dependence of the pion properties is included, the widths of the composing particles, namely, the quarks and gluons, have not yet been taken into account explicitly.

FIG. 2. Calculated temperature dependence of the masses (upper panel) and their widths (lower panel) of the σ - and ρ -channel resonances and that of the $4M_q$ (M_q , the constituent quark mass). The dashed and dotted-dashed lines in the lower panel refer to the results given by only the direct RE calculation ($\Gamma_R(T)$), and the solid and dotted lines stand for those including the contribution of the hard thermal loop resummation (pQCD-like) width accordingly.

To consider the effects missed in the direct RE calculation, analog to the pQCD calculation results, we modify the widths of the resonances by adding a T -dependent term so as to

$$\Gamma(T) = \Gamma_R(T) + \frac{ag^2 N_c}{24\pi} T,$$

with $\Gamma_R(T)$ the width given by the RE directly, and the additional term from the hard thermal loop resummation calculation with $N_c = 3$, $g^2 = 96/25$, and $a \simeq 6.64$ [74]. Consequently, we find that the width of σ still decreases,

FIG. 3. Calculated temperature and momentum dependence of the pion thermal width.

but that of ρ becomes generally increasing, as shown in the lower panel of Fig. 2. The obtained increasing feature of the ρ -meson width is definitely consistent with the empirical behavior (see, e.g., Refs. [75–79]).

With the above obtained resonance properties of the π - π scattering and Eq. (10), we get further the thermal width of the pion. The obtained momentum and temperature dependence of the width is displayed in Fig. 3. It is evident that the width behaves generally as a convex function with respect to both the momentum and the temperature, especially in low momentum region.

IV. NUMERICAL RESULTS OF THE VISCOSITIES

After obtaining the pion mass and its thermal width Γ_π^{TW} , we calculate the shear and bulk viscosities (η and ζ) of the matter. The obtained T -dependence of the η and ζ are shown in Fig. 4. The figure manifests obviously that the η ascends slightly in the low- T region and increases rapidly at the T for the chiral phase transition (χ PT, in fact a crossover, with a pseudocritical temperature $T_c^\chi \sim (154 \pm 9)$ MeV [80]) to occur, in particular, in case that the pQCD-like widths of the resonances of π - π scattering are not taken into account. Meanwhile, at low- T , the ζ decreases until the χ PT takes place, and the minimum of the ζ in the region is about 10% of the η . Moreover, in the region of the phase transitions (besides the χ PT, deconfinement phase transition happens at $T_c^d \approx 183$ MeV), the ζ demonstrates a tremendous bump in the case that the

FIG. 4. Calculated temperature dependence of the shear viscosity η (upper panel) and the bulk viscosity ζ (lower panel). The solid lines refer to the results given by the direct Roy equation calculation, and the dashed ones stand for those including the contribution of the pQCD-like width to the resonances of the π - π scattering accordingly.

FIG. 5. The same as Fig. 4 but for the viscosity to entropy density ratios.

pQCD-like widths of the π - π scattering are not considered. As the pQCD-like widths are included, the bump is still obvious even though the height is suppressed.

We compute further the ratios of η/s and ζ/s with the entropy density

$$s = \frac{\partial P}{\partial T} = \frac{3}{T} \int \frac{d^3 \vec{k}}{(2\pi)^3} \left(\omega_{|\vec{k}|}^\pi + \frac{\vec{k}^2}{3\omega_{|\vec{k}|}^\pi} \right) n_{|\vec{k}|}(\omega_{|\vec{k}|}^\pi).$$

The obtained T -dependence of the ratios η/s and ζ/s is illustrated in Fig. 5. One can observe obvious minima for both the η/s and the ζ/s around the T_c^χ . More concretely, the temperature for the η/s to have its minimum is $T = 164.3$ MeV, while for ζ/s it is $T = 162.1$ MeV. The minimal value of the η/s is 0.196, which is about $\frac{2.4}{4\pi}$. This manifests that if we deal with the temperature effect on the mesons involved carefully, the ratio η/s of strong interaction matter would not break the KSS bound. Such a small value still shows the strong-coupled property of the matter near the T_c^χ . After then, the η/s increases approximately linearly with respect to T with a rate almost the same as that given in the functional renormalization group (FRG) calculation [16] as the contribution of the pQCD-like width to the resonances of the π - π scattering is included. The monotonous decreasing or increasing behavior given in previous studies may then be attributed to that the respective calculation pays only attention to the behavior in different regions (before the phase transition or after the phase transition).

As for the ζ/s , it holds a minimum with value about 0.01 at $T \approx T_c^\chi$ and a maximum about 0.012 (0.1) at $T \approx 190$ MeV $\approx T_c^d$ in the case that the pQCD-like widths are included (not included) to the resonances of the π - π

scattering. It tends to vanish at very high T . Such a concavo-convex behavior looks quite distinct. However, combining the result of the η/s given in the FRG calculation [16] and the extracted sound speed squared, c_s^2 , from the equation of state given in lattice QCD simulation [81] with $\zeta/s \sim (1/3 - c_s^2)\eta/s$ or $15(1/3 - c_s^2)^2\eta/s$, it yields an obvious concavo-convex behavior for the ζ/s . If one combines the previous result in the low- T region (e.g., given in Ref. [47]) and that obtained recently in the improved fluid dynamical framework [49] in the transition and higher- T region, it also gives a concavo-convex behavior. Therefore, our presently obtained concavo-convex feature for the T -dependence of the ζ/s should be a demonstration of the phase transitions. It is also notable that our result leads to $\zeta/\eta \approx (0.85 \sim 1.04)(1/3 - c_s^2)$ and $\zeta/\eta \approx (0.80 \sim 1.82)15(1/3 - c_s^2)^2$ at $T \in [150, 200]$ MeV. It seems that our result near the phase transition region favors the estimation from the strong-coupled nonconformal theory (similar to that given in Ref. [53] for gluon plasma) than of the leading-order pQCD result [35].

V. SUMMARY

By developing a scheme that combines the DSEs of QCD with the Roy equations of π - π scattering, we studied the temperature dependence of the shear and bulk viscosities of QCD matter in this paper. We compute firstly the T -dependence of the pion mass and decay constant, then the properties of the σ - and ρ -channel resonances of the scattering, and obtain for the first time both the temperature and momentum dependence of the width of the pion. We gain eventually the T -dependence of the viscosities and their ratios to entropy density of the matter via a continuum QCD approach. The obtained ζ is about one order of magnitude smaller than the η . It is also apparent that our present study not only produces the concave behavior of the η/s in terms of T , but also reveals a novel feature for the ζ/s , namely, a concavo-convex function against T . Specifically, both the ratios hold their minima around the T_c^χ , and the ζ/s exhibits a bump behavior in the $T > T_c^\chi$ region with the maximal value located at $T \approx 1.2T_c^\chi \approx T_c^d$, independent of whether the pQCD-like widths are included or not. It indicates that the T -dependence of the ζ/s demonstrates distinctly not only the chiral phase transition but also the deconfinement. In addition, the method we developed in this work, combining the DSEs with the Roy equations, may provide a novel scheme for describing the composing particles' thermal properties and the transport properties of matters at other energy scales.

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