Neutrino mass in flavor dependent gauged lepton model

Takaaki Nomura^{1,*} and Hiroshi Okada^{2,†}¹*School of Physics, KIAS, Seoul 02455, Korea*²*Physics Division, National Center for Theoretical Sciences, Hsinchu 300, Taiwan*

(Received 27 July 2017; published 29 March 2018)

We study a neutrino model introducing an additional nontrivial gauged lepton symmetry where the neutrino masses are induced at two-loop level, while the first and second charged-leptons of the standard model are done at one-loop level. As a result of the model structure, we can predict one massless active neutrino, and there is a dark matter candidate. Then we discuss the neutrino mass matrix, muon anomalous magnetic moment, lepton flavor violations, oblique parameters, and relic density of dark matter, taking into account the experimental constraints.

DOI: [10.1103/PhysRevD.97.055044](https://doi.org/10.1103/PhysRevD.97.055044)

I. INTRODUCTION

A flavor dependent gauge model often plays an important role in explaining flavor specific measurements such as deviation from the standard model (SM) in the semileptonic B decay process $B \rightarrow K^{(*)}\mu^+\mu^-$ reported by LHCb [1,2], lepton flavor violations (LFVs) such as the $\mu \rightarrow e\gamma$ process [3], and the muon anomalous magnetic moment (Δa_μ) at Brookhaven National Laboratory [4]. Along these lines of thought, Ref. [5] has established a flavor dependent model with a larger gauge group, and a smaller group such as a $U(1)$ flavor dependent model can be found as a result of partially breaking the symmetry.

In particular, it is interesting to consider the active neutrino mass matrix with a lepton specific flavor dependent gauge symmetry $U(1)_L$ in Ref. [5]. The active neutrino masses are not allowed at the tree level and they should be related to the $U(1)_L$ gauge symmetry breaking. Then a radiative seesaw scenario is an attractive candidate to generate the mass, which can be achieved by introducing some exotic particles. Furthermore, we expect predictability in the active neutrino mass due to restriction from the gauge symmetry.

In this paper, we construct a neutrino model based on the lepton flavor symmetry $U(1)_L$, where the neutrino mass can be induced at two-loop level [6–44]. We first discuss the case of general charge assignment and the conditions needed to cancel gauge anomalies. Then phenomenological

analysis is carried out by fixing the charge assignment for our particle contents. As a result, we predict one massless active neutrino, and discuss a dark matter candidate calculating relic density. In addition, we discuss lepton flavor violations (LFVs) and muon $g - 2$ in the model.

This paper is organized as follows. In Sec. II, we show the mechanism of anomaly cancellations, review our model setup, formulate the Higgs sector, fermion sector including active neutrinos, muon anomalous magnetic moment, lepton flavor violations, and phenomenologies of a dark matter candidate where we show the allowed region of DM mass to satisfy the relic density without conflicting with the direct detection bound. In Sec. III, we have a globally numerical analysis, and investigate the allowed region to satisfy all the data that we will discuss. Finally, we devote Sec. IV to our conclusions.

II. MODEL SETUP AND PHENOMENOLOGIES

Here we construct our model with $U(1)_L$ symmetry and carry out a phenomenological analysis. In the fermion sector, we introduce three exotic singly charged fermions with different $U(1)_L$ charges; E_{L_a} and E_{R_a} ($a = 1, 2$), respectively, have $\frac{1}{2}$ and x_a , while E_3 , which is a vectorlike fermion, has $-x_1 - x_2$. Notice here that the charge of E_3 is arbitrary in general, but x_a are determined by several anomaly cancellations as discussed below. In the boson sector, we introduce three isospin singlet bosons with nonzero vacuum expectation values (VEVs), and φ_1 and φ_a ($a = 1, 2$), respectively, have 1 and $\frac{1}{2} - x_a$. In addition, we introduce a singly charged boson $h^\pm$, a doubly charged boson $k^{\pm\pm}$, and an isospin singlet (doublet) inert boson $s(\eta)$, and each of them has $U(1)_L$ charge of $-\frac{1}{2}$, -1 , and 0. H is identified as the SM-like Higgs. Furthermore, we impose Z_2 odd for new fields, except $\varphi_{1,a}$ and $k^{\pm\pm}$, in order to forbid the mixing between the SM fermions and exotic

*nomura@kias.re.kr

†macokada3hiroshi@cts.nthu.edu.tw

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

TABLE I. Field contents of fermions and bosons and their generic charge assignments under $SU(2)_L \times U(1)_Y \times U(1)_L \times Z_2$, where E_3 is a vectorlike singly charged fermion, $a(=1-2)$ are flavor indices, and x_a is a nonzero arbitral charge.

	L_{L_a}	L_{L_3}	e_{R_a}	e_{R_3}	E_{L_a}	E_{R_a}	E_3	H	φ_a	φ_1	η	h^+	k^{++}	s_0
$SU(2)_L$	2	2	1	1	1	1	1	2	1	1	2	1	1	1
$U(1)_Y$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	-1	-1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	2	0
$U(1)_L$	x_a	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	x_a	$-x_1 - x_2$	0	$\frac{1}{2} - x_a$	1	0	$-\frac{1}{2}$	-1	0
Z_2	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$-$	$+$	$-$

one and to assure the stability of the DM candidate: the neutral component of η . Thus, the neutrino masses are induced at two-loop level as shown below. These particle contents and their charge assignments are summarized in Table I.

Anomaly cancellation: We explore conditions for anomaly cancellations under the $SU(2)_L \times U(1)_Y \times U(1)_L$ gauge symmetry in Table I, where $[U(1)_L]^2 U(1)_Y$ is automatically zero under these symmetries. Thus, one should consider three types of triangle anomalies, including all the families as follows:

$$[SU(2)]^2 U(1)_L: x_1 + x_2 - 1 = 0, \quad (2.1)$$

$$[U(1)_Y]^2 U(1)_L: \frac{1}{2}(x_1 + x_2) - \frac{1}{2} = 0, \quad (2.2)$$

$$[U(1)_L]^3: -(x_1^3 + x_2^3) + 1 = 0, \quad (2.3)$$

$$U(1)_L: -(x_1 + x_2) + 1 = 0. \quad (2.4)$$

Therefore, one finds the following two conditions:

$$x_1 + x_2 = 1, \quad x_1^2 - x_1 x_2 + x_2^2 = 1. \quad (2.5)$$

One finds several simple solutions such as $(x_1, x_2) = (0, 1), (1, 0)$ that respectively correspond to $\mu - \tau$ and $e - \tau$ symmetry for lepton doublets.

Fixing charge assignments: Here we fix charge assignments of $U(1)_L$ to be $x_1 = 0$ and $x_2 = 1$ for simplicity. In this case, one can simply define $\varphi_a \equiv \varphi_0$ with a $U(1)_L$ charge of $\frac{1}{2}$. Under these symmetries in Table I, the renormalizable Lagrangian in the lepton sector and Higgs potential are respectively given by

$$-\mathcal{L}_{\text{lepton}} = y_{\ell_3} \bar{L}_{L_3} H e_{R_3} + f_a \bar{E}_{R_a} \tilde{\eta} L_{L_a} + g_{L_{ab}} \bar{E}_{L_a}^c E_{L_b} k^{++} + \kappa_a \bar{E}_{L_a} e_{R_a} s_0 \quad (2.6)$$

$$+ y_{\varphi_{a1}} \bar{E}_{L_a} E_{R_1} \varphi_0 + y_{\varphi_{a2}} \bar{E}_{L_a} E_{R_2} \varphi_0^* + y_{\varphi_{31}} \bar{E}_{L_3} E_{R_1} \varphi_1^* + M_{33} \bar{E}_{L_3} E_{R_3} \quad (2.7)$$

$$+ g_{ab} \bar{e}_{R_a}^c e_{R_b} k^{++} + g_{R_{12}} (\bar{E}_{R_1}^c E_{R_2} + \bar{E}_{R_2}^c E_{R_1}) k^{++} + \text{c.c.}, \quad (2.8)$$

$$V_{\text{nontrivial}} = \mu [H^\dagger \eta s_0 + \text{c.c.}] + \frac{\mu_{khh}}{2} [k^{++} (h^-)^2 + \text{c.c.}] + \frac{\mu_\varphi}{2} [\varphi_0^2 \varphi_1^* + \text{c.c.}] + \lambda_0 [(H^\dagger i \sigma_2 \eta) h^- \varphi_0^* + \text{c.c.}], \quad (2.9)$$

where α runs over 1 to 3, and (a, b) run over 1 to 2, $\tilde{H} \equiv (i\sigma_2)H^*$ with σ_2 being the second Pauli matrix. The first term gives the masses for the SM charged leptons, and $f, g_{L/R}$ as well as μ_{khh} and λ_0 contribute to the structure of neutrino masses. μ_φ forbids the massless goldstone boson (GB) arising from $\varphi_{0,1}$. On the other hand, g_{ab} does not contribute to the neutrino masses and $g_{R_{12}}$ itself cannot reproduce the experimental results for neutrinos. Thus, we just neglect these terms in our analysis.

Next we formulate the scalar sector, in which we parametrize as follows:

$$H = \begin{bmatrix} w^+ \\ \frac{v+h+iz}{\sqrt{2}} \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta^+ \\ \frac{\eta_0+i\eta_L}{\sqrt{2}} \end{bmatrix}, \quad \varphi_{0,1} = \frac{v'_{0,1} + \sigma_{0,1} + iz'_{\varphi_{0,1}}}{\sqrt{2}}, \quad (2.10)$$

where w^+ , two massless eigenstates among z , and $z'_{\varphi_{0,1}}$ are absorbed by the SM and $U(1)_L$ gauge bosons W^+ , Z , and Z' . Inserting tadpole conditions, the CP -even mass matrix in the basis of (h, σ_0, σ_1) can be formulated by $O_R M_R^2 O_R^T \equiv \text{Diag}[m_{h_1}^2, m_{h_2}^2, m_{h_3}^2]$, where h_1 is the SM Higgs and O_R is a three by three orthogonal mixing matrix. On the other hand, the inert boson mass matrix in the basis of (η_0, s_0) is formulated by $U M_{\text{inert}}^2 U^T \equiv \text{Diag}[m_\eta^2, m_s^2]$, where U is a two by two orthogonal matrix. Then one can parametrize the relation between flavor and the mass eigenstate as [22]

$$\begin{bmatrix} \eta_0 \\ s_0 \end{bmatrix} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} \eta \\ s \end{bmatrix}, \quad (2.11)$$

where $\sin\beta$ is proportional to μv , and we expect $\sin\beta \ll 1$ in our analysis below. On the other hand, the singly charged boson mass matrix in basis of $(\eta^\pm, h^\pm)$ is formulated by $O_\pm M_\pm^2 O_\pm^T \equiv \text{Diag}[m_{H_1^\pm}^2, m_{H_2^\pm}^2]$, where $O_\pm$ is two by two orthogonal matrix.¹ Then one can parametrize the relation between flavor and mass eigenstate as [22]

$$\begin{bmatrix} \eta^\pm \\ h^\pm \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} H_1^\pm \\ H_2^\pm \end{bmatrix}, \quad (2.12)$$

where $\sin\theta$ is proportional to $\lambda_0 v v'_0$.

Z' boson: A massive Z' boson appears after spontaneous symmetry breaking of $U(1)_L$. The mass of Z' is given by $m_{Z'} = g' \sqrt{v_0'^2 + 4v_1'^2}/2$ where g' is gauge coupling of $U(1)_L$. Since the Z' couples to electrons, the mass and gauge coupling are constrained by the LEP data. Here we simply assume the mass is around TeV scale and the value of g' satisfies the constraints. Note that Z' does not contribute to neutrino mass generation and our DM candidate has no direct interaction with the Z' since η does not have a $U(1)_L$ charge. Thus, we will not discuss Z' physics in our analysis.

Exotic charged-fermion masses: After the symmetry breaking, the exotic charged fermion mass matrix in Eq. (2.7) can be given in the basis $[E_1, E_2, E_3]^T$ as follows:

$$M_E \equiv \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & 0 \\ M_{13} & 0 & M_{33} \end{bmatrix}, \quad (2.13)$$

where we have assumed M_E to be a real symmetric matrix for simplicity and define $M_{11} \equiv y_{\varphi_{11}} v'_0/\sqrt{2}$, $M_{12} \equiv y_{\varphi_{21}} v'_0/\sqrt{2}$, $M_{22} \equiv y_{\varphi_{22}} v'_0/\sqrt{2}$, and $M_{13} \equiv y_{\varphi_{31}} v'_1/\sqrt{2}$. Then M_E is diagonalized by an orthogonal mixing matrix V ($VV^T = 1$) as

$$VM_E V^T = D_E \equiv \text{Diag}[M_e, M_\mu, M_\tau], \quad E_{1,2,3} = V^T E_{e,\mu,\tau}, \quad (2.14)$$

where $M_{e,\mu,\tau}$ is the mass eigenstate.

SM charged-fermion masses: Since the first and second charged leptons are not induced at the tree level, they are rather done at the one-loop level in Fig. 1. In order to formulate these masses, let us write down the relevant Lagrangian to the SM charged leptons in the mass eigenbasis as

¹In general, it should be unitary, but it can be orthogonal when all the couplings of the Higgs potential are real.

FIG. 1. The one-loop diagram to generate SM charged-lepton masses.

$$\begin{aligned} -\mathcal{L} &\sim \sum_i^{1-3} \sum_\alpha^{e,\mu,\tau} V_{ai} f_i \bar{E}_{R\alpha} \ell_{L_i} (\cos\beta\eta - \sin\beta s) \\ &+ \sum_j^{1,2} \sum_\alpha^{e,\mu,\tau} V_{aj} \kappa_j \bar{E}_{L\alpha} \ell_{R_j} (\sin\beta\eta + \cos\beta s) + m_{\ell_{33}} \bar{e}_{L3} e_{R3} \\ &\equiv \sum_i^{1-3} \sum_\alpha^{e,\mu,\tau} F_{ai} \bar{E}_{R\alpha} \ell_{L_i} (\cos\beta\eta - \sin\beta s) \\ &+ \sum_j^{1,2} \sum_\alpha^{e,\mu,\tau} H_{aj} \bar{E}_{L\alpha} \ell_{R_j} (\sin\beta\eta + \cos\beta s) + m_{\ell_{33}} \bar{e}_{L3} e_{R3}, \end{aligned} \quad (2.15)$$

where $m_{\ell_{33}} \equiv y_{\ell_3} v/\sqrt{2}$. Then the mass matrix for the charged leptons can be induced as follows [36,45,46]:

$$(m_\ell)_{ab} = \sum_j^{1,2} \sum_i^{1-3} (\delta m_\ell)_{ji} + m_{\ell_{33}}, \quad (2.16)$$

$$\begin{aligned} (\delta m_\ell)_{ji} &= \frac{\sqrt{2} \sin\beta \cos\beta}{(4\pi)^2} \sum_\alpha^{e,\mu,\tau} (H^\dagger)_{j\alpha} M_\alpha F_{ai} \\ &\times \int_0^1 dx \frac{x M_\alpha^2 + (1-x) m_s^2}{x M_\alpha^2 + (1-x) m_\eta^2}. \end{aligned} \quad (2.17)$$

$(m_\ell)_{ab}$ is generally diagonalized by biunitary matrices as $V_{\ell_R} (m_\ell)_{ab} V_{\ell_L}^\dagger \equiv D_\ell$, where D_ℓ is the mass eigenstate of charged leptons. Then the resulting mass eigenvalues for the SM charged leptons are generally given by

$$V_{\ell_L} m_\ell^\dagger m_\ell V_{\ell_L}^\dagger = \text{diag}(|m_e|^2, |m_\mu|^2, |m_\tau|^2), \quad (2.18)$$

Thus, the observed lepton mixing arises from the neutrino part only.

Active neutrinos: First of all, let us write down the relevant Lagrangian to the neutrinos in the mass eigenbasis as

$$\begin{aligned}
-\mathcal{L} &\sim \sum_{\alpha=e,\mu,\tau} V_{ai} f_i \bar{E}_\alpha P_L \nu_i (\cos \theta H_1^- - \sin \theta H_2^-) + \sum_{a,b=1}^2 V_{ia} (g_L)_{ab} (V^T)_{bj} \bar{E}_{L_i}^c E_{L_j} k^{++} + \text{c.c.} \\
&\equiv \sum_{\alpha=e,\mu,\tau} F_{ai} \bar{E}_\alpha P_L \nu_i (\cos \theta H_1^- - \sin \theta H_2^-) + (G_L)_{ij} \bar{E}_{L_i}^c E_{L_j} k^{++} + \text{c.c.},
\end{aligned} \tag{2.19}$$

where g_L is the rank two matrix. Then the neutrino mass matrix is induced at the two-loop level in Fig. 2, which is given by [22]

$$\begin{aligned}
(m_\nu)_{ij} &\approx \frac{\mu_{khh} \sin^2 2\theta}{2(4\pi)^4} \sum_{\alpha,\beta=e,\mu,\tau} \left[\frac{F_{ia}^T M_\alpha (G_L)_{\alpha\beta} M_\beta F_{\beta j}}{M_\alpha^2} \right] \\
&\times [\mathcal{F}_I(r_{H_1}^\alpha, r_k^\alpha, r_{H_1}^\alpha, r_\alpha^\beta) - \mathcal{F}_I(r_{H_2}^\alpha, r_k^\alpha, r_{H_1}^\alpha, r_\alpha^\beta) - \mathcal{F}_I(r_{H_1}^\alpha, r_k^\alpha, r_{H_2}^\alpha, r_\alpha^\beta) + \mathcal{F}_I(r_{H_2}^\alpha, r_k^\alpha, r_{H_2}^\alpha, r_\alpha^\beta)],
\end{aligned} \tag{2.20}$$

$$\mathcal{F}_I(r_1, r_2, r_3, r_4) \equiv \int_0^1 \frac{[dx][dx']}{z(z-1)(x'+y'r_1) - z'(xr_4 + yr_3 + zr_2)}, \tag{2.21}$$

where $r_\beta^\alpha \equiv \frac{M_\beta^2}{M_\alpha^2}$, $[dx] \equiv dx dy dz \delta(x+y+z-1)$, and $[dx'] \equiv dx' dy' dz' \delta(x'+y'+z'-1)$. At the same time, we also formulate the experimental neutrino mass matrix as $m_\nu^{\text{exp}} \approx |V_{MNS}^\dagger D_\nu V_{MNS}^*|$, which can be determined by neutrino oscillation data when numerical (Dirac and Majorana) phases are provided. *Notice here that one of the three active neutrino mass eigenstates is massless, since the matrix rank g_L is two; $|m_{\nu_{1(3)}}| = 0$ in case of normal(inverted) hierarchy. The structure of the mass matrix indicate that the neutrino mixing angles mainly arise from the mixing matrix V .* Then one finds the following ranges at 3σ confidential level [47] given by,² assuming the normal hierarchy:

$$m_\nu^{\text{exp}} \approx \begin{bmatrix} 0.0845 - 0.475 & 0.0629 - 0.971 & 0.0411 - 0.964 \\ * & 1.44 - 3.49 & 1.94 - 2.85 \\ * & * & 1.22 - 3.33 \end{bmatrix} \times 10^{-11} \text{ GeV}, \tag{2.22}$$

and Majorana phases $\alpha_{1,2}$ taken to be $\alpha_{1,2} \in [-\pi, \pi]$. In the numerical analysis, we impose the constraint $|m_\nu| \approx m_\nu^{\text{exp}}$.

Muon anomalous magnetic moment (Δa_μ): Δa_μ has been observed and its discrepancy is estimated by [48]

$$\Delta a_\mu = (26.1 \pm 8.0) \times 10^{-10}. \tag{2.23}$$

The relevant Yukawa Lagrangian contributing to Δa_μ as well as LFVs in the mass eigenbasis is given by

$$-\mathcal{L} \sim \sum_i^{1-3} \sum_{\alpha=e,\mu,\tau} F_{ai} \bar{E}_\alpha P_L \ell_i \eta + \sum_j^{1,2} \sum_{\alpha} H_{aj} \bar{E}_{L_a} \ell_{R_j} s + \text{c.c.}, \tag{2.24}$$

where $\eta \equiv \eta_R \simeq \eta_l$ and $\sin \beta \ll 1$. Then our Δa_μ is induced by interaction with F coupling as explained above, and its form is computed as

²Recently $\delta = -\pi/2$ is experimentally favored. However, our result does not change significantly, even if we fix to be $\delta = -\pi/2$.

$$\begin{aligned}
\Delta a_\mu^Y &\approx \frac{2m_\mu^2}{(4\pi)^2} \sum_{\alpha=e,\mu,\tau} (|F_{2\alpha}|^2 F_{II}(m_\eta, M_\alpha) \\
&+ |H_{2\alpha}|^2 F_{II}(m_s, M_\alpha)),
\end{aligned} \tag{2.25}$$

$$\begin{aligned}
F_{II}(m_a, m_b) &\equiv \frac{2m_a^6 + 3m_a^4 m_b^2 - 6m_a^2 m_b^4 + m_b^6 + 12m_a^4 m_b^2 \ln \frac{m_b}{m_a}}{12(m_a^2 - m_b^2)^4}.
\end{aligned} \tag{2.26}$$

Considering the neutrino oscillations and lepton flavor violations for the F term as will be discussed below, we find the maximal order of Δa_μ^Y to be 10^{-12} . On the other hand,

FIG. 2. The two-loop diagram to generate neutrino masses.

the term with $H_{2\alpha}$ provides the dominant contribution to Δa_μ , since it is not constrained by any phenomenologies once we take $H_{21}, H_{23} \ll H_{22}$.

In addition, the Z' gauge boson can contribute to Δa_μ and its form is approximately given by

$$\Delta a_\mu^{Z'} \approx \frac{g_{Z'}^2 m_\mu^2}{48\pi^2 M_{Z'}^2}, \quad (2.27)$$

where Z' is the new gauge vector boson. Since the right-handed electron couples to the Z' boson, we have the constraint $3.7 \text{ TeV} \lesssim M_{Z'}/g_{Z'}$ [49]; the maximal value of $\Delta a_\mu^{Z'}$ is $\Delta a_\mu^{Z'}(\text{Max}) = 1.72 \times 10^{-12}$. Combining Δa_μ^Y and $\Delta a_\mu^{Z'}$, we find the final result of muon $g-2$, $\Delta a_\mu \equiv \Delta a_\mu^Y + \Delta a_\mu^{Z'}$, where we will adapt the maximal value $\Delta a_\mu^{Z'}(\text{Max})$ in our numerical analysis below.

Lepton flavor violations: LFV processes of $\ell \rightarrow \ell' \gamma$ are given by the same term as the $(g-2)_\mu$, and their forms are given by

$$\text{BR}(\ell_i \rightarrow \ell_j \gamma) \approx \frac{48\pi^3 C_{ij} \alpha_{em}}{(4\pi)^4 G_F^2} \left| \sum_{\alpha=e,\mu,\tau} F_{j\alpha}^\dagger F_{ai} F_{II}(m_{\eta^0}, M_\alpha) \right|^2, \quad (2.28)$$

where $\alpha_{em} \approx 1/137$ is the fine-structure constant, $G_F \approx 1.17 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant, and $C_{21} \approx 1$, $C_{31} \approx 0.1784$, $C_{32} \approx 0.1736$. Experimental upper bounds are given by [3,50]:

$$\begin{aligned} \text{BR}(\mu \rightarrow e \gamma) &\lesssim 4.2 \times 10^{-13}, & \text{BR}(\tau \rightarrow e \gamma) &\lesssim 3.3 \times 10^{-8}, \\ \text{BR}(\tau \rightarrow \mu \gamma) &\lesssim 4.4 \times 10^{-8}, \end{aligned} \quad (2.29)$$

where we define $\ell_1 \equiv e$, $\ell_2 \equiv \mu$, and $\ell_3 \equiv \tau$.

Oblique parameter: Since we have an isospin doublet boson η , we have to consider the oblique parameter known as ΔS and ΔT [51]. In our case, one finds the following relations:

$$\begin{aligned} \Delta S &\approx \frac{1}{12\pi} \ln \left[\frac{m_\eta^2}{m_{H^\pm}^2} \right], \\ \Delta T &\approx \frac{1}{16\pi^2 v^2 \alpha_{em}} \left[\frac{m_\eta^2 + m_{H^\pm}^2}{2} - \frac{m_{H^\pm}^2 m_\eta^2}{m_{H^\pm}^2 - m_\eta^2} \ln \left[\frac{m_{H^\pm}^2}{m_\eta^2} \right] \right], \end{aligned} \quad (2.30)$$

where we recall the conditions $(\theta, \beta) \ll 1$; $m_{\eta^\pm} \approx m_{H^\pm}$ and $m_\eta \approx m_{\eta_R} \approx m_{\eta_L}$. Then the current bounds are given by [52]

$$-0.04 \lesssim \Delta S \lesssim 0.14, \quad 0.01 \lesssim \Delta T \lesssim 0.15. \quad (2.31)$$

Dark matter: In our model, we have two types of DM candidates: η and s . But let us here focus on the neutral

component of η , resymbolized as a DM candidate by $X \equiv \eta$, because η is more testable than s due to constraining the mass from experiments such as neutrino oscillations, LFVs, and oblique parameters. Although the general analysis has been done by Ref. [53], we impose a constraint of the DM mass, which is smaller than the mass of the W boson but greater than half of the Z boson mass, in order to forbid the invisible decay of the Z boson; therefore,

$$\frac{m_Z}{2} \lesssim M_X \lesssim m_W. \quad (2.32)$$

This region is favorable for getting sizable muon $g-2$, and well testable in the direct detection constraint such as LUX experiment [54] because it provides the most severe bound at around 50 GeV. Under the condition, we have two relevant annihilation cross sections to explain the relic density of DM. One mode arises from Yukawa coupling F that gives the d -wave dominance, and another one does from the s channel via SM Higgs with a final state of bottom pairs, where we assume mixing among the CP -even neutral bosons is negligible. The d -wave dominant cross section given by Eq. (2.24) is found to be [55]

$$\sigma_{v_{\text{rel}}} \approx \frac{M_X^6}{60\pi} \sum_{i,j=1}^3 \left| \sum_{\alpha=e,\mu,\tau} \frac{F_{i\alpha}^\dagger F_{aj}}{(M_\alpha^2 + M_X^2)^2} \right|^2. \quad (2.33)$$

In our estimation, however, this cross section reaches $10^{-10} [\text{GeV}]^{-2}$ at most, which is smaller than the cross section required to give the correct relic density by one order of magnitude. Thus, we have to rely on the Higgs portal interaction mode, and its dimensionless cross section W is found to be

$$W(s) = \frac{3\lambda_{XXh_1}^2 m_b^2 (s - 4m_b^2)}{4\pi |s - m_{h_1}^2 + im_h \Gamma_{\text{tot}}^{h_1}|^2} \sqrt{1 - \frac{4m_b^2}{s}}, \quad (2.34)$$

$$\begin{aligned} \Gamma_{\text{tot}}^{h_1} &\equiv \Gamma_{\text{SM}}^{h_1} + \Gamma_{\text{inv}}^h, \\ \Gamma_{\text{inv}}^h &= \frac{\mu_{XXh_1}^2 v^2}{64\pi} \sqrt{1 - \frac{4M_X^2}{m_{h_1}}} \Theta\left(\frac{m_{h_1}}{2} - M_X\right), \end{aligned} \quad (2.35)$$

where $\lambda_{XXh_1} (\equiv \lambda_{H\eta} + \lambda'_{H\eta} + 2\lambda''_{H\eta})$ is the trilinear couplings of $X - X - h_1$, arising from $\lambda_{H\eta} |\eta|^2 |H|^2 + \lambda'_{H\eta} |\eta^\dagger H|^2 + \lambda''_{H\eta} / 2 (\eta^\dagger H)^2 + \text{H.c.}$ Notice here that μ_{XXh_1} is restricted by the direct detection with spin independent scattering via the Higgs portal,³ and its bound is conservatively found to be [55]

³The constraint of invisible decay of the SM Higgs always gives milder than the one of direct detection in our parameter region. Thus, we will not discuss here.

FIG. 3. Plot line of relic density in terms of the DM mass, where we have used $\lambda_{XXh_1}^{\text{Max}} = 0.01$. The horizontal line represents the measured relic density ~ 0.12 .

$$\lambda_{XXh_1} \lesssim 0.01. \quad (2.36)$$

Here we apply the following formula to get the relic density of DM given by [56]:

$$\Omega h^2 \approx \frac{1.07 \times 10^9}{\sqrt{g_*(x_f)} M_{\text{Pl}} J(x_f) [\text{GeV}]}, \quad (2.37)$$

where $g^*(x_f \approx 25)$ is the degrees of freedom for relativistic particles at temperature $T_f = M_X/x_f$, $M_{\text{Pl}} \approx 1.22 \times 10^{19}$ GeV, and $J(x_f) (\equiv \int_{x_f}^{\infty} dx \frac{\langle \sigma v_{\text{rel}} \rangle}{x^2})$ is given by [57]

$$J(x_f) = \int_{x_f}^{\infty} dx \left[\frac{\int_{4M_X^2}^{\infty} ds \sqrt{s - 4M_X^2} W(s) K_1(\frac{\sqrt{s}}{M_X} x)}{16M_X^5 x [K_2(x)]^2} \right], \quad (2.38)$$

where $M_P \approx 1.22 \times 10^{19}$ GeV is the Planck mass, $g_* \approx 100$ is the total number of effective relativistic degrees of freedom at the time of freeze-out, and $x_f \approx 25$ is defined by M_X/T_f at the freeze-out temperature (T_f). Then one has

$$\begin{aligned} M_E &\in [100, 1000] \text{ GeV}, & m_{H_2^\pm} &\in [200, 500] \text{ GeV}, & m_{H_2^\pm} &\in [80, M_X + 140] \text{ GeV}, & m_{k^{++}} &\in [500, 600] \text{ GeV}, \\ \mu_{khh} &\in [1, 2] \text{ GeV}, & (\sin \theta, \sin \beta) &\in [0, \pi/4], & m_{k^{++}} &\in [500, 600] \text{ GeV}, & m_S &\in [1.2M_X, 150] \text{ GeV}, \\ |f_{e,\mu,\tau}, \kappa_3, g_{L_3}| &\in [0.001, 1], & (\rho, \sigma) &\in [0, \pi], & \delta &\in [\pi, 2\pi], & |s_\alpha| &\in [10^{-5}, 0.1], \end{aligned} \quad (3.1)$$

where the lower mass range for $m_{H_2^\pm}$ arises from the bound from LEP data [59], while the upper bound is from the oblique parameters, and we impose all the constraints as discussed above.

⁴In principle, all the Yukawa couplings could be solved by using all the components of these relations. However, it is technically difficult in our model.

FIG. 4. Scattering plots on the muon $g - 2$ and m_S plane, where each point satisfies experimental constraints.

to satisfy the current relic density of DM: $\Omega h^2 \approx 0.12$ [58]. In Fig. 3, we show the line of Ωh^2 in terms of M_X , where we have used the maximum value in Eq. (2.36): $\lambda_{XXh_1}^{\text{Max}} = 0.01$. Thus, one finds that the resulting allowed region is

$$54 \text{ GeV} \lesssim M_X \lesssim 62.5 \text{ GeV}, \quad \text{for } \lambda_{XXh_1} \lesssim 0.01, \quad (2.39)$$

where the upper bound of the DM mass, 62.5 GeV, arises from the pole mass of the half SM Higgs.

III. NUMERICAL ANALYSIS

In this section, we show a global analysis. Before the numerical analysis, we work on the diagonal basis of g_L by the phase redefinition of E_L ; $g_L = \text{Diag}(g_{L_1}, g_{L_2}, g_{L_3})$. Then we directly solve the couplings $\kappa_{1,2}$ and $g_{L_{1,2}}$ by using the relations $|V_{\ell_R} m_\ell V_{\ell_L}^\dagger|_{11(22)} = |m_{e(\mu)}|$ and $|m_\nu|_{11(12)} \approx (m_\nu^{\text{exp}})_{11(22)}$, respectively, where we impose perturbative bounds on these output parameters: $(\kappa_{1,2}, g_{L_{1,2}}) \lesssim \sqrt{4\pi}$.⁴ Now we randomly select the following range of reduced input parameters as

In Fig. 4, we show the scattering allowed plots in terms of muon $g - 2$ and m_S . It suggests that the typical value of muon $g - 2$ is of the order 10^{-12} that is smaller than the experimental value by a three-order magnitude.

In Fig. 5, we demonstrate the couplings of $\kappa_1 - \kappa_2$ in the left panel, and $g_{L_1} - g_{L_2}$ in the right panel. The left one implies $2 \lesssim \kappa_1$ requires rather large coupling, whereas κ_2 is of the order 0.001, and each of them has a weak correlation.

FIG. 5. Left panel: Allowed parameter points on the $k_1 - k_2$ plane. Right panel: Allowed parameter points on the $g_{L_1} - g_{L_2}$ plane.

While the right one suggests both of couplings run $-2.5 \sim -0.3$ with degeneracy to some extent.

IV. CONCLUSION

We have constructed a radiative neutrino mass model based on a gauged lepton flavor symmetry $U(1)_L$. The condition needed to cancel gauge anomalies is discussed by introducing some exotic leptons with general $U(1)_L$ charge. Then we discussed the phenomenology of the model by fixing the charge assignment.

The neutrino mass matrix can be induced at two-loop level where the exotic leptons and charged scalar bosons propagate inside the loop diagram. On the other hand, the first and second charged leptons of SM are induced at one-loop level. Due to the feature of flavor dependent symmetry, we have predicted one massless active neutrino and a bosonic dark matter candidate from an inert doublet. Calculating the relic density, we have found that an

observed value can be obtained via a Higgs portal interaction with a mass range of dark matter at $56\text{--}62.5$ GeV. In addition, we have also discussed lepton flavor violation and muon $g - 2$ in the model.

Then we have done a global numerical analysis to satisfy all the constraints such as charged-lepton masses, neutrino mass differences its mixing, LFVs, and oblique parameters, within the range of the DM mass. Then we have found that the typical value of muon $g - 2$ is of the order 10^{-12} , which is smaller than the experimental value by a three-order magnitude. Also, we have shown the typical Yukawa couplings of $\kappa_{1,2}$ and $g_{L_{1,2}}$, and have found typical ranges and their correlations.

ACKNOWLEDGMENTS

H. O. is sincerely grateful for the KIAS member's comments and all around support.

-
- [1] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **113**, 151601 (2014).
 - [2] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **08** (2017) 055.
 - [3] A. M. Baldini *et al.* (MEG Collaboration), *Eur. Phys. J. C* **76**, 434 (2016).
 - [4] G. W. Bennett *et al.* (Muon $g-2$ Collaboration), *Phys. Rev. D* **73**, 072003 (2006).
 - [5] A. Crivellin, J. Fuentes-Martin, A. Greljo, and G. Isidori, *Phys. Lett. B* **766**, 77 (2017).
 - [6] A. Zee, *Nucl. Phys.* **B264**, 99 (1986); K. S. Babu, *Phys. Lett. B* **203**, 132 (1988).
 - [7] K. S. Babu and C. Macesanu, *Phys. Rev. D* **67**, 073010 (2003).
 - [8] D. Aristizabal Sierra and M. Hirsch, *J. High Energy Phys.* **12** (2006) 052.
 - [9] M. Nebot, J. F. Oliver, D. Palao, and A. Santamaria, *Phys. Rev. D* **77**, 093013 (2008).
 - [10] D. Schmidt, T. Schwetz, and H. Zhang, *Nucl. Phys.* **B885**, 524 (2014).
 - [11] J. Herrero-Garcia, M. Nebot, N. Rius, and A. Santamaria, *Nucl. Phys.* **B885**, 542 (2014).
 - [12] H. N. Long and V. V. Vien, *Int. J. Mod. Phys. A* **29**, 1450072 (2014).
 - [13] V. Van Vien, H. N. Long, and P. N. Thu, *arXiv:1407.8286*.
 - [14] M. Aoki, S. Kanemura, T. Shindou, and K. Yagyu, *J. High Energy Phys.* **07** (2010) 084; **11** (2010) 49.
 - [15] M. Lindner, D. Schmidt, and T. Schwetz, *Phys. Lett. B* **705**, 324 (2011).
 - [16] S. Baek, P. Ko, H. Okada, and E. Senaha, *J. High Energy Phys.* **09** (2014) 153.
 - [17] M. Aoki, J. Kubo, and H. Takano, *Phys. Rev. D* **87**, 116001 (2013).
 - [18] Y. Kajiyama, H. Okada, and K. Yagyu, *Nucl. Phys.* **B874**, 198 (2013).

- [19] Y. Kajiyama, H. Okada, and T. Toma, *Phys. Rev. D* **88**, 015029 (2013).
- [20] S. Baek, H. Okada, and T. Toma, *J. Cosmol. Astropart. Phys.* **06** (2014) 027.
- [21] H. Okada, [arXiv:1404.0280](#).
- [22] H. Okada, T. Toma, and K. Yagyu, *Phys. Rev. D* **90**, 095005 (2014).
- [23] H. Okada, [arXiv:1503.04557](#).
- [24] C. Q. Geng and L. H. Tsai, *Ann. Phys. (Amsterdam)* **365**, 210 (2016).
- [25] S. Kashiwase, H. Okada, Y. Orikasa, and T. Toma, *Int. J. Mod. Phys. A* **31**, 1650121 (2016).
- [26] M. Aoki and T. Toma, *J. Cosmol. Astropart. Phys.* **09** (2014) 016.
- [27] S. Baek, H. Okada, and T. Toma, *Phys. Lett. B* **732**, 85 (2014).
- [28] H. Okada and Y. Orikasa, *Phys. Rev. D* **93**, 013008 (2016).
- [29] D. Aristizabal Sierra, A. Degee, L. Dorame, and M. Hirsch, *J. High Energy Phys.* **03** (2015) 040.
- [30] T. Nomura and H. Okada, *Phys. Lett. B* **756**, 295 (2016).
- [31] T. Nomura, H. Okada, and Y. Orikasa, *Eur. Phys. J. C* **77**, 103 (2017).
- [32] C. Bonilla, E. Ma, E. Peinado, and J. W. F. Valle, *Phys. Lett. B* **762**, 214 (2016).
- [33] M. Kohda, H. Sugiyama, and K. Tsumura, *Phys. Lett. B* **718**, 1436 (2013).
- [34] B. Dasgupta, E. Ma, and K. Tsumura, *Phys. Rev. D* **89**, 041702 (2014).
- [35] T. Nomura and H. Okada, *Phys. Rev. D* **94**, 075021 (2016).
- [36] T. Nomura and H. Okada, *Phys. Rev. D* **94**, 093006 (2016).
- [37] T. Nomura, H. Okada, and Y. Orikasa, *Phys. Rev. D* **94**, 115018 (2016).
- [38] Z. Liu and P. H. Gu, *Nucl. Phys.* **B915**, 206 (2017).
- [39] C. Simoes and D. Wegman, *J. High Energy Phys.* **04** (2017) 148.
- [40] S. Baek, H. Okada, and Y. Orikasa, [arXiv:1703.00685](#).
- [41] S. Y. Ho, T. Toma, and K. Tsumura, *J. High Energy Phys.* **07** (2017) 101.
- [42] T. Nomura and H. Okada, [arXiv:1706.01321](#).
- [43] S. Y. Guo, Z. L. Han, B. Li, Y. Liao, and X. D. Ma, *Nucl. Phys.* **B928**, 435 (2018).
- [44] F. del Aguila, A. Aparici, S. Bhattacharya, A. Santamaria, and J. Wudka, *J. High Energy Phys.* **05** (2012) 133.
- [45] T. Nomura and H. Okada, *Phys. Lett. B* **761**, 190 (2016).
- [46] T. Nomura and H. Okada, *Phys. Rev. D* **96**, 015016 (2017).
- [47] D. V. Forero, M. Tortola, and J. W. F. Valle, *Phys. Rev. D* **90**, 093006 (2014).
- [48] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, and T. Teubner, *J. Phys. G* **38**, 085003 (2011).
- [49] T. Nomura and H. Okada, *Phys. Rev. D* **97**, 015015 (2018).
- [50] J. Adam *et al.* (MEG Collaboration), *Phys. Rev. Lett.* **110**, 201801 (2013).
- [51] R. Barbieri, L. J. Hall, and V. S. Rychkov, *Phys. Rev. D* **74**, 015007 (2006).
- [52] K. Cheung, T. Nomura, and H. Okada, *Phys. Lett. B* **768**, 359 (2017).
- [53] T. Hambye, F.-S. Ling, L. Lopez Honorez, and J. Rocher, *J. High Energy Phys.* **07** (2009) 090; **05** (2010) 66.
- [54] D. S. Akerib *et al.* (LUX Collaboration), *Phys. Rev. Lett.* **118**, 021303 (2017).
- [55] A. Das, T. Nomura, H. Okada, and S. Roy, *Phys. Rev. D* **96**, 075001 (2017).
- [56] J. Edsjo and P. Gondolo, *Phys. Rev. D* **56**, 1879 (1997).
- [57] K. Nishiwaki, H. Okada, and Y. Orikasa, *Phys. Rev. D* **92**, 093013 (2015).
- [58] P. A. R. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **571**, A16 (2014).
- [59] G. Abbiendi *et al.* (OPAL Collaboration), *Eur. Phys. J. C* **72**, 2076 (2012).