Dark matter and electroweak phase transition in the mixed scalar dark matter model

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We study the electroweak phase transition in the framework of the scalar singlet-doublet mixed dark matter model, in which the particle dark matter candidate is the lightest neutral Higgs that comprises the CP -even component of the inert doublet and a singlet scalar. The dark matter can be dominated by the inert doublet or singlet scalar depending on the mixing. We present several benchmark models to investigate the two situations after imposing several theoretical and experimental constraints. An additional singlet scalar and the inert doublet drive the electroweak phase transition to be strongly first order. A strong first-order electroweak phase transition and a viable dark matter candidate can be accomplished in two benchmark models simultaneously, for which a proper mass splitting among the neutral and charged Higgs masses is needed.

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I. INTRODUCTION

The dark matter (DM) relic abundance and baryon asymmetry of the Universe (BAU) are two fundamental problems puzzling cosmologists and physicists. With the accumulation of direct and indirect detection experiments, the weakly interacting massive particle (WIMP) is still one of the most popular DM candidates, which requires new physics beyond the standard model (SM) of particle physics. On the other hand, the electroweak baryogenesis (EWBG) provides a very attractive mechanism to explain the BAU problem. One key ingredient of the EWBG mechanism is the strong first-order electroweak phase transition (EWPT), which prevents the baryon asymmetry from being washed out by the sphaleron process [1–3] and helps one understand the electroweak symmetry breaking. The model that can provide a strong first-order EWPT might drive deviation of the triple Higgs coupling from the SM prediction, which can be detected at a future high-energy proton-proton collider [3,4]. It could be very intriguing if new physics can explain DM relic abundance and accomplish a strong first-order EWPT.

In the simplest scalar extended dark matter models, the DM-Higgs quartic coupling leads to the interaction between

the DM and nucleon. Except the so-called *Higgs funnel regime* (with $m_{\text{DM}} \sim m_h/2$), the scalar singlet dark matter suffers from severe constraints from the upgrade of direct detection experiments, such as PandaX-II [5], LUX [6], XENON [7], etc. Another popular DM model is the inert doublet model (IDM), in which only one doublet receives a vacuum expectation value (VEV) in the two-Higgs doublet model (2HDM). The IDM has been studied extensively as a framework to provide scalar doublet DM [8–18]. In this model, the low-mass and intermediate-mass regions are almost excluded by the collider constraints and the DM direct detection limits, and the remnant space is the high-mass region (> 500 GeV), which is quite difficult to probe in the direct detection experiments [19]. Then, it is very natural to consider if extending of such models can offer the possibility of avoiding current and even future direct detection bounds. DM phenomenology aside, the strength of the EWPT can generally be enhanced by adding a scalar to the SM Higgs potential through cubic or quartic couplings with the SM-like Higgs field. The role played by the scalar dark matter during the Universe cooling down has been studied extensively, including singlet as well as inert doublet types of DM; e.g., see Refs. [20–31].

A scenario with the DM being the singlet-doublet mixed scalar is very appealing and has been studied previously [32–37].¹ In such a model, the IDM is extended with a SM singlet scalar. The inert doublet and the singlet are odd in a

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¹For the other 2HDM + Singlet studies, one can see, e.g., Refs. [38–40] and the references therein.

new $\mathbb{Z}_2$ discrete symmetry, whereas the SM sector is even. After electroweak symmetry breaking, the electroweak charged doublet and the extra singlet mix into the scalar DM. The mixing comes from the renormalizable couplings to the SM-like Higgs field. The DM-Higgs boson triple and quartic couplings in such a model are controlled by the singlet-doublet scalars mixing. As a consequence, the DM annihilation and spin-independent DM-nucleon scattering processes can be characterized by the mixing. The thermal DM relic abundance can be achieved with an appropriate magnitude of the mixing. The parameter space of this kind of model has the advantage of being less constrained by the spin-independent direct detection experiments.

In this work, we study dark matter phenomenology and the EWPT in the scalar singlet-doublet mixed DM model. Different from Refs. [34,35], although the DM self-interactions have no impact on the DM annihilation and DM-nuclear scattering, we keep them in our model since these terms might affect the vacuum barrier during the Universe cooling down. In the model, besides the contribution of the inert doublet to the vacuum barrier, there is an additional contribution that comes from the singlet scalar, which is also determined by the mixing. Therefore, a proper mixture also characterizes the realization of the strong first-order EWPT. In this model, we find that the additional singlet along with the mixing effects makes the situation of the EWPT different from the IDM; two benchmarks with moderate dark matter mass are allowed to gain the correct relic abundance and to achieve a strong first-order EWPT. Some features of the model are found and listed below:

- (1) The vacuum stability and T parameter constrain the singlet-doublet mixing nontrivially.
- (2) DM relic density and the DM-nuclear scattering cross section highly depend on the mixing of the singlet and doublet scalars.
- (3) Coannihilation effects play an important role near the degenerate masses region of the dark sector and lead to the cancellations between the annihilation processes to yield the correct DM relic density.
- (4) A proper mass splitting between the $\mathbb{Z}_2$ -odd neutral Higgs and charged Higgs is needed to obtain a strong first-order EWPT.

The paper is organized as follows. In Sec. II, we present the mixed scalar dark matter model. Theoretical constraints and Large Electron Positron collider (LEP) bounds on scalar masses are addressed in Sec. III. Section IV is devoted to the comments on DM relic density and direct detection in the model. In Sec. V, we present the approach of estimation of the strength of the EWPT with thermal

effective potential. In Sec. VI, we give a comprehensive analysis of DM phenomenology and the EWPT in several benchmark models. After that, we address the collider prospects of the benchmark models in Sec. VII. We conclude in Sec. VIII.

II. MIXED SINGLET-DOUBLET SCALAR DARK MATTER MODEL

We extend the inert doublet model with an additional real singlet scalar (S) and assume the $\mathbb{Z}_2$ symmetry to inert doublet Φ_I and the S ; the tree-level potential of the model is

$$\begin{aligned}
 V = & \mu_H^2 \Phi_H^\dagger \Phi_H + \mu_I^2 \Phi_I^\dagger \Phi_I + \mu_S^2 S^2 + \lambda_1 (\Phi_H^\dagger \Phi_H)^2 \\
 & + \lambda_2 (\Phi_I^\dagger \Phi_I)^2 + \lambda_3 (\Phi_H^\dagger \Phi_H) (\Phi_I^\dagger \Phi_I) \\
 & + \lambda_4 (\Phi_I^\dagger \Phi_H) (\Phi_H^\dagger \Phi_I) + \frac{1}{2} \lambda_5 [(\Phi_I^\dagger \Phi_H)^2 + (\Phi_H^\dagger \Phi_I)^2] \\
 & + \lambda_6 S^4 + \lambda_7 S^2 (\Phi_H^\dagger \Phi_H) + \lambda_8 S^2 (\Phi_I^\dagger \Phi_I) \\
 & + \mu_{\text{soft}} S (\Phi_H^\dagger \Phi_I + \Phi_I^\dagger \Phi_H),
 \end{aligned} \tag{1}$$

where Φ_H is the SM Higgs doublet and Φ_I is the inert doublet. The doublets are given as

$$\Phi_H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + iG^0) \end{pmatrix}, \quad \Phi_I = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H_0 + iA) \end{pmatrix}, \tag{2}$$

where Φ_H develops a VEV $v = 246$ GeV, whereas the inert doublet and the real singlet scalar S do not produce any VEV. The $\mathbb{Z}_2$ symmetry of Φ_I , S remains unbroken. Goldstones G^+ and G^0 are absorbed in $W^\pm$, Z bosons after spontaneous symmetry breaking. All the parameters in Eq. (1) are assumed to be real. By minimizing the potential in Eq. (1), we obtain the relation

$$\mu_H^2 = -\lambda_1 v^2, \tag{3}$$

and the mass terms for the SM Higgs and the charged scalars can be obtained as

$$\begin{aligned}
 m_h^2 &= 2\lambda_1 v^2, \\
 m_{H^\pm}^2 &= \mu_I^2 + \lambda_3 \frac{v^2}{2}.
 \end{aligned} \tag{4}$$

Working in the basis of real neutral scalars, (S, H_0, A) , where H_0 and A are the real and imaginary components of the neutral component of Φ_I , the mass squared matrix is

$$\mathbf{M}^2 = \begin{pmatrix} 2\mu_S^2 + v^2\lambda_7 & v\mu_{\text{soft}} & 0 \\ v\mu_{\text{soft}} & \mu_I^2 + v^2(\lambda_3 + \lambda_4 + \lambda_5)/2 & 0 \\ 0 & 0 & \mu_I^2 + v^2(\lambda_3 + \lambda_4 - \lambda_5)/2 \end{pmatrix}. \tag{5}$$

Considering one of the mixed states of the CP -even scalars S and H_0 is the lightest particle among the three eigenstates, then it could serve as the DM candidate, and the DM mixing is induced by the upper-left 2×2 block of the mass matrix in Eq. (5). The custodial $SU(2)$ symmetry could be violated slightly by the mixing of S and H_0 , which is important for the T parameter exclusion and therefore the choice of the parameters when we explore the EWPT and dark matter phenomenology. Usually, the relation between S , H_0 and mass eigenstates χ and H can be written as

$$\begin{pmatrix} S \\ H_0 \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \chi \\ H \end{pmatrix}, \quad (6)$$

with the mixing angle

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2v\mu_{\text{soft}}}{\tilde{M}_S^2 - \tilde{M}_{H_0}^2} \right), \quad (7)$$

and the two eigenvalues of the mass-squared matrix are

$$\frac{1}{2} [\tilde{M}_S^2 + \tilde{M}_{H_0}^2 \pm \sqrt{(\tilde{M}_S^2 - \tilde{M}_{H_0}^2)^2 + 4v^2\mu_{\text{soft}}^2}], \quad (8)$$

where $\tilde{M}_S^2 = 2\mu_S^2 + v^2\lambda_7$ and $\tilde{M}_{H_0}^2 = \mu_I^2 + v^2(\lambda_3 + \lambda_4 + \lambda_5)/2$. Either χ or H can be the DM candidate.

The masses and interactions of the scalar sector are parametrized by the scalar-potential parameters $\lambda_{1...8}$ and $\mu_{H,I,S,\text{soft}}$, which can be traded for five physical masses $\{m_h, m_\chi, m_H, m_A, m_{H^\pm}\}$ along with $\{\lambda_L, \lambda_{2,6,7,8}, \theta\}$, where $\lambda_L = (\lambda_3 + \lambda_4 + \lambda_5)/2$. Then, the other parameters in the scalar potential in terms of the physical ones are expressed as

$$\lambda_3 = \frac{2}{v^2} (-m_H^2 \cos^2 \theta + m_{H^\pm}^2 - m_\chi^2 \sin^2 \theta + \lambda_L v^2), \quad (9)$$

$$\lambda_4 = \frac{1}{v^2} (m_A^2 + m_H^2 \cos^2 \theta - 2m_{H^\pm}^2 + m_\chi^2 \sin^2 \theta), \quad (10)$$

$$\lambda_5 = \frac{1}{v^2} (-m_A^2 + m_H^2 \cos^2 \theta + m_\chi^2 \sin^2 \theta), \quad (11)$$

$$\mu_S^2 = \frac{1}{2} (m_\chi^2 \cos^2 \theta + m_H^2 \sin^2 \theta - \lambda_7 v^2), \quad (12)$$

$$\mu_I^2 = m_H^2 \cos^2 \theta + m_\chi^2 \sin^2 \theta - \lambda_L v^2, \quad (13)$$

$$\mu_{\text{soft}} = \frac{1}{2v} (m_\chi^2 - m_H^2) \sin 2\theta. \quad (14)$$

The associated DM-Higgs couplings that characterize the Higgs portal annihilation of the DM and the DM-nucleon scattering are given by

$$a_{h\chi\chi} = 2v\lambda_7 \cos^2 \theta + 2v\lambda_L \sin^2 \theta + \mu_{\text{soft}} \sin 2\theta \quad (15)$$

or

$$a_{hHH} = 2v\lambda_7 \sin^2 \theta + 2v\lambda_L \cos^2 \theta - \mu_{\text{soft}} \sin 2\theta. \quad (16)$$

III. THEORETICAL AND EXPERIMENTAL CONSTRAINTS

A. Perturbative unitarity

In the high-energy limit, the quartic contact interaction terms contribute to the tree-level scalar-scalar scattering matrix dominantly. The s -wave scattering amplitudes are constrained by the perturbative unitarity limits, which requires that the eigenvalues of the S matrix $\mathcal{M}$ must be smaller than the unitarity bound:

$$|\text{Re} \mathcal{M}| < \frac{1}{2}. \quad (17)$$

The perturbative unitarity of the general two-Higgs-doublet model was first studied in Refs. [41,42]. In this work, we extend the formalism of Refs. [43,44] for the IDM to states containing an extra singlet S . The initial states are classified according to their total hypercharge Y (0, 1 or 2), weak isospin σ (0, $\frac{1}{2}$ or 1), and discrete $\mathbb{Z}_2$ charge X .

For simplicity, we only list here the initial states with hypercharge $Y=0$ and $\sigma=0$, which differ from the 2HDM initial states [43,44],

$$\begin{aligned} & \frac{1}{\sqrt{2}} \Phi_H^\dagger \Phi_H, & \frac{1}{\sqrt{2}} \Phi_I^\dagger \Phi_I, & \frac{1}{\sqrt{2}} S^2, \\ & \frac{1}{\sqrt{2}} \Phi_H^\dagger \Phi_I, & \frac{1}{\sqrt{2}} \Phi_I^\dagger \Phi_H, & \end{aligned} \quad (18)$$

where the first three states are even under $\mathbb{Z}_2$ and the last two states are odd. In the following, we present all the scattering matrices of the model:

$$\begin{aligned} 8\pi S_{Y=2,\sigma=1} &= \begin{pmatrix} 2\lambda_1 & \lambda_5 & 0 \\ \lambda_5^* & 2\lambda_2 & 0 \\ 0 & 0 & \lambda_3 + \lambda_4 \end{pmatrix}, \\ 8\pi S_{Y=2,\sigma=0} &= \lambda_3 - \lambda_4, \end{aligned} \quad (19)$$

$$\begin{aligned} 8\pi S_{Y=0,\sigma=1} &= \begin{pmatrix} 2\lambda_1 & \lambda_4 & 0 & 0 \\ \lambda_4 & 2\lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & \lambda_5^* \\ 0 & 0 & \lambda_5 & \lambda_3 \end{pmatrix}, \\ 8\pi S_{Y=1,\sigma=1/2} &= \begin{pmatrix} 2\lambda_7 & 0 \\ 0 & 2\lambda_8 \end{pmatrix}, \end{aligned} \quad (20)$$

$$8\pi S_{Y=0,\sigma=0} = \begin{pmatrix} 6\lambda_1 & 2\lambda_3 + \lambda_4 & \sqrt{2}\lambda_7 & 0 & 0 \\ 2\lambda_3 + \lambda_4 & 6\lambda_2 & \sqrt{2}\lambda_8 & 0 & 0 \\ \sqrt{2}\lambda_7 & \sqrt{2}\lambda_8 & \lambda_6 & 0 & 0 \\ 0 & 0 & 0 & \lambda_3 + 2\lambda_4 & 3\lambda_5^* \\ 0 & 0 & 0 & 3\lambda_5 & \lambda_3 + 2\lambda_4 \end{pmatrix}. \quad (21)$$

Then, the eigenvalues $\Lambda_{Y\sigma i}^X$ of the above scattering matrices (where $i = \pm$ or 1, 2, 3) can be calculated as

$$\begin{aligned} \Lambda_{21\pm}^{\text{even}} &= \lambda_1 + \lambda_2 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + |\lambda_5|^2}, \\ \Lambda_{21}^{\text{odd}} &= \lambda_3 + \lambda_4, \quad \Lambda_{20}^{\text{odd}} = \lambda_3 - \lambda_4, \end{aligned} \quad (22)$$

$$\Lambda_{01\pm}^{\text{even}} = \lambda_1 + \lambda_2 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \lambda_4^2}, \quad \Lambda_{01\pm}^{\text{odd}} = \lambda_3 \pm |\lambda_5|, \quad (23)$$

$$\Lambda_{00\pm}^{\text{odd}} = \lambda_3 + 2\lambda_4 \pm 3|\lambda_5|, \quad (24)$$

and the $\Lambda_{001,2,3}^{\text{even}}$ correspond to the three roots of the polynomial equation

$$\begin{aligned} x^3 - x^2(6\lambda_1 + 6\lambda_2 + \lambda_6) + x(36\lambda_1\lambda_2 - 4\lambda_3^2 - 4\lambda_3\lambda_4 - \lambda_4^2 \\ + 6\lambda_1\lambda_6 + 6\lambda_2\lambda_6 - 2\lambda_7^2 - 2\lambda_8^2) + 12\lambda_1\lambda_8^2 + 12\lambda_2\lambda_7^2 \\ - 4\lambda_4\lambda_7\lambda_8 - 8\lambda_3\lambda_7\lambda_8 + \lambda_4^2\lambda_6 + 4\lambda_3\lambda_4\lambda_6 \\ + 4\lambda_3^2\lambda_6 - 36\lambda_1\lambda_2\lambda_6 = 0. \end{aligned} \quad (25)$$

B. Vacuum stability conditions

To make sure the potential is bounded from below in the limit of large field values, we impose copositivity criteria on the quartic couplings of the tree-level potential as explored in Ref. [45]. The vacuum stability conditions for the model are given as

$$\begin{aligned} \lambda_1, \lambda_2, \lambda_6 &> 0, \quad 2\sqrt{\lambda_1\lambda_6} + \lambda_7 > 0, \\ 2\sqrt{\lambda_2\lambda_6} + \lambda_8 &> 0, \end{aligned} \quad (26)$$

and for the $\lambda_4 > 0$ case,

$$2\sqrt{\lambda_1\lambda_2} + \lambda_3 > 0, \quad (27)$$

$$\begin{aligned} \sqrt{\lambda_1\lambda_2\lambda_6} + \sqrt{\lambda_1\lambda_8} + \sqrt{\lambda_2\lambda_7} + \sqrt{\lambda_6\lambda_3} \\ + \sqrt{(2\sqrt{\lambda_1\lambda_6} + \lambda_7)(2\sqrt{\lambda_2\lambda_6} + \lambda_8)(2\sqrt{\lambda_1\lambda_2} + \lambda_3)} > 0; \end{aligned} \quad (28)$$

if $\lambda_4 < 0$, one obtains

$$2\sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 - |\lambda_5| > 0, \quad (29)$$

$$\sqrt{\lambda_1\lambda_2\lambda_6} + \sqrt{\lambda_1\lambda_8} + \sqrt{\lambda_2\lambda_7} + \sqrt{\lambda_6(\lambda_3 + \lambda_4 - |\lambda_5|)} + \sqrt{(2\sqrt{\lambda_1\lambda_6} + \lambda_7)(2\sqrt{\lambda_2\lambda_6} + \lambda_8)(2\sqrt{\lambda_1\lambda_2} + \lambda_3 + \lambda_4 - |\lambda_5|)} > 0. \quad (30)$$

It should be noted that, with loop corrections being included, one may expect more viable parameter spaces allowed by the vacuum stability conditions.

C. Electroweak precision test

We should also make sure that electroweak precision test data are respected when we study the phenomenology of the model. As we know, the electroweak precision test can be expressed in terms of three measurable quantities, called S , T , and U , which parametrize the contributions from beyond standard model physics to electroweak radiative corrections [46]. The most relevant one is the T parameter, which characterizes the spoil extent of the global $SU(2)$ symmetry. Since the scalar singlet S mixes with the H_0 in

this model, the self-energies of the Goldstone bosons get corrected by the diagrams with virtual inert particle pairs and S . Following the method in Ref. [10], we first obtain

$$\begin{aligned} \Delta\rho &= \frac{(\lambda_4 + \lambda_5)^2}{2} (\cos^2\theta f(m_{H^\pm}, m_H) + \sin^2\theta f(m_{H^\pm}, m_\chi)) \\ &+ \frac{(\lambda_4 - \lambda_5)^2}{2} f(m_{H^\pm}, m_A) - 2(\lambda_5 \cos\theta \\ &- \mu_{\text{soft}} \sin\theta/v)^2 f(m_A, m_H) \\ &- 2(\lambda_5 \sin\theta + \mu_{\text{soft}} \cos\theta/v)^2 f(m_A, m_\chi), \end{aligned} \quad (31)$$

$$\text{with } f(m_1, m_2) = \frac{v^2}{32\pi^2} \int_0^1 \frac{dx x(1-x)}{xm_1^2 + (1-x)m_2^2}. \quad (32)$$

In the limit of custodial SU(2) symmetry, i.e., $m_{H^\pm} = m_A$, Eq. (31) can be recast as

$$\Delta\rho = \frac{(\lambda_4 + \lambda_5)^2}{2} (\cos^2\theta f(m_{H^\pm}, m_H) + \sin^2\theta f(m_{H^\pm}, m_\chi)) - 2(\lambda_5 \cos\theta - \mu_{\text{soft}} \sin\theta/v)^2 f(m_A, m_H) - 2(\lambda_5 \sin\theta + \mu_{\text{soft}} \cos\theta/v)^2 f(m_A, m_\chi), \quad (33)$$

which sets bounds on the magnitude of the mixing between S and H_0 as well as the mass splitting among neutral and charged Higgses in the model. The T parameter can be obtained as $T = \Delta\rho/\alpha_{\text{EM}}$ following the notation of Ref. [10]. The Gfitter fit to the electroweak data [47], $T = 0.09 \pm 0.13$, has been used to constrain the parameters.

D. LEP bounds

The LEP bounds on the scalar masses of this model can be considered the same as in the IDM, and the only difference is that the CP -even neutral Higgs H is an admixture of the doublet and singlet. The precise measurements of the W and Z widths lead to the lower limits on the scalar masses

$$m_{H/\chi} + m_{H^\pm} > m_{W^\pm}, \quad m_A + m_{H^\pm} > m_{W^\pm}, \\ m_{H/\chi} + m_A > m_Z, \quad 2m_{H^\pm} > m_Z, \quad (34)$$

to ensure that decay channels of $W^\pm \rightarrow H/\chi H^\pm$, $AH^\pm$ and $Z \rightarrow H/\chi A$, H^+H^- are kinematically forbidden. The production of the charge Higgs pairs $e^+e^- \rightarrow H^+H^-$ at LEP-II [48] sets

$$m_{H^\pm} > 70 \text{ GeV}, \quad (35)$$

which does not apply when the scalar mass is larger than $m_Z/2$, provided the mass splitting $m_{H^\pm} - m_H$ is smaller than 5 GeV [49]. For the neutral Higgses of the model, the constraints come from the pair production process of $e^+e^- \rightarrow H/\chi A$ followed by the cascade decay $A \rightarrow H/\chi Z \rightarrow H/\chi f\bar{f}$, which can be obtained through a reinterpretation of the neutralino production search at LEP-II. The analysis of Ref. [50] shows that the limit on the neutral Higgs is $\max(m_A, m_H) \geq 100 \text{ GeV}$ for the IDM, which can be roughly applied to our model when the mixing of H_0 and S is small or can be interpreted as $\max(m_A, m_\chi) \geq 100 \text{ GeV}$ when the mixing of H_0 and S is large, which can be figured out from Eq. (53) in Sec. VII D.

IV. DM RELIC DENSITY AND DIRECT DETECTION

The thermal DM relic density should match the observed DM density in the Universe of $\Omega h^2 = 0.1199 \pm 0.0022$ [51], or at least of not overproducing such density via

thermal production. To an approximation, the relic density is given by

$$\Omega_{\text{DM}} \simeq 1.07 \times 10^9 \frac{x_f}{\sqrt{g_*} M_{\text{Pl}} \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle} \text{GeV}^{-1}, \quad (36)$$

where $x_f = m_D/T_f \simeq 20$ with T_f being the typical freeze-out temperature of a WIMP [52], $m_D = m_\chi$ or m_H is the DM mass, M_{Pl} is the Planck mass, g_* is number of relativistic degrees of freedom, and $\langle \sigma v \rangle$ is the thermally averaged cross section for DM pair annihilation into the SM particles (i.e., $f\bar{f}$, W^+W^- , ZZ , hh). For the mixed DM scenario, the DM annihilation is characterized by the masses of dark matter and the mixing of DM constituents. To illustrate the DM annihilation properties with different channels in this model, we collect several benchmarks in distinct DM mass regions in the following subsection. The relic density is calculated by implementing package micrOMEGAs [53].

In the direct detection experiment of dark matter, the nuclear recoil spectrum is directly related to the DM-nuclei scattering cross section [54], which is given by

$$\sigma_{D\text{-nuc}} = \int_0^{4\mu_r^2 v^2} \frac{d\sigma(q=0)}{d|\mathbf{q}|^2} d|\mathbf{q}|^2 \\ = \frac{4\mu_r^2}{\pi} f_p^2 \left[Z + \frac{f_n}{f_p} (A - Z) \right]^2, \quad (37)$$

where $\mathbf{q}$ is the momentum transfer, $\mu_r = (m_{\text{nuc}} m_D)/(m_{\text{nuc}} + m_D)$, and v is the relative velocity. The couplings of DM to the proton and neutron, f_p and f_n , can be expressed as

$$f_N = \frac{m_N}{2m_D} \left(\sum_{q=u,d,s} f_{Tq}^N \frac{\lambda_{DDqq}}{m_q} + \frac{2}{27} f_{TG}^N \sum_{q=c,b,t} \frac{\lambda_{DDqq}}{m_q} \right), \\ \text{with } f_{TG}^N = 1 - \sum_{q=u,d,s} f_{Tq}^N, \quad (N=p,n), \quad (38)$$

where m_N is the nucleon mass, f_{Tq}^N is the form factor of the nucleon (see Table I), and λ_{DDqq} is the effective coupling of the DM particle to a q -flavor quark component in the nucleon. In this model, DM-quark interaction derives from t -channel exchange of the SM Higgs h . Thus, in the limit of zero momentum transfer, the Higgs can be integrated out, and the effective coupling becomes

TABLE I. Form factors extracted from micrOMEGAs.

q	u	d	s
f_{Tq}^p	0.0153	0.0191	0.0447
f_{Tq}^n	0.0110	0.0273	0.0447

$$\lambda_{DDqq} = \frac{a_{hDD} m_q}{m_h^2 v}, \quad (39)$$

with the coupling a_{hDD} given in Eqs. (15) and (16). The direct detection rates in our calculation also have been evaluated using the package micrOMEGAs.

Direct searches for dark matter by the LUX [6] and PandaX-II [5] collaborations² have recently come up with the most stringent limits on the spin-independent elastic scattering of DM off nucleons. In comparing the calculated scattering cross section to the limits from the experiments, we rescale the cross section by the ratio of the predicted and observed relic density to account for the reduced flux of dark matter particles in the detectors when the relic density is undersaturated, i.e., $\sigma_{\text{scaled}} = \sigma_{D-nucl} \cdot \Omega_{\text{DM}} h^2 / \Omega h^2$, with Ωh^2 being the observed relic density, and we take the central value 0.1199 in our calculations.

V. RESEARCH STRATEGY OF ELECTROWEAK PHASE TRANSITION

The strength of the electroweak phase transition can be parametrized by v_c/T_c , and generally the strong first-order condition $v_c/T_c > 1$ is needed to prevent the washout of baryon asymmetry [55].³ For more details on the electroweak baryogenesis, we refer to Ref. [3], and the interplay between the v_c/T_c and the generation of baryon asymmetry in the framework of EWBG can be found in Ref. [57].

The effective potential at finite temperature can be used to explore the Universe cooling history. To perform the numerical analysis of vacuum structure at finite temperature T , one needs the effective potential

$$V_{\text{eff}} = V_0 + V_{CW} + V_T, \quad (40)$$

with V_0 , V_{CW} , and V_T being the tree-level, one-loop temperature-independent and -dependent Coleman-Weinberg potentials, respectively. The tree-level potential V_0 could be obtained from Eq. (1) after the field expansion. There could be multistep phase transition depending on the vacuum structure [57–60]. In our case, we focus on the one-step phase transition for simplicity. More complicate scenarios are left to future studies. In the Landau gauge ($\xi = 0$) and $\overline{\text{MS}}$ scheme, the temperature-independent one-loop Coleman-Weinberg potential is given by [61–64]

$$V_{CW} = \sum_i \frac{n_i}{64\pi^2} m_i^4(h) \left(\ln \frac{m_i^2(h)}{Q^2} - C_i \right). \quad (41)$$

Here, all fields coupling to the Higgs are summed, n_i is the number of degrees of freedom for bosonic and fermionic

fields, and renormalization scheme-dependent constants $C_i = 1/2$ for transverse gauge bosons and $3/2$ for longitudinal polarizations of gauge bosons and other particles; $m_i^2(h)$ are the field-dependent squared masses for all particles. Here, the counterterms have been absorbed into V_{CW} implicitly.

The temperature-dependent effective potential is obtained as [64,65]

$$V_T = \frac{T^4}{2\pi^2} \left(\sum_{i=\text{bosons}} n_i J_B[m_i^2(h)/T^2] - \sum_{i=\text{fermions}} n_i J_F[m_i^2(h)/T^2] \right), \quad (42)$$

with the J functions defined as

$$J_B(x) = \int_0^\infty dt t^2 \ln [1 - \exp(-\sqrt{t^2 + x})], \quad (43)$$

$$J_F(x) = \int_0^\infty dt t^2 \ln [1 + \exp(-\sqrt{t^2 + x})]. \quad (44)$$

To accomplish the analysis of the phase structure as a function of T analytically, high-temperature expansions are always used,

$$T^4 J_B[m^2/T^2] = -\frac{\pi^4 T^4}{45} + \frac{\pi^2}{12} T^2 m^2 - \frac{\pi}{6} T (m^2)^{3/2} - \frac{1}{32} m^4 \ln \frac{m^2}{a_b T^2} + \mathcal{O}(m^2/T^2), \quad (45)$$

$$T^4 J_F[m^2/T^2] = \frac{7\pi^4 T^4}{360} - \frac{\pi^2}{24} T^2 m^2 - \frac{1}{32} m^4 \ln \frac{m^2}{a_f T^2} + \mathcal{O}(m^2/T^2), \quad (46)$$

where $a_b = 16a_f = 16\pi^2 \exp(3/2 - 2\gamma_E)$. It should be noted that the T^2 terms in the above expressions can drive symmetry restoration at high temperatures, and the non-analytic $(m^2)^{3/2}$ term in Eq. (45) is responsible for the barrier between the symmetric (at high temperature) and broken phases (at the critical temperature). To avoid the breakdown of perturbation theory induced by symmetry restoration at high temperatures, one needs to perform a resummation of daisy diagrams [66] by adding finite-temperature corrections to the boson masses in Eq. (43),

$$m^2 \rightarrow m^2 + m(T)^2, \quad (47)$$

with $m(T)^2$ computed from the infrared limit of the corresponding two-point function [67] and presented in the Appendix; this approach is usually called Parwani's method in the literature.

²We found the XENON1T [7] results published when this work was finalized. We just comment here that the benchmarks we take in the following also evade the new upper limits.

³The gauge-invariance problem needs to be kept in mind when interpreting the condition; see Ref. [56].

FIG. 1. The fraction $m_{SS}(T)^{3/2}/\sum_i(m_i^2)^{3/2}$ for two representative benchmarks; the top and bottom panels correspond to the BP5 and BP-c2 scenarios, which will be studied in the next section.

Here, the thermal masses contribution of $m_{SS}(T)$ gives an additional contribution to the $(m^2)^{3/2}$ term⁴ of Eq. (45) in comparison with the IDM case. In Fig. 1, we depict the fraction $m_{SS}(T)^{3/2}/\sum_i(m_i^2)^{3/2}$ to illustrate the contribution in various parameter spaces. We find that the ratio is more than 10%, from which one could expect the vacuum barrier can be altered. Therefore, the mixed dark matter and the mixing of CP -even neutral scalars might affect the Universe cooling process, which will be explored in the following section.

VI. DM AND EWPT PHENOMENOLOGY

In the low-mass region of dark matter, Refs. [25–27] found the dark matter relic abundance and the strong first-order EWPT can be accomplished in the Higgs funnel regime in the IDM. The bounds of the current direct detection on the high-mass region are much weaker, which is caused by the lower number density for high-mass DM. Therefore, we alternatively focus on the intermediate dark matter mass region. We investigate if the contributions of

dark matter and the other CP -even scalar to the vacuum barrier help to realize the strong first-order EWPT.

A. Benchmarks for DM from different mass regions

In this subsection, we present several benchmarks for distinct DM mass regions. After imposing the Higgs collider bounds by using HIGGSBOUNDS [68–70] and checking rate measurements with HIGGSSIGNALS [71], all the parameter settings in each benchmark model agree with the aforementioned constraints. The quartic couplings $\lambda_2, \lambda_6, \lambda_8$ are not involved in the DM annihilation or DM-nucleus scattering processes, so we fix them only by following the constraints from the unitarity and vacuum stability conditions.

1. Original IDM case

At the beginning, we briefly study the DM phenomenology in the original IDM by setting $\lambda_{6,7,8} = \theta = 0$, we restore to the IDM scenario. Two benchmark points are shown in Table II.

First, we take m_H to be smaller than the gauge bosons, and the main DM annihilation channel is $HH \rightarrow h \rightarrow b\bar{b}$. To avoid an overlarge cross section which leads to the relic underabundance, we expect the DM-Higgs coupling $a_{hHH} = 2v\lambda_L$ to be small enough. As the DM mass increases, a much smaller λ_L is required, as could be found

⁴It should be noted that the $(m^2)^{3/2}$ here means $(m^2 + m(T)^2)^{3/2}$.

TABLE II. Benchmarks for the original IDM; here, H is the DM candidate.

	m_H (GeV)	λ_L	$m_A = m_{H^\pm}$ (GeV)	$\Omega_{\text{DM}} h^2$	$\sigma_{D\text{-nucl}}$ (pb)
IDM-BP1	57.5	0.001	450	0.110	1.05×10^{-11}
IDM-BP2	60	0.00034	450	0.103	1.12×10^{-12}

TABLE III. χ as the DM candidate, with $\lambda_2 = \lambda_6 = \lambda_8 = 0.1$.

	m_χ (GeV)	m_H (GeV)	λ_L	λ_7	θ	$m_A = m_{H^\pm}$ (GeV)	$\Omega_{\text{DM}} h^2$	$\sigma_{D\text{-nucl}}$ (pb)
BP1	60	75	0.1	-0.001	0.1	450	0.119	1.01×10^{-12}
BP2	240	280	0.045	0.06	0.36	450	0.120	2.69×10^{-10}

TABLE IV. χ as the DM candidate, with $\lambda_2 = 0.3$, $\lambda_6 = \lambda_8 = 0.1$.

	m_χ (GeV)	m_H (GeV)	λ_L	λ_7	θ	$m_A = m_{H^\pm}$ (GeV)	$\Omega_{\text{DM}} h^2$	$\sigma_{D\text{-nucl}}$ (pb)
BP3	54	61	0.01	0.001	0.1	450	0.094	1.1×10^{-11}
BP4	240	370	0.1	0.12	0.36	450	0.101	3.77×10^{-10}
BP5	240	370	0.12	0.115	0.35	330	0.102	2.51×10^{-10}

TABLE V. H as the DM candidate, with $\lambda_2 = \lambda_6 = \lambda_8 = 0.1$.

	m_χ (GeV)	m_H (GeV)	λ_L	λ_7	θ	$m_A = m_{H^\pm}$ (GeV)	$\Omega_{\text{DM}} h^2$	$\sigma_{D\text{-nucl}}$ (pb)
BP6	75	60	-0.001	0.1	0.1	450	0.112	1.01×10^{-12}
BP7	280	240	0.06	0.03	1.1	450	0.117	2.45×10^{-10}
BP-c1	445	440	0.02	-0.07	0.6	450	0.102	1.11×10^{-10}
BP-c2	480	448	0.05	0.07	0.58	450	0.0979	3.93×10^{-10}

from the IDM-BP1 and IDM-BP2 models where the DM masses are in the Higgs funnel region. When the DM mass is larger than the gauge boson mass, the annihilation process of DM pairs to gauge bosons contributes a lot to the total DM annihilation cross section, and therefore the DM is underabundant even the coupling $\lambda_L \rightarrow 0$. Then, the degenerate case is the only viable scenario; for example, a parameter set can be $m_H = 550$ GeV, $m_A = m_{H^\pm} = 555$ GeV, and also $\lambda_L \rightarrow 0^5$.

2. χ as the DM candidate

In the singlet-doublet scalar mixed model, we first investigate the possibility of χ being the DM candidate. We identify two benchmarks given in Table III, where the correct DM relic density can be matched and the magnitude of the cross sections of scattering off nucleus are below the LUX (or PandaX-II) strictly upper limits. When $m_\chi < m_V (V = W^\pm, Z)$, the dominant annihilation channel is $\chi\chi \rightarrow h \rightarrow b\bar{b}$; then, the DM-Higgs coupling $a_{h\chi\chi}$ characterizes the relic density as well as the DM-nucleus scattering. For $m_\chi > m_V$ (BP2), the annihilations to gauge

bosons could be dominant, which would lead to the DM underabundance (as discussed in the original IDM case). Because the DM-gauge boson quartic couplings $\lambda_{\chi\chi VV}$ are proportional to $\sin^2 \theta$, we need a smaller θ in order to avoid the larger cross section of $\chi\chi \rightarrow VV$. Above the hh threshold with a small θ , the channel $\chi\chi \rightarrow hh$ is the dominant one.

Three benchmarks with conditions $m_h > m_H + m_\chi$ or $m_H > m_\chi + m_h$ are listed Table IV, where χ is the DM candidate. Here, considering the stability constraints, the λ_2 that does not take part in the DM phenomenology is taken to be a larger value in comparison with other benchmark scenarios in Tables III and V. For BP3, the Higgs invisible decay channel is open; the masses of both H and χ are smaller than $m_h/2$ in comparison with the BP1; and three decay channels $h \rightarrow \chi\chi$, $h \rightarrow H\chi$, and $h \rightarrow HH$ should be considered. For BP4 and BP5, the decay channel $H \rightarrow \chi h$ is involved in the decay chains of $e^+e^- \rightarrow HA$. We will discuss the constraints from collider experiments on parameter space and possible signals searching at colliders in Sec. VII.

3. H as the DM candidate

For a comparison between the scenarios of χ and H being the DM candidates, here, we investigate the case of $m_H < m_\chi$ by employing the benchmarks in Table V. The

⁵In the original IDM, the intermediate mass 80–500 GeV is excluded by direct detection [19].

anatomy of dark matter phenomenology in different benchmark models is discussed in the following.

BP6.—We exchange the two scalars' masses in comparison with BP1. The DM-Higgs triple coupling (here, it is a_{hHH}) plays an important role in the DM relic density estimation since the annihilations to the gauge bosons are kinematically forbidden. A simple exchange of λ_L and λ_7 compared to BP1 would satisfy the relic abundance and the direct detection cross section limits of DM, as indicated by Eqs. (14)–(16).

BP7.—When $m_H > m_V$, the pair production of $WW(ZZ)$ in the final state of DM pair annihilation processes becomes increasingly important, because the four-point interaction through gauge couplings and the s -channel process start contributing significantly. As a result, the cross section for a DM pair annihilating to gauge bosons becomes very large, such that the estimated thermal DM relic density may be systematically below the observed DM relic density with combinations of model parameters.

The DM-gauge boson quartic couplings λ_{HHVV} are proportional to $\cos^2 \theta$, so a smaller θ drives much more efficient annihilations to the gauge bosons, and therefore DM relic density is not abundant. We take a $\theta = 1.1$ in the benchmark model BP7. The main component of the DM is still the singlet scalar, which is the same as BP2 by taking the χ to be the DM candidate.

Degenerate case.—In the IDM scenario, when scalars H , A , and $H^\pm$ are nearly mass degenerate, the coannihilation processes become important. There is a cancellation taking place between the t/u -channel contributions and the four-vertex diagram [10,19]. In this manner, the WIMP depletion rate can be balanced by varying the mass splitting between DM and other scalars and the parameter λ_L to obtain the correct mixture of transverse and longitudinal gauge bosons in the final state. These solutions are always found for small mass splittings of the odd particles and require some tuning of the value of λ_L . In practice, the maximal allowed mass splitting is of the order $O(10)$ GeV [19].

In benchmark models of BP-c1 and BP-c2, the cancellation also happens, augmented by the mixing between the singlet and doublet components of the DM particle. The difference between the benchmark BP-c2 and the BP-c1 is that the odd particle χ does not degenerate with H . The mass splitting is much bigger, which varies μ_{soft} dramatically, so the λ_L should be tuned to adjust the DM-Higgs coupling a_{hhH} to be smaller enough, which is required to evade the direct detection limits.

B. Electroweak phase transition assisted by the mixed dark matter

We examine all the benchmarks given above to find which can yield a strong first-order EWPT. There are two benchmark models left, and they are summarized in Table VI along with the corresponding critical temperatures

TABLE VI. Benchmark models in which the EWPT can be strongly first order.

Benchmarks	v_c (GeV)	T_c (GeV)	v_c/T_c	$\Omega_{\text{DM}} h^2$	$\sigma_{D\text{-nucl}}$ (pb)
BP5	242.851	133.750	1.8157	0.102	2.51×10^{-10}
BP-c2	319.011	195.652	1.6305	0.0979	3.93×10^{-10}

and the strengths of the phase transition. In the following, we present the combined results of both DM and the EWPT by varying sensitive parameters near the benchmark points BP5 and BP-c2. The $\chi - H$ mixing effects are shown in Figs. 3 and 5, and other parameter effects are presented in Figs. 4 and 6.

Before devoting ourselves to the numerical analysis of DM and the EWPT, we first consider the electroweak precision test constraints. The T parameter exclusion of two benchmark scenarios is shown in Fig. 2. The two plots in top panel imply that the mixing angle should be small for the DM mass around 200 GeV. For BP-c2, parameter space is less constrained by the T parameter. We also perform the constraints from the stability conditions; the blue region in Figs. 3, 4, 5, and 6 are excluded. One can see that the parameter space is constrained severely. Moreover, a large variation of the quartic coupling $\lambda_2, \lambda_6, \lambda_8$ only leads to a tiny effect on the one-step phase transition case as numerically checked by us. Therefore, we focus on the survey of the phase transition in the parameter spaces of $\lambda_L, \lambda_7, \theta, m_{H,\chi}$ with others being fixed.

1. χ -DM benchmark model: BP5

We first analyze the combined results of benchmark BP5, which are presented in Figs. 3 and 4.

EWPT.—As mentioned before in Sec. V, the vacuum barrier can be affected by the DM mass slightly. We find that the DM mass dependence of the strength of EWPT is tiny in this benchmark, as explored in Ref. [57], and the whole mass range can provide a strongly first-order EWPT (red dotted region), as depicted in the parameter spaces of $m_\chi - \theta$ (bottom right of Fig. 3) and $m_\chi - \lambda_7$ (bottom right of Fig. 4). We observe that the strength of the EWPT subtly depends on the mass splitting $\Delta M = |m_H - m_{A,H^\pm}|$ and the mixing angle, and the larger θ leads to a strong first-order EWPT when the mass splitting ΔM is large. As depicted in the top-left panel of Fig. 3 and top panels of Fig. 4, the realization of the strongly first-order EWPT prefers a narrow range of m_H with the fixed values of θ, λ_7 . Furthermore, a limited range of the coupling λ_L is required by the condition $v_c/T > 1$ as shown in top-right panel of Fig. 3 and bottom-left panel of Fig. 4.

DM relic density.—In this benchmark model, the scalar χ plays the role of DM. As demonstrated in the $\lambda_L - \theta, \lambda_7 - \theta$, and $m_\chi - \theta$ planes of Fig. 3, the mixing angle θ

FIG. 2. 95% C.L. excluded regions from the fit to the T parameter for BP5 (top) and BP-c2 (bottom) benchmark scenarios.

FIG. 3. Combined results corresponding to BP5 by varying the mixing angle θ and other parameters. The red dotted regions are the parameter space where strong first-order EWPT occurs, the green regions stand for $0.09 < \Omega h^2 < 0.12$. The gray regions are excluded by LUX limits. The blue shaded regions are excluded by the vacuum stability conditions.

FIG. 4. For BP5 with fixed mixing angle θ . The convention of the colors in the plots is the same as in Fig. 3.

FIG. 5. For BP-c2 with varying θ . The green regions stand for $0.09 < \Omega h^2 < 0.12$. The red dotted regions satisfies the condition $v_c/T_c > 1$. The gray regions are excluded by LUX. The blue shaded regions are excluded by the vacuum stability conditions.

FIG. 6. For BP-c2 with fixed θ . The red dotted regions stand for the parameter space can satisfy the condition $v_c/T_c > 1$. The gray regions are excluded by LUX. The blue shaded regions are excluded by the vacuum stability conditions.

induces an efficient change of the magnitude of the DM relic density; this is because the main annihilation channel is $\chi\chi \rightarrow hh$ with a θ -dependent four-vertex coupling $2\lambda_7 \cos^2 \theta + 2\lambda_L \sin^2 \theta$. We find that the DM relic requires the mixing angle to be smaller than 0.5, and then we notice that the main contribution comes from the $2\lambda_7 \cos^2 \theta$ part. Thus, the DM relic density sensitively depends on the parameters λ_7 .

In the top-left panel of Fig. 3 and top panels of Fig. 4, when m_H becomes larger than about 400 GeV, the main annihilation channel changes to be $\chi\chi \rightarrow t\bar{t}$, which is proportional to the DM-Higgs coupling $a_{h\chi\chi}$. Because the coupling $a_{h\chi\chi}$ is a function of μ_{soft} and μ_{soft} is a function of m_H , one can expect the predicted DM relic abundance is affected by the variation of m_H . In the top-left panel of Fig. 3 (top panels of Fig. 4), with the increasing of m_H , one gets an increasing (decreasing) $a_{h\chi\chi}$ and therefore a narrower (broader) viable region for the correct DM relic density.

DM direct detection.—Considering the current restrictive limit from LUX (and PandaX-II), we study the constraints on the parameter space of the model. The gray regions in the figures are excluded by the DM direct detection limits.

In the intermediate mass region (around 200 GeV), we find that LUX limits constrain the parameter θ and λ_7 rigorously, due to the DM-Higgs coupling $a_{h\chi\chi} \sim 2v\lambda_7 \cos^2 \theta$. Only a small region evades the constraint in each plane of Figs. 3 and 4. The narrow allowed parameter spaces are due to the cancellation effects in the DM-Higgs

coupling $a_{h\chi\chi}$, as can be seen from Eq. (15) when $m_\chi < m_H$. In the bottom-right plots of Figs. 3 and 4, we preform the exclusion of θ , λ_7 by varying the DM mass. Although the limit is much stricter, there is still room for obtaining the correct DM relic density and the strong first-order EWPT.

2. H-DM benchmark model: BP-c2

The combined results of benchmark BP-c2 are presented in the Figs. 5 and 6. In this benchmark model, the scalars H , A , and $H^\pm$ are nearly mass degenerate.

EWPT.—To obtain the correct DM relic density, the scalars in the inert doublet should be degenerate when the DM mass m_H is larger than the gauge bosons', arising from the cancellation between different annihilation channels. However, the strength of the EWPT $v_c/T_c > 1$ cannot be obtained due to the small mass spitting ΔM , as found in Ref. [25]. In this work, the existence of the extra singlet scalar χ provides an additional contribution to the vacuum barrier at finite temperature, as explored in Sec. V. Therefore, we get the chance to yield a strong enough EWPT as well as the correct DM relic abundance simultaneously.

We find that with the increase of the mass of the singlet scalar the mixing angle needs to be smaller to meet the strong first-order EWPT condition, as shown in the top-left plot of Fig. 5. Also, the criteria severely constrain the coupling λ_L as shown in the top-right plots of Fig. 5 and bottom plots of Fig. 6.

DM relic density.—In the top-left panel of Fig. 5 and top panels of Fig. 6, the mass difference of $m_\chi^2 - m_H^2$ changes the DM-Higgs coupling a_{hHH} efficiently [see Eqs. (14) and (16)], leading to the variation of relic abundance. The correct DM relic density constrains the mass of the singlet scalar χ to be smaller than 500 GeV.

From the top-right of Fig. 5 and bottom panels of Fig. 6, we find that the relic density highly constrains the parameters λ_L to be small (around zero). The reason is that the coupling $a_{hHH} \sim 2v\lambda_L \cos^2 \theta$ in this case, and the annihilation cross section of the DM pair can be suppressed by a smaller λ_L in order to avoid the relic underabundance (as also discussed in Sec. VI A 3).

In the large mass spitting region of m_χ and m_H , the term $-\mu_{\text{soft}} \sin^2 2\theta$ becomes the dominant part of the coupling a_{hHH} . So, as the a_{hHH}^2 increases, the annihilation cross section of DM becomes larger, which causes the DM relic density deficiency; see the $m_\chi - \theta$, $m_\chi - \lambda_7$ and $m_H - \lambda_7$ plots in Figs. 5 and 6. Also, the DM relic density behaviors in the $m_\chi - \lambda_7$ and $m_H - \lambda_7$ planes are symmetrical with

respect to λ_7 ; this is because the annihilation cross section $HH \rightarrow hh$ is proportional to λ_7^2 .

DM direct detection.—At the relative large DM mass region, the direct detection is less constraining. However, one still needs small λ_L to suppress the Higgs portal coupling a_{hHH} , and LUX results rule out the most space of parameter λ_L in the $\lambda_L - \theta$, $m_\chi - \lambda_L$, $\lambda_L - \lambda_7$ spaces. The exclusion in the bottom-right panels of Figs. 5 and 6 with respect to the varying DM mass tells us that plenty parameter space is needed to probe the underground experiments of the next generation.

VII. COMMENTS ON LEPTONIC AND HADRONIC COLLIDER SEARCH

A. Invisible Higgs decay

For the case of $m_h > 2m_{H,\chi}$, the invisible decay of the SM-like Higgs needs to be taken into account:

$$\Gamma(h \rightarrow H\chi) = \frac{\sqrt{m_h^4 - 2m_h^2(m_H^2 + m_\chi^2) + (m_H^2 - m_\chi^2)^2} (2\mu_{\text{soft}} \cos 2\theta + 2(\lambda_L - \lambda_7)v \sin 2\theta)^2}{64\pi m_h^3}, \quad (48)$$

$$\Gamma(h \rightarrow HH) = \frac{\sqrt{m_h^4 - 4m_h^2 m_H^2} (-2\mu_{\text{soft}} \sin 2\theta + 2(\lambda_L - \lambda_7)v \cos 2\theta + 2(\lambda_L + \lambda_7)v)^2}{128\pi m_h^3}, \quad (49)$$

$$\Gamma(h \rightarrow \chi\chi) = \frac{\sqrt{m_h^4 - 4m_h^2 m_\chi^2} (2\mu_{\text{soft}} \sin 2\theta - 2(\lambda_L - \lambda_7)v \cos 2\theta + 2(\lambda_L + \lambda_7)v)^2}{128\pi m_h^3}. \quad (50)$$

The ATLAS search [72] sets bounds on the invisible Higgs decay branching ratio $\text{Br}(h \rightarrow \text{inv}) < 28\%$ at the 95% C.L., which has been considered in our choice of DM scenarios with $m_{\chi,H} \leq m_h/2$, i.e., in the benchmark models BP1, BP3, and BP6.

B. Diphoton rate deviation

As in the IDM, the charged state $H^\pm$ leads to an additional contribution to the loop-induced $h \rightarrow \gamma\gamma$ and γZ rates [73,74]. The $h \rightarrow \gamma\gamma$ rate can be obtained as [73–78]

$$\Gamma(h \rightarrow \gamma\gamma) = \frac{\alpha^2 G_F m_h^3}{128\sqrt{2}\pi^3} \left| \mathcal{A}_{\text{SM}} + \frac{\lambda_3 v^2}{2m_{H^\pm}^2} \mathcal{A}_0 \left(\frac{m_h^2}{4m_{H^\pm}^2} \right) \right|^2; \quad (51)$$

here, $\mathcal{A}_{\text{SM}} \approx -6.56 + 0.08i$ is the leading contribution coming from W bosons and top quarks for $m_h = 125$ GeV. The contribution coming from $H^\pm$ can enhance or suppress the $h \rightarrow \gamma\gamma$ rate relative to the SM depending on the sign of λ_3 , which is characterized by the mass splitting

of $m_H - m_{H^\pm}$ as well as the mixing of the singlet and inert doublet scalars. For our benchmarks where strong first-order EWPT and correct DM relic abundance can be accomplished, a tiny enhancement of $h \rightarrow \gamma\gamma$ [around $\mathcal{O}(10^{-3})$] is obtained due to $\lambda_3 < 0$. Other benchmark scenarios favor $\lambda_3 > 0$, which may lead to a large suppression of the rate, around $\mathcal{O}(10^{-2})$. And the rate can be diluted by the invisible decay of SM-like Higgs when $m_{H,\chi} < m_h$, as also noticed in Refs. [25,26,30].

C. Triple Higgs couplings v.s. EWPT

As mentioned in the Sec. I, the EWBG stands out from many baryogenesis mechanisms. Based on the mechanism, the triple Higgs couplings might be modified, which can be probed at the future hadron colliders, such as Super-pp-Collider (SppC) [4]. However, the strong first-order phase transition does not necessarily require the enhancement or suppression of the triple Higgs coupling compared to SM prediction, as studied in the singlet assisted case in Ref. [79]. Since there is no mixing between the SM-like

FIG. 7. Triple Higgs coupling ratio for BP5 and BP-c2 with respect to the mixing parameters θ , λ_L , and λ_7 .

Higgs and other neutral Higgses, we expect the model does not cause large modification of the triple Higgs couplings in the allowed benchmark scenarios. Generally, we can expect a tiny loop correction to the triple Higgs couplings (C_{hhh}) from new scalar sectors. The ratio of C_{hhh} of our model with respect to the SM can be obtained as

$$r_{3h} = \frac{C_{hhh}}{C_{hhh}^{\text{SM}}}, \quad (52)$$

with $C_{hhh} = d^3 V_{\text{eff}}(T=0)/dh^3$. The plots of ratio r_{3h} in Fig. 7 indicate that the deviation from the SM prediction is around $\mathcal{O}(10^{-2}-10^{-1})$ in the benchmark models BP5 and BP-c2. When the triple Higgs vertices are modified by the magnitude of $\mathcal{O}(10^{-1})$, which falls in the projection of the ILC [80], one can expect the benchmark modes to be detected through the process of $e^+e^- \rightarrow Zh h$. When the deviation is $\sim \mathcal{O}(10^{-2})$, one may expect the benchmark models to be probed by the SPPC [4,81] through the process of $pp \rightarrow hh$.

D. Lepton collider and hadron collider prospects

At first, because of the mixing of χ and H in the model, one can expect the pair production process of $H(\chi)A$ at leptonic colliders [10,82], and the corresponding cross section is

$$\sigma(e^+e^- \rightarrow H(\chi)A) = g_{H(\chi)AZ}^2 \frac{2\pi\alpha_{em}^2((-1/4)^2 + (-1/4 + \sin^2\theta_w)^2)}{3\sin^4\theta_w \cos^4\theta_w} \times \frac{\lambda_{H(\chi)A}^3}{\sqrt{s}((s-m_Z^2)^2 + m_Z^2\Gamma_Z^2)}, \quad (53)$$

where $g_{H(\chi)AZ} = \cos\theta(\sin\theta)$ and $\lambda_{ij} = [(s-M_i^2-M_j^2)^2 - 4M_i^2M_j^2]/4s^{1/2}$ is the space function of the two-body phase space. Unfortunately, for our two benchmark models (BP5 and BP-c2) that can address both the strongly first-order EWPT and DM relic abundance, the future leptonic colliders, such as Circular electron-positron Collider (CEPC) [83] and Future e^+e^- Circular Collider (FCC-ee) [84], lack the ability to search the $H(\chi)A$ pair productions, since such processes are kinematically inaccessible for the projected design of the center-of-mass energy. We plot the cross section predictions at $\sqrt{s} = 1000$ GeV at the International Linear Collider (ILC) for the two benchmark models in Fig. 8. With the increasing of the mixing angle θ , one obtains the increasing (decreasing) of the cross section of $e^+e^- \rightarrow \chi(H)A$.

As in the IDM, the added scalars may alleviate the Higgs naturalness problem since more bosonic field contributions

FIG. 8. The cross section of $\sigma(e^+e^- \rightarrow H(\chi)A)$ for BP5 and BP-c2 with respect to the mixing parameter θ .

are added to the Veltman condition [10,85], while for the naturalness problem to be probed at leptonic colliders, one needs to expect a sizable modification of wave function renormalization of the SM Higgs field after these extra Higgses and dark matter fields (H, χ, A) are integrated out [86–88]. We leave the detailed study on the interplay between the naturalness probe and EWPT to the future study.

At the Hadronic Collider, the dominant Higgs pairs production processes are⁶

$$pp \rightarrow W^* \rightarrow H^\pm A \quad \text{or} \quad H^\pm H(\chi), \quad (54)$$

$$pp \rightarrow Z^*(\gamma^*) \rightarrow H(\chi)A \quad \text{or} \quad H^+H^-, \quad (55)$$

followed by the decays of channels being

$$H \rightarrow h\chi, \quad (56)$$

$$H^\pm \rightarrow \chi W^\pm, \quad (57)$$

$$A \rightarrow \chi Z^*. \quad (58)$$

We explore the possible prospects of the future search of the two benchmark models: BP5 and BP-c2. We use MADGRAPH5 [89] to estimate cross sections of different channels; the cross sections of the $H^\pm A$, $H^\pm H$, and $H^\pm \chi$ channels of Eq. (54) are estimated to be 2.9, 2.1, and 0.6 fb at 14 TeV; and the cross sections of the HA , χA , and H^+H^- channels of Eq. (55) are estimated to be 1.6, 0.5, and 0.7 fb. As in the usual IDM models, the multilepton plus missing energy signatures [18,90,91] can be expected in the final states at the detector level, while, due to the smallness of the cross sections, these channels might be unreachable by LHC Run 2 at 14 TeV or even SPPC. More detailed studies are left to the future projects.

As for the BP-c2, the scalar mass relation of $m_{H^\pm} = m_A > m_H$ indicates that the followed decay of Eq. (57) is replaced by $H^\pm \rightarrow HW^*$, the macroscopic decay lengths of

⁶The production channel involving an off-shell h is negligible here.

which can be $c\tau \sim 1$ mm [60] when the mass splitting between $m_{H^\pm(A)}$ and m_H is 2 GeV. And the bottom-right panels of Figs. 5 and 6 illustrate that m_H can be very close to and even degenerate with the $m_{H^\pm A}$ for $\theta > 0.5$ and $\lambda_7 > 0.07$, which means one can expect $c\tau > 1$ mm, and therefore the displaced decay of $H^\pm \rightarrow HW^*$ and $A \rightarrow HZ^*$ can be searched through a monojet with a soft displaced vertex [92–94]. This signature can make the benchmark model different from the IDM case [60]. When the $\lambda_L \leq -0.2$, one has $m_H \sim m_{A,H^\pm}$ for the dark matter relic abundance undersaturated case of BP5. Then, one can also expect the displaced vertex signature.

VIII. CONCLUSION AND DISCUSSION

In this work, we study the dark matter and electroweak phase transition in the framework of mixed scalar dark matter model. In the model, the dark matter candidate is the lightest $\mathbb{Z}_2$ -odd mixed particle coming from the mixing of the singlet scalar and the CP -even neutral component of the inert doublet. We investigate the scenarios in which the dark matter particle is mostly the CP -even neutral component of the inert doublet (the benchmark scenario with H being the DM candidate) or the singlet (with χ being the DM candidate), which is determined by the mixing of the two. After imposing theoretical constraints of unitarity, stability, and electroweak precision tests (in particular, the T parameter), we explore several benchmark models classified by the mass of the dark matter particle. The dark matter annihilation process can be affected a lot by the mixing and coannihilation effects. Two benchmark models in which the strongly first-order electroweak phase transition can be accomplished have been investigated.

The effects of quartic scalar couplings (λ_L, λ_7), DM mass, mass splittings among different extra scalars, and the mixing angle on the phase transition as well as dark matter phenomenology have been studied. Certain mass differences between the mixed scalars χ, H and other scalars in the inert doublet ($H^\pm$ and A) are required to successfully realize the strongly first-order electroweak phase transition. The DM direct detection limits can be evaded through tuning the mixing parameter. Lepton and

hadron collider prospects have also been addressed in the model.

The unbroken $\mathbb{Z}_2$ symmetry in the model precludes spontaneous and explicit CP violation arising from the Higgs potential, because of which the model fails to accommodate successful electroweak baryogenesis to explain the baryon asymmetry of the Universe. One remedy method could be the introduction of effective high-dimensional operators [95,96], which is beyond the scope of this study.

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APPENDIX: THERMAL FIELD MASSES

The transverse components of the W boson do not receive thermal corrections, whereas the longitudinally polarized W bosons are corrected as

$$m_{W_L}^2 = m_W^2 + 2g^2 T^2. \quad (\text{A1})$$

The masses of the longitudinal Z and A are determined by diagonalizing the matrix

$$\frac{1}{4}h^2 \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} + \begin{pmatrix} 2g^2 T^2 & 0 \\ 0 & 2g'^2 T^2 \end{pmatrix}. \quad (\text{A2})$$

The eigenvalues can be written as

$$m_{Z_L, A_L}^2 = \frac{1}{2}m_Z^2 + (g^2 + g'^2)T^2 \pm \Delta, \quad (\text{A3})$$

where

$$\Delta^2 = \left(\frac{1}{2}m_Z^2 + (g^2 + g'^2)T^2 \right)^2 - g^2 g'^2 T^2 (h^2 + 4T^2). \quad (\text{A4})$$

Following the approach of Ref. [67], the field-dependent thermal masses of scalar masses are calculated to be

$$\begin{aligned} m_{hh}(T) &= (m_h^2 - 3\lambda_1 v^2) + \frac{T^2}{12}(6\lambda_1 + 2\lambda_3 + \lambda_4 \\ &\quad + (9g^2 + 3g'^2)/4 + 3g_t^2 + \lambda_7) + 3[m_h^2/(2v^2)]h^2, \\ m_{GG}(T) &= (m_h^2 - 3\lambda_1 v^2) + \frac{T^2}{12}(6\lambda_1 + 2\lambda_3 + \lambda_4 \\ &\quad + (9g^2 + 3g'^2)/4 + 3g_t^2 + \lambda_7) + 3[m_h^2/(2v^2)]h^2, \\ m_{SS}(T) &= 2\mu_S^2 + \lambda_7 h^2 + \frac{T^2}{24}(12\lambda_6 + 4\lambda_7 + 4\lambda_8), \\ m_{H_0}(T) &= \mu_I^2 + 2\lambda_L h^2 + \frac{T^2}{12}(6\lambda_2 + 2\lambda_3 + \lambda_4 + \lambda_8), \\ m_{A_0}(T) &= \mu_I^2 + \frac{T^2}{12}(6\lambda_2 + 2\lambda_3 + \lambda_4 + (9g^2 + 3g'^2)/4 + \lambda_8) \\ &\quad + \frac{1}{2}\lambda_A h^2, \\ m_{H^\pm}(T) &= \mu_I^2 + \frac{T^2}{12}(6\lambda_2 + 2\lambda_3 + \lambda_4 + (9g^2 + 3g'^2)/4 + \lambda_8) \\ &\quad + \frac{1}{2}\lambda_3 h^2, \end{aligned} \quad (\text{A5})$$

with $\mu_{I,S}^2$ given by Eq. (14), and $g_t = \sqrt{2}m_t/v$.

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