Trimaximal mixing with a texture zero

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We analyze neutrino mass matrices having one texture zero, assuming that the neutrino mixing matrix has either its first (TM₁) or second (TM₂) column identical to that of the tribimaximal mixing matrix. We found that all six possible one texture-zero neutrino mass matrices are compatible with the present neutrino oscillation data when combined with TM₁ or TM₂ mixing. These textures have interesting predictions for the presently unknown parameters such as the effective Majorana neutrino mass, the Dirac *CP* violating phase, and the neutrino mass scale. We also present a way to theoretically realize some of these textures using *A*₄ symmetry within the framework of the type-I + II seesaw mechanism.

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I. INTRODUCTION

Various neutrino oscillation experiments in the last decade or so have measured the three lepton mixing angles, and it is now clear that the flavor mixing in the lepton sector is quite large as compared to the quark sector. Non-Abelian discrete flavor symmetries have been extensively utilized to explain the large mixing in the lepton sector [1]. Among the most widely studied lepton mixing patterns obtained using discrete non-Abelian symmetries is the tribimaximal (TBM) mixing pattern [2],

$$U_{\text{TBM}} = \begin{pmatrix} -\frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

which predicts the reactor mixing angle θ_{13} to be zero, a maximal atmospheric mixing angle, i.e., $\theta_{23} = \frac{\pi}{4}$, and the solar mixing angle $\theta_{12} = \sin^{-1}(1/\sqrt{3})$. However, recent neutrino oscillation experiments have found θ_{13} to be nonzero [3], necessitating some modifications to the TBM mixing scheme to make it compatible with the present experimental data. In this context, it has been observed that the first or the second column of TBM mixing matrix can still accommodate the recent neutrino oscillation data [4]. When the second column of TBM mixing matrix remains intact while the other columns

deviate from TBM values, we denote that mixing matrix as TM₂ [5]. Similarly, if the first column of TBM remains intact and other columns deviate from TBM, we denote it by TM₁ [5]. When either of these columns of TBM remain intact in the lepton mixing matrix, we can parametrize the mixing matrix in terms of one mixing angle and one *CP* violating Dirac phase along with two Majorana phases [4,5].

There are many other approaches which have been used to explain neutrino mixing; some of these are texture zeros [6], vanishing cofactors [7], hybrid textures [8], etc. Out of these, texture zeros have been quite successful in explaining mixing in both the quark and the lepton sectors. In the basis where the charged lepton mass matrix is diagonal there are seven patterns of two texture zeros which are compatible with the current neutrino oscillation data [6]. There are six possible patterns of one texture zero in the neutrino mass matrix which are shown in Table I. All of these are compatible with the present experimental data [9].

Neutrino mass matrices with two texture zeros in combination with TM₂ mixing have been studied in Ref. [10], where it has been found that only two patterns, namely *A*₁ and *A*₂ (with texture zeros at (1,1), (1,2), and (1,1), (1,3) entries, respectively), can satisfy the present

TABLE I. Six allowed patterns of one texture zero in the neutrino mass matrix.

Pattern	Constraining equation
I	$M_{ee} = 0$
II	$M_{e\mu} = 0$
III	$M_{e\tau} = 0$
IV	$M_{\mu\mu} = 0$
V	$M_{\mu\tau} = 0$
VI	$M_{\tau\tau} = 0$

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experimental data when we combine two texture zeros with TM_2 mixing. The combination of TM_1 mixing with two texture zeros has been studied in Ref. [11] and, similar to the TM_2 case, it has been found that only patterns A_1 and A_2 of two texture zeros are compatible with the present neutrino oscillation data when combined with TM_1 mixing. This approach of combining texture zeros with TM_1/TM_2 mixing turns out to be very fruitful as it leads to very predictive texture structures of the neutrino mass matrix. In the present work, we study neutrino mass matrices having one texture zero in combination with TM_2/TM_1 mixing.

II. TM_2 MIXING AND ONE TEXTURE ZERO

A. TM_2 mixing

A neutrino mass matrix with TBM mixing can be diagonalized as

$$M_{\text{diag}} = U_{\text{TBM}}^T M_{\text{TBM}} U_{\text{TBM}}, \quad (2)$$

where U_{TBM} contains the eigenvectors $t = (-\frac{\sqrt{2}}{\sqrt{3}} \frac{1}{\sqrt{6}} \frac{1}{\sqrt{6}})^T$, $u = (\frac{1}{\sqrt{3}} \frac{1}{\sqrt{3}} \frac{1}{\sqrt{3}})^T$, and $v = (0 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}})^T$. The diagonal matrix M_{diag} is given as

$$M_{\text{diag}} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & e^{2i\alpha} m_2 & 0 \\ 0 & 0 & e^{2i\beta} m_3 \end{pmatrix}, \quad (3)$$

where m_1 , m_2 , and m_3 are the three neutrino masses and α and β are the two Majorana phases. The mass matrix M_{TBM} is invariant under the transformations G_t , G_u , and G_v ; i.e., $G_t^T M_{\text{TBM}} G_t = M_{\text{TBM}}$, $G_u^T M_{\text{TBM}} G_u = M_{\text{TBM}}$, and $G_v^T M_{\text{TBM}} G_v = M_{\text{TBM}}$ with $G_t = 1 - 2tt^T$, $G_u = 1 - 2uu^T$, and $G_v = 1 - 2vv^T$. The transformation matrices G_u and G_v correspond to the magic symmetry [12] and the $\mu - \tau$ symmetry [12], respectively. G_i ($i = t, u, v$) are generators of the Z_2 group. A 3×3 neutrino mass matrix with TBM symmetry is invariant under the $Z_2' \times Z_2^u \times Z_2^v$ group. Recently, neutrino oscillation experiments have confirmed a nonzero θ_{13} ; thus, the neutrino mass matrix M_ν cannot remain invariant under the $\mu - \tau$ symmetry transformation G_v . However, the neutrino mass matrix can still be invariant under the magic symmetry transformation G_u as the experimental data are still compatible with the magic symmetry. The mixing matrix which corresponds to the magic symmetry is known as the trimaximal mixing (TM_2) matrix and can be parametrized as [4,5,12–14]

$$U_{\text{TM}_2} = \begin{pmatrix} \sqrt{\frac{2}{3}} \cos \theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \sin \theta \\ -\frac{\cos \theta}{\sqrt{6}} + \frac{e^{-i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin \theta}{\sqrt{6}} - \frac{e^{-i\phi} \cos \theta}{\sqrt{2}} \\ -\frac{\cos \theta}{\sqrt{6}} - \frac{e^{-i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\sin \theta}{\sqrt{6}} + \frac{e^{-i\phi} \cos \theta}{\sqrt{2}} \end{pmatrix}. \quad (4)$$

The TM_2 mixing matrix has its middle column fixed to the TBM value (u), which leaves only two free parameters

(θ and ϕ) in U_{TM_2} after we take into account the unitarity constraints. The neutrino mass matrix corresponding to TM_2 mixing is given as

$$M_{\text{TM}_2} = U_{\text{TM}_2}^* M_{\text{diag}} U_{\text{TM}_2}^\dagger. \quad (5)$$

B. One zero in M_{TM_2}

A neutrino mass matrix with TM_2 mixing can be parametrized as [15]

$$M_{\text{TM}_2} = \begin{pmatrix} a & b & c \\ b & d & a + c - d \\ c & a + c - d & b - c + d \end{pmatrix}. \quad (6)$$

The constraint equations for all the patterns with one texture zero and TM_2 mixing can be obtained by substituting the respective texture zero constraints from Table I into Eq. (6). In the diagonal charged lepton mass matrix basis, all the six patterns of one texture zero in the neutrino mass matrix are compatible with the present experimental data. The combination of these one texture zero patterns with TM_2 mixing is bound to produce very predictive forms of neutrino mass matrices.

The neutrino mass matrices with one texture zero and TM_2 mixing are given below:

$$M_{\text{TM}_2}^{\text{I}} = \begin{pmatrix} 0 & b & c \\ b & d & c - d \\ c & c - d & b - c + d \end{pmatrix} \quad (7)$$

$$M_{\text{TM}_2}^{\text{II}} = \begin{pmatrix} a & 0 & c \\ 0 & d & a + c - d \\ c & a + c - d & -c + d \end{pmatrix} \quad (8)$$

$$M_{\text{TM}_2}^{\text{III}} = \begin{pmatrix} a & b & 0 \\ b & d & a - d \\ 0 & a - d & b + d \end{pmatrix} \quad (9)$$

$$M_{\text{TM}_2}^{\text{IV}} = \begin{pmatrix} a & b & c \\ b & 0 & a + c \\ c & a + c & b - c \end{pmatrix} \quad (10)$$

$$M_{\text{TM}_2}^{\text{V}} = \begin{pmatrix} a & b & c \\ b & a + c & 0 \\ c & 0 & a + b \end{pmatrix} \quad (11)$$

$$M_{\text{TM}_2}^{\text{VI}} = \begin{pmatrix} a & b & c \\ b & c - b & a + b \\ c & a + b & 0 \end{pmatrix}. \quad (12)$$

The above mass matrices [Eqs. (7) to (12)] can be rewritten as

$$M_{\text{TM}_2}^{\text{I}} = \begin{pmatrix} 0 & b & c \\ b & c - \Delta & \Delta \\ c & \Delta & b - \Delta \end{pmatrix} \quad \text{where } \Delta = c - d \quad (13)$$

$$M_{\text{TM}_2}^{\text{II}} = \begin{pmatrix} a & 0 & c \\ 0 & c - \Delta & a + \Delta \\ c & a + \Delta & -\Delta \end{pmatrix} \quad \text{where } \Delta = c - d \quad (14)$$

$$M_{\text{TM}_2}^{\text{III}} = \begin{pmatrix} a & b & 0 \\ b & d & a - d \\ 0 & a - d & b + d \end{pmatrix} \quad (15)$$

$$M_{\text{TM}_2}^{\text{IV}} = \begin{pmatrix} \Lambda - c & b & c \\ b & 0 & \Lambda \\ c & \Lambda & b - c \end{pmatrix} \quad \text{where } \Lambda = a + c \quad (16)$$

$$M_{\text{TM}_2}^{\text{V}} = \begin{pmatrix} a & \Omega - a & c \\ \Omega - a & a + c & 0 \\ c & 0 & \Omega \end{pmatrix} \quad \text{where } \Omega = a + b \quad (17)$$

$$M_{\text{TM}_2}^{\text{VI}} = \begin{pmatrix} \Omega - b & b & c \\ b & c - b & \Omega \\ c & \Omega & 0 \end{pmatrix} \quad \text{where } \Omega = a + b \quad (18)$$

All the six patterns from Eqs. (13) to (18) can be written as a linear combination of following matrices

$$\begin{aligned} p_{12} &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & p_{13} &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\ p_{23} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & b_{12} &= \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ b_{13} &= \begin{pmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{pmatrix}, & b_{23} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix}, \end{aligned} \quad (19)$$

where the first three are the symmetric permutation matrices and the last three are in block diagonal form, e.g. $M_{\text{TM}_2}^{\text{I}}$ is obtained as a linear combination of p_{12} , p_{13} , and b_{23} :

$$M_{\text{TM}_2}^{\text{I}} = \Delta b_{23} + b p_{12} + c p_{13}. \quad (20)$$

Similarly, we can construct other patterns. This representation brings all the patterns on equal footing; i.e., all the one texture zero patterns with TM_2 mixing are made

up of simple combinations of two symmetric permutation matrices and a block diagonal matrix. The above decomposition into permutation and block diagonal matrices also helps in the symmetry realization of these patterns.

A neutrino mass matrix with TM_2 mixing is diagonalized by the mixing matrix $U = U_{\text{TM}_2}$ given in Eq. (4).

$$U_{\text{TM}_2}^T M_{\text{TM}_2} U_{\text{TM}_2} = M_{\text{diag}}. \quad (21)$$

We can calculate the neutrino mixing angles from a given mixing matrix U by using the following relations:

$$s_{12}^2 = \frac{|U_{12}|^2}{1 - |U_{13}|^2}, \quad s_{23}^2 = \frac{|U_{23}|^2}{1 - |U_{13}|^2}, \quad \text{and} \quad s_{13}^2 = |U_{13}|^2. \quad (22)$$

The mixing angles for TM_2 mixing in terms of parameters θ and ϕ are

$$\begin{aligned} s_{12}^2 &= \frac{1}{3 - 2\sin^2\theta}, & s_{13}^2 &= \frac{2}{3}\sin^2\theta, \\ s_{23}^2 &= \frac{1}{2} \left(1 + \frac{\sqrt{3}\sin 2\theta \cos \phi}{3 - 2\sin^2\theta} \right). \end{aligned} \quad (23)$$

The Dirac CP violating phase δ can be obtained from the Jarlskog rephasing invariant (J_{CP}) [16]

$$J_{CP} = \text{Im}(U_{11}U_{12}^*U_{21}^*U_{22}). \quad (24)$$

In the standard parametrization,

$$J_{CP} = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2 \sin \delta. \quad (25)$$

For the TM_2 mixing matrix,

$$J_{CP} = \frac{1}{6\sqrt{3}} \sin 2\theta \sin \phi. \quad (26)$$

From Eqs. (25) and (26), we obtain

$$\tan \delta = \frac{\cos 2\theta + 2}{2 \cos 2\theta + 1} \tan \phi. \quad (27)$$

The effective Majorana mass term relevant for neutrinoless double beta decay is given by

$$|M_{ee}| = |m_1 U_{e1}^2 + m_2 e^{2i\alpha} U_{e2}^2 + m_3 e^{2i\beta} U_{e3}^2|. \quad (28)$$

For TM_2 mixing, the above expression takes the following form:

$$|M_{ee}| = \frac{1}{3} |2m_1 \cos^2\theta + m_2 e^{2i\alpha} + 2m_3 e^{2i\beta} \sin^2\theta|. \quad (29)$$

There are many ongoing and forthcoming experiments such as GERDA [17], CUORE [18], EXO [19], NEXT [20], MAJORANA [21], SuperNEMO [22], which aim to achieve a sensitivity up to 0.01 eV for $|M_{ee}|$. Cosmological

observations put an upper bound on the sum of neutrino masses:

$$\Sigma = \sum_{i=1,2,3}^3 m_i. \quad (30)$$

Planck satellite data [23] combined with WMAP, CMB, and BAO experiments limit the sum of neutrino masses $\sum m_i \leq 0.23$ eV at 95% confidence level (CL). In the present work, we assume a more conservative limit of $\sum m_i \leq 1$ eV. The existence of one texture zero in the neutrino mass matrix with TM_2 mixing implies

$$(M_{\text{TM}_2})_{ij} = 0. \quad (31)$$

This condition yields a complex equation viz.

$$m_1 A + m_2 B + m_3 C = 0, \quad (32)$$

where $A = U_{a1}U_{b1}$, $B = U_{a2}U_{b2}e^{2i\alpha}$, $C = U_{a3}U_{b3}e^{2i\beta}$ and a, b can take values e, μ and τ . The above complex equation yields two mass ratios:

$$\frac{m_1}{m_2} = \frac{\text{Re}(C)\text{Im}(B) - \text{Re}(B)\text{Im}(C)}{\text{Re}(A)\text{Im}(C) - \text{Re}(C)\text{Im}(A)} \quad (33)$$

and

$$\frac{m_1}{m_3} = \frac{\text{Re}(C)\text{Im}(B) - \text{Re}(B)\text{Im}(C)}{\text{Re}(B)\text{Im}(A) - \text{Re}(A)\text{Im}(B)}, \quad (34)$$

where Re (Im) denotes the real (imaginary) part. These mass ratios can be used to obtain the expression for the parameter R_ν , which is the ratio of mass squared differences ($\Delta m_{ij}^2 = m_i^2 - m_j^2$),

$$R_\nu \equiv \frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} = \frac{(\frac{m_2}{m_1})^2 - 1}{|(\frac{m_3}{m_1})^2 - 1|}, \quad (35)$$

where $m_1 > m_3$ for an inverted mass ordering (IO) and $m_1 < m_3$ for the normal mass ordering (NO). For a texture

zero to be compatible with the present neutrino oscillation data, the parameter R_ν should lie within its experimentally allowed range. The phenomenological predictions of patterns $M_{\text{TM}_2}^{\text{II}}$ and $M_{\text{TM}_2}^{\text{III}}$ are related, and one can obtain the predictions for pattern $M_{\text{TM}_2}^{\text{III}}$ by making the following transformations,

$$\theta_{23} \rightarrow \frac{\pi}{2} - \theta_{23}, \quad \delta = \pi - \delta, \quad (36)$$

on the predictions of pattern $M_{\text{TM}_2}^{\text{II}}$ and vice versa. This is because patterns $M_{\text{TM}_2}^{\text{III}}$ and $M_{\text{TM}_2}^{\text{II}}$ are related via 2–3 symmetry: $M_{\text{TM}_2}^{\text{III}} = p_{23}^T M_{\text{TM}_2}^{\text{II}} p_{23}$ where p_{23} is the 2–3 permutation matrix given in Eq. (19). Similarly, patterns $M_{\text{TM}_2}^{\text{IV}}$ and $M_{\text{TM}_2}^{\text{VI}}$ are related to each other by above transformations. Thus, we need to study in detail only one of the 2–3 symmetry related patterns.

In the numerical analysis, the neutrino mass matrix is reconstructed using Eq. (5), which takes into account the constraint of TM_2 mixing. For the numerical analysis we generate 10^8 points (10^{10} for pattern $M_{\text{TM}_2}^{\text{V}}$ with NO). The mass squared differences Δm_{21}^2 and $|\Delta m_{31}^2|$ are varied randomly within their 3σ experimental ranges. Parameters θ, ϕ, α and β are varied randomly within their full possible ranges. The texture zero constraint is imposed by requiring that the parameter R_ν in Eq. (35), written in terms of mass ratios $\frac{m_2}{m_1}$ and $\frac{m_3}{m_1}$ should lie within its 3σ experimental range. In addition to above constraints, we require the allowed points to lie within the 3σ experimental ranges of mixing angles θ_{12}, θ_{13} and θ_{23} where the neutrino mixing angles are extracted by using the relations given in Eq. (22). The experimental ranges of various neutrino oscillation parameters with their 1, 2, 3 σ ranges are given in Table II.

The numerical results for one texture zero in the neutrino mass matrix with TM_2 mixing are presented below. The main observations are

- (i) All the six possible patterns of one texture zero in the neutrino mass matrix with TM_2 mixing are consistent with the present neutrino oscillation data.

TABLE II. Current neutrino oscillation parameters from global fits [24].

Parameter	Normal ordering		Inverted ordering	
	best fit $\pm 1\sigma, \pm 2\sigma$	3σ range	best fit $\pm 1\sigma, \pm 2\sigma$	3σ range
θ_{12}°	$34.5_{-1.0, -2.0}^{+1.1, +2.3}$	31.5–38.0	$34.5_{-1.0, -2.0}^{+1.1, +2.3}$	31.5–38.0
θ_{23}°	$41.0_{-1.1, -1.9}^{+1.1, +2.7}$	38.3–52.8	$50.5_{-1.0, -2.3}^{+1.0, +1.7}$	38.5–53.0
θ_{13}°	$8.44_{-0.15, -0.34}^{+0.18, +0.26}$	7.9–8.9	$8.41_{-0.17, -0.41}^{+0.16, +0.29}$	7.9–8.93
δ_{CP}°	$252_{-36, -99}^{+56, +99}$	0.0–360	$259_{-41, -77}^{+47, +88}$	$0-31 \oplus 142-360$
$\Delta m_{21}^2/10^{-5} \text{ eV}^2$	$7.56_{-0.19, -0.36}^{+0.19, +0.39}$	7.05–8.14	$7.56_{-0.19, -0.36}^{+0.19, +0.39}$	7.05–8.14
$ \Delta m_{3l}^2 /10^{-3} \text{ eV}^2$	$2.55_{-0.04, -0.08}^{+0.04, +0.08}$	2.43–2.67	$2.49_{-0.04, -0.08}^{+0.04, +0.08}$	2.37–2.61

- (ii) The pattern $M_{\text{TM}_2}^{\text{I}}$ is consistent with normal mass ordering only.
- (iii) All the viable patterns except $M_{\text{TM}_2}^{\text{I}}$, allow a quasi-degenerate mass spectrum.
- (iv) In case of NO, the parameter $|M_{ee}|$ can vanish for patterns $M_{\text{TM}_2}^{\text{I}}$, $M_{\text{TM}_2}^{\text{II}}$ and $M_{\text{TM}_2}^{\text{III}}$. For the remaining patterns, $|M_{ee}|$ is found to be bounded from below.
- (v) The smallest neutrino mass cannot vanish except for patterns $M_{\text{TM}_2}^{\text{II}}$ and $M_{\text{TM}_2}^{\text{III}}$ with IO.
- (vi) The parameter J_{CP} cannot vanish for patterns $M_{\text{TM}_2}^{\text{II}}$ with IO and $M_{\text{TM}_2}^{\text{V}}$ with NO, implying that these patterns are necessarily CP violating.
- (vii) The atmospheric neutrino mixing angle θ_{23} remains below (above) 45° for pattern $M_{\text{TM}_2}^{\text{IV}}$ ($M_{\text{TM}_2}^{\text{VI}}$) with NO.

The numerical predictions for the presently unknown neutrino parameters have been summarized in Table III. The allowed ranges (at 3σ CL) of parameters θ_{12} , θ and J_{CP} are $(35.65^\circ\text{--}35.76^\circ)$, $(9.68^\circ\text{--}10.93^\circ)$ and $(-0.0363\text{--}0.0363)$, respectively, for all the allowed patterns except for patterns $M_{\text{TM}_2}^{\text{II,III}}$ with IO and $M_{\text{TM}_2}^{\text{V}}$ with NO, for which J_{CP} cannot vanish and has the allowed ranges $\pm(0.006\text{--}0.036)$ and $\pm(0.0165\text{--}0.036)$, respectively. Some of the interesting correlation plots are given in Figs. 1, 2, and 3. Figure 1(a) shows that the Majorana phases α and β are strongly correlated with each other for pattern $M_{\text{TM}_2}^{\text{I}}$ with NO. One can see from Fig. 1(c) that the

Dirac phase δ and phase ϕ are linearly correlated and are almost identical to each other. From Eq. (27) we can see that the ratio $(\frac{\cos 2\theta+2}{2\cos 2\theta+1})$ multiplying with $\tan \phi$ remains ≈ 1 for the allowed values of θ . This leads to the feature $\delta \approx \phi$, for all the neutrino mass matrices with a texture zero and TM_2 mixing.

For pattern $M_{\text{TM}_2}^{\text{II}}$, the Dirac phase δ is restricted to two regions [Fig. 2(a)]. The correlation between mixing angles θ_{13} and θ_{12} is shown in Fig. 2(c). This is a generic feature of TM_2 mixing arising from Eq. (23). Since the TBM value of θ_{12} is already above its experimental best-fit value, an increase in θ drives θ_{12} further away from the best-fit experimental value. Thus, TM_2 mixing leads to some tension with mixing angle θ_{12} . Figure 3 shows the 2–3 symmetry between patterns $M_{\text{TM}_2}^{\text{IV}}$ and $M_{\text{TM}_2}^{\text{VI}}$.

C. Symmetry realization

The neutrino mass matrices with one texture zero and TM_2 mixing can be realized in the framework of type-I+II seesaw mechanism [25,26] using A_4 [27] symmetry. In addition to the three standard model lepton $SU(2)_L$ doublets D_{lL} (where $l = e, \mu$ and τ) and three right-handed charged lepton singlets l_R , we need a $SU(2)_L$ singlet right-handed neutrino ν_R , six $SU(2)_L$ doublet Higgs fields ψ_i and φ_i ($i = 1, 2, 3$), and two $SU(2)_L$ triplet Higgs fields Δ_1, Δ_2 . We also impose an additional Z_2 symmetry, to prevent couplings between charged leptons (neutrinos) and

FIG. 1. Correlation plots for patterns $M_{\text{TM}_2}^{\text{I}}$ (a), $M_{\text{TM}_2}^{\text{II}}$ (b), $M_{\text{TM}_2}^{\text{IV}}$ (c) and $M_{\text{TM}_2}^{\text{V}}$ (d) with NO.

FIG. 2. Correlation plots for patterns $M_{TM_2}^{II}$ (a,b), $M_{TM_2}^{IV}$ (c) and $M_{TM_2}^V$ (d) with IO.

scalars φ_i (ψ_i). Below we discuss in detail the symmetry realization of pattern $M_{TM_2}^I$. The transformation properties of various fields under A_4 and Z_2 corresponding to pattern I are given in Table IV. These transformation properties lead to the following Yukawa Lagrangian which is invariant under A_4 and Z_2 :

$$\begin{aligned}
 -\mathcal{L}_{\text{Yukawa}} = & y_1(\bar{D}_{e_L}\psi_1 + \bar{D}_{\mu_L}\psi_2 + \bar{D}_{\tau_L}\psi_3)_{\underline{1}}e_{R_{\underline{1}}} + y_2(\bar{D}_{e_L}\psi_1 + \omega^2\bar{D}_{\mu_L}\psi_2 + \omega\bar{D}_{\tau_L}\psi_3)_{\underline{1}'}\tau_{R_{\underline{1}''}} \\
 & + y_3(\bar{D}_{e_L}\psi_1 + \omega\bar{D}_{\mu_L}\psi_2 + \omega^2\bar{D}_{\tau_L}\psi_3)_{\underline{1}''}\mu_{R_{\underline{1}'}} + y_4(\bar{D}_{e_L}\tilde{\varphi}_1 + \bar{D}_{\mu_L}\tilde{\varphi}_2 + \bar{D}_{\tau_L}\tilde{\varphi}_3)_{\underline{1}}\nu_{R_{\underline{1}}} \\
 & - y_{\Delta_1}(D_{e_L}^T C^{-1}D_{e_L} + \omega D_{\mu_L}^T C^{-1}D_{\mu_L} + \omega^2 D_{\tau_L}^T C^{-1}D_{\tau_L})_{\underline{1}'}i\tau_2\Delta_{1_{\underline{1}'}} \\
 & - y_{\Delta_2}(D_{e_L}^T C^{-1}D_{e_L} + \omega^2 D_{\mu_L}^T C^{-1}D_{\mu_L} + \omega D_{\tau_L}^T C^{-1}D_{\tau_L})_{\underline{1}'}i\tau_2\Delta_{2_{\underline{1}''}} - m_R(\nu_R^T C^{-1}\nu_R) + \text{H.c.}, \quad (37)
 \end{aligned}$$

where $\tilde{\varphi} = i\tau_2\varphi^*$. Assuming that ψ_i Higgs fields acquire nonzero vacuum expectation values (VEVs) along the direction $\langle\psi\rangle_o = v_\psi(1, 1, 1)^T$ leads to the following form of the charged lepton mass matrix:

FIG. 3. Correlation plots for patterns $M_{TM_2}^{IV}$ (a) and $M_{TM_2}^{VI}$ (b) with NO.

TABLE III. Numerical predictions (at 3σ CL) for patterns having one texture zero in M_ν with TM_2 mixing.

Pattern	Mass ordering	$ M_{ee} $ (eV)	Lightest neutrino mass m (eV)	$\sum m_i$ (eV)	δ°
I	NO	0.0	0.0026–0.0084	0.061–0.073	0–360
II	NO	0.0–0.172	0.002–0.183	0.056–0.57	0–360
	IO	0.047–0.275	0.0–0.272	0.097–0.825	21–154 $\oplus$ 200–345
III	NO	0.0–0.172	0.002–0.183	0.056–0.57	0–360
	IO	0.047–0.275	0.0–0.272	0.097–0.825	29–159 $\oplus$ 200–332
IV	NO	0.034–0.237	0.035–0.3	0.13–0.9	90–270
	IO	0.015–0.126	0.007–0.18	0.106–0.553	0–360
V	NO	0.14–0.34	0.147–0.332	0.45–1	32–150 $\oplus$ 209–333
	IO	0.015–0.305	0.017–0.31	0.127–0.94	0–360
VI	NO	0.034–0.237	0.035–0.3	0.13–0.9	0–90 $\oplus$ 270–360
	IO	0.015–0.126	0.007–0.18	0.106–0.553	0–360

$$m_l = \begin{pmatrix} y_1 v_\psi & y_2 v_\psi & y_3 v_\psi \\ y_1 v_\psi & y_2 \omega v_\psi & y_3 \omega^2 v_\psi \\ y_1 v_\psi & y_2 \omega^2 v_\psi & y_3 \omega v_\psi \end{pmatrix}. \quad (38)$$

$$m_\nu^I = a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \quad \text{where } a = y_4^2 v_\varphi^2 / m_R. \quad (40)$$

The φ_i fields which couple to neutrinos are assumed to have the VEV alignment: $\langle \varphi \rangle_o = v_\varphi(0, -1, 1)^T$. Similar vacuum alignment has been obtained earlier in Ref. [28] for A_4 and $SU(2)_L$ triplet scalars. The above VEV alignment leads to the following Dirac neutrino mass matrix:

$$m_D = y_4 v_\varphi(0, -1, 1)^T. \quad (39)$$

The type-II seesaw contribution to the effective neutrino mass matrix has the following form which is obtained when the $SU(2)_L$ triplet Higgses Δ_1 and Δ_2 acquire nonzero and small VEVs:

$$m_\nu^{\text{II}} = \begin{pmatrix} b+c & 0 & 0 \\ 0 & \omega b + \omega^2 c & 0 \\ 0 & 0 & \omega^2 b + \omega c \end{pmatrix}, \quad (41)$$

With only one right-handed neutrino with mass m_R , the effective neutrino mass matrix obtained using the type-I seesaw mechanism $m_\nu^I \approx m_D m_R^{-1} m_D^T$, has the form

where $b = y_{\Delta_1} v_{\Delta_1}$ and $c = y_{\Delta_2} v_{\Delta_2}$. The collective effective neutrino mass matrix $m_\nu = m_\nu^I + m_\nu^{\text{II}}$ from the type-I + II seesaw mechanism becomes

TABLE IV. Transformation properties of various fields under A_4 and Z_2 . The VEV alignments for ψ and φ are $\langle \psi \rangle_o = v_\psi(1, 1, 1)^T$ and $\langle \varphi \rangle_o = v_\varphi(0, -1, 1)^T$.

Pattern	Symmetry	D_{l_L}	e_R	μ_R	τ_R	ν_R	ψ	φ	Δ_1	Δ_2	D_L triplet representation under A_4
$M_{\text{TM}_2}^{\text{I}}$	$SU(2)_L$	2	1	1	1	1	2	2	2	3	$3 \sim \begin{pmatrix} D_{e_L} \\ D_{\mu_L} \\ D_{\tau_L} \end{pmatrix}$
	A_4	3	1	1'	1''	1	3	3	1'	1''	
	Z_2	1	1	1	1	-1	1	-1	1	1	
$M_{\text{TM}_2}^{\text{II}}$	$SU(2)_L$	2	1	1	1	1	2	2	3	3	$3 \sim \begin{pmatrix} D_{e_L} \\ D_{\mu_L} \\ D_{\tau_L} \end{pmatrix}$
	A_4	3	1	1'	1''	1	3	3	1	1''	
	Z_2	1	1	1	1	-1	1	-1	1	1	
$M_{\text{TM}_2}^{\text{III}}$	$SU(2)_L$	2	1	1	1	1	2	2	3	3	$3 \sim \begin{pmatrix} D_{e_L} \\ D_{\mu_L} \\ D_{\tau_L} \end{pmatrix}$
	A_4	3	1	1'	1''	1	3	3	1	1'	
	Z_2	1	1	1	1	-1	1	-1	1	1	
$M_{\text{TM}_2}^{\text{IV}}$	$SU(2)_L$	2	1	1	1	1	2	2	3	3	$3 \sim \begin{pmatrix} D_{\mu_L} \\ D_{\tau_L} \\ D_{e_L} \end{pmatrix}$
	A_4	3	1''	1	1'	1	3	3	1'	1''	
	Z_2	1	1	1	1	-1	1	-1	1	1	
$M_{\text{TM}_2}^{\text{V}}$	$SU(2)_L$	2	1	1	1	1	2	2	3	3	$3 \sim \begin{pmatrix} D_{\tau_L} \\ D_{e_L} \\ D_{\mu_L} \end{pmatrix}$
	A_4	3	1'	1''	1	1	3	3	1	1'	
	Z_2	1	1	1	1	-1	1	-1	1	1	
$M_{\text{TM}_2}^{\text{VI}}$	$SU(2)_L$	2	1	1	1	1	2	2	3	3	$3 \sim \begin{pmatrix} D_{\tau_L} \\ D_{e_L} \\ D_{\mu_L} \end{pmatrix}$
	A_4	3	1'	1''	1	1	3	3	1''	1'	
	Z_2	1	1	1	1	-1	1	-1	1	1	

$$m_\nu = \begin{pmatrix} b+c & 0 & 0 \\ 0 & a+\omega b+\omega^2 c & -a \\ 0 & -a & a+\omega^2 b+\omega c \end{pmatrix}. \quad (42)$$

In the present basis, the charged lepton mass matrix is nondiagonal. We move to the diagonal charged lepton mass matrix basis using the transformation $M_l = U_L^\dagger m_l U_R$, where

$$U_L = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (43)$$

and U_R is a unit matrix. In this basis, the effective neutrino mass matrix becomes

$$M_\nu = \begin{pmatrix} 0 & b & c \\ b & -a+c & a \\ c & a & -a+b \end{pmatrix}, \quad (44)$$

which is the pattern $M_{\text{TM}_2}^I$ of one texture zero with TM_2 mixing. The symmetry realization of other patterns can be obtained in a similar way. We have summarized the desired transformation properties of various leptonic and scalar fields (under A_4 and Z_2) which lead to neutrino mass matrices with one texture zero and TM_2 mixing, in Table IV.

For the symmetry realization of above patterns, we require many Higgs $SU(2)_L$ doublets. It should be noted that such multi-Higgs models generally lead to flavor-changing neutral currents which can contribute to charged lepton flavor violating decays. However, an explicit calculation of such effects is beyond the scope of the present work.

III. TM_1 MIXING AND ONE TEXTURE ZERO

A. TM_1 mixing

In section II we studied the one texture zero patterns having TM_2 mixing. For these patterns the allowed values of θ_{12} lie in the 2σ upper limit and values within the 1σ experimental range are not allowed. This leads to some tension with the present neutrino oscillation data. However, this is a generic feature of TM_2 mixing. One possible way to resolve this discrepancy is to consider charged lepton corrections to the neutrino mixing matrix.

Alternatively, instead of considering TM_2 mixing one may also consider TM_1 mixing where θ_{12} can take values which are in good agreement with the present neutrino oscillation data. In this section we explore the neutrino mass matrices having one texture zero along with TM_1 mixing. The TM_1 mixing matrix can be parametrized as [4,5,12,14]

$$U_{\text{TM}_1} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} \cos \theta & \frac{1}{\sqrt{3}} \sin \theta \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \cos \theta - \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} \sin \theta + \frac{e^{i\phi} \cos \theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \cos \theta + \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} \sin \theta - \frac{e^{i\phi} \cos \theta}{\sqrt{2}} \end{pmatrix}; \quad (45)$$

here, the first column of the neutrino mixing matrix is identical with TBM mixing matrix and the other two columns have been parametrized in terms of two free parameters (θ and ϕ) after taking into consideration the unitarity constraints on the mixing matrix. The corresponding neutrino mass matrix for TM_1 mixing is given as

$$M_{\text{TM}_1} = U_{\text{TM}_1}^* M_{\text{diag}} U_{\text{TM}_1}^\dagger. \quad (46)$$

B. One zero in M_{TM_1}

A neutrino mass matrix with TM_1 mixing can be written as

$$M_{\text{TM}_1} = \begin{pmatrix} a & 2b & 2c \\ 2b & 4b+d & a-b-c-d \\ 2c & a-b-c-d & 4c+d \end{pmatrix}. \quad (47)$$

All the neutrino mass matrices with one texture zero and TM_1 mixing can be obtained by substituting the respective constraints from Table I in Eq. (47):

$$M_{\text{TM}_1}^I = \begin{pmatrix} 0 & 2b & 2c \\ 2b & 4b+d & -b-c-d \\ 2c & -b-c-d & 4c+d \end{pmatrix} \quad (48)$$

$$M_{\text{TM}_1}^{\text{II}} = \begin{pmatrix} a & 0 & 2c \\ 0 & d & a-c-d \\ 2c & a-c-d & 4c+d \end{pmatrix} \quad (49)$$

$$M_{\text{TM}_1}^{\text{III}} = \begin{pmatrix} a & 2b & 0 \\ 2b & 4b+d & a-b-d \\ 0 & a-b-d & d \end{pmatrix} \quad (50)$$

$$M_{\text{TM}_1}^{\text{IV}} = \begin{pmatrix} a & 2b & 2c \\ 2b & 0 & a+3b-c \\ 2c & a+3b-c & 4c-4b \end{pmatrix} \quad (51)$$

$$M_{\text{TM}_1}^{\text{V}} = \begin{pmatrix} a & 2b & 2c \\ 2b & a+3b-c & 0 \\ 2c & 0 & a+3c-b \end{pmatrix} \quad (52)$$

$$M_{\text{TM}_1}^{\text{VI}} = \begin{pmatrix} a & 2b & 2c \\ 2b & 4b - 4c & a + 3c - b \\ 2c & a + 3c - b & 0 \end{pmatrix}. \quad (53)$$

A neutrino mass matrix with TM_1 mixing can be diagonalized by the mixing matrix $U = U_{\text{TM}_1}$ given in Eq. (45),

$$U_{\text{TM}_1}^T M_{\text{TM}_1} U_{\text{TM}_1} = M_{\text{diag}}. \quad (54)$$

The mixing angles for TM_1 mixing in terms of parameters θ and ϕ are

$$\begin{aligned} s_{13}^2 &= \frac{1}{3} \sin^2 \theta, & s_{12}^2 &= 1 - \frac{2}{3 - \sin^2 \theta}, \\ s_{23}^2 &= \frac{1}{2} \left(1 + \frac{\sqrt{6} \sin 2\theta \cos \phi}{3 - \sin^2 \theta} \right). \end{aligned} \quad (55)$$

We see from Eq. (55) that the solar mixing angle θ_{12} is smaller than its TBM value $s_{12}^2 = 1/3$. In contrast, for TM_2 mixing, the value of θ_{12} is larger than the TBM value. Since the experimental best-fit value of θ_{12} is towards the lower side of the TBM value, TM_1 mixing is more appealing than TM_2 mixing. The Dirac CP violating phase δ can be obtained from the Jarlskog rephasing invariant (J_{CP}) [16]:

$$J_{CP} = \text{Im}(U_{11}U_{12}^*U_{21}^*U_{22}). \quad (56)$$

For the TM_1 mixing matrix,

$$J_{CP} = \frac{1}{6\sqrt{6}} \sin 2\theta \sin \phi. \quad (57)$$

Using Eqs. (25) and (57), we obtain

$$\tan \delta = \frac{\cos 2\theta + 5}{5 \cos 2\theta + 1} \tan \phi. \quad (58)$$

The effective Majorana mass for TM_1 mixing is given by

$$|M_{ee}| = \frac{1}{3} |2m_1 + m_2 e^{2i\alpha} \cos^2 \theta + m_3 e^{2i\beta} \sin^2 \theta|. \quad (59)$$

The existence of one texture zero in the neutrino mass matrix implies

$$(M_{\text{TM}_1})_{ij} = 0. \quad (60)$$

Following the same procedure as we did for TM_2 mixing, we analyze the phenomenological predictions of neutrino mass matrices having one texture zero and TM_1 mixing.

The main results of the numerical analysis are

- (i) All six patterns of one texture zero in the neutrino mass matrix with TM_1 mixing are consistent with the present neutrino oscillation data.
- (ii) The pattern $M_{\text{TM}_1}^{\text{I}}$ is consistent with normal mass ordering only.
- (iii) All the allowed patterns except for $M_{\text{TM}_1}^{\text{I}}$, allow a quasidegenerate mass spectrum.
- (iv) In case of NO, vanishing values of the parameter $|M_{ee}|$ are allowed for patterns $M_{\text{TM}_1}^{\text{I}}$, $M_{\text{TM}_1}^{\text{II}}$ and $M_{\text{TM}_1}^{\text{III}}$. For the remaining patterns $|M_{ee}|$ is found to be bounded from below.
- (v) The smallest neutrino mass can have vanishing values for patterns $M_{\text{TM}_1}^{\text{II}}$ and $M_{\text{TM}_1}^{\text{III}}$ with IO.
- (vi) The parameter J_{CP} cannot vanish for any of the allowed patterns implying that these patterns are necessarily CP violating.
- (vii) The atmospheric neutrino mixing angle θ_{23} remains below (above) 45° for pattern $M_{\text{TM}_1}^{\text{IV}}$ ($M_{\text{TM}_1}^{\text{VI}}$) with NO.

Numerical results for the presently unknown neutrino parameters have been summarized in Table V. The allowed ranges (at 3σ CL) of parameters θ_{12} , θ and J_{CP} are $(34.26^\circ - 34.48^\circ)$, $(13.77^\circ - 15.55^\circ)$ and $\pm(0.022 - 0.035)$, respectively, for all the allowed patterns. Some of the correlation plots are given in Figs. 4 and 5. Figure 4(a) depicts the correlation plot between Dirac phase δ and mixing angle θ_{23} for patten $M_{\text{TM}_1}^{\text{II}}$ with NO. The CP violating Dirac

TABLE V. Numerical predictions (at 3σ CL) for patterns having one texture zero in M_ν with TM_1 mixing.

Pattern	Mass ordering	$ M_{ee} $ (eV)	m (eV)	Σm_i (eV)	δ°
I	NO	0.0	0.0022–0.0071	0.06–0.07	$46-127 \oplus 233-314$
II	NO	0.0–0.124	0.0017–0.124	0.06–0.382	$46-127 \oplus 233-314$
	IO	0.047–0.258	0.0–0.273	0.097–0.83	$46-127 \oplus 233-314$
III	NO	0.0–0.124	0.0017–0.124	0.06–0.382	$46-127 \oplus 233-314$
	IO	0.047–0.258	0.0–0.273	0.097–0.83	$46-127 \oplus 233-314$
IV	NO	0.03–0.3	0.033–0.3	0.13–0.91	$90-127 \oplus 233-270$
	IO	0.017–0.24	0.0037–0.28	0.1–0.85	$45-122 \oplus 235-313$
V	NO	0.14–0.311	0.15–0.331	0.46–1	$48-125 \oplus 235-313$
	IO	0.018–0.24	0.016–0.26	0.12–0.78	$45-125 \oplus 235-314$
VI	NO	0.029–0.3	0.031–0.3	0.12–0.9	$45-90 \oplus 270-314$
	IO	0.017–0.24	0.0037–0.28	0.1–0.85	$52-125 \oplus 235-314$

FIG. 4. Correlation plots for patterns $M_{TM_1}^I$ (a), $M_{TM_1}^{II}$ (b), and $M_{TM_1}^{IV}$ (c),(d) with NO.

FIG. 5. Correlation plots for patterns $M_{TM_1}^{II}$ (a),(b), $M_{TM_1}^{IV}$ (c), and $M_{TM_1}^V$ (d) with IO.

phase δ is restricted to two regions around 90° and 270° . This result holds for all the allowed patterns and is independent of the mass ordering. In the numerical analysis, we have varied the Dirac phase δ within its full possible range of 0° – 360° . Recent long baseline neutrino oscillation experiments such as MINOS and T2K [29] have shown a preference for the CP violating phase δ to be around 270° . Particularly, recent global analysis in Ref. [24] rules out δ from 32° to 141° at 3σ CL for inverted mass ordering. If we take into account, the limits on δ as given in Ref. [24], the region of δ around 90° is ruled out and only the second region around 270° remains compatible with IO. Figure 4(c) shows the correlation plot between δ and J_{CP} for pattern $M_{TM_1}^{IV}$ with NO. It is clear that a vanishing J_{CP} is not allowed for this pattern; in fact, all the patterns with one texture zero and TM_1 mixing predict a nonzero J_{CP} which implies that these patterns are necessarily CP violating. This is because these patterns do not allow values 0° and 180° for the Dirac phase δ , and since all the mixing angles are nonzero, the CP invariant J_{CP} cannot vanish.

Phases ϕ and δ are found to have almost identical values [Fig. 5(c)] which is similar to the TM_2 case. The correlation between mixing angles $\theta_{13} - \theta_{12}$ is shown in Fig. 5(d). In contrast to the TM_2 case, here, the value of θ_{12} decreases with increasing θ . This is a generic feature of TM_1 mixing arising from Eq. (55). This brings θ_{12} near its best-fit experimental value. Thus TM_1 mixing is phenomenologically more appealing than TM_2 mixing.

IV. SUMMARY

We studied the implications of having one texture zero in the neutrino mass matrix along with TM_1/TM_2 mixing. Considering neutrinos to be Majorana fermions, there are six possible patterns of one texture zero in the neutrino mass matrix. All six patterns are found to be phenomenologically allowed when combined with TM_1/TM_2 mixing. The presence of a texture zero in the neutrino mass matrix leads to relations between neutrino masses and mixing matrix elements, whereas TM_1/TM_2 mixing implies relations between mixing angles. Thus, the combination of one texture zero patterns with TM_1/TM_2 mixing leads to very predictive neutrino mass matrices. For the pattern where the texture zero is at (1,1) position in the neutrino mass matrix, only normal mass ordering is experimentally allowed. Since TM_2 mixing predicts values of θ_{12} away from its best-fit value, TM_1 mixing is phenomenologically more appealing. We have obtained predictions for the unknown parameters such as the effective Majorana neutrino mass, the Dirac CP violating phase and the neutrino mass scale. The Dirac phase δ has been found to be strongly correlated with the phase parameter ϕ for both TM_1 as well as TM_2 mixing. We have also constructed neutrino mass models which lead to

patterns of one texture zero with TM_2 mixing. To realize these patterns we have employed the A_4 symmetry within the framework of type-I+II seesaw mechanism.

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APPENDIX: GROUP A_4

A_4 has four inequivalent irreducible representations (IRs) which are three singlets $\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{1}''$, and one triplet $\mathbf{3}$. The group A_4 is generated by two generators S and T such that

$$S^2 = T^3 = (ST)^3 = 1. \quad (A1)$$

The one-dimensional unitary IRs are

$$\mathbf{1}S = 1 \quad T = 1, \quad \mathbf{1}'S = 1 \quad T = \omega, \quad \mathbf{1}''S = 1 \quad T = \omega^2. \quad (A2)$$

The three-dimensional unitary IR in the S diagonal basis is

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \quad (A3)$$

The multiplication rules of the IRs are as follows:

$$\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'', \quad \mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}', \quad \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}. \quad (A4)$$

The product of two $\mathbf{3}$'s gives

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'' \oplus \mathbf{3}_s \oplus \mathbf{3}_a, \quad (A5)$$

where s, a denote the symmetric, anti-symmetric products, respectively. Let (x_1, x_2, x_3) and (y_1, y_2, y_3) denote the basis vectors of two $\mathbf{3}$'s. IRs obtained from their products are

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}} = x_1y_1 + x_2y_2 + x_3y_3 \quad (A6)$$

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}'} = x_1y_1 + \omega^2x_2y_2 + \omega x_3y_3 \quad (A7)$$

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{1}''} = x_1y_1 + \omega x_2y_2 + \omega^2x_3y_3 \quad (A8)$$

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{3}_s} = (x_2y_3 + x_3y_2, x_3y_1 + x_1y_3, x_1y_2 + x_2y_1) \quad (A9)$$

$$(\mathbf{3} \otimes \mathbf{3})_{\mathbf{3}_a} = (x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1). \quad (A10)$$

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