Prediction on neutrino Dirac and Majorana phases and absolute mass scale from the CKM matrix

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In the type-I seesaw model, the lepton-flavor-mixing matrix (Pontecorvo-Maki-Nakagawa-Sakata matrix) and the quark-flavor-mixing matrix [Cabibbo-Kobayashi-Maskawa (CKM) matrix] may be connected implicitly through a relation between the neutrino Dirac Yukawa coupling Y_D and the quark Yukawa couplings. In this paper, we study whether Y_D can satisfy—in the flavor basis where the charged lepton Yukawa and right-handed neutrino Majorana mass matrices are diagonal—the relation $Y_D \propto \text{diag}(y_d, y_s, y_b)V_{\text{CKM}}^T$ or $Y_D \propto \text{diag}(y_u, y_c, y_t)V_{\text{CKM}}^*$ without contradicting the current experimental data on quarks and neutrino oscillations. We search for sets of values of the neutrino Dirac CP phase δ_{CP} , Majorana phases α_2, α_3 , and the lightest active neutrino mass that satisfy either of the above relations, with the normal or inverted hierarchy of neutrino masses. In performing the search, we consider renormalization group evolutions of the quark masses and CKM matrix and the propagation of their experimental errors along the evolutions. We find that only the former relation $Y_D \propto \text{diag}(y_d, y_s, y_b)V_{\text{CKM}}^T$ with the normal neutrino mass hierarchy holds, based on which we make predictions for δ_{CP} , α_2, α_3 , and the lightest active neutrino mass.

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The two flavor mixing matrices—i.e., the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix for leptons [1,2] and the Cabibbo-Kobayashi-Maskawa (CKM) matrix for quarks [3,4]—are seemingly irrelevant to each other, since the former includes two large mixing angles $\theta_{23} \sim 45^\circ$ and $\theta_{12} \sim 30^\circ$, while the mixing angles of the latter are all below 15° . However, if the type-I seesaw mechanism [5] is operative, there can be a connection between them, because the right-handed neutrino Majorana mass matrix that enters into the seesaw mass formula distorts the flavor structure of the neutrino Dirac Yukawa coupling, so that the active neutrino mass matrix (in the basis where the charged lepton mass is diagonal) may have large mixings even when the neutrino Dirac Yukawa coupling (in the same basis) only contains small mixings.

In this paper, we consider the Standard Model (SM) extended with the type-I seesaw mechanism, for which the Yukawa interaction and Majorana mass terms read

$$\begin{aligned}
 -\mathcal{L} = & (Y_u)_{ij} \bar{q}_L^i i\sigma_2 H^* u_R^j + (Y_d)_{ij} \bar{q}_L^i H d_R^j \\
 & + (Y_e)_{ij} \bar{\ell}_L^i H e_R^j + (Y_D)_{ij} \bar{\ell}_L^i i\sigma_2 H^* \nu_R^j \\
 & + \frac{1}{2} (M_N)_{ij} \nu_R^i \nu_R^j + \text{H.c.}, \quad (1)
 \end{aligned}$$

where Y_u, Y_d, Y_e , and Y_D denote the up-type quark, down-type quark, charged lepton, and neutrino Dirac Yukawa couplings, respectively, and $i, j = 1, 2, 3$ are flavor indices. The mass matrix for active neutrinos $(m_\nu)_{ij}$ is derived as $m_\nu = -Y_D^* (M_N^*)^{-1} Y_D^\dagger$. On the other hand, in the flavor basis where the charged lepton mass is diagonal, $(m_\nu)_{ij}$ is parametrized in terms of the PMNS matrix U_{PMNS} and the active neutrino masses m_1, m_2 , and m_3 as

$$m_\nu = U_{\text{PMNS}}^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\text{PMNS}}^\dagger, \quad (2)$$

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with

$$U_{\text{PMNS}} \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & e^{i\alpha_3/2} \end{pmatrix},$$

$$s_{ab} \equiv \sin \theta_{ab}, \quad c_{ab} \equiv \cos \theta_{ab} \quad (a, b = 1, 2, 3), \quad (3)$$

where θ_{12} , θ_{23} , and θ_{13} are the neutrino mixing angles, δ_{CP} is the Dirac CP phase, and α_2 and α_3 are the Majorana CP phases.

We propose the following hypothesis. In the flavor basis where Y_e and M_N are diagonal,

$$Y_D = z \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} V_{\text{CKM}}^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\psi_2} & 0 \\ 0 & 0 & e^{i\psi_3} \end{pmatrix}, \quad (4)$$

$$\text{or } Y_D = z \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix} \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} V_{\text{CKM}}^* \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\psi_2} & 0 \\ 0 & 0 & e^{i\psi_3} \end{pmatrix}, \quad (5)$$

either of which holds at one renormalization scale μ located somewhere between the TeV scale and the Planck scale. Here, y_d , y_s , y_b , y_u , y_c , and y_t denote the Yukawa couplings (taken to be real and positive) for d , s , b , u , c , and t quarks, respectively. z is an unspecified complex number and ϕ_2 , ϕ_3 , ψ_2 , and ψ_3 are unspecified phases. We do not focus on the different hypotheses in which $V_{\text{CKM}}^\dagger$ replaces V_{CKM}^T in Eq. (4), or V_{CKM} replaces V_{CKM}^* in Eq. (5). This is because the combinations

$V_{\text{CKM}} \text{diag}(y_d, y_s, y_b)$ and $V_{\text{CKM}}^\dagger \text{diag}(y_u, y_c, y_t)$ enter into the Yukawa couplings Y_d and Y_u , respectively, and the corresponding operators $\bar{q}_L i\sigma_2 H^* u_R$ and $\bar{q}_L H d_R$ have the same chirality as the operator $\bar{\ell}_L i\sigma_2 H^* \nu_R$ that is associated with Y_D . This nice feature is spoiled if we take the complex conjugate of one side of Eq. (4) and/or Eq. (5).

The hypothesis (4)–(5) leads to the following relation between the PMNS and CKM matrices. If Eq. (4) holds, we obtain

$$U_{\text{PMNS}}^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\text{PMNS}}^\dagger = -(z^*)^2 \begin{pmatrix} y_d & 0 & 0 \\ 0 & e^{-i\phi_2} y_s & 0 \\ 0 & 0 & e^{-i\phi_3} y_b \end{pmatrix} V_{\text{CKM}}^\dagger \begin{pmatrix} \frac{1}{M_{N_1}^*} & 0 & 0 \\ 0 & \frac{e^{-2i\psi_2}}{M_{N_2}^*} & 0 \\ 0 & 0 & \frac{e^{-2i\psi_3}}{M_{N_3}^*} \end{pmatrix} V_{\text{CKM}}^* \begin{pmatrix} y_d & 0 & 0 \\ 0 & e^{-i\phi_2} y_s & 0 \\ 0 & 0 & e^{-i\phi_3} y_b \end{pmatrix} \frac{v^2}{2}, \quad (6)$$

where M_{N_j} ($j = 1, 2, 3$) are the components of the diagonalized Majorana mass matrix M_N , and $v \simeq 246$ GeV. If Eq. (5) holds instead, we find

$$\begin{aligned}
& U_{\text{PMNS}}^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\text{PMNS}}^\dagger \\
&= -(z^*)^2 \begin{pmatrix} y_u & 0 & 0 \\ 0 & e^{-i\phi_2} y_c & 0 \\ 0 & 0 & e^{-i\phi_3} y_t \end{pmatrix} V_{\text{CKM}} \begin{pmatrix} \frac{1}{M_{N_1}} & 0 & 0 \\ 0 & \frac{e^{-2i\psi_2}}{M_{N_2}} & 0 \\ 0 & 0 & \frac{e^{-2i\psi_3}}{M_{N_3}} \end{pmatrix} V_{\text{CKM}}^T \begin{pmatrix} y_u & 0 & 0 \\ 0 & e^{-i\phi_2} y_c & 0 \\ 0 & 0 & e^{-i\phi_3} y_t \end{pmatrix} \frac{v^2}{2}. \quad (7)
\end{aligned}$$

Experimentally, the Dirac phase δ_{CP} , the Majorana phases α_2 and α_3 , and the absolute scale of the active neutrino mass have not been measured conclusively. Equation (6) or (7) hence contains 12 real undetermined variables, which are

$$\delta_{CP}, \quad \alpha_2, \quad \alpha_3, \quad \phi_2, \quad \phi_3, \quad m_1, \quad \frac{z^2}{M_{N_1}} (\text{complex}), \quad \frac{z^2 e^{2i\psi_2}}{M_{N_2}} (\text{complex}), \quad \frac{z^2 e^{2i\psi_3}}{M_{N_3}} (\text{complex}), \quad (8)$$

where it should be noted that z , M_{N_i} , and ψ_i appear only in the above combination. On the other hand, Eq. (6) or (7) yields six complex equations, since both sides are complex symmetric matrices. Therefore, Eq. (6) or (7) can, in principle, fix the 12 undetermined variables. As a matter of fact, some undetermined variables are phases and hence it is highly nontrivial that the solution to Eq. (6) or (7) exists. In the rest of the paper, we study whether the solution to Eq. (6) or (7) exists for the normal ($m_3 > m_2 > m_1$) and inverted hierarchies ($m_2 > m_1 > m_3$) of the active neutrino masses, and if it does, we draw a prediction for δ_{CP} , α_2 , α_3 , and the lightest active neutrino mass. We pay attention to the fact that some of the quantities that enter into Eqs. (6)–(7) are subject to sizable experimental errors, which causes ambiguity in the solution. We also note that different solutions may be obtained depending on the scale at which Eq. (4) or (5) holds, due to renormalization group (RG) evolutions of the quark Yukawa couplings and CKM matrix. Therefore, we scrutinize their RG evolutions and how the experimental errors of the quark masses, mixing angles, and Kobayashi-Maskawa phase propagate along the evolutions. In contrast, we directly use the values of neutrino mixing angles and mass differences measured in neutrino oscillation experiments; this is justified by assuming that all of the components of Y_D are much below 1, or equivalently $|z| \ll 1$, so that terms like $Y_D Y_D^\dagger Y_D$ in RG equations are negligible and the RG evolutions change Y_D only by an overall constant that can be absorbed into the number z in Eqs. (4)–(5).

Our calculation of RG evolutions of quark Yukawa couplings and V_{CKM} involves the following steps. All of the renormalization scales are in the $\overline{\text{MS}}$ scheme.

(I) Below a scale $\mu_{\text{EW}} \sim M_Z$, we work in five- or four-flavor QCD $\times$ QED theory [the decoupling of b at $\mu = m_b(m_b)$ is properly taken into account [6]]. We solve QCD

three-loop and QED one-loop RG equations [7] for the QCD coupling α_s in the range $\mu_{\text{EW}} > \mu > 2$ GeV and that for the QED coupling α_{em} in the range $\mu_{\text{EW}} > \mu > 13$ GeV (we ignore QED effects below 13 GeV), with the initial values of $\alpha_s^{(5)}(M_Z) = 0.1182$ and $\alpha(M_Z) = 1/127.950$ quoted from the Particle Data Group [8].

(II) We solve QCD three-loop and QED one-loop RG equations [9] for u , d , s , c , and b quark masses up to $\mu = \mu_{\text{EW}}$. For the average u - d mass and s mass, we quote the results of lattice calculations [10,11]: $\frac{1}{2}(m_u + m_d)(2 \text{ GeV}) = 3.373(80) \text{ MeV}$ and $m_s(2 \text{ GeV}) = 92.0(2.1) \text{ MeV}$. For the u - d mass ratio, we refer to an estimate in Ref. [11]: $m_u/m_d = 0.46(3)$. For the c and b masses, we adopt the results of QCD sum rule analyses [12]: $m_c(3 \text{ GeV}) = 0.986 - 9(\alpha_s^{(5)}(M_Z) - 0.1189)/0.002 \pm 0.010 \text{ GeV}$ and $m_b(m_b) = 4.163 + 7(\alpha_s^{(5)}(M_Z) - 0.1189)/0.002 \pm 0.014 \text{ GeV}$.

(III) We match five-flavor QCD $\times$ QED theory with the full SM at $\mu = \mu_{\text{EW}}$. For the t quark mass, we adopt the pole mass obtained from the exclusive t pair production cross section at the LHC [13] ($M_t = 173.7_{-2.1}^{+2.3} \text{ GeV}$), and for the W , Z , and Higgs boson masses and G_F we use Particle Data Group values [8]. We evaluate QCD two-loop threshold corrections of the t quark on α_s [6] and QED one-loop threshold corrections of the t quark and W boson on α_{em} [14] to obtain QCD and QED gauge couplings in the SM. We employ the results of Refs. [15–19] implemented in the code [20] to compute the t -quark Yukawa coupling $y_t(\mu_{\text{EW}})$ with QCD four-loop and QED two-loop threshold corrections, and to compute the Higgs quartic coupling $\lambda_H(\mu_{\text{EW}})$, running Higgs vacuum expectation value (VEV) $v(\mu_{\text{EW}})$, and running weak mixing angle $\sin^2 \theta_W(\mu_{\text{EW}})$ with QED two-loop and QCD one-loop

threshold corrections. We reconstruct the CKM matrix $\bar{V}_{\text{CKM}}$ from up-to-date values of the Wolfenstein parameters reported by the CKMfitter group [21]: $A=0.8250^{+0.0071}_{-0.0111}$, $\lambda = 0.22509^{+0.00029}_{-0.00028}$, $\bar{\rho} = 0.1598^{+0.0076}_{-0.0072}$ and $\bar{\eta} = 0.3499^{+0.0063}_{-0.0061}$. Finally, we derive the running Yukawa matrices for quarks by neglecting threshold corrections for the CKM matrix as

$$\begin{aligned} y_i(\mu_{\text{EW}}) &= \sqrt{2}m_i(\mu_{\text{EW}})/v(\mu_{\text{EW}}) \quad (i = u, d, s, c, b), \\ Y_d(\mu_{\text{EW}}) &= \bar{V}_{\text{CKM}} \begin{pmatrix} y_d(\mu_{\text{EW}}) & 0 & 0 \\ 0 & y_s(\mu_{\text{EW}}) & 0 \\ 0 & 0 & y_b(\mu_{\text{EW}}) \end{pmatrix}, \\ Y_u(\mu_{\text{EW}}) &= \begin{pmatrix} y_u(\mu_{\text{EW}}) & 0 & 0 \\ 0 & y_c(\mu_{\text{EW}}) & 0 \\ 0 & 0 & y_t(\mu_{\text{EW}}) \end{pmatrix}. \end{aligned} \quad (9)$$

Although it is insignificant in our analysis, we further derive the running Yukawa matrix for charged leptons from the Particle Data Group values of lepton masses, by adding one-loop threshold corrections and then dividing them by the running Higgs VEV $v(\mu_{\text{EW}})$.

(IV) We solve the full three-loop RG equations of the SM [22] in the range $\mu_{\text{EW}} \leq \mu \leq 10^{18}$ GeV.

(V) At various scales μ , we derive the running Yukawa couplings (taken to be real and positive), $y_i(\mu)$ ($i = u, c, t, d, s, b$), and the running CKM matrix $V_{\text{CKM}}(\mu)$ in the following manner. We diagonalize the Yukawa matrices at the scale μ as

$$\begin{aligned} V_{uL}(\mu)Y_u(\mu)Y_u^\dagger(\mu)V_{uL}^\dagger(\mu) &= \begin{pmatrix} y_u(\mu)^2 & 0 & 0 \\ 0 & y_c(\mu)^2 & 0 \\ 0 & 0 & y_t(\mu)^2 \end{pmatrix}, \\ V_{dL}(\mu)Y_d(\mu)Y_d^\dagger(\mu)V_{dL}^\dagger(\mu) &= \begin{pmatrix} y_d(\mu)^2 & 0 & 0 \\ 0 & y_s(\mu)^2 & 0 \\ 0 & 0 & y_b(\mu)^2 \end{pmatrix}, \end{aligned} \quad (10)$$

where $V_{uL}(\mu)$ and $V_{dL}(\mu)$ are unitary matrices depending on μ . Then we calculate $V_{\text{CKM}}(\mu) = V_{uL}(\mu)V_{dL}^\dagger(\mu)$, and further decompose it into three physical mixing angles $\theta_{ij}^{ckm}(\mu)$ and one CP phase $\delta_{\text{km}}(\mu)$ as

$$\begin{aligned} \sin \theta_{13}^{ckm}(\mu) &= |V_{ub}(\mu)|, \quad \sin \theta_{12}^{ckm}(\mu) = \frac{|V_{us}(\mu)|}{\sqrt{1 - |V_{ub}(\mu)|^2}}, \quad \sin \theta_{23}^{ckm}(\mu) = \frac{|V_{cb}(\mu)|}{\sqrt{1 - |V_{ub}(\mu)|^2}}, \\ e^{i\delta_{\text{km}}(\mu)} &= \left(\frac{V_{us}V_{cb}V_{ub}^*V_{cs}^*}{|V_{ub}||V_{us}||V_{cb}|} + \frac{|V_{ub}||V_{us}||V_{cb}|}{1 - |V_{ub}|^2} \right) \frac{1 - |V_{ub}|^2}{\sqrt{1 - |V_{ub}|^2 - |V_{us}|^2}\sqrt{1 - |V_{ub}|^2 - |V_{cb}|^2}}. \end{aligned} \quad (11)$$

We estimate the uncertainties of the Yukawa couplings and CKM matrix at each scale μ as follows:

- (1) For each running Yukawa coupling $y_i(\mu)$ ($i = u, c, t, d, s, b$), we consider the propagation of the experimental error of its corresponding mass *only* and estimate its uncertainty $\Delta y_i(\mu)$ as

$$\begin{aligned} (\Delta y_i(\mu))^2 &= \left(\frac{\partial y_i(\mu)}{\partial (m_u + m_d)(2 \text{ GeV})} \right)^2 \Delta(m_u + m_d)(2 \text{ GeV})^2 + \left(\frac{\partial y_i(\mu)}{\partial m_u/m_d} \right)^2 \Delta(m_u/m_d)^2 (i = u, d), \\ \Delta y_s(\mu) &= \left| \frac{\partial y_s(\mu)}{\partial m_s(2 \text{ GeV})} \right| \Delta m_s(2 \text{ GeV}), \quad \Delta y_c(\mu) = \left| \frac{\partial y_c(\mu)}{\partial m_c(3 \text{ GeV})} \right| \Delta m_c(3 \text{ GeV}), \\ \Delta y_b(\mu) &= \left| \frac{\partial y_b(\mu)}{\partial m_b(m_b)} \right| \Delta m_b(m_b), \quad \Delta y_t(\mu) = \left| \frac{\partial y_t(\mu)}{\partial M_t} \right| \Delta M_t, \end{aligned} \quad (12)$$

where we take $\Delta M_t = 2.3$ GeV.

- (2) For the running CKM mixing angles $\theta_{ij}^{ckm}(\mu)$ and CP phase $\delta_{\text{km}}(\mu)$, we estimate their uncertainties, $\Delta \theta_{ij}(\mu)$ and $\Delta \delta_{\text{km}}(\mu)$, by assuming that experimental errors of the Wolfenstein parameters are maximally correlated, and thereby linearly adding errors propagating from these experimental errors as

$$\Delta \theta_{ij}(\mu) = \sum_{Q=A, \lambda, \bar{\rho}, \bar{\eta}} \left| \frac{\partial \theta_{ij}^{ckm}(\mu)}{\partial Q} \right| \Delta Q, \quad \Delta \delta_{\text{km}}(\mu) = \sum_{Q=A, \lambda, \bar{\rho}, \bar{\eta}} \left| \frac{\partial \delta_{\text{km}}(\mu)}{\partial Q} \right| \Delta Q, \quad (13)$$

where we take $\Delta A = 0.0111$, $\Delta \lambda = 0.00029$, $\bar{\rho} = 0.0076$, and $\bar{\eta} = 0.0063$.

TABLE I. Running Yukawa couplings, CKM mixing angles and phase, and their uncertainties at various scales μ in the $\overline{\text{MS}}$ scheme, evaluated by taking the matching scale as $\mu_{\text{EW}} = M_Z$. For each Yukawa coupling y_i , the uncertainty is estimated by considering the propagation of the experimental error of its corresponding mass only [for u and d quarks, we consider both errors of $(m_u + m_d)(2 \text{ GeV})$ and m_u/m_d]. For the CKM mixing angles and phase, we assume maximal correlation among the errors of the Wolfenstein parameters, thereby linearly adding uncertainties that propagate from the experimental errors, as in Eq. (13).

scale μ	10^4 GeV	10^6 GeV	10^{12} GeV	10^{15} GeV	10^{18} GeV
$10^6 y_u$	5.43(27)	4.56(23)	3.22(16)	2.83(14)	2.53(13)
$10^3 y_c$	2.794(28)	2.347(24)	1.657(17)	1.457(15)	1.301(13)
y_t	0.777(13)	0.668(13)	0.493(13)	0.440(12)	0.398(12)
$10^6 y_d$	11.89(37)	10.02(31)	7.17(22)	6.36(20)	5.72(18)
$10^3 y_s$	0.2366(54)	0.1995(46)	0.1428(33)	0.1265(29)	0.1140(26)
y_b	0.012 195(43)	0.010 042(35)	0.006 879(24)	0.006 008(21)	0.005 349(19)
$\theta_{12}^{ckm} \text{ (rad)}$	0.22 705(30)	0.22 705(30)	0.22 707(30)	0.22 707(30)	0.22 707(30)
$10^3 \theta_{13}^{ckm} \text{ (rad)}$	3.85(16)	3.94(16)	4.12(17)	4.18(17)	4.22(17)
$10^2 \theta_{23}^{ckm} \text{ (rad)}$	4.332(69)	4.436(71)	4.635(74)	4.703(75)	4.758(76)
$\delta_{\text{km}}/(2\pi)$	0.1819(39)	0.1819(39)	0.1819(39)	0.1819(39)	0.1819(39)

In Table I, we present the running quark Yukawa couplings, CKM mixing angles and phase, and their uncertainties at the scales $\mu = 10^4, 10^6, 10^{12}, 10^{15}$, and 10^{18} GeV in the $\overline{\text{MS}}$ scheme, evaluated by taking the matching scale as $\mu_{\text{EW}} = M_Z$. In Table II, we present the results obtained by taking $\mu_{\text{EW}} = 160 \text{ GeV}$ instead. The sizable discrepancy in the Yukawa couplings for $\mu_{\text{EW}} = M_Z$ and $\mu_{\text{EW}} = 160 \text{ GeV}$ is due to the keen μ dependence of the running Higgs VEV $v(\mu)$, which originates from tadpole contributions to $v(\mu)$ [23] that are enhanced by $(N_c M_t^4 / (M_W^2 M_h^2))^l$ as the number of loops l increases.

For the neutrino mixing angles and mass differences, we employ Particle Data Group values [8]: $\sin^2 \theta_{12} = 0.307 \pm 0.013$, $\sin^2 \theta_{13} = 0.0210 \pm 0.0011$, $\Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$, $\sin^2 \theta_{23} = 0.51 \pm 0.04$ (normal hierarchy), $\sin^2 \theta_{23} = 0.50 \pm 0.04$ (inverted hierarchy), $|\Delta m_{32}^2| = (2.45 \pm 0.05) \times 10^{-3} \text{ eV}^2$ (normal hierarchy), $|\Delta m_{32}^2| = (2.52 \pm 0.05) \times 10^{-3} \text{ eV}^2$ (inverted hierarchy). We neglect

RG evolutions of Y_D and directly use the above experimental values in our analysis.

Using the running quark Yukawa couplings, running CKM matrix, neutrino mass differences, and mixing angles obtained above, we search for the solution to Eq. (6) or (7), and further estimate its uncertainty that stems from experimental errors of the quark masses, Wolfenstein parameters, and neutrino oscillation parameters. We test four cases:

- (A) Equation (4) holds and the neutrino mass hierarchy is normal.
- (B) Equation (4) holds and the neutrino mass hierarchy is inverted.
- (C) Equation (5) holds and the neutrino mass hierarchy is normal.
- (D) Equation (5) holds and the neutrino mass hierarchy is inverted.

For Case (A), we search for a solution by reformulating Eq. (6) into the equation below:

TABLE II. Same as Table I, except that we take the matching scale as $\mu_{\text{EW}} = 160 \text{ GeV}$.

scale μ	10^4 GeV	10^6 GeV	10^{12} GeV	10^{15} GeV	10^{18} GeV
$10^6 y_u$	5.11(26)	4.29(22)	3.03(15)	2.67(13)	2.38(12)
$10^3 y_c$	2.626(27)	2.206(22)	1.558(16)	1.370(14)	1.224(12)
y_t	0.778(13)	0.669(13)	0.494(13)	0.441(13)	0.399(12)
$10^6 y_d$	11.17(35)	9.42(30)	6.75(21)	5.98(19)	5.39(17)
$10^3 y_s$	0.2225(51)	0.1876(43)	0.1343(31)	0.1190(27)	0.1072(24)
y_b	0.011 525(37)	0.009 491(31)	0.006 504(21)	0.005 681(18)	0.005 058(16)
$\theta_{12}^{ckm} \text{ (rad)}$	0.22 704(30)	0.22 705(30)	0.22 706(30)	0.22 707(30)	0.22 707(30)
$10^3 \theta_{13}^{ckm} \text{ (rad)}$	3.83(16)	3.92(16)	4.09(17)	4.15(17)	4.20(17)
$10^2 \theta_{23}^{ckm} \text{ (rad)}$	4.310(69)	4.413(71)	4.612(74)	4.680(75)	4.735(76)
$\delta_{\text{km}}/(2\pi)$	0.1819(39)	0.1819(39)	0.1819(39)	0.1819(39)	0.1819(39)

$$\begin{aligned}
 & \begin{pmatrix} 1/y_d & 0 & 0 \\ 0 & 1/y_s & 0 \\ 0 & 0 & 1/y_b \end{pmatrix} U_{\text{PMNS}}^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\text{PMNS}}^\dagger \begin{pmatrix} 1/y_d^2 & 0 & 0 \\ 0 & 1/y_s^2 & 0 \\ 0 & 0 & 1/y_b^2 \end{pmatrix} U_{\text{PMNS}} \\
 & \times \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\text{PMNS}}^T \begin{pmatrix} 1/y_d & 0 & 0 \\ 0 & 1/y_s & 0 \\ 0 & 0 & 1/y_b \end{pmatrix} \\
 & = |z|^2 \frac{v^4}{4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi_2} & 0 \\ 0 & 0 & e^{-i\phi_3} \end{pmatrix} V_{\text{CKM}}^\dagger \begin{pmatrix} \frac{1}{M_{N_1}^2} & 0 & 0 \\ 0 & \frac{1}{M_{N_2}^2} & 0 \\ 0 & 0 & \frac{1}{M_{N_3}^2} \end{pmatrix} V_{\text{CKM}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}. \quad (14)
 \end{aligned}$$

Then, we take the following steps [the procedures are analogous for Cases (B), (C), and (D)]:

FIG. 1. Each dot represents a solution to Eq. (6) that fits within the 2σ range of $\theta_{12}^{\text{ckm}}, \theta_{13}^{\text{ckm}}, \theta_{23}^{\text{ckm}}, \delta_{\text{km}}$ at the scale $\mu = 10^{18}$ GeV evaluated by taking $\mu_{\text{EW}} = M_Z$. These solutions are obtained from 1.6×10^{10} random sets of values of $(\delta_{\text{CP}}, \alpha_2, \alpha_3, \log(m_1))$ in the range (16). The input values of y_d, y_s, y_b and $\theta_{12}, \theta_{13}, \theta_{23}, |\Delta m_{32}^2|, \Delta m_{21}^2$ are as shown in Table III, with the left plots corresponding to $\sin^2 \theta_{23} = 0.43$ and the right plots to $\sin^2 \theta_{23} = 0.47$. The plots in the first row are in the $(\theta_{13,\text{test}}^{\text{ckm}}, \delta_{\text{CP}})$ plane, those in the second row with black dots are in the $(\delta_{\text{CP}}, \alpha_2)$ plane, those in the second row with red dots are in the $(\delta_{\text{CP}}, \alpha_3)$ plane, those in the third row with black dots are in the $(\delta_{\text{CP}}/(2\pi), m_1)$ plane, and those in the third row with red dots are in the $(\delta_{\text{CP}}/(2\pi), m_{ee})$ plane.

- (i) We select a set of “input values” of y_d, y_s, y_b and $\theta_{12}, \theta_{13}, \theta_{23}, |\Delta m_{32}^2|, \Delta m_{21}^2$ from the 2σ range. For y_d, y_s, y_b , we quote the values at $\mu = 10^{18}$ GeV evaluated by taking $\mu_{EW} = M_Z$ in Table I. Namely, we take

$$\begin{aligned} y_d &\in [5.36, 6.08] \times 10^{-6}, \\ y_s &\in [1.088, 1.192] \times 10^{-4}, \\ y_b &\in [5.293, 5.405] \times 10^{-3}, \\ \sin^2 \theta_{12} &\in [0.281, 0.333], \\ \sin^2 \theta_{13} &\in [0.0188, 0.0232], \\ \sin^2 \theta_{23} &\in [0.43, 0.59], \\ \Delta m_{21}^2 &\in [7.17, 7.89] \times 10^{-5} \text{ eV}^2, \\ |\Delta m_{32}^2| &\in [2.35, 2.55] \times 10^{-3} \text{ eV}^2. \end{aligned} \quad (15)$$

- (ii) We randomly generate a set of “trial values” of the neutrino Dirac CP phase and Majorana CP phases and the logarithm of the lightest neutrino mass, $(\delta_{CP}, \alpha_2, \alpha_3, \log(m_1))$, which vary in the following ranges:

$$\begin{aligned} 2\pi > \delta_{CP} &\geq 0, \quad 2\pi > \alpha_2 \geq 0, \\ 2\pi > \alpha_3 &\geq 0, \quad -1 > \log_{10}(m_1/\text{eV}) > -3, \end{aligned} \quad (16)$$

where the maximum of m_1 corresponds to the quasidegenerate case with $m_1 \simeq m_2 \simeq m_3$, and the minimum corresponds to the case in which the lightest neutrino mass is negligible, $m_1 \ll m_2$. [For Cases (B) and (D), m_1 should be replaced with m_3 .]

- (iii) We insert the above “input values” and “trial values” into the left-hand side of Eq. (14), numerically diagonalize the left-hand side with a unitary matrix V_{test} , and decompose V_{test} into three mixing angles $\theta_{12,\text{test}}^{ckm}, \theta_{13,\text{test}}^{ckm}, \theta_{23,\text{test}}^{ckm}$ and one CP phase $\delta_{km,\text{test}}$ in a way analogous to Eq. (11). When decomposing V_{test} , we rearrange the rows of V_{test} so that $\theta_{12,\text{test}} > \theta_{23,\text{test}} > \theta_{13,\text{test}}^{ckm}$ holds, which is justifiable because the ordering of $M_{N_1}, M_{N_2}, M_{N_3}$ is arbitrary. We study whether the mixing angles $\theta_{12,\text{test}}^{ckm}, \theta_{13,\text{test}}^{ckm}, \theta_{23,\text{test}}^{ckm}$ and CP phase $\delta_{km,\text{test}}$ fit within the 2σ range of $\theta_{12}^{ckm}, \theta_{13}^{ckm}, \theta_{23}^{ckm}, \delta_{km}$ at $\mu = 10^{18}$ GeV evaluated with $\mu_{EW} = M_Z$ (Table I); namely, we check

FIG. 2. Same as Fig. 1, except that the different input values as shown in Table III are used, with the left plots corresponding to $\sin^2 \theta_{12} = 0.281$ and the right plots to $\sin^2 \theta_{12} = 0.333$.

$$\begin{aligned}
 0.22767 &\geq \theta_{12,\text{test}}^{ckm} \geq 0.22647, \\
 4.59 \times 10^{-3} &\geq \theta_{13,\text{test}}^{ckm} \geq 3.88 \times 10^{-3}, \\
 4.910 \times 10^{-2} &\geq \theta_{23,\text{test}}^{ckm} \geq 4.606 \times 10^{-2}, \\
 0.1897 &\geq \delta_{km,\text{test}} \geq 0.1741.
 \end{aligned} \tag{17}$$

If the above inequalities all hold, the corresponding set of “trial values” $(\delta_{CP}, \alpha_2, \alpha_3, \log(m_1))$ is regarded as a solution to Eq. (6).

The results are as follows.

- (1) In Cases (B), (C), and (D), we have generated 1.6×10^{10} random sets of “trial values” of $(\delta_{CP}, \alpha_2, \alpha_3, \log(m_1))$ for each set of “input values” of y_d, y_s, y_b and $\theta_{12}, \theta_{13}, \theta_{23}, |\Delta m_{32}^2|, \Delta m_{21}^2$, and found no solution to Eq. (6) or (7). There is no solution even when we loosen the criteria of (iii) and allow $\theta_{12,\text{test}}^{ckm}, \theta_{13,\text{test}}^{ckm}, \theta_{23,\text{test}}^{ckm}, \delta_{km,\text{test}}$ to fit within the 5σ range.
- (2) In Case (A), we have generated 1.6×10^{10} random sets of “trial values” of $(\delta_{CP}, \alpha_2, \alpha_3, \log(m_1))$ for

each set of “input values,” and found solutions to Eq. (6) for $\sin^2 \theta_{23} = 0.43, 0.47$, whereas no solution is found for $\sin^2 \theta_{23} = 0.51, 0.55, 0.59$. The value of δ_{CP} in the solutions exhibits a correlation with $\theta_{13,\text{test}}^{ckm}$, so we plot the solutions in the $(\theta_{13,\text{test}}^{ckm}, \delta_{CP})$ plane. Since the values of α_2, α_3, m_1 in the solutions are strongly correlated with δ_{CP} , we further plot the solutions in the $(\delta_{CP}, \alpha_2), (\delta_{CP}, \alpha_3)$, and (δ_{CP}, m_1) planes. Additionally, for individual solutions we calculate m_{ee} (the quantity measured in neutrinoless double β -decay experiments) as

$$m_{ee} = \left| \left[U_{\text{PMNS}}^* \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{\text{PMNS}}^\dagger \right]_{ee} \right|, \tag{18}$$

and plot the solutions in the (δ_{CP}, m_{ee}) plane. The results are displayed in Figs. 1–6, whose corresponding “input values” are listed in Table III. Note

FIG. 3. Same as Fig. 1, except that different input values as shown in Table III are used, with the left plots corresponding to $\sin^2 \theta_{13} = 0.0188$ and the right plots to $\sin^2 \theta_{13} = 0.0232$.

FIG. 4. Same as Fig. 1, except that different input values as shown in Table III are used, with the left plots corresponding to $y_d = 5.36 \times 10^{-6}$ and the right plots to $y_d = 6.08 \times 10^{-6}$.

that the scattering of dots in each figure represents the uncertainty of the solution due to the uncertainties of θ_{12}^{ckm} , θ_{13}^{ckm} , θ_{23}^{ckm} , and δ_{km} .

- (3) We have varied the values of Δm_{12}^2 and $|\Delta m_{23}^2|$ within the 2σ experimental range, and found no significant change in the plots.
- (4) In Fig. 7, we have used the values of y_d , y_s , y_b and θ_{12}^{ckm} , θ_{13}^{ckm} , θ_{23}^{ckm} , δ_{km} at a different renormalization scale $\mu = 10^4$ GeV (evaluated by taking $\mu_{EW} = M_Z$ and shown in Table I) in Eqs. (15) and (17). The neutrino oscillation parameters are identical to those for the right plots of Fig. 1.
- (5) To study the dependence of the results on the matching scale μ_{EW} , we have used the values of y_d , y_s , y_b and θ_{12}^{ckm} , θ_{13}^{ckm} , θ_{23}^{ckm} , δ_{km} at $\mu = 10^{18}$ GeV evaluated by taking $\mu_{EW} = 160$ GeV, shown in Table II. No significant difference is observed in the plots for $\mu_{EW} = M_Z$ and $\mu_{EW} = 160$ GeV.

From the above results, the following observations are made.

- (1) Only $Y_D \propto \text{diag}(y_d, y_s, y_b) V_{CKM}^T$ [Eq. (4)] can be consistent with the experimental data, and this is the case exclusively with the normal hierarchy of neutrino masses and for smaller values of $\sin^2 \theta_{23}$ in the current bound.
- (2) In most cases, the value of δ_{CP} satisfying Eq. (4) is in the range $\pi > \delta_{CP} > 0$ and hence is incompatible with the value hinted by the T2K Collaboration, $\delta_{CP} \sim 3\pi/2$ [24]. Nevertheless, for cases with $\sin^2 \theta_{12} = 0.333$, $y_d = 5.36 \times 10^{-6}$, and $y_s = 1.192 \times 10^{-4}$, there exist solutions with $\delta_{CP} \simeq 1.2\pi$. Precise measurements of θ_{12} and improved evaluations of the s quark mass m_s and u - d quark mass ratio m_u/m_d [the average $(m_u + m_d)/2$ is well calculated compared to m_u/m_d] are required to falsify or corroborate our hypothesis.
- (3) If we associate operators with opposite chiralities and consider the different hypothesis

FIG. 5. Same as Fig. 1, except that different input values as shown in Table III are used, with the left plots corresponding to $y_s = 1.088 \times 10^{-4}$ and the right plots to $y_s = 1.192 \times 10^{-4}$.

$$Y_D = z \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix} \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} V_{\text{CKM}}^\dagger \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\psi_2} & 0 \\ 0 & 0 & e^{i\psi_3} \end{pmatrix}, \quad (19)$$

then the signs of $\delta_{CP}, \alpha_2, \alpha_3$ in Figs. 1–6 are simply flipped. This hypothesis is in good agreement with the T2K data on δ_{CP} .

- (4) The values of α_2, α_3, m_1 satisfying Eq. (4) are strongly correlated with δ_{CP} . For $\delta_{CP} \simeq 1.2\pi$, m_1 is predicted to be about 0.005 eV, which may be tested in future cosmological observations (for forecasts, see, e.g., Ref. [25]). m_{ee} is suppressed below 0.002 eV due to cancellation of the active neutrino masses, and thus there is absolutely no chance to detect neutrinoless double β -decay in the near future [26].

- (5) The pattern of the correlation between δ_{CP} and $\theta_{13, \text{test}}^{\text{CKM}}$ is similar for the cases with $\mu = 10^4$ GeV and $\mu = 10^{18}$ GeV. Since the running mixing angle θ_{13}^{CKM} depends linearly on the measured value of $|V_{ub}|$, we conclude that the correlation between δ_{CP} and $|V_{ub}|$ is nearly the same for $\mu = 10^4$ GeV and $\mu = 10^{18}$ GeV. The plots for $\alpha_2, \alpha_3, m_1, m_{ee}$ are likewise the same for $\mu = 10^4$ GeV and $\mu = 10^{18}$ GeV. Because the two distinctively different assumptions on the scale at which Eq. (4) holds lead to similar results, we infer that our prediction is almost independent of the scale of Eq. (4).

FIG. 6. Same as Fig. 1, except that different input values as shown in Table III are used, with the left plots corresponding to $y_b = 5.293 \times 10^{-3}$ and the right plots to $y_b = 5.405 \times 10^{-3}$.

- (6) We have confirmed that the above results are insensitive to the choice of the matching scale μ_{EW} , which is reasonable because the ratio $y_d:y_s:y_b$ and the CKM mixing angles are intact with the change of μ_{EW} , as read from Tables I and II.

To summarize, we have investigated whether the relation $Y_D \propto \text{diag}(y_d, y_s, y_b)V_{\text{CKM}}^T$ or $Y_D \propto \text{diag}(y_u, y_c, y_t)V_{\text{CKM}}^*$ (in the flavor basis where the charged lepton Yukawa coupling and right-handed neutrino Majorana mass are diagonal) can be consistent with the current experimental data on the quark masses, CKM mixing

TABLE III. Input values of y_d, y_s, y_b and $\theta_{12}, \theta_{13}, \theta_{23}, |\Delta m_{32}^2|, \Delta m_{21}^2$ which correspond to Figs. 1–6.

Input values	Fig. 1	Fig. 2	Fig. 3	Fig. 4	Fig. 5	Fig. 6
$10^6 y_d$	5.72	5.72	5.72	5.36 or 6.08	5.72	5.72
$10^4 y_s$	1.140	1.140	1.140	1.140	1.088 or 1.192	1.140
$10^3 y_b$	5.349	5.349	5.349	5.349	5.349	5.293 or 5.405
$\sin^2 \theta_{12}$	0.307	0.281 or 0.333	0.307	0.307	0.307	0.307
$10^2 \sin^2 \theta_{13}$	2.10	2.10	1.88 or 2.32	2.10	2.10	2.10
$\sin^2 \theta_{23}$	0.43 or 0.47	0.47	0.47	0.47	0.47	0.47
$10^5 \Delta m_{21}^2 / \text{eV}^2$	7.53	7.53	7.53	7.53	7.53	7.53
$10^3 \Delta m_{32}^2 / \text{eV}^2$	2.45	2.45	2.45	2.45	2.45	2.45

FIG. 7. Same as the right plots of Fig. 1, except that the values of y_d, y_s, y_b and $\theta_{12}^{ckm}, \theta_{13}^{ckm}, \theta_{23}^{ckm}, \delta_{km}$ at $\mu = 10^4$ GeV (shown in Table I) are used in the analysis. The same values for the neutrino oscillation parameters as in the right plots of Fig. 1 are employed.

angles and phase, and neutrino mixing angles, and thus hopefully unveil an implicit connection between the PMNS and CKM matrices. We have found sets of values of $(\delta_{CP}, \alpha_2, \alpha_3, m_1)$ that satisfy $Y_D \propto \text{diag}(y_d, y_s, y_b)V_{CKM}^T$ with the normal neutrino mass hierarchy, while there are no such sets for $Y_D \propto \text{diag}(y_u, y_c, y_t)V_{CKM}^*$ and/or with the inverted hierarchy. δ_{CP} is predicted to be in the range $1.2\pi \gtrsim \delta_{CP} > 0$ and is hence in tension with the latest T2K data. However, since the prediction crucially depends on the neutrino mixing angles and d and s quark masses, their future precise measurement or evaluation is necessary to draw any conclusion about our hypothesis. We have made a prediction for m_1 that may be tested in future cosmological observations, whereas m_{ee} is smaller than 0.002 eV and is far below the reach of near-future experiments.

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