Production and sequential decay of charmed hyperons

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We investigate production and decay of the Λ_c^+ hyperon. The production considered is through the e^+e^- annihilation channel, $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$, with summation over the $\bar{\Lambda}_c^-$ antihyperon spin directions. It is in this situation that the Λ_c^+ decay chain is identified. Two kinds of sequential decays are studied. The first one is the doubly weak decay $B_1 \rightarrow B_2 M_2$, followed by $B_2 \rightarrow B_3 M_3$. The other one is the mixed weak-electromagnetic decay $B_1 \rightarrow B_2 M_2$, followed by $B_2 \rightarrow B_3 \gamma$. In both schemes B denotes baryons and M mesons. We should also mention that the initial state of the Λ_c^+ hyperon is polarized.

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I. INTRODUCTION

We shall investigate properties of certain sequential decays of the Λ_c^+ hyperon, but in order to do so we first need to produce them. To this end we consider the reaction $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$, which is analyzed in detail in Refs. [1,2]. In order to describe such an annihilation process two hadronic form factors are needed, commonly denoted G_E and G_M . They can be parametrized by two parameters, α and $\Delta\Phi$, with $-1 \leq \alpha \leq 1$. For their precise definitions we refer to Ref. [2]. At the $\Lambda_c \bar{\Lambda}_c$ threshold $G_E = G_M$ leading to a vanishing $\Delta\Phi$.

The general cross-section distribution of this annihilation reaction depends on six structure functions which themselves are functions of α , $\Delta\Phi$, and θ , the scattering angle. In our application, however, we sum over the decay products of the antihyperon $\bar{\Lambda}_c^-$, but identify the decay chain of the hyperon Λ_c^+ , so called single tag events. In this simplified case only two structure functions are relevant,

$$\mathcal{R} = 1 + \alpha \cos^2 \theta, \quad (1.1)$$

$$\mathcal{S} = \sqrt{1 - \alpha^2} \sin \theta \cos \theta \sin(\Delta\Phi). \quad (1.2)$$

The scattering distribution function for the Λ_c^+ hyperon production becomes, according to Refs. [1,2], proportional to

$$W(\mathbf{n}) = \mathcal{R} + \mathcal{S} \mathbf{N} \cdot \mathbf{n}, \quad (1.3)$$

where $\mathbf{n}$ is the direction of the hyperon spin vector in the hyperon rest system, $\mathbf{N}$ the normal to the scattering plane,

$$\mathbf{N} = \frac{1}{\sin \theta} \hat{\mathbf{p}} \times \hat{\mathbf{k}}, \quad (1.4)$$

and $\cos \theta = \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}$. The momenta $\mathbf{k}$ and $\mathbf{p}$ are the relative momenta in the initial and final states in the c.m. (center of momentum) system, i.e.,

$$\mathbf{k} = \mathbf{k}_1 = -\mathbf{k}_2 \quad (1.5)$$

$$\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2. \quad (1.6)$$

The spin four-vector s of a particle of four-momentum p satisfies $s \cdot p = 0$ [3], and in the particle rest system it simplifies to $s = (0, \mathbf{n})$, with the three-vector $\mathbf{n}$ a unit vector, $\mathbf{n} \cdot \mathbf{n} = 1$.

From Eq. (1.3) we deduce for the spin-density distribution function,

$$S(\mathbf{P}) = 1 + \mathbf{P} \cdot \mathbf{n} \quad (1.7)$$

and $\mathbf{P}$ the hyperon polarization,

$$\mathbf{P} = (\mathcal{S}/\mathcal{R})\mathbf{N}, \quad (1.8)$$

subject to the restriction $|\mathbf{P}| \leq 1$. At threshold, $\Delta\Phi$ vanishes and so does $\mathcal{S}$ and the polarization $\mathbf{P}$. For an unpolarized initial-state hyperon $\mathbf{P} = 0$.

II. WEAK HYPERON DECAYS

The weak hyperon decay $c \rightarrow d\pi$, of which $\Lambda \rightarrow p\pi^-$ is an example, is described by two amplitudes, one S-wave and one P-wave amplitude. The spin-density distribution function of the decay is commonly characterized by three

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parameters, denoted $\alpha\beta\gamma$. They are not independent but fulfill the relation

$$\alpha^2 + \beta^2 + \gamma^2 = 1. \quad (2.1)$$

This parametrization is discussed in detail in Ref. [4] and also in Ref. [1].

We denote by $G_c(c, d)$ the joint spin-density distribution function for the weak hyperon decay $c \rightarrow d\pi$, given the spin vectors $\mathbf{n}_c$ and $\mathbf{n}_d$,

$$G_c(c, d) = 1 + \alpha_c \mathbf{n}_c \cdot \mathbf{l}_d + \alpha_c \mathbf{n}_d \cdot \mathbf{l}_d + \mathbf{n}_c \cdot \mathbf{L}_c(\mathbf{n}_d, \mathbf{l}_d), \quad (2.2)$$

with

$$\mathbf{L}_c(\mathbf{n}_d, \mathbf{l}_d) = \gamma_c \mathbf{n}_d + [(1 - \gamma_c) \mathbf{n}_d \cdot \mathbf{l}_d] \mathbf{l}_d + \beta_c \mathbf{n}_d \times \mathbf{l}_d. \quad (2.3)$$

The vector $\mathbf{l}_d$ is a unit vector in the direction of motion of the decay baryon d in the rest system of baryon c . The indices on the $\alpha\beta\gamma$ parameters remind us they characterize hyperon c . We repeat that $\mathbf{n}_c$ and $\mathbf{n}_d$ are the directions of the spin vectors of baryons c and d in their respective rest systems.

Since the spin of baryon d is often not measured, the relevant spin-density distribution function of hyperon c is obtained by averaging over the spin directions $\mathbf{n}_d$,

$$W_c(\mathbf{n}_c; \mathbf{l}_d) = \langle G_c(c, d) \rangle_d = U_c + \mathbf{n}_c \cdot \mathbf{V}_c, \quad (2.4)$$

with

$$U_c = 1, \quad \mathbf{V}_c = \alpha_c \mathbf{l}_d. \quad (2.5)$$

The notation $\langle -- \rangle_d$ is a shorthand notation for average over the directions of the spin vector $\mathbf{n}_d$, following the prescription of Ref. [1],

$$\langle 1 \rangle_{\mathbf{n}} = 1, \quad \langle \mathbf{n} \rangle_{\mathbf{n}} = 0, \quad \langle \mathbf{n} \cdot \mathbf{k} \mathbf{n} \cdot \mathbf{l} \rangle_{\mathbf{n}} = \mathbf{k} \cdot \mathbf{l}. \quad (2.6)$$

Hyperons we study are produced in some reaction, and their states are described by some spin-density distribution function, Eq. (1.7),

$$S_c(\mathbf{P}_c) = 1 + \mathbf{P}_c \cdot \mathbf{n}_c. \quad (2.7)$$

The spin-density distribution function for the production of a hyperon followed by its decay is obtained by a contraction of the products of the spin-densities for the production and decay steps. The contraction involves averages over initial and final hyperon spin directions $\mathbf{n}_c$ and $\mathbf{n}_d$,

$$W_c(\mathbf{P}_c; \mathbf{l}_d) = \langle S_c(\mathbf{P}_c) G_c(c, d) \rangle_{cd} = 1 + \mathbf{P}_c \cdot \mathbf{V}_c, \quad (2.8)$$

where $\mathbf{V}_c = \alpha_c \mathbf{l}_d$, from Eq. (2.5). The cd double index in Eq. (2.8) indicates averages over both $\mathbf{n}_c$ and $\mathbf{n}_d$ according to the prescription (2.6).

From Eq. (2.8) it is clear that if the polarization is known the asymmetry parameter α_c can be measured, but not the β_c or γ_c parameters. For that to be possible we must measure the polarization of the decay baryon d . If hyperon c is produced within a $c\bar{c}$ pair in e^+e^- annihilation then its polarization can be determined from the cross-section distribution.

III. ELECTROMAGNETIC HYPERON TRANSITIONS

Electromagnetic transitions such as $\Sigma^0 \rightarrow \Lambda\gamma$ and $\Xi^0 \rightarrow \Lambda\gamma$ can also be studied in Λ_c^+ decays.

An electromagnetic transition $c \rightarrow d\gamma$ is described by a spin-density distribution function similar to that of the weak decay, Eq. (2.2). However, the special feature of the electromagnetic interaction is the photon helicity which can take only two values, $\lambda_\gamma = \pm 1$.

The electromagnetic transition distribution function corresponding to Eq. (2.2) is

$$G_\gamma(cd; \lambda_\gamma) = (1 - \mathbf{n}_c \cdot \mathbf{l}_d \mathbf{l}_d \cdot \mathbf{n}_d) - \lambda_\gamma (\mathbf{n}_c \cdot \mathbf{l}_d - \mathbf{n}_d \cdot \mathbf{l}_d), \quad (3.1)$$

where $\mathbf{l}_d$ is a unit vector in the direction of motion of hyperon d in the rest system of hyperon c .

Averaging over photon polarizations the transition distribution takes a simpler form,

$$G_\gamma(c, d) = 1 - \mathbf{n}_c \cdot \mathbf{l}_d \mathbf{l}_d \cdot \mathbf{n}_d. \quad (3.2)$$

We notice that when both hadron spins are parallel or antiparallel to the photon momentum, then the transition probability vanishes, a property of angular-momentum conservation. We also notice that expression (3.2) cannot be written in the $\alpha\beta\gamma$ representation of Eq. (2.2).

IV. TWO-STEP WEAK HYPERON DECAY

Now, we apply the above technique to hyperons decaying in two steps, such as $b \rightarrow c \rightarrow d$, accompanied by pions. An example of this decay mode is $\Lambda_c^+ \rightarrow \Lambda\pi^+$ followed by $\Lambda \rightarrow p\pi^-$.

We denote by $G_b(b, c)$ the spin-density distribution function describing the hyperon decay $b \rightarrow c\pi$ pertaining to spin vectors $\mathbf{n}_b$ and $\mathbf{n}_c$,

$$G_b(b, c) = 1 + \alpha_b \mathbf{n}_b \cdot \mathbf{l}_c + \alpha_b \mathbf{n}_c \cdot \mathbf{l}_c + \mathbf{n}_b \cdot \mathbf{L}_b(\mathbf{n}_c, \mathbf{l}_c), \quad (4.1)$$

with

$$\mathbf{L}_b(\mathbf{n}_c, \mathbf{l}_c) = \gamma_b \mathbf{n}_c + [(1 - \gamma_b) \mathbf{n}_c \cdot \mathbf{l}_c] \mathbf{l}_c + \beta_b \mathbf{n}_c \times \mathbf{l}_c. \quad (4.2)$$

The vector $\mathbf{l}_c$ is a unit vector in the direction of motion of baryon c in the rest system of baryon b .

By folding the two spin-density distribution functions $G_b(b, c)$ and $G_c(c, d)$, i.e., averaging their product over the spin vectors $\mathbf{n}_c$ and $\mathbf{n}_d$ according to prescription (2.6), we get the decay-density distribution function

$$W_b(\mathbf{n}_b; \mathbf{l}_c, \mathbf{l}_d) = \langle G_b(b, c) G_c(c, d) \rangle_{cd} = U_b + \mathbf{n}_b \cdot \mathbf{V}_b, \quad (4.3)$$

with

$$U_b = 1 + \alpha_b \alpha_c \mathbf{l}_c \cdot \mathbf{l}_d, \quad (4.4)$$

$$\mathbf{V}_b = \alpha_b \mathbf{l}_c + \alpha_c \mathbf{L}_b(\mathbf{l}_d, \mathbf{l}_c). \quad (4.5)$$

The result is interesting. In many cases the asymmetry parameter α_c for the c hyperon and the polarization $\mathbf{P}_b$ for the initial-state b hyperon are known. Then, just as in the single-step case of Eq. (2.7), the initial state is described by a spin-density distribution function

$$S_b(\mathbf{P}_b) = 1 + \mathbf{P}_b \cdot \mathbf{n}_b. \quad (4.6)$$

For the decay distribution of a polarized hyperon, we obtain

$$W_b(\mathbf{P}_b; \mathbf{l}_c, \mathbf{l}_d) = \langle S_b(\mathbf{P}_b) G_b(b, c) G_c(c, d) \rangle_{bcd} = U_b + \mathbf{P}_b \cdot \mathbf{V}_b. \quad (4.7)$$

This is equivalent to making the replacement $\mathbf{n}_b \rightarrow \mathbf{P}_b$ in Eq. (4.3).

We conclude that by determining U_b and $\mathbf{V}_b$ of Eqs. (4.4) and (4.5), we should be able to determine all three decay parameters α_b , β_b , and γ_b , for the b hyperon, and α_c for the c hyperon.

It is now clear how to get the cross-section distribution for production of Λ_c^+ in e^+e^- annihilation and its subsequent decay $\Lambda_c^+ \rightarrow \Lambda \pi^+$ and $\Lambda \rightarrow p \pi^-$. Starting from the expressions for the scattering distribution function, Eq. (1.3), and the polarization, Eq. (1.8), we obtain

$$d\sigma \propto [\mathcal{R} U_{\Lambda_c} + \mathcal{S} \mathbf{N} \cdot \mathbf{V}_{\Lambda_c}] d\Omega_{\Lambda_c} d\Omega_{\Lambda} d\Omega_p, \quad (4.8)$$

with $\mathbf{N}$, Eq. (1.4), the normal to the scattering plane. The functions $\mathcal{R}$ and $\mathcal{S}$ are defined in Eqs. (1.1) and (1.2) and depend among other things on the Λ_c^+ scattering angle θ ($= \theta_{\Lambda_c}$). In Eqs. (4.4) and (4.5) indices are interpreted as; $b = \Lambda_c^+$, $c = \Lambda$, $d = p$.

When integrating over the decay angles Ω_{Λ} and Ω_p in Eq. (4.8) we observe that the term involving the polarization $\mathbf{N} \cdot \mathbf{V}_{\Lambda_c}$ vanishes, as does the term involving the angular dependent part of U_{Λ_c} . This results is the cross-section distribution of Eq. (1.1),

$$d\sigma \propto [1 + \alpha \cos^2 \theta_{\Lambda_c}] d\Omega_{\Lambda_c}, \quad (4.9)$$

describing the annihilation reaction $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$.

It is more interesting to perform a partial integration. Let us integrate over the angles Ω_{Λ} and Ω_p keeping $\cos \theta_{\Lambda p}$

$$\cos \theta_{\Lambda p} = \mathbf{l}_{\Lambda} \cdot \mathbf{l}_p \quad (4.10)$$

constant. Also in this case does the contribution involving the polarization vanish. We are left with

$$d\sigma \propto [1 + \alpha \cos^2 \theta_{\Lambda_c}] [1 + \alpha_{\Lambda_c} \alpha_{\Lambda} \cos \theta_{\Lambda p}] \times d(\cos \theta_{\Lambda_c}) d(\cos \theta_{\Lambda p}). \quad (4.11)$$

The cross-section distribution of Eq. (4.8) applies also to the decay chain, $\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$ and $\Sigma^+ \rightarrow p \pi^0$, with the corresponding identification of indices b , c , and d .

V. DIFFERENTIAL DISTRIBUTIONS

The cross-section distribution (4.8) is a function of two unit vectors $\mathbf{l}_1 = \mathbf{l}_{\Lambda}$, the direction of motion of the Lambda hyperon in the rest system of the charmed-Lambda hyperon, and $\mathbf{l}_2 = \mathbf{l}_p$ the direction of motion of the proton in the rest system of the Lambda hyperon. In order to handle these vectors we need a common coordinate system, Fig. 1, which we define as follows.

The scattering plane of the reaction $e^+e^- \rightarrow \Lambda_c^+ \bar{\Lambda}_c^-$ is spanned by the unit vectors $\hat{\mathbf{p}} = \mathbf{l}_{\Lambda_c}$ and $\hat{\mathbf{k}} = \mathbf{l}_{e^+}$, as measured in the c.m. system. We assume the scattering to be to the left, with scattering angle $\theta \geq 0$. If the scattering is to the right we rotate such an event 180° around the $\mathbf{k}$ -axis, so that the scattering appears to be to the left. The

FIG. 1. Momentum vectors in the scattering plane in the c.m. system, with $\mathbf{p}$ the hyperon Λ_c momentum and $\mathbf{k}$ the positron e^+ momentum. The e^- momentum is $-\mathbf{k}$ and the $\bar{\Lambda}_c$ momentum $-\mathbf{p}$. Theta θ is the scattering angle, and $\cos \theta = \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}$. The normal to the scattering plane, $\mathbf{e}_y$, points downwards.

scattering plane makes up the xz -plane, with the y -axis along the normal to the scattering plane. We choose a right-handed coordinate system with basis vectors

$$\mathbf{e}_z = \hat{\mathbf{p}}, \quad (5.1)$$

$$\mathbf{e}_y = \frac{1}{\sin \theta} (\hat{\mathbf{p}} \times \hat{\mathbf{k}}), \quad (5.2)$$

$$\mathbf{e}_x = \frac{1}{\sin \theta} (\hat{\mathbf{p}} \times \hat{\mathbf{k}}) \times \hat{\mathbf{p}}. \quad (5.3)$$

Expressed in terms of them the initial-state momentum

$$\hat{\mathbf{k}} = \sin \theta \mathbf{e}_x + \cos \theta \mathbf{e}_z. \quad (5.4)$$

This coordinate system is used for defining the directional angles of the Lambda and the proton. The directional angles of the Lambda hyperon in the charmed-Lambda hyperon rest system are,

$$\mathbf{I}_1 = (\cos \phi_1 \sin \theta_1, \sin \phi_1 \sin \theta_1, \cos \theta_1), \quad (5.5)$$

whereas the directional angles of the proton in the Lambda hyperon rest system are

$$\mathbf{I}_2 = (\cos \phi_2 \sin \theta_2, \sin \phi_2 \sin \theta_2, \cos \theta_2). \quad (5.6)$$

An event of the reaction $e^+e^- \rightarrow \bar{\Lambda}_c \Lambda_c$; $\Lambda_c \rightarrow \Lambda \pi$; $\Lambda \rightarrow p \pi$ is specified by the five dimensional vector $\xi = (\theta, \Omega_1, \Omega_2)$, and the differential-cross-section distribution as summarized by Eq. (4.8) reads,

$$d\sigma \propto \mathcal{W}(\xi) d\cos \theta d\Omega_1 d\Omega_2.$$

At the moment, we are not interested in absolute normalizations. The differential-distribution function $\mathcal{W}(\xi)$ is obtained from Eqs. (1.1), (1.2), (4.4), (4.5), (4.8) and can be expressed as,

$$\begin{aligned} \mathcal{W}(\xi) = & \mathcal{F}_0(\xi) + \alpha \mathcal{F}_1(\xi) + \alpha_1 \alpha_2 (\mathcal{F}_2(\xi) + \alpha \mathcal{F}_3(\xi)) + \sqrt{1 - \alpha^2} \cos(\Delta\Phi) (\mathcal{F}_7(\xi) \\ & + \alpha_1 \mathcal{F}_4(\xi) + \beta_1 \mathcal{F}_6(\xi) + \gamma_1 (\mathcal{F}_5(\xi) - \mathcal{F}_7(\xi))), \end{aligned} \quad (5.7)$$

using a set of eight angular functions $\mathcal{F}_k(\xi)$ defined as:

$$\begin{aligned} \mathcal{F}_0(\xi) &= 1, \\ \mathcal{F}_1(\xi) &= \cos^2 \theta, \\ \mathcal{F}_2(\xi) &= \sin \theta_1 \sin \theta_2 \cos(\phi_1 - \phi_2) + \cos \theta_1 \cos \theta_2, \\ \mathcal{F}_3(\xi) &= \cos^2 \theta \mathcal{F}_2(\xi), \\ \mathcal{F}_4(\xi) &= \sin \theta \cos \theta \sin \theta_1 \sin \phi_1, \\ \mathcal{F}_5(\xi) &= \sin \theta \cos \theta \sin \theta_2 \sin \phi_2, \\ \mathcal{F}_6(\xi) &= \sin \theta \cos \theta (\cos \theta_2 \sin \theta_1 \cos \phi_1 - \cos \theta_1 \sin \theta_2 \cos \phi_2), \\ \mathcal{F}_7(\xi) &= \sin \theta \cos \theta \sin \theta_1 \sin \phi_1 \mathcal{F}_2(\xi). \end{aligned} \quad (5.8)$$

The differential distribution of Eq. (5.7) involves two parameters related to the $e^+e^- \rightarrow \Lambda_c \bar{\Lambda}_c$ reaction that can be determined by data: the ratio of form factors α , and the relative phase of form factors $\Delta\Phi$. In addition, the distribution function $\mathcal{W}(\xi)$ depends on the weak-decay parameters $\alpha_1 \beta_1 \gamma_1$ of the charmed-hyperon decay $\Lambda_c \rightarrow \Lambda \pi$, and on the weak-decay parameters $\alpha_2 \beta_2 \gamma_2$ of the hyperon decay $\Lambda \rightarrow p \pi^-$. However, the dependency on β_2 and γ_2 drops out. Similarly, integrating over $d\Omega_2$ we get

$$\begin{aligned} d\sigma \propto & [1 + \alpha \cos^2 \theta \\ & + \alpha_1 \sqrt{1 - \alpha^2} \cos(\Delta\Phi) \sin \theta \cos \theta \sin \theta_1 \sin \phi_1] \\ & \times d\Omega d\Omega_1, \end{aligned} \quad (5.9)$$

where now the dependency on β_1 and γ_1 also drops out. The last term in this equation originates with the scalar $\mathbf{P}_{\Lambda_c} \cdot \mathbf{N}$. The charmed-hyperon polarization vanishes at $\theta = 0^\circ, 90^\circ$ and 180° .

The distributions presented here will hopefully be of value in the analysis of BESIII data.

VI. MIXED WEAK-ELECTROMAGNETIC HYPERON DECAY

Now, we extend the formalism to hyperons decaying in two steps, with one being electromagnetic. An example of such a decay chain is $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$ followed by $\Sigma^0 \rightarrow \Lambda \gamma$. As before we employ indices b, c , and d for variables belonging to Λ_c^+, Σ^0 , and Λ .

The spin-density distribution functions for the weak and electromagnetic transitions are given in Eqs. (4.1) and (3.2),

$$G_b(b, c) = 1 + \alpha_b \mathbf{n}_b \cdot \mathbf{l}_c + \alpha_b \mathbf{n}_c \cdot \mathbf{l}_c + \mathbf{n}_b \cdot \mathbf{L}_b(\mathbf{n}_c, \mathbf{l}_c), \quad (6.1)$$

$$G_\gamma(c, d) = 1 - \mathbf{n}_c \cdot \mathbf{l}_d \mathbf{l}_d \cdot \mathbf{n}_d. \quad (6.2)$$

By folding their product as done for the corresponding product of Eq. (4.3), we get

$$\begin{aligned} W_b(\mathbf{n}_b; \mathbf{l}_c, \mathbf{l}_d) &= \langle G_b(b, c) G_\gamma(c, d) \rangle_{cd} \\ &= U_b + \mathbf{n}_b \cdot \mathbf{V}_b, \end{aligned} \quad (6.3)$$

with

$$U_b = 1, \quad \mathbf{V}_b = \alpha_b \mathbf{l}_c. \quad (6.4)$$

These expressions for U_b and $\mathbf{V}_b$ are noteworthy. They are in fact the same as those of a one-step $b \rightarrow c\pi$ decay, Eq. (2.5). Hence, the electromagnetic decay does not add any structure, Eq. (6.4) is independent of $\mathbf{l}_d$.

The initial state spin distribution function for hyperon b produced in e^+e^- annihilation is as above, Eq. (4.6),

$$S_b(\mathbf{P}_b) = 1 + \mathbf{P}_b \cdot \mathbf{n}_b. \quad (6.5)$$

Folding this distribution function with the decay distribution function of Eq. (6.3), we obtain

$$\begin{aligned} W_b(\mathbf{P}_b; \mathbf{l}_c, \mathbf{l}_d) &= \langle S_b(\mathbf{P}_b) G_b(b, c) G_\gamma(c, d) \rangle_{bcd} \\ &= U_b + \mathbf{P}_b \cdot \mathbf{V}_b. \end{aligned} \quad (6.6)$$

As noted earlier this is equivalent to making the replacement $\mathbf{n}_b \rightarrow \mathbf{P}_b$ in Eq. (6.3). We also notice if we manage to determine U_b and $\mathbf{V}_b$ of Eq. (6.6), the only parameter that can be fixed is α_b , a meager return.

The expression for the cross-section distribution for Λ_c^+ production and subsequent decays $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$ and $\Sigma^0 \rightarrow \Lambda \gamma$ is

$$d\sigma \propto [\mathcal{R}U_{\Lambda_c} + \mathcal{S}\mathbf{N} \cdot \mathbf{V}_{\Lambda_c}] d\Omega_{\Lambda_c} d\Omega_{\Sigma} d\Omega_{\Lambda}, \quad (6.7)$$

with $\mathbf{N}$, Eq. (1.4), the normal to the scattering plane, and $U_{\Lambda_c} = 1$, $\mathbf{V}_{\Lambda_c} = \alpha_{\Lambda_c} \mathbf{l}_{\Sigma}$, from Eq. (6.6). The functions $\mathcal{R}$ and $\mathcal{S}$ are defined in Eqs. (1.1) and (1.2) and depend among other things on the Λ_c^+ scattering angle θ .

Finally, we mention that it is possible to extend the Λ_c^+ decay chain by adding the decay $\Lambda \rightarrow p\pi^-$.

VII. FINAL REMARKS

Production and decay of hyperons in e^+e^- annihilation is being vigorously pursued at BESIII. But also at Fermilab sequential-hyperon decays have been investigated Ref. [5]. In many applications the initial-hyperon polarization can be or is ignored. We have chosen to illustrate our formalism with the decay chains $\Lambda_c^+ \rightarrow \Lambda \pi^+$; $\Lambda \rightarrow p\pi^-$, and $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$; $\Sigma^0 \rightarrow \Lambda \gamma$, but the formulas are more general than that. A recent study of similar decay chains, but in the helicity formalism, can be found in Ref. [6].

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APPENDIX: ANGULAR INTEGRALS

In this Appendix we detail the angular integration leading to Eq. (4.11).

Consider two unit vectors $\mathbf{l}_c$ and $\mathbf{l}_d$. We want to integrate over the angles Ω_c and Ω_d keeping $\cos \theta_{cd} = \mathbf{l}_c \cdot \mathbf{l}_d$ fixed. To this end we put the vectors in the xy -plane of the coordinate system O' ,

$$\mathbf{l}_c = (1, 0, 0), \quad (A1)$$

$$\mathbf{l}_d = (\cos \theta_{cd}, \sin \theta_{cd}, 0), \quad (A2)$$

$$\mathbf{l}_c \times \mathbf{l}_d = (0, 0, \sin \theta_{cd}). \quad (A3)$$

We then rotate the coordinate system O' with respect to the space fixed coordinate system O , where the normal to the scattering plane is along the Z -direction. The rotation matrix which transforms the column vector $\bar{r}_b$ in O' into the column vector $\bar{r}_s$ in O is the matrix

$$R^{-1}(\alpha, \beta, \gamma) = \begin{pmatrix} \cos \alpha \cos \beta \cos \gamma - \sin \alpha \sin \gamma & \cos \gamma \cos \beta \sin \alpha + \sin \gamma \cos \alpha & -\sin \beta \cos \gamma \\ -\sin \gamma \cos \beta \cos \alpha - \cos \gamma \sin \alpha & -\sin \gamma \cos \beta \sin \alpha + \cos \gamma \cos \alpha & \sin \beta \sin \gamma \\ \cos \alpha \sin \beta & \sin \gamma \sin \beta & \cos \beta \end{pmatrix}, \quad (A4)$$

with $\bar{r}_s = R^{-1}(\alpha, \beta, \gamma) \bar{r}_b$ and $\alpha\beta\gamma$ the Euler angles.

The angular integrations can be expressed in terms of the Euler angles, as

$$d\Omega_c d\Omega_d = d(\cos \theta_{cd}) d\alpha d(\cos \beta) d\gamma. \quad (A5)$$

The expression to be integrated, Eq. (4.5), reads

$$U_b + \mathbf{P} \cdot \mathbf{V}_b = 1 + \alpha_b \alpha_c \mathbf{l}_c \cdot \mathbf{l}_d + \alpha_b \mathbf{P} \cdot \mathbf{l}_c + \gamma_b \mathbf{P} \cdot \mathbf{l}_d \\ + (1 - \gamma_b) \mathbf{P} \cdot \mathbf{l}_c \mathbf{l}_d \cdot \mathbf{l}_c + \beta_b \mathbf{P} \cdot (\mathbf{l}_d \times \mathbf{l}_c), \quad (\text{A6})$$

with $\mathbf{P}$ along the Z-direction.

Now, we note that terms proportional to $\mathbf{P} \cdot \mathbf{l}_c$ or $\mathbf{P} \cdot \mathbf{l}_d$ vanish upon integration over angles α or γ . Therefore,

$$\int d\Omega_c d\Omega_d (U_b + \mathbf{P} \cdot \mathbf{V}_b) \\ = 4\pi^2 \int d(\cos \theta_{cd}) d(\cos \beta) (1 + \alpha_b \alpha_c \cos \theta_{cd} \\ - \beta_b \alpha_c P \sin \theta_{cd} \cos \beta) \\ = 8\pi^2 \int d(\cos \theta_{cd}) (1 + \alpha_b \alpha_c \cos \theta_{cd}). \quad (\text{A7})$$

This result leads to Eq. (4.11).

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