Supersymmetric interactions of a six-dimensional self-dual tensor and fixed-shape second quantized strings

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“Curvepole (2,0)-theory” is a deformation of the (2,0)-theory with nonlocal interactions. A curvepole is defined as a two-dimensional generalization of a dipole. It is an object of fixed two-dimensional shape of which the boundary is a charged curve that interacts with a 2-form gauge field. Curvepole theory was previously only defined indirectly via M-theory. Here, we propose a supersymmetric Lagrangian, constructed explicitly up to quartic terms, for an “Abelian” curvepole theory, which is an interacting deformation of the free (2,0) tensor multiplet. This theory contains fields of which the quanta are curvepoles (i.e., fixed-shape strings). Supersymmetry is preserved (at least up to quartic terms) if the shape of the curvepoles is (two-dimensional) planar. This nonlocal six-dimensional quantum field theory may also serve as a UV completion for certain (local) five-dimensional gauge theories.

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I. INTRODUCTION

Gaining a better understanding of the six-dimensional (6D) (2,0)-theory, originally discovered in Ref. [1], has become a major objective in contemporary theoretical high-energy physics. The (2,0)-theory’s relation to the low-energy limit of M5-branes [2], as well as its power to elucidate the strong coupling dynamics of four-dimensional and lower-dimensional gauge theories, has been realized at the outset [1], and further developed in Refs. [3–5], with a remarkably unified picture emerging more recently in Refs. [6–11] and other works. These developments generate a strong motivation for finding a fundamental description of the (2,0)-theory. Several promising approaches have been proposed over the years (see for example Refs. [12–22]), but the problem is still open. Part of the difficulty stems from the problem that (at least on the Coulomb branch) the theory describes an interacting anti-self-dual 2-form gauge field, and the charged objects that couple to 2-forms are strings rather than point particles.

It is therefore interesting to explore the various possibilities that arise from interacting 2-form field theories, and in this paper, we take first steps to construct a supersymmetric field theory describing a deformation of a free (2,0) tensor multiplet (containing a 2-form gauge field) where some of the scalars and spinors take the form of extended stringlike

objects of which the boundary is charged under the 2-form field.

The sort of theory we are looking for was put forward in Ref. [23] (referred to by another moniker) as an M-theoretic construction. In order to better explain what kind of 6D field theory we seek to construct, it is useful to recall the “dipole theories” in four dimensions. In Refs. [24–26], motivated by the construction of Yang-Mills theory on a noncommutative space [27–29], a theory of fundamental dipoles interacting with a gauge field was constructed and embedded in string theory. The term “fundamental” here means that the dipoles of dipole theory are quanta of a fundamental field, rather than composites of other fields. For example, a scalar fundamental dipole field ϕ couples to a gauge field $A_\mu dx^\mu$ via the covariant derivative

$$D_\mu^{[l]} \phi(x) \equiv \partial_\mu \phi(x) + iq \left[A_\mu \left(x + \frac{1}{2} l \right) - A_\mu \left(x - \frac{1}{2} l \right) \right] \phi(x), \quad (1)$$

where l is a constant vector from the $(-q)$ -charged end point to the $(+q)$ -charged end point of the dipole and x is taken at the center of the dipole. Such theories arise naturally in string theory and in the context of noncommutative geometry, as explained in Refs. [24,25].

The goal of this paper is to construct a 6D counterpart of the dipole theories where the role of the gauge field $A_\mu dx^\mu$ is played by a gauge 2-form B with anti-self-dual field strength $H = -^*H$ (and at zeroth order $H = dB$). The role of the dipole fields will then be played by “curvepole” fields. Let us define a *curvepole* to be a nonpointlike particle that has the shape of a fixed oriented closed curve

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$C \subset \mathbb{R}^6$ and that interacts with a given 2-form field $B = \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu$ ($\mu, \nu, \dots = 1, \dots, 6$ label the coordinates on Euclidean $\mathbb{R}^6$) via the action $i \int_{(x+C)} B$, where $(x+C)$ denotes the result of translating C by the spacetime vector x . We would also like the role of the covariant derivative (1) to be played by [30]

$$\partial_\mu \phi(x) + i\phi(x) \int_C B_{\mu\nu}(x+y) dy^\nu. \quad (2)$$

However, Eq. (2) is incompatible with anti-self-duality and will require a small modification, to be explained in Sec. III.

We also wish to preserve some amount of supersymmetry, and in fact, we will require (1,0)-supersymmetry (SUSY). The 2-form B will be part of a tensor multiplet (containing also a real scalar and two Weyl spinors), and the curvepole field ϕ will be part of a hypermultiplet (containing two complex scalars and a complex Weyl spinor). The interaction also modifies the anti-self-duality relation $dB = -*dB$, as we shall see.

Before proceeding to the details, let us discuss another reason to be interested in the curvepole deformation of the (2,0)-theory. As was suggested in Refs. [23,24,31], such theories arise naturally in M-theory. Following Ref. [23], we probe with an M2-brane a flat M-theory background of the form $\mathbb{R}^{1,2} \times T^2 \times \mathbf{W}_\Omega$, where $\mathbf{W}_\Omega$ is the six-dimensional flat space formed by fibering $\mathbb{R}^5$ [parametrized by $\mathbf{y} = (x^6, \dots, x^{10})$] over S^1 (of radius r and parametrized by x^3) with a twist $\Omega \in \text{spin}(5)$. (That is, we identify $x^3 \simeq x^3 + 2\pi r$ together with $\mathbf{y} \simeq \Omega \mathbf{y}$.) For simplicity, we also take T^2 to be a product of two circles of radius r (parametrized by x^4, x^5). We then set $\Omega = \exp(i\mathbf{h})$ [for $\mathbf{h}$ a fixed element of the Lie algebra $so(5)$], insert an M2-brane probe at $x^3 = \dots = x^{10} = 0$, and take the limit $\mathbf{h} \rightarrow 0$, (Planck length) $\ell_p \rightarrow 0$, $r/\ell_p \rightarrow 0$, with $\mathbf{h}\ell_p^4/r^2$ kept finite. Note that at the location of the M2-brane directions 3, 4, and 5 form a small T^3 of volume $(2\pi r)^3 \rightarrow 0$. If also $\mathbf{h} = 0$, we can apply U-duality to turn the M2-brane into an M5-brane wrapped on a large T^3 , which in turn is described at low energy by a free tensor multiplet. It was argued in Refs. [23,31] that for nonzero $\mathbf{h}$ the dynamics has features of a curvepole theory. We will now proceed with a purely field theoretic construction, which we conjecture is applicable to the case that $\mathbf{h}$ preserves half the supersymmetry, i.e., annihilates a spinor of $\text{spin}(5)$.

II. NOTATION

We work in Euclidean signature on $\mathbb{R}^6$ and denote coordinates by x^μ (with greek indices taking values $\alpha, \beta, \dots = 1, \dots, 6$). Components of a 2-form B are denoted by $B_{\mu\nu} = -B_{\nu\mu}$ with $B = \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu$. The 3-form field strength $H = dB$ has components $H_{\mu\nu\sigma} = 3\partial_{[\mu} B_{\nu\sigma]}$.

Its self-dual $H_{\mu\nu\sigma}^{(+)}$ and anti-self-dual $H_{\mu\nu\sigma}^{(-)}$ parts are given by $H_{\mu\nu\sigma}^{(\pm)} = \frac{1}{2} (H_{\mu\nu\sigma} \pm \frac{i}{3!} \epsilon_{\mu\nu\sigma\alpha\beta\gamma} H^{\alpha\beta\gamma})$, where $\epsilon_{\alpha\beta\gamma\delta\mu\nu}$ is the Levi-Civita symbol.

A. Spinor conventions

The 6D spin group is $\text{spin}(6) \cong SU(4)$, and we use lowercase latin indices ($\mathbf{a}, \mathbf{b}, \mathbf{c}, \dots = 1, \dots, 4$) to denote components of a chiral spinor, with lower indices for the fundamental representation 4 of $SU(4)$ (e.g., $\psi_{\mathbf{a}}$) and upper indices for the dual representation $\bar{\mathbf{4}}$ (e.g., $\bar{\psi}^{\mathbf{a}}$). A vector in the representation 6 with components V_μ will be represented by an antisymmetric $V_{\mathbf{ab}} = -V_{\mathbf{ba}}$, normalized so that the scalar product of two vectors V and U is $V_\mu U^\mu = -\frac{1}{4} \epsilon^{\mathbf{abcd}} V_{\mathbf{ab}} U_{\mathbf{cd}}$, where $\epsilon^{\mathbf{abcd}}$ is totally antisymmetric with $\epsilon^{1234} = 1$. We also define $V^{\mathbf{ab}} \equiv \frac{1}{2} \epsilon^{\mathbf{abcd}} V_{\mathbf{cd}}$ so that $V_\mu U^\mu = -\frac{1}{2} V_{\mathbf{ab}} U^{\mathbf{ab}}$.

Components of a 2-form $B_{\mu\nu}$ are written in spinor notation as traceless $B_{\mathbf{a}}^{\mathbf{b}}$ (with $B_{\mathbf{a}}^{\mathbf{a}} = 0$) and normalized so that $B_{\mu\nu} = U_{[\mu} V_{\nu]}$ translates to $B_{\mathbf{a}}^{\mathbf{b}} = \frac{1}{4} (V_{\mathbf{ac}} U^{\mathbf{bc}} - U_{\mathbf{ac}} V^{\mathbf{bc}})$. We also note that $B_{\mu\nu} B^{\mu\nu} = -2B_{\mathbf{a}}^{\mathbf{b}} B_{\mathbf{b}}^{\mathbf{a}}$, and if W^μ and T^μ are vectors, then $T_\mu = B_{\mu\nu} W^\mu$ translates to $T^{\mathbf{ab}} = 2W^{\mathbf{c[a}} B_{\mathbf{c}}^{\mathbf{b}]}$.

An anti-self-dual 3-form with tensor components $H_{\mu\nu\sigma}^{(-)}$ is expressed in spinor notation as a symmetric $H_{\mathbf{ab}} = H_{\mathbf{ba}}$, while a self-dual 3-form is expressed as $H^{\mathbf{ab}} = H^{\mathbf{ba}}$. They are normalized so that $H_{\mathbf{ab}} H^{\mathbf{ab}} = \frac{1}{2} H_{\mu\nu\sigma} H^{\mu\nu\sigma}$. Moreover, $B_{\mu\nu} = H_{\mu\nu\sigma}^{(-)} V^\sigma$ translates to $B_{\mathbf{a}}^{\mathbf{b}} = -\frac{1}{2} H_{\mathbf{ac}} V^{\mathbf{bc}}$, and $B_{\mu\nu} = H_{\mu\nu\sigma}^{(+)} V^\sigma$ translates to $B_{\mathbf{a}}^{\mathbf{b}} = -\frac{1}{2} H^{\mathbf{bc}} V_{\mathbf{ac}}$. The relation $H_{\mu\nu\sigma} = 3\partial_{[\mu} B_{\nu\sigma]}$ translates to $H_{\mathbf{ab}} = 2\partial_{\mathbf{c[a}} B_{\mathbf{b}}^{\mathbf{c}]}$ and $H^{\mathbf{ab}} = 2\partial^{\mathbf{c[a}} B_{\mathbf{b}}^{\mathbf{c}]}$.

B. Curvepole integrals

The action presented below depends on the choice of a fixed (two-dimensional) open surface $\Sigma \subset \mathbb{R}^6$ with boundary $C = \partial\Sigma$. For a (either fundamental or composite) field $\Phi(x)$, we define the *curvepole integral*,

$$[\Phi(x)]^{\mu\nu} \equiv \int_\Sigma \Phi(x+y) dy^\mu \wedge dy^\nu, \quad (3)$$

and the *anticurvepole integral*,

$$[\Phi(x)]^{\mu\nu} \equiv \int_\Sigma \Phi(x-y) dy^\mu \wedge dy^\nu. \quad (4)$$

We call Σ the *curvepole surface* and say that it is *balanced* if for arbitrary Φ and Ψ

$$0 = \int ([\Phi(x)]_{\alpha\beta} [\Psi(x)]_{\gamma\delta} - [\Phi(x)]_{\gamma\delta} [\Psi(x)]_{\alpha\beta}) d^6x. \quad (5)$$

It can also be expressed as $[\Phi]_{\gamma\delta} [\Psi]_{\alpha\beta} = [\Phi]_{\alpha\beta} [\Psi]_{\gamma\delta}$. We will call Σ *n-planar* if it can be embedded in some $\mathbb{R}^n$ subspace of $\mathbb{R}^6$.

It is not hard to see that a 2-planar curvepole is balanced [in which case the integrand of (5) vanishes], but the reverse is not necessarily true. In fact, any parity invariant Σ is balanced.

III. CURVEPOLE DEFORMATION

We will start with the action I_0 of the free 6D tensor theory and add interaction terms $I_1 + I_2 + \dots$, where I_n is a polynomial of order $(n + 2)$ in the fundamental fields and each of its terms contains a total of n curvepole integrals. At each order, we will modify the supersymmetry variation $\delta = \delta_0 + \delta_1 + \delta_2 + \dots$, where δ_0 is the free SUSY variation that is linear in the fields, δ_1 is quadratic in the fields with terms that have one curvepole integral, etc. The calculations have been performed up to quartic order, so the results presented below satisfy

$$0 = \delta_0 I_0 = \delta_0 I_1 + \delta_1 I_0 = \delta_2 I_0 + \delta_1 I_1 + \delta_2 I_0. \quad (6)$$

The curvepole deformation preserves a $U(1) \times SU(2)$ subgroup of the $Sp(2)$ R-symmetry of the free tensor multiplet. $SU(2)$ doublets will carry indices $i, j, k, \dots = 1, 2$. They will be lowered with the antisymmetric ϵ_{ij} . The surviving anticommuting (1,0) SUSY parameters, which are $U(1)$ neutral $SU(2)$ doublets and chiral spinors, will be denoted by η^{ai} . The $U(1)$ -symmetry will be further discussed in Sec. III B.

A. Quadratic terms

The fields of the (2,0) tensor multiplet are a 2-form gauge field B_a^b , an $SU(2)$ singlet scalar φ , an $SU(2)$ doublet anticommuting spinor χ_a^i , an $SU(2)$ doublet scalar ϕ^i and its complex conjugate $\bar{\phi}_i$, and an $SU(2)$ singlet anticommuting spinor ψ_a and its complex conjugate $\bar{\psi}_a$. The first three fields form a (1,0) tensor multiplet, and the last four form a hypermultiplet. The SUSY transformations are

$$\begin{aligned} \delta_0 \phi^i &= \eta^{ai} \psi_a, & \delta_0 \psi_a &= -2i\eta_i^b \partial_{ab} \phi^i, \\ \delta_0 \bar{\phi}_i &= \eta_i^a \bar{\psi}_a, & \delta_0 \bar{\psi}_a &= -2i\eta_i^b \partial_{ab} \bar{\phi}^i, \\ \delta B_a^b &= \eta_i^b \chi_a^i - \frac{1}{4} \delta_a^b \eta_i^c \chi_c^i, & \delta_0 \varphi &= \eta_i^a \chi_a^i, \\ \delta_0 \chi_a^i &= i\eta^{bi} \partial_{ab} \varphi + i\eta^{bi} H_{ab}. \end{aligned}$$

The action is

$$\begin{aligned} I_0 &= \int d^6x \left(\frac{1}{12\pi} H_{\mu\nu\sigma} H^{\mu\nu\sigma} + \frac{1}{4\pi} \partial_\mu \varphi \partial^\mu \varphi + \partial_\mu \bar{\phi}_i \partial^\mu \phi^i \right. \\ &\quad \left. - \frac{i}{2\pi} \chi_{ia} \partial^{ab} \chi_b^i + \bar{\psi}_a \partial^{ab} \psi_b \right). \end{aligned} \quad (7)$$

Note that the coupling constant of the 3-form field strength has the self-dual value $\sqrt{2\pi}$. The self-dual part of the

3-form field strength $H_{\mu\nu\sigma}^{(+)}$ decouples, and we will make sure below that interaction terms are always expressed in terms of the anti-self-dual $H_{\mu\nu\sigma}^{(-)}$ only.

B. Cubic terms

At zeroth order, the $U(1) \subset Sp(2)$ current that the curvepole deformation preserves is given by

$$J_{ab} \equiv i\bar{\phi}_i \partial_{ab} \phi^i - i\partial_{ab} \bar{\phi}_i \phi^i + \bar{\psi}_a \psi_b - \bar{\psi}_b \psi_a. \quad (8)$$

The fields that will become curvepoles are those that are charged under this $U(1)$. However, the covariant derivative (2) is not satisfactory, because interaction terms in our action are allowed to depend on B only through $H^{(-)}$. To overcome this, we recall that for dipole theories there is a way to rewrite (1) so that only the field strength $F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]}$ appears [24]. One performs a field redefinition $\tilde{\Phi}(x) \equiv \exp(iq \int_{(x+P)} A_\mu dx^\mu) \Phi(x)$, where P is some fixed path from $-\frac{1}{2}l$ to $\frac{1}{2}l$ and $x + P$ is the translation of the path by x . Then, $e^{iq \int_{(x+P)} A_\mu dx^\mu} D_\mu^{[l]} \phi(x)$ is

$$\partial_\mu \tilde{\Phi}(x) - iq \tilde{\Phi}(x) \int_P F_{\mu\nu}(x+y) dy^\nu.$$

For curvepole interactions, guided by the above example, we replace $q \int_P F_{\mu\nu}(x+y) dy^\nu$ with $\pm \boxed{H_{\mu\nu\sigma}^{(-)}}^{\nu\sigma}$. We thus add to I_0 the interaction term $-\frac{1}{2} J^\mu \boxed{H_{\mu\nu\sigma}^{(-)}}^{\nu\sigma}$ and look for a supersymmetric completion. The results are described below.

We define a vector field,

$$\mathbf{V}_\mu \equiv \frac{1}{2} \boxed{H_{\mu\nu\sigma}^{(-)}}^{\nu\sigma} - \frac{1}{2} \boxed{\partial^\nu \varphi}_{\mu\nu}, \quad (9)$$

which will play the role of an effective gauge field. We also define an effective gaugino field ρ^{ia} and a composite field S_a^i by

$$\rho^{ia} \equiv \boxed{\partial^{ab} \chi_c^i}_{b}^{c} - \boxed{\partial^{bc} \chi_b^i}_{b}^{a}, \quad S_a^i \equiv \phi^i \bar{\psi}_a + \bar{\phi}^i \psi_a. \quad (10)$$

Then, the cubic interactions in the action are given by

$$I_1 = \int d^6x (-J^\mu \mathbf{V}_\mu + i S_{ia} \rho^{ia}). \quad (11)$$

To be consistent with the equations of motion, the anti-self-duality condition is modified to

$$H_{\mu\nu\sigma}^{(+)} = -\frac{3\pi i}{4} \boxed{J_{[\mu}}_{\nu\sigma]} + \frac{\pi}{8} \epsilon_{\mu\nu\sigma\alpha\beta\gamma} \boxed{J^\alpha}_{\beta\gamma}.$$

To write the correction to the SUSY transformation at this order, we define $\lambda \equiv \eta_i^a \chi_{\mathbf{c}}^i$ and set $\delta_1 \varphi = 0$,

$$\begin{aligned} \delta_1 \phi^i &= i\lambda \phi^i, \quad \delta_1 \psi_{\mathbf{a}} = 2\eta_i^b \mathbf{V}_{\mathbf{ab}} \phi^i + i\lambda \psi_{\mathbf{a}}, \\ \delta_1 \bar{\phi}^i &= -i\lambda \bar{\phi}^i, \quad \delta_1 \bar{\psi}_{\mathbf{a}} = -2\eta_i^b \mathbf{V}_{\mathbf{ab}} \bar{\phi}^i - i\lambda \bar{\psi}_{\mathbf{a}}, \\ \delta_1 \chi_{\mathbf{a}}^i &= \frac{\pi i}{2} \eta^{ci} \overline{[\bar{\psi}_{\mathbf{b}} \psi_{\mathbf{a}} - \bar{\psi}_{\mathbf{a}} \psi_{\mathbf{b}}]}_{\mathbf{c}} + \frac{\pi i}{2} \eta^{ci} \overline{[\bar{\psi}_{\mathbf{b}} \psi_{\mathbf{c}} - \bar{\psi}_{\mathbf{c}} \psi_{\mathbf{b}}]}_{\mathbf{a}} \\ &\quad - \frac{\pi}{2} \eta_j^c \overline{[3(\bar{\phi}^j \partial_{\mathbf{bc}} \phi^j + \phi^j \partial_{\mathbf{bc}} \bar{\phi}^j) + \bar{\phi}^j \partial_{\mathbf{bc}} \phi^j + \phi^j \partial_{\mathbf{bc}} \bar{\phi}^j]}_{\mathbf{a}}^{\mathbf{b}} \\ &\quad + \frac{\pi}{2} \eta_j^c \overline{[\phi^j \partial_{\mathbf{ba}} \bar{\phi}^j + \bar{\phi}^j \partial_{\mathbf{ba}} \phi^j + 3(\phi^j \partial_{\mathbf{ba}} \bar{\phi}^j + \bar{\phi}^j \partial_{\mathbf{ba}} \phi^j)]}_{\mathbf{c}}^{\mathbf{b}}, \\ \delta_1 B_{\mathbf{a}}^{\mathbf{b}} &= \pi i \eta_i^a \overline{[S_{\mathbf{c}}^i]}_{\mathbf{b}} - \frac{\pi i}{2} \eta_i^c \overline{[S_{\mathbf{c}}^i]}_{\mathbf{b}}^{\mathbf{a}} - 2 \overline{[S_{\mathbf{b}}^i]}_{\mathbf{c}}^{\mathbf{a}} + \delta_{\mathbf{b}}^{\mathbf{a}} \overline{[S_{\mathbf{d}}^i]}_{\mathbf{c}}^{\mathbf{d}}. \end{aligned} \quad (12)$$

C. Quartic terms

We define an $SU(2)$ triplet “meson” composite field

$$M_i^j \equiv \bar{\phi}_i \phi^j - \frac{1}{2} \delta_i^j \bar{\phi}_k \phi^k, \quad (M_i^i = 0).$$

We then find that

$$\begin{aligned} I_2 &= \int d^6 x (\mathbf{V}_{\mu} \mathbf{V}^{\mu} \bar{\phi}_i \phi^i \\ &\quad - \frac{3\pi}{16} \overline{[J_{\mu}^{\sigma\nu}]}_{\sigma\nu} \overline{[J_{\mu}^{\sigma\nu}]}_{\sigma\nu} + \frac{\pi}{2} \overline{[\partial^{\mu} M_i^j]}_{\mu\sigma} \overline{[\partial_{\nu} M_j^i]}^{\nu\sigma}) \end{aligned} \quad (13)$$

satisfies (6) provided the *balanced curvepole* condition (5) holds. (We will not present δ_2 here, but we claim that $\delta_0 I_2 + \delta_1 I_1$ vanishes up to zeroth order equations of motion, which is equivalent to the existence of a δ_2 such that $\delta_0 I_2 + \delta_1 I_1 + \delta_2 I_0 = 0$.) The balanced curvepole condition is needed to cancel terms of the form $\overline{[\phi \partial \psi \bar{\psi} \psi]}$.

The first term $\mathbf{V}_{\mu} \mathbf{V}^{\mu} \bar{\phi}_i \phi^i$ in (13) combines with $-J^{\mu} \mathbf{V}_{\mu}$ of (11) and the kinetic term of (7) to give the action of a scalar that is minimally coupled to the effective gauge field $\mathbf{V}_{\mu}$. The second term of (13) is part of an effective magnetic coupling. It can also be derived by formally dualizing an electrically coupled $H_{\mu\nu\sigma}$ using the standard technique of treating H as the independent field and introducing a Lagrange multiplier for the Bianchi identity $dH = 0$, then integrating over H . (See Ref. [32] for a thorough explanation on how to couple a theory of anti-self-dual fields to a 3-form source, and see Refs. [33–35] for related calculations in the context of other nonlocal gauge theories.)

The terms in I_2 were determined by requiring supersymmetry up to (and including) quartic terms. But SUSY alone is not sufficient to uniquely determine I_2 , because it turns out that there is a term, $I'_2 = \int d^6 x (\frac{1}{2} \overline{[J_{\sigma\mu}]}_{\mu\nu} \overline{[J^{\sigma\mu}]}^{\mu\nu} - \overline{[\partial_{\sigma} M_j^i]}_{\mu\nu} \overline{[\partial^{\sigma} M_i^j]}^{\mu\nu} + i \overline{[S_{\mathbf{a}}^i]}_{\mu\nu} \partial^{\mathbf{ab}} \overline{[S_{\mathbf{b}}^i]}^{\mu\nu})$, that satisfies $\delta_0 I'_2 + \delta'_2 I_0$ for an appropriate δ'_2 . Thus, $I_2 \rightarrow I_2 + c I'_2$, together with $\delta_2 \rightarrow \delta_2 + c \delta'_2$, will also be a solution to (6) for any

arbitrary constant c . Nevertheless, Eq. (13) is uniquely determined by imposing the “hollowness” requirement to be introduced next.

IV. HOLLOW CURVEPOLES

We would like the curvepole theory defined by $I = I_0 + I_1 + I_2 + \dots$ to depend only on the curve $C = \partial\Sigma$, and not on the way we fill it with the curvepole surface Σ . We will refer to terms that only depend on $\partial\Sigma$ as *hollow*. As it stands, however, Eqs. (11) and (13) appear to depend on the bulk of Σ . Upon closer inspection, we see that the last term of (13) and the last term of (11) in fact do depend only on C . This is because $\overline{[\partial_{\nu} \Phi]}^{\nu\sigma} = \int_C \Phi(x+y) dy^{\sigma}$, and therefore $\overline{[\partial_{\nu} M_j^i]}^{\nu\sigma}$ only depends on C . Moreover, ρ^{ia} , which was defined in (10), can be expressed as the contraction $\mathbf{c} = \mathbf{d}$ of the expression $\overline{[\partial^{\mathbf{ab}} \chi_{\mathbf{d}}^i]}_{\mathbf{b}}^{\mathbf{c}} - \overline{[\partial^{\mathbf{cb}} \chi_{\mathbf{d}}^i]}_{\mathbf{b}}^{\mathbf{a}}$, and for fixed $\mathbf{d}$, this is antisymmetric in $\mathbf{a}, \mathbf{b}$, and hence a vector that is proportional to $\overline{[\partial_{\mu} \chi_{\mathbf{d}}^i]}^{\mu\nu}$, which depends only on C . The terms in the action that are not explicitly hollow are

$$\int d^6 x \left(-\frac{1}{2} J^{\mu} \overline{[H_{\mu\nu\sigma}^{(-)}]}^{\nu\sigma} + \mathbf{V}_{\mu} \mathbf{V}^{\mu} \bar{\phi}_i \phi^i - \frac{3\pi}{16} \overline{[J_{\mu}^{\sigma\nu}]}_{\sigma\nu} \overline{[J_{\mu}^{\sigma\nu}]}_{\sigma\nu} \right). \quad (14)$$

The first term of (14) could have been hollow if $H^{(-)}$ were a closed 3-form. But this is only true at zeroth order. The equations of motion that follow from $I_0 + I_1$ are

$$\partial_{[\alpha} H_{\mu\nu\sigma]}^{(-)} = -\frac{3\pi i}{32} \epsilon_{\alpha\mu\nu\sigma\gamma\delta} \overline{[\partial_{\tau} J^{\gamma\delta}]}^{\delta\tau} + \frac{3\pi}{4} \overline{[\partial_{[\alpha} J_{\mu}]}_{\nu\sigma]}^{\delta\tau}. \quad (15)$$

However, it turns out that the total action is hollow up to a field redefinition, provided Σ is 3-planar. To see this, consider a deformation of Σ given by $y^{\mu} \rightarrow y^{\mu} + \xi^{\mu}$, where $\xi: \Sigma \rightarrow \mathbb{R}^6$ is an infinitesimal vector that vanishes on C . Then, for any field Φ , the variation of the curvepole integral is

$$\delta_{\xi} \overline{[\Phi]}^{\mu\nu} = \overline{[\xi^{\sigma} \partial_{\sigma} \Phi]}^{\mu\nu} + \overline{[\xi^{\mu} \partial_{\sigma} \Phi]}^{\nu\sigma} - \overline{[\xi^{\nu} \partial_{\sigma} \Phi]}^{\mu\sigma}, \quad (16)$$

where $\overline{[\xi^{\sigma} \dots]}$ is defined as the integral (3) with $\xi^{\sigma}(y)$ inserted in the integrand, and similarly $\overline{[\xi^{\sigma} \dots]}$ is given by (4) with $\xi^{\sigma}(-y)$. Next, we augment (16) with the field redefinition $\delta B_{\gamma\delta} = \frac{\pi i}{4} \epsilon_{\alpha\mu\nu\sigma\gamma\delta} \overline{[\xi^{\alpha} J^{\mu\nu}]}^{\sigma\tau}$ (note that i will be absent in Minkowski signature), as well as $\delta \phi^i = i \epsilon \phi^i$, $\delta \bar{\phi}^i = -i \epsilon \bar{\phi}^i$, $\delta \psi^i = i \epsilon \psi^i$, $\delta \bar{\psi}^i = -i \epsilon \bar{\psi}^i$, where $\epsilon \equiv \frac{1}{2} \overline{[\xi^{\alpha} H_{\alpha\mu\nu}^{(-)}]}^{\mu\nu}$. Then, a straightforward calculation shows that

$$\delta_{\xi} (I_0 + I_1 + I_2) = \frac{\pi i}{32} \int \epsilon_{\alpha\beta\gamma\mu\nu\sigma} \overline{[J^{\alpha\beta\gamma}]}_{\mu\nu\sigma} \overline{[\xi^{\tau} \partial_{\tau} J^{\mu\nu}]}^{\nu\sigma} d^6 x. \quad (17)$$

The rhs of (17) vanishes for a 3-planar Σ , because in a suitable coordinate system, there are only three possible values for the indices $\beta, \gamma, \nu, \sigma$ for which the curvepole integral does not vanish. Thus, if C is 2-planar, we can deform Σ in the direction of an arbitrary 3-plane without affecting the theory. This demonstrates that the bulk of Σ is “incorporeal.” Recall that a 2-planar curvepole Σ is balanced. We now see that, even if only C is 2-planar, as long as Σ is 3-planar the theory is supersymmetric (up to quartic order), although perhaps not manifestly so.

V. DISCUSSION

We have constructed, up to quartic order, a supersymmetric Lagrangian that describes the interactions of a curvepole field with a tensor gauge field $B_{\mu\nu}$ of which the field strength is anti-self-dual. Requiring, in addition to supersymmetry, that the interactions be only through the boundary $C = \partial\Sigma$, in the sense of Sec. IV, determines the quartic terms uniquely.

The question of quintic and higher terms is open. Terms of order n (within I_{n-2}) have a total of $(n-2)$ $\square$'s and $\bar{\square}$'s. Basic dimensional analysis shows that no such terms are explicitly hollow for $n \geq 5$ (since each $\square$ would require an additional ∂ in order to be expressible as an integral over C). Thus, to preserve hollowness, we would expect the only modification to be that J^μ in (13) should be corrected from its zeroth order form (8) to include the contribution of interactions. It is easy to derive it order by order as the Noether current of the $U(1)$ -symmetry.

At order $n = 6$, possible anomalies should also be considered. The SUSY transformations (12) involve anomalous chiral rotations of fermions. However, since $\mathbf{V}$ is not a true gauge field, this is not expected to be a problem. In fact, without the chiral rotation, the SUSY algebra closes only up to an anomalous $U(1)$ gauge transformation, and hence supersymmetry would be anomalous itself (see Ref. [36]). The chiral rotation ensures that the anticommutator of two SUSY transformations is a translation, and hence is expected to be anomaly free. The fermionic field redefinitions of Sec. IV also have an anomaly proportional to $\int \epsilon(d\mathbf{V})^3$, and it is possible that the hollowness condition suffers from anomalies at $n \geq 6$. An anomaly would manifest itself as a nonzero δ_ϵ variation of a four-point correlator of tensor fields, of the form $\langle HHHH \rangle$. This variation, however, also receives a one-loop contribution from the field redefinition $\delta B_{\gamma\delta} = \frac{\pi i}{4} \epsilon_{\alpha\mu\nu\sigma\gamma\delta} \bar{\Sigma}^{\alpha} J^{\mu\nu\sigma}$, introduced in Sec. IV. The complete result, and in particular the question of whether a

hollowness anomaly exists or not, would require further analysis.

We expect the action constructed here to describe the low-energy degrees of freedom of the M-theory setup described at the end of the Introduction when the twist $\mathbf{h}$ has a nonzero kernel in the spinor representation (and hence preserves SUSY). It would be interesting to determine Σ from the twist. That setup has a formal dual with an M5-brane probing a background with strong 4-form flux [25,31]. The limit of small Σ might therefore be related to the order-by-order expansion of the interaction of an M5-brane with 4-form flux (see also Refs. [21,37–41]). It would also be interesting to connect the M-theory construction of Refs. [24,31] with the field theory construction of Ref. [42] (using Bagger-Lambert-Gustavsson theory [43,44]) and to generalize to other 6D theories [45–49].

Curvepole theories can be viewed as higher-dimensional analogs of dipole theories, and the latter have interesting supergravity duals [25] that have found applications in the study of rotating black holes [50,51]. We would expect a curvepole deformation to exist for any number of M5-branes, and it would be interesting to identify deformations of $\text{AdS}_7 \times S^4$ [52] that depend on Σ as a parameter.

Curvepole theories can also serve as novel kinds of UV completion of five-dimensional theories. Consider curvepole theory on $\mathbb{R}^5 \times S^1$, with $\Sigma = I \times S^1$, where $I \subset \mathbb{R}^5$ is a straight segment. Σ is 2-planar and hence balanced. We can take one end of the segment to ∞ and the other end at the origin. According to Sec. IV, it does not matter in which direction we take the segment. The theory is therefore local on $\mathbb{R}^5$, and at low energy, it becomes a $U(1)$ gauge field interacting with a charged hypermultiplet. (See Ref. [53] for a recent analysis of UV completions of supersymmetric five-dimensional theories.)

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