Equality of e^+e^- production amplitudes for scalar-vector and pseudoscalar-axial heavy meson-antimeson pairs

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The production of heavy meson-antimeson pairs of the type SV and PA in e^+e^- annihilation is considered, with P and V being the ground-state $J^P = 0^-$ and $J^P = 1^-$ (anti)mesons from the $(1/2)^-$ doublet and S and A standing for the excited $J^P = 0^+$ and $J^P = 1^+$ (anti)mesons from the $(1/2)^+$ doublet. It is argued that the production amplitudes in these two channels should be equal up to a higher (than one) order in the heavy quark mass (Λ_{QCD}/M_Q) expansion, $A(e^+e^- \rightarrow S\bar{V}) = A(e^+e^- \rightarrow A\bar{P})$, including both the S -wave and the D -wave amplitudes. Given that the SV and PA thresholds are extremely close, the production cross section in both channels should be the same to a high degree of accuracy. In practice, this behavior can be studied for the processes $e^+e^- \rightarrow D_{s0}(2317)\bar{D}_s^* + \text{c.c.}$ and $e^+e^- \rightarrow D_{s1}(2460)\bar{D}_s + \text{c.c.}$ in the charm sector and $e^+e^- \rightarrow B_{s0}\bar{B}_s^* + \text{c.c.}$ and $e^+e^- \rightarrow B_{s1}\bar{B}_s + \text{c.c.}$ in the B sector.

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Recently, the BESIII experiment has reported [1] an observation and analysis of the final state $D_{s0}(2317)\bar{D}_s^* + \text{c.c.}$ produced in the e^+e^- annihilation at a c.m. energy up to 4.6 GeV. In addition to other results of the study, this is in fact the first experimental evidence of a mixed $(1/2)^+ + (1/2)^-$ heavy meson pair production. The final states of this type are singled out among other combinations of heavy meson-antimeson pairs by the fact that only for these is a direct S -wave production in the e^+e^- annihilation allowed, and the observed angular distribution [1] is consistent with the S wave. Another well-known [2] peculiarity of the charmed-strange $(1/2)^+$ meson doublet is that the mass splitting between its heavier and lighter components $D_{s1}(2460) - D_{s0}(2317)$, $141.8 \pm 0.8 \text{ MeV}$, is almost exactly the same as in the ground-state $(1/2)^-$ doublet $D_s^* - D_s$: $143.8 \pm 0.4 \text{ MeV}$. For this reason, the thresholds in the channels $D_{s0}(2317)\bar{D}_s^* + \text{c.c.}$ and $D_{s1}(2460)\bar{D}_s + \text{c.c.}$ are split by only about 2 MeV and are thus almost degenerate, and a very similar (approximate) degeneracy is also expected for the bottom-strange mesons [3–5]. [For the nonstrange heavy mesons, such degeneracy is rather moot, since the excited $(1/2)^+$ mesons have a large width (up to 250–400 MeV) due to S -wave decay to a pion and their $(1/2)^-$ counterpart, e.g., $D_0^*(2400) \rightarrow D\pi$.] Furthermore, it

has been argued [6] on the basis of the heavy quark spin symmetry (HQSS) that in the limit of exact degeneracy only one coherent combination of the two channels is produced in e^+e^- annihilation: $(V_i\bar{S} - \bar{V}_iS + A_i\bar{P} - \bar{A}_iP)$, where P and V stand for the pseudoscalar and vector mesons, respectively, in the ground-state doublet and S and A denote the scalar and axial mesons, respectively, in the excited doublet. The coherence between the channels can be important in near-threshold “molecular” resonances, if such states exist, but the predictions from HQSS for the relative production of various meson-antimeson channels are usually strongly violated, as has been known since long ago [7], at the energy distance from the relevant thresholds that is comparable with the spin splittings between the meson masses [8]. The purpose of the present paper is to point out that the equality of the yield in the channels SV and PA is protected, in addition to the HQSS, by the chiral symmetry and parity conservation. The additional constraint arises in a scheme [3] where the observed equality of the mass splittings in the doublets is understood by considering the mesons in the $(1/2)^+$ and $(1/2)^-$ doublets as “parity doubles.”

Namely, if the mesons are described by the heavy spin multiplets (S, A_μ) and (P, V_μ) in the matrix form

$$\begin{aligned}\mathcal{H} &= \frac{1}{2}(i\gamma^5 P + \gamma^\mu V_\mu)(1 + \gamma \cdot v), \\ \mathcal{H}' &= \frac{1}{2}(i\gamma^5 S + \gamma^\mu A_\mu)(1 + \gamma \cdot v)\end{aligned}\quad (1)$$

with v being the 4-velocity, the combinations

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$$\mathcal{H}_L = \frac{1}{\sqrt{2}}(\mathcal{H} - i\mathcal{H}'), \quad \mathcal{H}_R = \frac{1}{\sqrt{2}}(\mathcal{H} + i\mathcal{H}') \quad (2)$$

transform as (3,1) and (1,3) representations, respectively, under the chiral $SU(3)_L \times SU(3)_R$ symmetry, where the flavor $SU(3)$ indices for the heavy mesons are understood. Clearly, the latter property implies that the spinless and spin-one mesons of the appropriate chirality, $H_L = (P - iS)/\sqrt{2}$, $H_R = (P + iS)/\sqrt{2}$, $H_L^\mu = (V^\mu - iA^\mu)/\sqrt{2}$, and $H_R^\mu = (V^\mu + iA^\mu)/\sqrt{2}$, have definite transformation properties under the chiral symmetry.

The meson-antimeson pairs under the present consideration can be produced in e^+e^- annihilation in the S and D waves. The data [1] indicate that the production in the channel $D_{s0}(2317)\bar{D}_s^* + \text{c.c.}$ at 4.6 GeV (i.e., approximately 170 MeV above the threshold) is mostly contributed by an S wave. However, there can be some contribution from a D wave whose relative significance can grow at a higher energy. Thus, one should generally consider both terms in the amplitude of the production of the meson pairs. The heavy meson pairs are produced by the electromagnetic current J_μ of the corresponding heavy quark (charm or bottom), which is a singlet under the chiral $SU(3)_L \times SU(3)_R$ symmetry as well as being parity negative. These constraints result in a unique expression for the amplitude that can be written in terms of the wave functions of the chiral combinations of the mesons. In the center of mass of the production process, where the current has only spatial components $\vec{J}$, this expression has the form

$$\begin{aligned} J_\ell &= i \left[C_S \delta_{\ell j} + C_D \left(p_\ell p_j - \frac{1}{3} \delta_{\ell j} \vec{p}^2 \right) \right] \\ &\quad \times [(\bar{H}_L H_{Lj} - \bar{H}_R H_{Rj}) - (\bar{H}_{Lj} H_L - \bar{H}_{Rj} H_R)] \\ &= \left[C_S \delta_{\ell j} + C_D \left(p_\ell p_j - \frac{1}{3} \delta_{\ell j} \vec{p}^2 \right) \right] \\ &\quad \times (V_j \bar{S} - \bar{V}_j S + A_j \bar{P} - \bar{A}_j P), \end{aligned} \quad (3)$$

where $\vec{p}$ is the vector of the c.m. momentum and C_S and C_D are scalar S - and D -wave amplitude factors, respectively, that are (generally complex) functions of the c.m. energy. Clearly, the presence in Eq. (3) of the $\vec{L} \times \vec{L}$ and $\vec{R} \times \vec{R}$ products results from the requirement for the current to be a chiral singlet, the relative coefficient (with the minus sign) is due to the parity, and the relative sign for the charge conjugate channels is due to the C conjugation property of the current. It is also clear that those are the first two requirements that set the relative coefficient of the $\bar{S}V$ and $\bar{P}A$ production amplitudes determining the relative yield in the two discussed channels not related by the charge conjugation. It should be noted that the expression in Eq. (3) does not rely on any assumptions about the full form of the current written in terms of full heavy symmetry multiplets from Eq. (1). The latter assumptions would relate

the amplitude in the considered channels to that in the heavier channel $A\bar{V} - \bar{A}V$ with any such relations being fully dependent on HQSS and generally affected by a significant violation in the threshold energy region. Thus, it should be expected that the relations between the production amplitudes, following from Eq. (3):

$$\begin{aligned} A(e^+e^- \rightarrow D_{s0}(2317)\bar{D}_s^*) &= A(e^+e^- \rightarrow D_s\bar{D}_{s1}(2460)), \\ A(e^+e^- \rightarrow B_{s0}\bar{B}_s^*) &= A(e^+e^- \rightarrow B_s\bar{B}_{s1}), \end{aligned} \quad (4)$$

should hold with accuracy beyond the first order in the HQSS breaking. In particular, these relations do not receive a first-order correction due to the term in the effective chiral-symmetric Lagrangian describing the HQSS-violating mass splittings in both doublets [3]:

$$\mathcal{L}_{\text{hyperfine}} = \frac{\kappa \Lambda_{\text{QCD}}^2}{12M_Q^2} \text{Tr}(\bar{\mathcal{H}}_L \sigma_{\mu\nu} \mathcal{H}_L \sigma^{\mu\nu} + \bar{\mathcal{H}}_R \sigma_{\mu\nu} \mathcal{H}_R \sigma^{\mu\nu}), \quad (5)$$

where κ is a coefficient of the order of one.

It is clear, however, that the HQSS-violating effects do modify in a higher order the degeneracy between the two discussed channels. One such effect is a purely kinematical one due to the mass splitting in the doublets. It lifts the degeneracy both at the level of the amplitudes and at the level of the cross section due to a (slightly) different c.m. momentum p in the two channels. Indeed, assuming that the mass splitting δ is the same in the two doublets, $M_V = M_P + \delta$, $M_A = M_S + \delta$, and using the notation $\Delta = M_S - M_P$, one readily finds at $E_{\text{c.m.}}^2 = s$ the ratio of the c.m. momenta in the two channels:

$$\begin{aligned} \frac{p_{\text{c.m.}}(SV)}{p_{\text{c.m.}}(AP)} &= \left[\frac{s - (M_S - M_V)^2}{s - (M_A - M_P)^2} \right]^{1/2} \\ &= \left[\frac{s - (\Delta - \delta)^2}{s - (\Delta + \delta)^2} \right]^{1/2} \approx 1 + \frac{2\Delta\delta}{s} \approx 1 + O\left(\frac{\Lambda_{\text{QCD}}^3}{M_Q^3}\right), \end{aligned} \quad (6)$$

where in the last transition it is taken into account that $\Delta \sim \Lambda_{\text{QCD}}$, $\delta \sim \Lambda_{\text{QCD}}^2/M_Q$, and $s \sim 4M_Q^2$ (or larger).

A potentially larger than the purely kinematical effect may arise in the order $\Lambda_{\text{QCD}}^2/M_Q^2$ from the mixing of the axial components of $(1/2)^+$ and $(3/2)^+$ heavy meson doublets (a discussion of such mixing for nonstrange B mesons can be found, e.g., in Ref. [9]). The HQSS-violating mixing, whose amplitude is generally of the order of Λ_{QCD}/M_Q , would enter quadratically in the relations (4) between the production amplitudes. Indeed, if $\theta \sim \Lambda_{\text{QCD}}/M_Q$ is the mixing angle, the coefficient of the axial A in Eq. (3) is modified by the factor $\cos\theta \sim 1 - O(\Lambda_{\text{QCD}}^2/M_Q^2)$. It is not clear at present how to estimate numerically a deviation from the equalities (4). However,

one may notice that the same mixing slightly shifts down (also in the second order) the mass of the axial meson in the $(1/2)^+$ doublet. Thus, if the (approximately) 2 MeV difference between the mass splittings in the $(1/2)^+$ and $(1/2)^-$ charmed doublets is attributed to this effect, its significance, in the scale of Λ_{QCD} , can be estimated as being of the order of 1%. It is then reasonable to expect a modification of the relations (4) by about the same amount in the charm sector.

As discussed, the equalities (4) between the production amplitudes rely on the chiral symmetry scheme [3], which symmetry is also only approximate and is violated by the light quark masses. This violation is especially significant for the strange heavy mesons due to a larger strange quark mass. However, the effect of this mass in the first order describes the flavor $SU(3)$ -breaking differences between the nonstrange and strange heavy mesons and apparently is absent (in this order) from the degeneracy relation for the intramultiplet mass splittings in the $(1/2)^+$ and $(1/2)^-$ charmed strange meson doublets, a relation that holds experimentally to a high accuracy. It appears not unreasonable to expect a similar significance of the strange quark mass in the production amplitude by the vector current of the heavy quarks; i.e., the effect can be substantial in the difference between the channels with and without hidden strangeness but significantly suppressed for the relations between the production of amplitudes for components of the same multiplets with the same hidden strangeness. It is clear, however, that a reliable theoretical understanding of the effects due to $m_s \neq 0$ is lacking, and an experimental test of the relations (4) can be quite instrumental in filling this gap.

The production amplitudes in Eq. (4) are necessarily complex, e.g., due to a rescattering between channels with and without the hidden strangeness. It should thus be emphasized that the equalities between them claimed here

imply that their absolute values should be the same (equal cross sections) and also their complex phases are the same. It can be noticed that both discussed channels end up in the same final state after strong and electromagnetic decays of excited heavy mesons; e.g., the meson pair $D_{s0}(2317)\bar{D}_s^*$ end up in $D_s\bar{D}_s\pi^0\gamma$ after the decays $D_{s0}(2317) \rightarrow D_s\pi^0$ and $\bar{D}_s^* \rightarrow \bar{D}_s\gamma$, while the pair $D_s\bar{D}_{s1}(2460)$ results in the same set of particles after $\bar{D}_{s1}(2460) \rightarrow \bar{D}_s^*\pi^0 \rightarrow \bar{D}_s\gamma\pi^0$. Thus, a study of the relative phase of the production amplitudes by interference effects is possible, in principle; however, in practice, the interference effects are greatly reduced by a very small region of the $D_s\bar{D}_s\pi^0\gamma$ phase space where the products from the two process chains kinematically overlap.

In summary, it is argued that within the chiral symmetry scheme, previously developed [3] for the description of the spectra of the ground-state and excited heavy mesons, the amplitudes of production in e^+e^- annihilation of mixed scalar-vector and axial-pseudoscalar pairs of heavy mesons are equal as described by Eq. (4). In particular, this implies the equality of the cross sections $\sigma[e^+e^- \rightarrow D_{s0}(2317)\bar{D}_s^* + \text{c.c.}] = \sigma[e^+e^- \rightarrow D_s\bar{D}_{s1}(2460) + \text{c.c.}]$. The relation should hold with an accuracy better than the first order in HQSS breaking and likely better than the first order in chiral symmetry breaking. An estimate of a specific numerical accuracy of the relation is not quite clear at present, but it can be as low as “in the ballpark” of a few percent. An observation of the former production process was recently reported [1] by BESIII. Thus, a test of the equality discussed here appears to be within the reach of present experiments.

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