S -wave $K\pi$ contributions to the hadronic charmonium B decays in the perturbative QCD approach

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We extend our recent works on the two-pion S -wave resonance contributions to the kaon-pion ones in the B meson hadronic charmonium decay modes based on the perturbative QCD approach. The S -wave $K\pi$ timelike form factor in its distribution amplitudes is described by the LASS parametrization, which consists of the $K_0^*(1430)$ resonant state together with an effective range nonresonant component. The predictions for the decays $B \rightarrow J/\psi K\pi$ in this work agree well with the experimental results from the *BABAR* and *Belle* collaborations. We also discuss theoretical uncertainties, indicating that the results of this work, which can be tested by the LHCb and *Belle-II* experiments, are reasonably accurate.

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I. INTRODUCTION

Within the quasi-two-body approximation, the B meson decays to the hadronic three-body final states can be restricted to the specific kinematical configurations, in which the three daughter mesons are quasialigned in the rest frame of the B meson. The related processes can be denoted as $B \rightarrow M_1(M_2M_3)$, where M_1 is the bachelor particle, and the remaining M_2M_3 pair proceeds via intermediate resonant states. The final state interactions are expected to be suppressed in such conditions. According to the kinematics and angular momentum conservation, the resonant states are predominantly found in the scalar (S -wave), vector (P -wave), or tensor (D -wave) meson spectrum, etc. Studies of the quasi-two-body decays will help us to clarify the nature of the resonances involved. The final state phase space can be represented in a Dalitz plot (DP), which provides information about the weak and strong phases in the decay processes.

Among the numerous three-body B decay channels, the category including a vector charmonium state and one kaon-pion pair via $b \rightarrow sc\bar{c}$ and $b \rightarrow dc\bar{c}$ transitions is particularly interesting. For instance, the interference between the S - and P -waves of the $K\pi$ systems produced in $B^0 \rightarrow J/\psi K\pi$ decays [1] allows one to resolve the sign ambiguity of $\cos(\beta)$, where β is the

Cabibbo-Kobayashi-Maskawa (CKM) phase. These decays have also been regarded as a source of information about the composition of the scalar-meson-like κ , which may exist as four-quark states in the low invariant mass regions [2]. Of course, the $K\pi$ pair is not the only resonance source. In recent years, many charmoniumlike resonant structures (a minimal quark content must be the exotic combination $c\bar{c}u\bar{d}$), such as $Z^+(4430)$, $Z^+(4050)$, and $Z^+(4250)$, have also been observed in the $\psi\pi$ [$\psi = J/\psi, \psi(2S)$] invariant mass spectrum in the $B \rightarrow \psi K\pi$ decays [3–8], which are not easy to accommodate in the quark model of hadrons [9,10]. DP analyses of such processes provide opportunities for the studies of the spectroscopy of these new structures.

Using the sample corresponds to an integrated luminosity of 413 fb^{-1} , the *BABAR* Collaboration studied the resonant structures in the $B \rightarrow J/\psi K\pi$ and $B \rightarrow \psi(2S)K\pi$ decays in Ref. [4]. The corresponding DP analyses in the two channels show important contributions in the $K\pi S$ -wave systems. Subsequently, the *Belle* Collaboration revealed a rich resonance spectrum in the $K\pi$ mass distribution based on a 711 fb^{-1} data sample collected at the asymmetric-energy e^+e^- collider KEKB [8,11]. They found clear evidence for the $K_0^*(1430)$ resonance with a 22.0σ significance for the decay modes including J/ψ , but only a 1.6σ signal for that of $\psi(2S)$ analogues. Furthermore, the $K_0^*(1430)$ fit fraction in $B \rightarrow \psi(2S)K\pi$ was comparable with its previous measurements [6] and the LHCb's data [12]. Recently, the $B_s \rightarrow \psi(2S)K\pi$ decay mode was first observed by the LHCb Collaboration; the fit fractions of the S -wave component reach 0.339 ± 0.052 , with statistical uncertainty only [13].

On the theoretical side, several approaches have been used for describing charmless three-body B decays involving $K\pi$ systems. For example, in Refs. [14–18], the authors

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predicted the branching ratios and direct CP violations in charmless three-body decays $B \rightarrow K\pi\pi$ and $B \rightarrow KK\pi$ using a model based on the factorization approach. The method was extended further in charmless three-body B_s decays in Ref. [19]. In Ref. [20], the CP violation and the contributions of the strong kaon-pion interactions have been studied in $B \rightarrow K\pi\pi$ decays using an approximate construction of relevant scalar and vector form factors. The K^* resonance effects on direct CP violation have been taken into account based on the QCD factorization scheme [21], while the three-body B meson decays with the charmonium mesons in the final states have not received much attention in the literature.

In our previous works, the decays $B_{(s)} \rightarrow (J/\psi, \eta_c)\pi\pi$ [22,23], as well as the corresponding $\psi(2S), \eta_c(2S)$ modes [24–26], with the pion pair in S -wave resonant states, have been studied in the perturbative QCD (PQCD) [27–29] framework by introducing two-pion distribution amplitudes for the resonances [30,31]. These processes have been well described by a series of scalar resonances such as $f_0(500)$, $f_0(980)$, $f_0(1500)$, $f_0(1790)$, and so on. In the present paper, motivated by the recent detailed DP analyses of the $K\pi$ invariant mass spectrum by the BABAR [4], Belle [8,11], and LHCb [12] collaborations, we will work on the decays of $B_{(s)} \rightarrow (J/\psi, \psi(2S))K\pi$, and we will focus on the $K\pi$ pair originating from a scalar quark-antiquark state, while other partial wave and charmoniumlike resonances are beyond the scope of the present analysis. The S -wave contributions are parametrized into the timelike scalar form factors involved in the kaon-pion distribution amplitudes. For these form factors, we will adopt the LASS parametrization in Ref. [32], which consists of a linear combination of the $K_0^*(1430)$ resonance and a nonresonant term coming from elastic scattering. By introducing the kaon-pion distribution amplitudes, the S -wave contributions of the related three-body B decays can be simplified into the quasi-two-body processes $B \rightarrow \psi(K\pi)_{S\text{-wave}} \rightarrow \psi K\pi$. Following the steps of Refs. [22,26], the decay amplitude of $B \rightarrow \psi(K\pi)_{S\text{-wave}}$ can be written as the convolution

$$\mathcal{A}(B \rightarrow \psi(K\pi)_{S\text{-wave}}) = \Phi_B \otimes H \otimes \Phi_{K\pi}^{S\text{-wave}} \otimes \Phi_\psi, \quad (1)$$

where the hard kernel H includes the leading-order contributions plus next-to-leading-order (NLO) vertex corrections.

FIG. 1. The leading-order Feynman diagrams for the quasi-two-body decays $B \rightarrow \psi K_0^* \rightarrow \psi K\pi$. The first two are factorizable and the last two are nonfactorizable, and K_0^* is the S -wave intermediate state.

The B meson (charmonium, S -wave $K\pi$ pair) distribution amplitude ϕ_B (ϕ_ψ , $\phi_{K\pi}^{S\text{-wave}}$) absorbs the nonperturbative dynamics in the hadronization processes.

The layout of this paper is as follows. In Sec. II, elementary kinematics, meson distribution amplitudes, and the required timelike scalar form factor are described. In Sec. III, we present a discussion following the presentation of the significant results on branching ratios. Finally, Sec. IV will be the conclusion of this work.

II. FRAMEWORK

In the light-cone coordinates, the kinematic variables of the decay $B(p_B) \rightarrow \psi(p_\psi)(K\pi)(p)$ can be described in the B meson rest frame as

$$p_B = \frac{M}{\sqrt{2}}(1, 1, \mathbf{0}_T), \quad p_3 = \frac{M}{\sqrt{2}}(r^2, 1 - \eta, \mathbf{0}_T),$$

$$p = \frac{M}{\sqrt{2}}(1 - r^2, \eta, \mathbf{0}_T), \quad (2)$$

with the mass ratio $r = m/M$, and where $m(M)$ is the mass of the charmonium (B) meson, the variable $\eta = \omega^2/(M^2 - m^2)$, and the invariant mass squared $\omega^2 = p^2$ for the kaon-pion pair. As usual, we also define the kaon momentum p_1 and pion momentum p_2 as

$$p_1 = \frac{M}{\sqrt{2}}((1 - r^2)\xi, \eta(1 - \xi), \mathbf{p}_{1T}),$$

$$p_2 = \frac{M}{\sqrt{2}}((1 - r^2)(1 - \xi), \eta\xi, \mathbf{p}_{2T}), \quad (3)$$

with ξ being the kaon momentum fraction. These momenta satisfy the momentum conservation $p = p_1 + p_2$. The three-momenta of the kaon and charmonium in the $K\pi$ center of mass are given by

$$|\vec{p}_1| = \frac{\sqrt{\lambda(\omega^2, m_K^2, m_\pi^2)}}{2\omega}, \quad |\vec{p}_3| = \frac{\sqrt{\lambda(M^2, m^2, \omega^2)}}{2\omega}, \quad (4)$$

respectively, with m_K (m_π) the kaon (pion) mass and the Källén function $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. For the valence quarks, momenta k_B, k_3, k , whose notations are displayed in Fig. 1, are chosen as

$$\begin{aligned}
k_B &= \left(0, \frac{M}{\sqrt{2}}x_B, \mathbf{k}_{BT}\right), \\
k_3 &= \left(\frac{M}{\sqrt{2}}r^2x_3, \frac{M}{\sqrt{2}}(1-\eta)x_3, \mathbf{k}_{3T}\right), \\
k &= \left(\frac{M}{\sqrt{2}}z(1-r^2), 0, \mathbf{k}_T\right),
\end{aligned} \quad (5)$$

where k_{iT} , x_i represent the transverse momentum and longitudinal momentum fraction of the quark inside the meson, respectively.

The B meson can be treated as a heavy-light system, whose wave function in impact coordinate space can be expressed by [27]

$$\Phi_B(x, b) = \frac{i}{\sqrt{2N_c}} [(\not{p}_B + M)\gamma_5 \phi_B(x, b)], \quad (6)$$

where b is the conjugate variable of the transverse momentum of the valence quark of the meson, and N_c is the color factor. The distribution amplitude $\phi_B(x, b)$ is adopted in the same form as it was in Refs. [27,33]:

$$\phi_B(x, b) = Nx^2(1-x)^2 \exp\left[-\frac{x^2 M^2}{2\omega_b^2} - \frac{\omega_b^2 b^2}{2}\right], \quad (7)$$

with shape parameter $\omega_b = 0.40 \pm 0.04$ GeV for the $B_{u,d}$ mesons and $\omega_b = 0.50 \pm 0.05$ GeV for the B_s meson. The normalization constant N is related to the decay constant f_B through

$$\int_0^1 \phi_B(x, b=0) dx = \frac{f_B}{2\sqrt{2N_c}}. \quad (8)$$

For the considered decays, the vector charmonium meson is longitudinally polarized. The longitudinal polarized component of the wave function is defined as [34,35]

$$\Phi_\psi^L = \frac{1}{2\sqrt{N_c}} [m \not{\epsilon}_L \phi^L(x, b) + \not{\epsilon}_L \not{p}_3 \phi^t(x, b)], \quad (9)$$

with the longitudinal polarization vector $\epsilon_L = \frac{1}{\sqrt{2r}}(-r^2, 1-\eta, \mathbf{0}_T)$. For the twist-2 (twist-3) distribution amplitudes $\phi^L(\phi^t)$, the same form and parameters are adopted as in Refs. [34,35].

The S -wave kaon-pion distribution amplitudes are introduced in analogy with the case of two-pion ones [22,30], which are organized into

$$\begin{aligned}
\Phi_{K\pi}^{S\text{-wave}} &= \frac{1}{\sqrt{2N_c}} [\not{p} \phi_{v\mu=-}^{I=1/2}(z, \xi, \omega^2) + \omega \phi_s^{I=1/2}(z, \xi, \omega^2) \\
&\quad + \omega (\not{p} \not{p} - 1) \phi_{\mu=+}^{I=1/2}(z, \xi, \omega^2)],
\end{aligned} \quad (10)$$

where $n = (1, 0, \mathbf{0}_T)$ and $v = (0, 1, \mathbf{0}_T)$ are two dimensionless vectors. For $I = \frac{1}{2}$, $\phi_{v\mu=-}^{I=1/2}$ contributes at twist-2, while

$\phi_s^{I=1/2}$ and $\phi_{\mu=+}^{I=1/2}$ contribute at twist-3. It is worthwhile to stress that this kaon-pion system has similar asymptotic distribution amplitudes (DAs) as the ones for a light scalar meson [36], but we replace the scalar decay constants with the timelike form factor:

$$\begin{aligned}
\phi_{v\mu=-}^{I=1/2}(z, \xi, \omega^2) &= \phi^0 = \frac{3}{\sqrt{2N_c}} F_s(\omega^2) z(1-z) \\
&\quad \times \left[\frac{1}{\mu_S} + B_1 3(2z-1) \right], \\
\phi_s^{I=1/2}(z, \xi, \omega^2) &= \phi^s = \frac{1}{2\sqrt{2N_c}} F_s(\omega^2), \\
\phi_{\mu=+}^{I=1/2}(z, \xi, \omega^2) &= \phi^t = \frac{1}{2\sqrt{2N_c}} F_s(\omega^2)(1-2z),
\end{aligned} \quad (11)$$

where $\mu_S = \frac{m_S}{m_2 - m_1}$ and m_S and $m_{1,2}$ are the scalar meson mass and running current quark masses, respectively; their values can be found in Refs. [37–39]. B_1 is the first odd Gegenbauer moment for the light scalar mesons. According to Refs. [37,38,40], there are two scenarios for the scalar mesons. In scenario I, all scalar mesons are viewed as the conventional two-quark states. In scenario II, the light scalar mesons below or near 1 GeV are treated as the four-quark states, while those above 1 GeV scalar mesons such as $f_0(1370)$, $K_0^*(1430)$, $a_0(1450)$, and so on are regarded as the ground states of $q\bar{q}$. As noticed, scenario II is more favored for explaining the $B^+ \rightarrow K_0^*(1430)\pi^+$ data measured by both BABAR [41] and Belle [42]. Besides, scenario II is also supported by a lattice calculation [43] and the recent Regge trajectory calculation [44]. Hence, we prefer to use the Gegenbauer moments $B_1 = -0.57 \pm 0.13$ at the 1 GeV scale in scenario II obtained using the QCD sum rule method [37,38].

As is known, the relativity Breit-Wigner (RBW) model is unsuitable for describing the $K\pi S$ -wave contributions because the broad κ and $K_0^*(1430)$ resonance interferes strongly with a slowly varying nonresonant (NR) component. Detailed discussions of the S -wave $K\pi$ systems in the isobar model, K -matrix model, and model-independent partial wave analysis method can be found in Refs. [45–48]. In this work, we parametrize the timelike scalar form factor $F_s(\omega^2)$ for the S -wave $K\pi$ systems by the LASS line shape [32], which has been widely adopted in the experimental data analysis; its expression is given as [4,49]

$$F_s(\omega^2) = \frac{\omega}{|\vec{p}_1|[\cot(\delta_B) - i]} + e^{2i\delta_B} \frac{m_0^2 \Gamma_0 / |\vec{p}_{10}|}{m_0^2 - \omega^2 - im_0^2 \frac{\Gamma_0}{\omega} \frac{|\vec{p}_{10}|}{|\vec{p}_1|}}, \quad (12)$$

where the first term is an empirical term from inelastic scattering and the second term is the resonant contribution with a phase factor to retain unitarity. $m_0 = 1.435$ GeV and $\Gamma_0 = 0.279$ GeV [4] are the pole mass and width of the

$K_0^*(1430)$ resonance state. $|\vec{p}_{10}|$ is $|\vec{p}_1|$ evaluated at the $K\pi$ pole mass. The phase factor $\cot(\delta_B)$ is defined as

$$\cot(\delta_B) = \frac{1}{a|\vec{p}_1|} + \frac{r|\vec{p}_1|}{2}, \quad (13)$$

with the shape parameters $a = 1.94$ and $r = 1.76$ [4].

The differential branching ratio for the $B \rightarrow \psi(K\pi)_{S\text{-wave}}$ decay takes the explicit form

$$\frac{d\mathcal{B}}{d\omega} = \frac{\tau\omega|\vec{p}_1||\vec{p}_3|}{32\pi^3 M^3} |\mathcal{A}|^2. \quad (14)$$

Since the S -wave kaon-pion distribution amplitude in Eq. (10) has the same Lorentz structure as that of two-pion ones in Ref. [22], the decay amplitude $\mathcal{A}$ here can be straightforwardly obtained just by replacing the twist-2 or twist-3 DAs of the $\pi\pi$ system with the corresponding twists of the $K\pi$ one in Eq. (11). In addition, we also consider the NLO vertex corrections to the factorizable diagrams in Fig. 1,

whose effects are included by the modifications to the Wilson coefficients as usual [50–52].

III. RESULTS

In the numerical calculations, parameters such as the meson mass, the Wolfenstein parameters, the decay constants, and the lifetime of B_s mesons are presented in Table I. Other parameters relevant to the kaon-pion DAs have been given in the second section.

By using Eq. (14), integrating separately for the $K_0^*(1430)$ resonant and nonresonant components, as well as their coherent sum, we obtained the CP -averaged branching ratios for the considered decays, which are shown in Table II together with some of the experimental measurements. Since the charged and neutral decay modes differ only in the lifetimes of B^0 and B^+ in our formalism, we can obtain the branching ratios of charged decay modes by multiplying the neutral ones by the lifetime ratio τ_{B^+}/τ_{B^0} . Some dominant uncertainties are considered in

TABLE I. The decay constants of the J/ψ and $\psi(2S)$ meson are from [34,35], while the other parameters are adopted in PDG 2016 [53] in our numerical calculations.

Mass (GeV)	$M_B = 5.28$ $m_{\psi(2S)} = 3.686$	$M_{B_s} = 5.37$ $m_{J/\psi(2S)} = 3.097$	$m_b = 4.66$ $m_K = 0.494$	$m_c = 1.275$ $m_\pi = 0.14$
Wolfenstein parameters	$\lambda = 0.22506,$	$A = 0.811,$	$\bar{\rho} = 0.124,$	$\bar{\eta} = 0.356$
Decay constants (MeV)	$f_B = 190.9 \pm 4.1$	$f_{B_s} = 227.2 \pm 3.4$	$f_{\psi(2S)} = 296_{-2}^{+3}$	$f_{J/\psi} = 405 \pm 14$
Lifetime (ps)	$\tau_{B_s} = 1.51$	$\tau_{B_0} = 1.52$		$\tau_{B^+} = 1.638$

TABLE II. The PQCD predictions for the CP -averaged branching ratios from various components together with the S -wave contribution for the considered decays. The theoretical errors correspond to the uncertainties due to the shape parameters ω_b in the wave function of the $B_{(s)}$ meson, the heavy quark masses m_b and m_c , the Gegenbauer moment B_1 , and the hard scale t , respectively. The experimental results are obtained by multiplying the fit fractions by the measured three-body branching ratios, where all errors are combined in quadrature.

Components	$B^0 \rightarrow J/\psi K^+ \pi^- \quad (10^{-5})$		$B^0 \rightarrow \psi(2S) K^+ \pi^- \quad (10^{-5})$	
	This work	Data	This work	Data
$K_0^*(1430)$	$6.1_{-0.8-0.5-0.8-0.0}^{+1.4+1.1+1.5+0.4}$	$6.8_{-0.6}^{+0.8} \text{ }^a$	$0.9_{-0.1-0.1-0.1-0.0}^{+0.1+0.3+0.1+0.1}$	$1.7 \pm 0.5 \text{ }^b$
LASS NR	$7.2_{-1.0-0.3-0.9-0.5}^{+2.2+1.1+1.2+1.0}$	...	$1.5_{-0.3-0.2-0.2-0.1}^{+0.3+0.3+0.3+0.1}$	...
LASS S -wave	$14.8_{-2.7-1.1-2.7-1.1}^{+3.7+2.2+3.2+1.8}$	$16.9_{-0.8}^{+0.9} \text{ }^c$	$3.4_{-0.5-0.5-0.7-0.2}^{+0.6+0.6+0.6+0.3}$	$14.1 \pm 1.4 \text{ }^c$
Components	$B_s^0 \rightarrow J/\psi K^- \pi^+ \quad (10^{-6})$		$B_s^0 \rightarrow \psi(2S) K^- \pi^+ \quad (10^{-6})$	
	This work	Data	This work	Data
$K_0^*(1430)$	$4.1_{-0.5-0.3-0.9-0.3}^{+0.7+0.6+1.1+0.8}$	...	$0.8_{-0.1-0.1-0.2-0.0}^{+0.1+0.2+0.2+0.1}$	...
LASS NR	$4.4_{-0.7-0.3-0.8-0.4}^{+0.6+0.6+1.0+0.6}$	...	$1.0_{-0.2-0.1-0.2-0.1}^{+0.3+0.3+0.2+0.2}$	...
LASS S -wave	$8.3_{-2.7-1.1-2.7-1.1}^{+3.7+2.2+3.2+1.8}$	...	$2.2_{-0.4-0.2-0.4-0.2}^{+0.6+0.6+0.4+0.4}$	$10.5 \pm 2.1 \text{ }^d$

^aThe fit fractions and the measured values for $\mathcal{B}(B^0 \rightarrow J/\psi K^+ \pi^-)$ are given in [8].

^bThe fit fraction is obtained from a weighted average of three measurements by Belle [6,11] and LHCb [12], while the measured value for $\mathcal{B}(B^0 \rightarrow \psi(2S) K^+ \pi^-)$ is given in PDG [53].

^cThe fit fractions and the measured values for $\mathcal{B}(B^0 \rightarrow (J/\psi, \psi(2S)) K^+ \pi^-)$ are given in [4].

^dThe fit fractions and the measured values for $\mathcal{B}(B_s^0 \rightarrow \psi(2S) K^- \pi^+)$ are given in [13].

our calculations. The first error is caused by the shape parameter ω_b in the $B_{(s)}$ meson wave function. The second error comes from the uncertainty of the heavy quark masses. In the evaluation, we vary the values of $m_{c(b)}$ within a 20% range. The third error is induced by the Gegenbauer moment $B_1 = -0.57 \pm 0.13$ [37,38]. The last one is caused by the variation of the hard scale from $0.75t$ to $1.25t$, which characterizes the size of the NLO QCD contributions. The first three errors are comparable and contribute the main uncertainties in our approach. While the last scale-dependent uncertainty is less than 20% due to the inclusion of the NLO vertex corrections. The errors from the uncertainty of the CKM matrix elements and the decay constants of charmonia are very small and have been neglected. We have checked the sensitivity of our results to the choice of the shape parameters a and r [see Eq. (13)] in the LASS parametrization. Some experimental groups [41,49] prefer to choose another set of solutions with $a = 2.07$ and $r = 3.32$. Using the above values, we find that the branching ratios displayed in Table II decrease by only a few percent.

From Table II, we find that the $K_0^*(1430)$ resonance accounts for 41% of the branching fraction and the LASS NR term accounts for 49%. The constructive interference between them is responsible for the remaining 10% in the $B \rightarrow J/\psi K\pi$ decays. For the corresponding $\psi(2S)$ modes, since the $K_0^*(1430)$ resonance region is very close to the upper limits of the $K\pi$ invariant mass spectra, the resonance contribution is suppressed to 25% of the total S -wave decay fraction. A similar situation also exists in the Cabbibo suppressed B_s^0 decay modes. All these channels receive a relatively large contribution from the LASS NR, which involved the component of κ resonance as mentioned in [12]. In fact, the κ fit fractions from both the Belle [6,8,11] and LHCb [12] measurements are larger than that of $K_0^*(1430)$ resonance.

As for the data, the fit fractions determined from the Dalitz plot analyses can be converted into quasi-two-body branching fractions by multiplying the corresponding branching fractions of the three-body decays. Taking the $B^0 \rightarrow J/\psi K^+ \pi^-$ decay as an example, based on the fit fraction of the $K_0^*(1430)$ component, which was measured to be $f_{K_0^*(1430)} = (5.9_{-0.4}^{+0.6})\%$ with a significance of 22.0σ by the Belle Collaboration [8], we have the center value of the quasi-two-body branching fraction

$$\begin{aligned} \mathcal{B}(B^0 \rightarrow J/\psi K_0^*(1430) \rightarrow J/\psi K^+ \pi^-) \\ = f_{K_0^*(1430)} \times \mathcal{B}(B^0 \rightarrow J/\psi K^+ \pi^-) = 6.8 \times 10^{-5}. \end{aligned} \quad (15)$$

Other available fit fractions are also converted into branching fraction measurements, which are listed in Table II. It is shown that the model calculations presented here are described reasonably well for the J/ψ mode, but less so for the case of $\psi(2S)$, especially for the S -wave contributions, which fall short by a large factor. It is worth noting

that the fit fractions for the $\psi(2S)$ modes have much larger relative errors because of limited statistics. For instance, the previous Belle Collaboration gives the fit $f_{K_0^*(1430)} = (5.3 \pm 2.6)\%$ [6], while the subsequent measurements from the Belle and LHCb collaborations are $(1.1 \pm 1.4)\%$ [11] and $(3.6 \pm 1.1)\%$ [12], respectively. Including the errors, all three measurements agree with one another. In Table II, we calculate a weighted average and error from them as $f_{K_0^*(1430)} = (2.9 \pm 0.8)\%$, which is closer to the LHCb data [12]. On the other hand, comparing with the ground state J/ψ modes, the branching ratio of the radially excited charmonium modes should be relatively small, owing to the phase space suppression and smaller decay constants. In Table II, our prediction of the S -wave branching ratio for $B^0 \rightarrow \psi(2S) K^+ \pi^-$ is a few times smaller than that of $B^0 \rightarrow J/\psi K^+ \pi^-$. However, the data from BABAR show the same order of magnitude between the two channels. Such a difference should be clarified in the forthcoming experiments based on much larger data samples.

The B_s^0 decay modes can be theoretically related to the counterpart B^0 decays since they have identical topology and similar kinematic properties in the limit of $SU(3)$ flavor symmetry. The relative ratios of the branching fractions for B_s^0 and B^0 decay modes are dominated by a Cabibbo suppression factor of $|V_{cd}|^2/|V_{cs}|^2 \sim \lambda^2$ under the naive factorization approximation. From Table II, one can see that the B_s channels have relatively small branching ratios (10^{-6}). Experimentally, the fraction of $B_s^0 \rightarrow \psi(2S) K^- \pi^+$ decay proceeding via an S -wave is measured to be $f_{S\text{-wave}} = 0.339 \pm 0.052$ by the LHCb experiment [13], with statistical uncertainty only. Although the signal $B_s^0 \rightarrow J/\psi K^- \pi^+$ is found with a 4.7σ significance by the LHCb experiment [54] using a mass window of ± 150 MeV around the nominal K^{*0} mass, the small size of the data sample does not permit the determination of $K_0^*(1430)$ and the S -wave fraction itself.

In Fig. 2, we plot the differential branching ratios as functions of the $K\pi$ invariant mass ω for the considered decays. The red (solid) curve denotes the total S -wave contribution, while individual terms are given by the blue (dotted) curve for $K_0^*(1430)$ resonance and green (dashed) curve for LASS NR contributions. Note that the $J/\psi - \psi(2S)$ mass difference causes significant differences in the range spanned in the respective decay modes. As expected, the contributions from LASS NR and the $K_0^*(1430)$ resonance are of comparable size. For the J/ψ modes, the dip region near 1.6 GeV is caused by strongly destructive interference between the resonance and non-resonant part of the LASS parametrization. From Eq. (12), one can estimate that the magnitude of these two terms is approximately equal, and the phase difference is roughly π around the 1.6 GeV regions. Experimentally, it is usually interpreted as resulting from interference between the $K_0^*(1430)$ and its first radial excitation [4]. However, the

FIG. 2. The ω dependence of the differential decay rates $dB/d\omega$ for the decay modes (a) $B^0 \rightarrow J/\psi K^+ \pi^-$, (b) $B^0 \rightarrow \psi(2S) K^+ \pi^-$, (c) $B_s^0 \rightarrow J/\psi K^- \pi^+$, and (d) $B_s^0 \rightarrow \psi(2S) K^- \pi^+$ with a logarithmic y-axis scale. The resonance $K_0^*(1430)$ and LASS nonresonant components are shown by the dotted blue and dashed green curves, respectively, while the solid red curves represent the total S -wave contributions.

dip is not seen in Figs. 2(b) and 2(d) because its region is beyond the $K\pi$ invariant mass spectra for the $\psi(2S)$ modes. Comparing with the $K\pi$ mass distributions obtained by *BABAR* (Fig. 11 of Ref. [4]) and *LHCb* (Fig. 2 of Ref. [55]), our distribution for the S -wave contribution agrees fairly well, showing a similar behavior.

IV. CONCLUSION

Motivated by the phenomenological importance of the hadronic charmonium B decays, in the present work we have carried out analyses of the $B_{(s)}^0 \rightarrow \psi K \pi$ decays within the framework of the PQCD factorization approach by introducing the kaon-pion distribution amplitudes. Both the S -wave resonant and nonresonant components are parametrized into the timelike scalar form factors, which can be described by the LASS line shape. It is worth noting that fractions of the resonant and nonresonant components in these decays are comparable in size. Our predicted S -wave decay spectrum in the kaon-pion pair invariant mass show a

similar behavior as the experiment. In particular, the $K_0^*(1430)$ production in the $B^0 \rightarrow J/\psi K \pi$ decay agrees well with the results of a recent Dalitz plot analysis by the Belle Collaboration. Nevertheless, for the case of $\psi(2S)$ modes, our results for the S -wave branching ratios turn out to be lower than the data. For the B_s decays, an amplitude analysis to determine the fraction of decays proceeding via an intermediate $K_0^*(1430)$ meson is still missing. We expect the relevant results could be tested by future experimental measurements.

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