Gauge and parametrization ambiguity in quantum gravity

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The gauge and parametrization dependence is discussed in quantum gravity in an arbitrary dimension D . Explicit one-loop calculations are performed within the most general parametrization of quantum metric with seven arbitrary parameters. On the other hand, some of the gauge fixing parameters are fixed to make the calculations relatively simple. We confirm the general theorem stating that the on shell local terms in the one-loop effective action are independent of the gauge and parametrization ambiguity.

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I. INTRODUCTION

Loop calculations traditionally play an important role in the understanding of quantum gravity (QG). The famous pioneer works in this direction were done by 'tHooft and Veltman [1], and Deser and van Nieuwenhuisen [2] for quantum general relativity (GR), including the interaction with scalar and vector quantum fields. It was shown that the one-loop divergences in pure quantum gravity do vanish on shell, but the interaction with matter fields always destroys this nice feature. The dependence on the choice of the gauge fixing conditions was first explored by Kallosh *et al.* [3]. This complicated calculation has been performed with a general two-parameter gauge condition. It was shown that, by means of the gauge-fixing choice, the one-loop divergences can be reduced to the single topological term which does not affect the S matrix for gravitons. Of course, this result is completely consistent with the one [1] for the pure quantum gravity without matter fields or sources.

It is clear that the derivation of divergences, beta functions, and alike in QG is only the first step, which has not much sense without taking care of the ambiguities

concerning the gauge fixing and, most difficult, the dependence on the parametrization of the quantum field.

The two-loop calculations in quantum GR [4,5] (see also the recent verification by more advanced methods in [6]) confirmed that even the theory of pure QG is nonrenormalizable. In particular, the two-loop S matrix can not be done finite in a consistent way. At the same time, the main attention was always attracted by the one-loop results, since they have especially interesting applications. In this respect, one can mention the asymptotic safety program in QG [7,8] and effective quantum gravity approach [9,10]. In the last case, the analysis based on the gauge independence of the S -matrix elements proved to be useful [11]. After all, we can state that it is important to know the level of ambiguity for the one-loop divergences in quantum GR, both the logarithmic and quadratic ones.

The general algorithm to explore the gauge-fixing ambiguities in the effective action of gauge theories is well-known [12] (see also [13] for a simplified one-loop version). And since QG is a particular example of gauge theories, one can easily establish how the effective action depends on the gauge fixing condition at the general level and also for the particular gauge fixing schemes (see, e.g., [14]). At the general level, the issue was elaborated in the paper of Fradkin and Tseytlin devoted mainly to the fourth-derivative models of QG [15] (see also [16,17] and finally, [18]). In brief, we know that the one-loop divergences (and also leading divergences at higher loops) are gauge-fixing independent on the classical mass shell (we call it simply *on shell* in what follows). In principle, the same should be true

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for the reparametrization ambiguity. At the same time, it is sometimes useful to verify the general statements by a direct calculations, and in the case of QG, this was done in several publications, at different levels of generality and consistency. After the pioneer work [3] which explored the gauge-fixing dependence, there were further publications [19–21] exploring also the parametrization dependence. In [19,20], this was done by the direct and extremely cumbersome calculation, based on the heavy use of a computer. The disadvantage of this approach is, in particular, the fact that this algebra is rather difficult to reproduce. Contrary to this, in the work of our group [21], qualitatively the same result was achieved by a relatively simple handmade approach, which will be essentially generalized below. In both cases, it was confirmed that the parametrization dependence vanishes on shell.

Recently, there were some works published which again reconsider the issue of parametrization and gauge dependence in quantum GR [22,23]. The main difference with the previous papers [3,19–21] is that in the publications [22,23], the background is not assumed to satisfy the classical equations of motion. Instead, the background metric has a special form which is motivated by the arguments of simplicity. In some cases, it is claimed that there is a gauge-fixing independence for these special backgrounds. At the same time, the general statements about ambiguities in gauge theories [12,15] tell us that this independence can be hardly achieved for the most general choice of parametrization and gauge fixing. Motivated by these recent works, we extend the previous analysis of [21] and consider the most general possible parametrization of a quantum metric, while the background metric is not constrained. In principle, our results can be used to reproduce the calculations on any particular background, being motivated by simplicity, physical arguments, etc. At the same time, our calculations include a strong control of correctness, by verifying the general statement of an on shell universality of the results.

The paper is organized as follows. In Sec. II, we present a simple introductory-style analysis of ambiguities of the one-loop divergences in quantum GR with a cosmological constant. In Sec. III, one can find the details of the background field method in QG, including the most general parametrization of a quantum metric and the most general linear gauge-fixing condition. In Sec. IV, the conformal symmetry fixing and the particular form of the general gauge fixing conditions are described, which makes calculations less complicated while maintaining almost general choice of parametrization of quantum variables. The derivation of divergences is reported in Sec. V. Furthermore, Sec. VI is about the on shell limit of the result for the D -dimensional quantities which become logarithmic and quadratic divergences at $D = 4$. Finally, in Sec. VII, we draw our conclusions.

II. GAUGE AND PARAMETRIZATION AMBIGUITIES IN ONE-LOOP GR

Consider the one-loop effects in GR with a cosmological constant. For the sake of generality, the calculations will be performed in a generic D -dimensional space-time. The metric is supposed to have a Minkowski signature $(+, -, -, \dots)$. Let us note that the use of heat-kernel methods require the usual analytic continuation to Euclidean space. We assume this operation without special explanations. The Einstein-Hilbert action has the form

$$S = -\frac{1}{\kappa^2} \int d^D x \sqrt{-g} (R + 2\Lambda), \quad (1)$$

where $\kappa^2 = 16\pi G$. The equations of motion are

$$\epsilon^{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} = R^{\mu\nu} - \frac{1}{2} (R + 2\Lambda) g^{\mu\nu}. \quad (2)$$

Let us use the general statement about gauge-fixing and parametrization independence on shell for the local part of the effective action. For the sake of simplicity, we consider the application of this rule to the divergences in $D = 4$. Then the power-counting arguments tell us that the divergent part of the one-loop effective action is

$$\Gamma_{\text{div}}^{(1)} = \frac{1}{\epsilon} \int d^4 x \sqrt{-g} \{ c_1 R_{\mu\nu\alpha\beta}^2 + c_2 R_{\alpha\beta}^2 + c_3 R^2 + c_4 \square R + c_5 R + c_6 \}, \quad (3)$$

where $1/\epsilon$ is the divergent coefficient.

According to the Weinberg theorem [24], the ambiguity in $\Gamma_{\text{div}}^{(1)}$ leaves this expression local. Then the mentioned feature of on shell universality tells us that the ambiguity has the form

$$\begin{aligned} \delta\Gamma_{\text{div}}^{(1)} &= \Gamma_{\text{div}}^{(1)}(\alpha_i) - \Gamma_{\text{div}}^{(1)}(\alpha_i^0) \\ &= \frac{1}{\epsilon} \int d^4 x \sqrt{-g} (b_1 R_{\mu\nu} + b_2 R g_{\mu\nu} + b_3 g_{\mu\nu} \Lambda \\ &\quad + b_4 g_{\mu\nu} \square + b_5 \nabla_\mu \nabla_\nu) \epsilon^{\mu\nu}, \end{aligned} \quad (4)$$

where α_i represent the full set of arbitrary parameters which characterize the ambiguity in the choice of gauge-fixing and parametrization of quantum metric. The special values α_i^0 correspond to some special choice of these parameters, e.g., to the ones which was used in the original paper of 'tHooft and Veltman [1].

The parameters $b_{1,2,\dots,5}$ in (4) depend on the choice of α_i , and the explicit form of the dependence can be known only after the explicit calculations. However, one can learn a lot about gauge fixing ambiguity just assuming that the dependence takes place. In the simplest case without the cosmological constant term, Eq. (4) tells us that only

the topological Gauss-Bonnet counterterm can not be set to zero by the special choice of the gauge fixing condition. This is exactly the result which was first discovered by direct calculation in the pioneer work [3]. The S matrix corresponds to the on shell limit of effective action, and hence, it is finite in the theory with $\Lambda = 0$.

In the general case of the theory with $\Lambda \neq 0$, the situation is more complicated. It is easy to see that the parameter b_5 makes no effect on divergences due to the third Bianchi identity. Therefore, there is a four-parameter $b_{1,2,3,4}$ ambiguity for the six coefficients $c_{1,2,\dots,6}$. As a result, only two combinations of these six coefficients can be expected to be gauge-fixing independent.

Let us elaborate a little bit more on the gauge fixing ambiguity. Direct calculations show that the parameters of the expression (3) vary according to

$$\begin{aligned} c_1 &\rightarrow c_1, & c_2 &\rightarrow c_2 + b_1, \\ c_3 &\rightarrow c_3 - \left(b_2 + \frac{1}{2}b_1\right), & c_4 &\rightarrow c_4 - b_4, \\ c_5 &\rightarrow c_5 - (b_1 + 4b_2 + b_3)\Lambda, & c_6 &\rightarrow c_6 - 4b_3\Lambda^2. \end{aligned} \quad (5)$$

Then, simple linear analysis shows that the two gauge-fixing invariant quantities are

$$c_1 \quad \text{and} \quad c_{\text{inv}} = c_6 - 4\Lambda c_5 + 4\Lambda^2 c_2 + 16\Lambda^2 c_3. \quad (6)$$

These two quantities do not modify under the change of the gauge fixing parameters α_i . It is interesting that the on shell expressions for the classical action and divergences read

$$\begin{aligned} S|_{\text{on shell}} &= \frac{6\Lambda}{\kappa^2} \int d^4x \sqrt{-g}, \\ \Gamma_{\text{div}}^{(1)}|_{\text{on shell}} &= \frac{1}{\epsilon} \int d^4x \sqrt{-g} \{c_1 R_{\mu\nu\alpha\beta}^2 + c_{\text{inv}}\}, \end{aligned} \quad (7)$$

and consist only from the gauge-fixing invariant quantities. This fact is the source of the so-called on shell renormalization group equation, as noticed in the seminal paper by Fradkin and Tseytlin [15]. The idea can be extended to the Einstein-Cartan model with a cosmological constant and external spinor current, as was discussed in [25,26].

The general considerations (see, e.g., [27]) show that the expression (4) should also apply to the parametrization ambiguity, which is in general much more difficult to trace. However, in this case, the statement is not proved at the same level of safety as in the case of gauge-fixing dependence [12], especially in the situation when two ambiguities are present at the same time. Therefore, it makes sense to perform explicit calculations and check whether the property explained above holds in this case. Because of the continuous interest in the quantum gravity in different dimensions, we perform this calculation for an arbitrary D .

III. BACKGROUND-FIELD METHOD FOR GRAVITY: GENERAL SETTING

Our purpose is to perform a derivation of the first two nontrivial Schwinger-DeWitt coefficients in the most general parametrization of quantum metric. To this end, using the background field method, let us consider the following splitting of the metric:

$$\begin{aligned} g_{\alpha\beta} &\rightarrow g'_{\alpha\beta} = e^{2\kappa r\sigma} [g_{\alpha\beta} + \kappa(\gamma_1 \phi_{\alpha\beta} + \gamma_2 \phi g_{\alpha\beta}) \\ &\quad + \kappa^2(\gamma_3 \phi_{\alpha\beta} \phi_{\beta}^{\rho} + \gamma_4 \phi_{\rho\omega} \phi^{\rho\omega} g_{\alpha\beta} + \gamma_5 \phi \phi_{\alpha\beta} \\ &\quad + \gamma_6 \phi^2 g_{\alpha\beta})], \end{aligned} \quad (8)$$

where $g_{\alpha\beta}$ is the background metric and $\phi_{\alpha\beta}$ and σ are the quantum fields. We also introduce a definition for the trace,

$$\phi = \phi_{\mu}^{\mu}. \quad (9)$$

In what follows, the indexes are lowered and raised with the metric background $g_{\alpha\beta}$ and its inverse $g^{\alpha\beta}$.

Finally, $\gamma_{1,2,\dots,6}$ and r are arbitrary coefficients which parametrize the choice of the quantum variables. A comment is in order. As far as the one-loop calculations require only a bilinear form in the quantum fields part of the action, it is easy to check that Eq. (8) represents the most general possible parametrization of the quantum metric for the sake of one-loop calculations.

A. Bilinear form in quantum fields

By using (8), the bilinear form in the quantum fields of action (1) reads

$$\begin{aligned} S^{(2)} &= - \int d^Dx \sqrt{-g} \left\{ \phi_{\alpha\beta} \left[\frac{d_1}{4} \delta^{\alpha\beta,\mu\nu} \square - \frac{d_2}{4} g^{\alpha\beta} g^{\mu\nu} \square \right. \right. \\ &\quad + \frac{d_3}{4} (g^{\mu\nu} \nabla^{\alpha} \nabla^{\beta} + g^{\alpha\beta} \nabla^{\mu} \nabla^{\nu}) - \frac{d_4}{2} g^{\beta\nu} \nabla^{\alpha} \nabla_{\mu} \\ &\quad \left. \left. - 2L^{\alpha\beta,\mu\nu} \Lambda + \gamma_1^2 M^{\alpha\beta,\mu\nu} \right] \phi_{\mu\nu} \right. \\ &\quad + \phi_{\alpha\beta} [l_0 \nabla^{\alpha} \nabla^{\beta} + l_1 g^{\alpha\beta} \square + l_2 g^{\alpha\beta} \Lambda + l_3 R^{\alpha\beta} + l_4 g^{\alpha\beta} R] \sigma \\ &\quad \left. + \sigma [s_1 \square + s_2 \Lambda + s_3 R] \sigma \right\}, \end{aligned} \quad (10)$$

where the coefficients are as follows:

$$\begin{aligned} d_1 &= d_4 = \gamma_1^2, \\ d_2 &= \gamma_1^2 + 2(D-2)\gamma_1\gamma_2 + (D-2)(D-1)\gamma_2^2, \\ d_3 &= \gamma_1^2 + (D-2)\gamma_1\gamma_2, \quad l_0 = (D-2)\gamma_1 r, \\ l_1 &= -(D-2)[\gamma_1 + (D-1)\gamma_2]r, \quad l_2 = D(\gamma_1 + D\gamma_2)r, \\ l_3 &= -(D-2)\gamma_1 r, \quad l_4 = \frac{(D-2)}{2}[\gamma_1 + (D-2)\gamma_2]r \end{aligned} \quad (11)$$

$$\text{and } s_1 = -(D-2)(D-1)r^2, \quad s_2 = D^2 r^2, \\ s_3 = \frac{(D-2)^2}{2} r^2. \quad (12)$$

In the formula (10), the relevant tensor objects are

$$\delta^{\alpha\beta,\mu\nu} = \frac{1}{2}(g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\beta\mu}), \quad (13)$$

which is the identity matrix in the space of the symmetric second-rank fields, and

$$M^{\alpha\beta,\mu\nu} = \frac{1}{2}R^{\alpha\mu\beta\nu} - \frac{1+x_1}{4}\delta^{\alpha\beta,\mu\nu}R + \frac{1+x_2}{2}R^{\alpha\mu}g^{\beta\nu} \\ - \frac{1+x_3}{4}(R^{\alpha\beta}g^{\mu\nu} + R^{\mu\nu}g^{\alpha\beta}) + \frac{1+x_4}{8}g^{\alpha\beta}g^{\mu\nu}R, \quad (14)$$

where

$$x_1 = -\frac{2}{\gamma_1^2}[\gamma_3 + (D-2)\gamma_4], \quad x_2 = -2\frac{\gamma_3}{\gamma_1^2}, \\ x_3 = (D-4)\frac{\gamma_2}{\gamma_1} + 2\frac{\gamma_5}{\gamma_1^2}, \\ x_4 = 2(D-4)\frac{\gamma_2}{\gamma_1} + (D-2)(D-4)\frac{\gamma_2^2}{\gamma_1^2} + \frac{4}{\gamma_1^2}[\gamma_5 + (D-2)\gamma_6] \quad (15)$$

and

$$L^{\alpha\beta,\mu\nu} = K^{\alpha\beta,\mu\nu} - \frac{1}{2}(\gamma_3 + D\gamma_4)\delta^{\alpha\beta,\mu\nu} \\ - \frac{1}{2}(\gamma_5 + D\gamma_6)g^{\alpha\beta}g^{\mu\nu}. \quad (16)$$

Let us explain the condensed notations which were used in these formulas. In Eq. (16), there is a K tensor

$$K^{\alpha\beta,\mu\nu} = \frac{1}{4}\left\{\gamma_1^2\delta^{\alpha\beta,\mu\nu} - \frac{1}{2}[\gamma_1^2 + 2(D-2)\gamma_1\gamma_2] \right. \\ \left. + D(D-2)\gamma_2^2 g^{\alpha\beta}g^{\mu\nu}\right\}. \quad (17)$$

The K tensor is an important object and deserves special attention. After the introduction of gauge fixing (GF), with a minimal choice of parameters, the structure (17) will represent the generalized DeWitt metric in the space of the fields for our model, see Eq. (25).

Furthermore, in the above formulas and in the following, we used a special condensed way to write formulas, which enables us to present the expressions in a relatively compact form. The idea of this condensed notation is that all the algebraic symmetries are implicit, including the

symmetrization in the couple of indexes $(\alpha\beta) \leftrightarrow (\mu\nu)$ and inside each couple, $(\alpha \leftrightarrow \beta)$, $(\mu \leftrightarrow \nu)$. In order to obtain the complete formulas explicitly, one has to restore all the symmetries. For example, it is necessary to trade

$$R^{\alpha\mu\beta\nu} \rightarrow \frac{1}{4}(R^{\alpha\mu\beta\nu} + R^{\alpha\nu\beta\mu} + R^{\beta\nu\alpha\mu} + R^{\beta\mu\alpha\nu})$$

and

$$R^{\alpha\mu}g^{\beta\nu} \rightarrow \frac{1}{4}(R^{\alpha\mu}g^{\beta\nu} + R^{\alpha\nu}g^{\beta\mu} + R^{\beta\nu}g^{\alpha\mu} + R^{\beta\mu}g^{\alpha\nu}) \quad (18)$$

implying that the mentioned symmetries are restored.

B. Gauge fixing action

Let us introduce the gauge fixing action for the diffeomorphism invariance in the form

$$S_{GF} = -\frac{1}{\alpha} \int d^D x \sqrt{-g} \chi_\mu \chi^\mu, \quad (19)$$

where

$$\chi_\mu = \nabla_\rho \phi_\mu^\rho - \beta_1 \nabla_\mu \phi - \beta_2 \nabla_\mu \sigma \quad (20)$$

is the linear background gauge. In the last formulas, α , β_1 , and β_2 are the gauge fixing parameters. The bilinear form of the GF action is the following:

$$S_{GF}^{(2)} = \int d^D x \sqrt{-g} \left\{ \phi_{\alpha\beta} \left[\frac{1}{\alpha} g^{\beta\nu} \nabla^\alpha \nabla_\nu \right. \right. \\ \left. \left. - \frac{\beta_1}{\alpha} (g^{\mu\nu} \nabla^\alpha \nabla_\beta + g^{\alpha\beta} \nabla^\mu \nabla_\nu) + \frac{\beta_1^2}{\alpha} g^{\alpha\beta} g^{\mu\nu} \square \right] \phi_{\mu\nu} \right. \\ \left. + \phi_{\alpha\beta} \left[\frac{2\beta_1\beta_2}{\alpha} g^{\alpha\beta} \square - \frac{2\beta_2}{\alpha} \nabla^\alpha \nabla_\beta \right] \sigma + \frac{\beta_2^2}{\alpha} \sigma \square \sigma \right\}. \quad (21)$$

By comparing Eqs. (10) and (21), let us note that for the values

$$\alpha = -\frac{2}{\gamma_1^2}, \quad (22)$$

$$\beta_1 = \frac{1}{2} \left[1 + (D-2) \frac{\gamma_2}{\gamma_1} \right], \quad (23)$$

$$\beta_2 = (D-2) \frac{r}{\gamma_1}, \quad (24)$$

the bilinear operator is minimal. The last means that for these values of gauge parameters, this operator contains the derivatives only in the combination $\square = g^{\mu\nu} \nabla_\mu \nabla_\nu$. Then

$$\begin{aligned}
(S + S_{GF})^{(2)} = & - \int d^D x \sqrt{-g} \{ \phi_{\alpha\beta} [K^{\alpha\beta,\mu\nu} \square \\
& - 2L^{\alpha\beta,\mu\nu} \Lambda + \gamma_1^2 M^{\alpha\beta,\mu\nu}] \phi_{\mu\nu} \\
& + \phi_{\alpha\beta} [\tilde{l}_1 g^{\alpha\beta} \square + l_2 g^{\alpha\beta} \Lambda + l_3 R^{\alpha\beta} + l_4 g^{\alpha\beta} R] \sigma \\
& + \sigma [\tilde{s}_1 \square + s_2 \Lambda + s_3 R] \sigma \}, \quad (25)
\end{aligned}$$

where the new coefficients, $\tilde{l}_1$ and $\tilde{s}_1$, are

$$\begin{aligned}
\tilde{l}_1 &= -\frac{(D-2)}{2}(\gamma_1 + D\gamma_2)r, \\
\tilde{s}_1 &= -\frac{D(D-2)}{2}r^2.
\end{aligned}$$

It is remarkable and certainly very useful that we could provide the simplest minimal form of a bilinear in a

quantum fields operator for an arbitrary parametrization of the quantum metric. After that instant, the calculation becomes pretty much standard, but we shall present them in full detail, which may be useful for eventual verifications.

C. Trace and traceless decomposition

It proves useful to separate the field $\phi_{\alpha\beta}$ into trace (9) and the traceless tensor field,

$$\bar{\phi}_{\alpha\beta} = \phi_{\alpha\beta} - \frac{1}{D} g_{\alpha\beta} \phi. \quad (26)$$

In the new variables, the bilinear form (25) becomes

$$\begin{aligned}
(S + S_{GF})^{(2)} = & - \int d^D x \sqrt{-g} \left\{ \bar{\phi}_{\alpha\beta} \left[\frac{\gamma_1^2}{4} \bar{\delta}^{\alpha\beta,\mu\nu} (\square - 2(1+z_1)\Lambda) + \gamma_1^2 \bar{M}^{\alpha\beta,\mu\nu} \right] \bar{\phi}_{\mu\nu} \right. \\
& + \bar{\phi}_{\alpha\beta} [-2z_2 R^{\alpha\beta}] \phi + \bar{\phi}_{\alpha\beta} [l_3 R^{\alpha\beta}] \sigma + \phi [y_1 \square + y_2 \Lambda + y_3 R] \phi + \phi [\tilde{l}_1 \square + l_2 \Lambda + \tilde{l}_3 R] \sigma \\
& \left. + \sigma [\tilde{s}_1 \square + s_2 \Lambda + s_3 R] \sigma \right\}, \quad (27)
\end{aligned}$$

where the new coefficients $z_{1,2}$, $y_{1,2,3}$, and $\tilde{l}_3$ are

$$\begin{aligned}
z_1 &= -\frac{2}{\gamma_1^2}(\gamma_3 + D\gamma_4), \\
z_2 &= \frac{(D-4)}{4D}\gamma_1(\gamma_1 + D\gamma_2) + \frac{\gamma_3}{D} + \frac{\gamma_5}{2}, \\
\tilde{l}_3 &= \frac{(D-2)^2}{2D}(\gamma_1 + D\gamma_2)r, \\
y_1 &= -\frac{(D-2)}{8D}(\gamma_1 + D\gamma_2)^2, \\
y_2 &= \frac{(D-2)}{4D}(\gamma_1 + D\gamma_2)^2 + \frac{1}{D}(\gamma_3 + D\gamma_4) + (\gamma_5 + D\gamma_6), \\
y_3 &= \frac{(D-2)}{8D^2} \{ (D-4)(\gamma_1 + D\gamma_2)^2 + 4(\gamma_3 + D\gamma_4) \\
& + 4D(\gamma_5 + D\gamma_6) \}. \quad (28)
\end{aligned}$$

Also, the projector onto the traceless states is

$$\bar{\delta}^{\alpha\beta,\mu\nu} = \delta^{\alpha\beta,\mu\nu} - \frac{1}{D} g^{\alpha\beta} g^{\mu\nu}, \quad (29)$$

and the last notation is

$$\begin{aligned}
\bar{M}^{\alpha\beta,\mu\nu} &= \frac{1}{2} R^{\alpha\mu\beta\nu} - \frac{(1+x_1)}{4} \bar{\delta}^{\alpha\beta,\mu\nu} R \\
& + \frac{(1+x_2)}{2} R^{\alpha\mu} g^{\beta\nu}. \quad (30)
\end{aligned}$$

IV. CONFORMAL GAUGE FIXING

In order to remove the remaining degeneracy, let us implement the conformal gauge fixing in the form

$$\sigma = \beta_3 \phi, \quad (31)$$

with β_3 being a new free gauge fixing parameter. Let us note that the conformal gauge fixing does not require Faddeev-Popov ghosts, because the conformal symmetry transformation has no derivatives [15]. Thus, (27) becomes

$$\begin{aligned}
(S + S_{GF})^{(2)} = & \int d^D x \sqrt{-g} \left\{ \bar{\phi}_{\alpha\beta} \left[\frac{\gamma_1^2}{4} \bar{\delta}^{\alpha\beta,\mu\nu} \square \right. \right. \\
& - \frac{2\gamma_1^2}{4} (1+z_1) \Lambda \bar{\delta}^{\alpha\beta,\mu\nu} + \gamma_1^2 \bar{M}^{\alpha\beta,\mu\nu} \left. \right] \bar{\phi}_{\mu\nu} \\
& \left. + \bar{\phi}_{\alpha\beta} [-2c R^{\alpha\beta}] \phi + \phi [b_1 \square + 2b_2 \Lambda + b_3 R] \phi \right\}, \quad (32)
\end{aligned}$$

where

$$c = \frac{D-4}{4D} \gamma_1(\gamma_1 + D\gamma_2) + \frac{\gamma_3}{D} + \frac{\gamma_5}{2} + \frac{D-2}{2} \gamma_1 r \beta_3$$

and

$$\begin{aligned}
 b_1 &= -\frac{D-2}{8D}[\gamma_1 + D(\gamma_2 + 2r\beta_3)]^2, \\
 b_2 &= \frac{D-2}{8D}(\gamma_1 + D\gamma_2)^2 + \frac{1}{2D}(\gamma_3 + D\gamma_4) \\
 &\quad + \frac{1}{2}(\gamma_5 + D\gamma_6) + \frac{D}{2}(\gamma_1 + D\gamma_2)r\beta_3 + \frac{D^2}{2}r^2\beta_3^2, \\
 b_3 &= (D-2)\left\{\frac{D-4}{8D^2}(\gamma_1 + D\gamma_2)^2 \right. \\
 &\quad + \frac{1}{2D^2}[(\gamma_3 + D\gamma_4) + D(\gamma_5 + D\gamma_6)] \\
 &\quad \left. + \frac{D-2}{2D}(\gamma_1 + D\gamma_2)r\beta_3 + \frac{D-2}{2}r^2\beta_3^2\right\}. \quad (33)
 \end{aligned}$$

A. Bilinear operator in quantum fields

Now we are in a position to write down the bilinear in a quantum fields operator in (32)

$$(S + S_{GF})^{(2)} = - \int d^D x \sqrt{-g} (\bar{\phi}_{\alpha\beta} \quad \phi) \hat{H} \begin{pmatrix} \bar{\phi}_{\mu\nu} \\ \phi \end{pmatrix}, \quad (34)$$

where

$$\begin{aligned}
 \hat{H} &= \begin{pmatrix} \hat{H}_{\bar{\phi}\bar{\phi}} & \hat{H}_{\bar{\phi}\phi} \\ \hat{H}_{\phi\bar{\phi}} & \hat{H}_{\phi\phi} \end{pmatrix}, \quad \text{and} \\
 \hat{H}_{\bar{\phi}\bar{\phi}} &= \frac{\gamma_1^2}{4} \bar{\delta}^{\alpha\beta,\mu\nu} [\square - 2(1 + z_1)\Lambda] + \gamma_1^2 \bar{M}^{\alpha\beta,\mu\nu} \\
 \hat{H}_{\bar{\phi}\phi} &= \hat{H}_{\phi\bar{\phi}} = -cR^{\alpha\beta} \\
 \hat{H}_{\phi\phi} &= b_1\square + 2b_2\Lambda + b_3R. \quad (35)
 \end{aligned}$$

In order to reduce the bilinear form (35) into the standard expression for the minimal operator, $\hat{1}\square + \hat{\Pi}$, consider a new operator, $\hat{H}' = \hat{C} \cdot \hat{H}$, where $\hat{C}$ is a c -number matrix. Since

$$\text{Tr} \ln \hat{H}' = \text{Tr} \ln(\hat{C} \cdot \hat{H}) = \text{Tr} \ln \hat{C} + \text{Tr} \ln \hat{H}, \quad (36)$$

and the contribution of $\text{Tr} \ln \hat{C}$ does not produce divergences, i.e., the divergent part satisfies

$$\text{Tr} \ln \hat{H}'|_{\text{div}} = \text{Tr} \ln \hat{H}|_{\text{div}}. \quad (37)$$

By choosing

$$\hat{C} = \begin{pmatrix} \frac{4}{\gamma_1^2} \bar{\delta}^{\alpha\beta,\mu\nu} & 0 \\ 0 & \frac{1}{b_1} \end{pmatrix}, \quad (38)$$

we found

$$\hat{H}' = \hat{1}\square + \hat{\Pi}, \quad (39)$$

where

$$\hat{1} = \begin{pmatrix} \bar{\delta}^{\alpha\beta,\mu\nu} & 0 \\ 0 & 1 \end{pmatrix} \quad (40)$$

and

$$\hat{\Pi} = \begin{pmatrix} 4\bar{M}^{\alpha\beta,\mu\nu} - 2(1 + z_1)\Lambda \bar{\delta}^{\alpha\beta,\mu\nu} & -\frac{4c}{\gamma_1^2} R^{\alpha\beta} \\ -\frac{c}{b_1} R^{\mu\nu} & \frac{2b_2\Lambda + b_3R}{b_1} \end{pmatrix}. \quad (41)$$

The last expression (39) has a standard form, and we can use known algorithm for the Schwinger-DeWitt technique.

V. ONE-LOOP DIVERGENCES

The one-loop effective action is given by the well-known formula

$$\Gamma^{(1)} = \frac{i}{2} \text{Tr} \ln \hat{H} - i \text{Tr} \ln \hat{H}_{GH}, \quad (42)$$

where $\hat{H}$ was defined in previously section and $\hat{H}_{GH}$ is the Faddeev-Popov ghost operator, which will be described in the next section.

In $D = 2$, the logarithmic divergences in (42) are given by the traces $\hat{a}_1$ of the coincidence limits of the Schwinger-DeWitt coefficients $\hat{a}_1(x, x')$ of the corresponding operators. In the $D = 4$ dimension, $\hat{a}_1$ gives the quadratic divergence, which is relevant for the applications to asymptotic safety [7], while the traces $\hat{a}_2$ of the coincidence limits of the Schwinger-DeWitt coefficients $\hat{a}_2(x, x')$ provide logarithmic operators. For the sake of generality, we will perform calculations for an arbitrary dimension D , which can be also useful for $2 - \epsilon$ and $4 - \epsilon$ approaches and other applications.

A. Derivation of metric contributions

The next step is to consider the calculation of each term of Eq. (42) separately. According to the Schwinger-DeWitt technique [28]

$$\begin{aligned}
 \hat{a}_2 &\equiv \text{tr} \lim_{x \rightarrow x'} \hat{a}_2(x, x') \\
 &= \text{tr} \left\{ \frac{\hat{1}}{180} (R_{\mu\nu\alpha\beta}^2 - R_{\alpha\beta}^2 + \square R) + \frac{1}{2} \hat{P}^2 + \frac{1}{6} \square \hat{P} + \frac{1}{12} \hat{S}_{\rho\omega}^2 \right\}, \quad (43)
 \end{aligned}$$

where $\hat{P} = \hat{\Pi} + \frac{1}{6}R$ and, in our case,

$$\hat{S}_{\rho\omega} = [\nabla_\rho, \nabla_\omega] \hat{1} = \begin{pmatrix} 2g^{\rho\beta} R_{\cdot\rho\omega}^{\mu\alpha} & 0 \\ 0 & 0 \end{pmatrix}. \quad (44)$$

Consequently,

$$\hat{P} = \begin{pmatrix} P_{\bar{\phi}\bar{\phi}}^{\alpha\beta,\mu\nu} & P_{\bar{\phi}\phi}^{\alpha\beta} \\ P_{\phi\bar{\phi}}^{\mu\nu} & P_{\phi\phi} \end{pmatrix}, \quad (45)$$

where

$$\begin{aligned} P_{\bar{\phi}\bar{\phi}}^{\alpha\beta,\mu\nu} &= - \left[\left(x_1 + \frac{5}{6} \right) R + 2(1 + z_1) \Lambda \right] \bar{\delta}^{\alpha\beta,\mu\nu} \\ &\quad + 2(1 + x_2) R^{\mu\alpha} g^{\nu\beta} + 2R^{\alpha\mu\beta\nu}, \\ P_{\bar{\phi}\phi}^{\alpha\beta} &= -4 \frac{c}{\gamma_1^2} R^{\alpha\beta}, \quad P_{\phi\bar{\phi}}^{\mu\nu} = -\frac{c}{b_1} R^{\mu\nu}, \\ P_{\phi\phi} &= \frac{2b_2}{b_1} \Lambda + \left(\frac{b_3}{b_1} + \frac{1}{6} \right) R. \end{aligned} \quad (46)$$

In order to evaluate (43), let us start from the $\text{tr} \hat{P}^2$ term. Using (45), one can write down

$$\begin{aligned} \text{tr} \hat{P}^2 &= \left[\left(x_1 + \frac{5}{6} \right) R + 2(1 + z_1) \Lambda \right]^2 \text{tr}(k_1 \cdot k_1) + 4(1 + x_2)^2 \text{tr}(k_2 \cdot k_2) + 4\text{tr}(k_3 \cdot k_3) \\ &\quad - 4 \left[\left(x_1 + \frac{5}{6} \right) R + 2(1 + z_1) \Lambda \right] [(1 + x_2) \text{tr}(k_1 \cdot k_2) + \text{tr}(k_1 \cdot k_3)] \\ &\quad + 8(1 + x_2) \text{tr}(k_2 \cdot k_3) + \frac{8c^2}{b_1 \gamma_1^2} R_{\alpha\beta} \bar{\delta}^{\alpha\beta,\mu\nu} R_{\mu\nu} + \left[\frac{2b_2}{b_1} \Lambda + \left(\frac{b_3}{b_1} + \frac{1}{6} \right) R \right]^2. \end{aligned} \quad (50)$$

It is not difficult to construct the following multiplication table for the basic traces:

$$\begin{aligned} \text{tr}(k_1 \cdot k_1) &= \frac{(D-1)(D+2)}{2}, \\ \text{tr}(k_2 \cdot k_2) &= \frac{(D-2)(D+4)}{4D} R_{\alpha\beta}^2 + \frac{(D^2+4)}{4D^2} R^2, \\ \text{tr}(k_3 \cdot k_3) &= \frac{3}{4} R_{\mu\alpha\beta}^2 - \frac{2}{D} R_{\alpha\beta}^2 + \frac{1}{D^2} R^2, \\ \text{tr}(k_1 \cdot k_2) &= \frac{(D-1)(D+2)}{2D} R, \\ \text{tr}(k_1 \cdot k_3) &= -\frac{(D+2)}{2D} R, \\ \text{tr}(k_2 \cdot k_3) &= -\frac{(D+4)}{2D} R_{\alpha\beta}^2 + \frac{1}{D^2} R^2. \end{aligned} \quad (51)$$

$$\hat{P}^2 = \begin{pmatrix} \hat{P}_{\bar{\phi}\bar{\phi}} \cdot \hat{P}_{\bar{\phi}\bar{\phi}} + \hat{P}_{\bar{\phi}\phi} \cdot \hat{P}_{\phi\bar{\phi}} & \cdots \\ \cdots & \hat{P}_{\phi\phi} \cdot \hat{P}_{\phi\phi} + \hat{P}_{\phi\bar{\phi}} \cdot \hat{P}_{\bar{\phi}\phi} \end{pmatrix}. \quad (47)$$

In this formula, we do not write indexes to clear the notations and do not show explicitly the irrelevant off diagonal terms. From (47), it follows

$$\text{tr} \hat{P}^2 = \text{tr} \hat{P}_{\bar{\phi}\bar{\phi}}^2 + 2\text{tr}(\hat{P}_{\bar{\phi}\phi} \cdot \hat{P}_{\phi\bar{\phi}}) + \text{tr} \hat{P}_{\phi\phi}^2, \quad (48)$$

where the traces are taken in different subspaces of the quantum metric space.

Introducing the compact notations

$$k_1 = \bar{\delta}^{\alpha\beta,\mu\nu}, \quad k_2 = R^{\mu\alpha} g^{\nu\beta}, \quad k_3 = R^{\alpha\mu\beta\nu}, \quad (49)$$

we obtain, for the formula (48),

We will also need the trace

$$R_{\alpha\beta} \bar{\delta}^{\alpha\beta,\mu\nu} R_{\mu\nu} = R_{\alpha\beta}^2 - \frac{1}{D} R^2. \quad (52)$$

Using the table [Eqs. (51)], Eq. (50) can be evaluated by using MATHEMATICA [29]. The result has the form

$$\begin{aligned} \text{tr} \hat{P}^2 &= p_1(D) R_{\mu\nu\alpha\beta}^2 + p_2(D) R_{\alpha\beta}^2 + p_4(D) R^2 \\ &\quad + p_5(D) \Lambda R + p_6(D) \Lambda^2, \end{aligned} \quad (53)$$

where

$$\begin{aligned} p_1(D) &= 3, \quad p_2(D) = \frac{D^2 - 2D - 32}{D} + \frac{8c^2}{b_1 \gamma_1^2} + \frac{D+4}{D} [(D-2)x_2^2 + 2(D-4)x_2], \\ p_4(D) &= \frac{25D^4 - 95D^3 + 24D^2 + 480D + 1152}{72D^2} + \frac{b_3}{3b_1} + \frac{b_3^2}{b_1^2} - \frac{8c^2}{Db_1 \gamma_1^2} + \frac{(D^2+4)}{D^2} x_2^2 \\ &\quad + (D+2) \left[\frac{D-1}{2} x_1^2 - \frac{2(D-1)}{D} x_1 x_2 + \frac{5D^2 - 17D + 24}{6D} x_1 \right] - \frac{(5D^3 - D^2 - 10D - 48)}{3D^2} x_2, \end{aligned}$$

$$\begin{aligned}
p_5(D) &= \frac{D+2}{D} \left[\frac{5D^2 - 17D + 24}{3} + 2(D-1)(Dx_1 - 2x_2) \right] (1+z_1) + \frac{2(b_1 + 6b_3)b_2}{3b_1^2}, \\
p_6(D) &= 2(D-1)(D+2)(1+z_1)^2 + \frac{4b_2^2}{b_1^2}.
\end{aligned} \tag{54}$$

Using formula (53) and the relations

$$\begin{aligned}
\text{tr} \hat{1} &= \frac{D(D+1)}{2}, \\
\text{tr} \hat{S}_{\alpha\beta}^2 &= -(D+2)R_{\mu\nu\alpha\beta}^2,
\end{aligned} \tag{55}$$

we arrive at the result for the expression (43),

$$\begin{aligned}
\hat{a}_2 &= h_1(D)R_{\mu\nu\alpha\beta}^2 + h_2(D)R_{\alpha\beta}^2 + h_3(D)\square R \\
&\quad + h_4(D)R^2 + h_5(D)\Lambda R + h_6(D)\Lambda^2,
\end{aligned} \tag{56}$$

where

$$\begin{aligned}
h_1(D) &= \frac{(D^2 - 29D + 480)}{360}, \\
h_2(D) &= -\frac{D^3 - 179D^2 + 360D + 5760}{360D} + \frac{4c^2}{b_1\gamma_1^2} + \frac{D+4}{D} \left[\frac{D-2}{2}x_2^2 + (D-4)x_2 \right], \\
h_3(D) &= -\frac{2D^3 - 3D^2 - 5D + 20}{30D} - \frac{(D-1)(D+2)}{12D}(Dx_1 - 2x_2) + \frac{b_3}{6b_1}, \\
h_4(D) &= \frac{25D^4 - 95D^3 + 24D^2 + 480D + 1152}{144D^2} + \frac{b_3}{6b_1} + \frac{b_3^2}{2b_1^2} - \frac{4c^2}{Db_1\gamma_1^2} \\
&\quad + (D+2) \left(\frac{D-1}{4}x_1^2 + \frac{5D^2 - 17D + 24}{12D}x_1 - \frac{D-1}{D}x_1x_2 \right) + \frac{(D^2+4)}{2D^2}x_2^2 - \frac{(5D^3 - D^2 - 10D - 48)}{6D^2}x_2, \\
h_5(D) &= \frac{(b_1 + 6b_3)b_2}{3b_1^2} + \frac{D+2}{D} \left[\frac{5D^2 - 17D + 24}{6} + (D-1)(Dx_1 - 2x_2) \right] (1+z_1), \\
h_6(D) &= (D-1)(D+2)(1+z_1)^2 + \frac{2b_2^2}{b_1^2}.
\end{aligned} \tag{57}$$

A much simpler task is to evaluate

$$\begin{aligned}
\hat{a}_1 &\equiv \text{tr} \lim_{x \rightarrow x'} \hat{a}_1(x, x') = \text{tr} \hat{P} \\
&= - \left[\left(x_1 + \frac{5}{6} \right) R + 2(1+z_1)\Lambda \right] \text{tr}(k_1 \cdot k_1) + 2(1+x_2)\text{tr}(k_1 \cdot k_2) + 2\text{tr}(k_1 \cdot k_3) + 2 \left(\frac{b_2}{b_1} \right) \Lambda + \left(\frac{b_3}{b_1} + \frac{1}{6} \right) R.
\end{aligned} \tag{58}$$

After a small amount of algebra, we find

$$\hat{a}_1 = \left[\frac{b_3}{b_1} - \frac{5D^3 - 7D^2 - 12D + 48}{12D} - \frac{(D-1)(D+2)(Dx_1 - 2x_2)}{2D} \right] R + \left[\frac{2b_2}{b_1} - (D-1)(D+2)(1+z_1) \right] \Lambda. \tag{59}$$

Let us give the expression for divergences in dimensional regularization for $D \rightarrow 4$,

$$\frac{i}{2} \text{Tr} \ln \hat{H}|_{\text{div}} = -\frac{\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} \hat{a}_2, \tag{60}$$

where $\epsilon = (4\pi)^2(D-4)$ and μ is the dimensional parameter of renormalization. Consequently,

$$\frac{i}{2} \text{Tr} \ln \hat{H}|_{\text{div}} = -\frac{\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} \{ h_1(4)R_{\mu\nu\alpha\beta}^2 + h_2(4)R_{\alpha\beta}^2 + h_3(4)\square R + h_4(4)R^2 \}, \tag{61}$$

where

$$\begin{aligned} h_1(4) &= \frac{19}{18}, & h_2(4) &= -\frac{55}{18} + 2x_2^2 + \frac{4c^2}{b_1\gamma_1^2}, & h_3(4) &= -\frac{2}{3} + \frac{b_3}{6b_1} - \frac{3}{4}(2x_1 - x_2), \\ h_4(4) &= \frac{59}{36} + \frac{b_3}{6b_1} + \frac{b_3^2}{2b_1^2} - \frac{c^2}{b_1\gamma_1^2} + \frac{9}{2}(x_1^2 - x_1x_2 + x_1) - \frac{9}{4}x_2 + \frac{5x_2^2}{8}, \\ h_5(4) &= 9 + \frac{b_2}{3b_1} + \frac{2b_2b_3}{b_1^2} + 9[(2x_1 - x_2)(1 + z_1) + z_1], & h_6(4) &= 18(1 + z_1)^2 + \frac{2b_2^2}{b_1^2}. \end{aligned} \quad (62)$$

B. Faddeev-Popov ghost term

Let us now evaluate the contribution of gauge ghosts. The Faddeev-Popov ghost operator is defined by

$$\hat{H}_{GH} = \frac{\delta\chi^\mu}{\delta\phi_{\alpha\beta}} R_{\alpha\beta}^\nu + \frac{\delta\chi^\mu}{\delta\sigma} R^\nu \Big|_{\phi_{\alpha\beta} \rightarrow 0, \sigma \rightarrow 0}, \quad (63)$$

where χ^μ is the background gauge, defined in Eq. (20), and $R_{\alpha\beta}^\nu$, R^ν are the gauge generators with respect to the quantum fields $\phi_{\alpha\beta}$ and σ , respectively. For the diffeomorphism symmetry, we have

$$\delta\phi_{\alpha\beta} = R_{\alpha\beta}^\mu \xi_\mu, \quad \delta\sigma = R^\mu \xi_\mu, \quad (64)$$

where

$$\begin{aligned} R_{\alpha\beta}^\mu &= -\frac{1}{\gamma_1} \left(\delta_\alpha^\mu \nabla_\beta + \delta_\beta^\mu \nabla_\alpha - \frac{2\gamma_2}{\gamma_1 + D\gamma_2} g_{\alpha\beta} \nabla^\mu \right), \\ R^\mu &= \nabla^\mu \sigma. \end{aligned} \quad (65)$$

The details of the derivation of gauge generator (65) can be found in Appendix A. The variational derivatives are

$$\begin{aligned} \frac{\delta\chi^\mu}{\delta\phi_{\alpha\beta}} &= \frac{1}{2} (g^{\mu\alpha} \nabla^\beta + g^{\mu\beta} \nabla^\alpha) - \beta_1 g^{\alpha\beta} \nabla^\mu \quad \text{and} \\ \frac{\delta\chi^\mu}{\delta\sigma} &= -\beta_2 \nabla^\mu. \end{aligned} \quad (66)$$

Consequently,

$$\begin{aligned} \hat{H}_{GH} &= -\frac{1}{\gamma_1} (g^{\mu\nu} \square + \tau \nabla^\mu \nabla^\nu + R^{\mu\nu}), \\ \text{where } \tau &= \frac{\gamma_1}{\gamma_1 + D\gamma_2} \left[1 - 2\beta_1 + (D-2) \frac{\gamma_2}{\gamma_1} \right]. \end{aligned} \quad (67)$$

Let us note that the contribution of σ in (67) is irrelevant due to the limit which has to be taken in Eq. (63). Now, by using the formula (23) for the parameter β_1 , we get

$$\hat{H}_{GH} = -\frac{1}{\gamma_1} (g^{\mu\nu} \square + R^{\mu\nu}). \quad (68)$$

Indeed, we are lucky enough, that the same choice of gauge fixing which makes the tensor operator minimal, also makes minimal the vector operator in the ghost sector.

Using the same logic which was explained for the tensor gravitational sector, the divergent contribution of the operator (68) is equivalent to

$$\hat{H}'_{GH} = g^{\mu\nu} \square + R^{\mu\nu}. \quad (69)$$

The expression (69) is the minimal vector field operator; hence, the divergences calculations can be derived, once again by the standard Schwinger-DeWitt algorithm,

$$(\hat{a}_1)_{GH} = \text{tr} \hat{P}_{GH}, \quad (70)$$

$$\begin{aligned} (\hat{a}_2)_{GH} &= \text{tr} \left[\frac{1}{180} \hat{1}_{GH} (R_{\mu\nu\alpha\beta}^2 - R_{\alpha\beta}^2 + \square R) \right. \\ &\quad \left. + \frac{1}{2} \hat{P}_{GH}^2 + \frac{1}{6} \square \hat{P}_{GH} + \frac{1}{12} (\hat{S}_{GH})_{\alpha\beta}^2 \right], \end{aligned} \quad (71)$$

where

$$\begin{aligned} \hat{1}_{GH} &= g^{\mu\nu}, & \hat{P}_{GH} &= R^{\mu\nu} + \frac{1}{6} g^{\mu\nu} R, \\ (\hat{S}_{GH})_{\alpha\beta} &= R_{\alpha\beta}^{\mu\nu}. \end{aligned} \quad (72)$$

Thus,

$$\begin{aligned} \text{tr} \hat{1}_{GH} &= D, & \text{tr} \hat{P}_{GH} &= \frac{(D+6)}{6} R, \\ \text{tr} \hat{P}_{GH}^2 &= R_{\alpha\beta}^2 + \frac{(D+12)}{36} R^2, \\ \text{tr} (\hat{S}_{GH})_{\alpha\beta}^2 &= -R_{\mu\nu\alpha\beta}^2. \end{aligned} \quad (73)$$

Finally, we arrive at

$$(\hat{a}_1)_{GH} = \frac{(D+6)}{6} R \quad (74)$$

and

$$(\hat{a}_2)_{GH} = \frac{(D-15)}{180} R_{\mu\nu\alpha\beta}^2 - \frac{(D-90)}{180} R_{\mu\nu}^2 + \frac{(D+5)}{30} \square R + \frac{(D+12)}{72} R^2. \quad (75)$$

In the limit $D \rightarrow 4$, we meet

$$-i\text{Tr} \ln \hat{H}_{GH}|_{\text{div}} = -\frac{\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} \left\{ \frac{11}{90} R_{\mu\nu\alpha\beta}^2 - \frac{43}{45} R_{\mu\nu}^2 - \frac{3}{5} \square R - \frac{4}{9} R^2 \right\}. \quad (76)$$

Changing the basis, we arrive at

$$-i\text{Tr} \ln \hat{H}_{GH}|_{\text{div}} = \frac{\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} \left\{ -\frac{11}{90} E_4 + \frac{7}{15} R_{\mu\nu}^2 + \frac{3}{5} \square R + \frac{17}{30} R^2 \right\}, \quad (77)$$

where $E_4 = R_{\mu\nu\alpha\beta}^2 - 4R_{\alpha\beta}^2 + R^2$ is the 4D Gauss-Bonnet integrand.

C. Divergent part of effective action

In order to obtain the total value of the $\hat{a}_2$ coefficient, we need to replace Eq. (56) and Eq. (75) into the general expression (42). The final answer is similar to (3),

$$(\hat{a}_2)_{\text{total}} = f_1(D) R_{\mu\nu\alpha\beta}^2 + f_2(D) R_{\alpha\beta}^2 + f_3(D) \square R + f_4(D) R^2 + f_5(D) \Lambda R + f_6(D) \Lambda^2, \quad (78)$$

where

$$\begin{aligned} f_1(D) &= \frac{(D^2 - 33D + 540)}{360}, & f_2(D) &= -\frac{D^3 - 183D^2 + 720D + 5760}{360D} + \frac{4c^2}{b_1\gamma_1^2} + \frac{D+4}{D} \left[\frac{D-2}{2} x_2^2 + (D-4)x_2 \right], \\ f_3(D) &= -\frac{2D^3 - D^2 + 5D + 20}{30D} - \frac{(D-1)(D+2)}{12D} (Dx_1 - 2x_2) + \frac{b_3}{6b_1}, \\ f_4(D) &= \frac{25D^4 - 99D^3 - 24D^2 + 480D + 1152}{144D^2} + \frac{b_3}{6b_1} + \frac{b_3^2}{2b_1^2} - \frac{4c^2}{Db_1\gamma_1^2} \\ &\quad + (D+2) \left(\frac{D-1}{4} x_1^2 + \frac{5D^2 - 17D + 24}{12D} x_1 - \frac{D-1}{D} x_1 x_2 \right) + \frac{(D^2 + 4)}{2D^2} x_2^2 - \frac{(5D^3 - D^2 - 10D - 48)}{6D^2} x_2, \\ f_5(D) &= \frac{D+2}{D} \left[\frac{5D^2 - 17D + 24}{6} + (D-1)(Dx_1 - 2x_2) \right] (1+z_1) + \frac{(b_1 + 6b_3)b_2}{3b_1^2}, \\ f_6(D) &= (D-1)(D+2)(1+z_1)^2 + \frac{2b_2^2}{b_1^2}. \end{aligned} \quad (79)$$

In the $D \rightarrow 4$ limit, we obtain the divergences,

$$\Gamma_{\text{div}}^{(1)} = -\frac{\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} (\hat{a}_2)_{\text{total}}. \quad (80)$$

One can rewrite (80) in terms of the 4D Gauss-Bonnet term and the square of the Weyl tensor,

$$C^2 = E_4 + 2 \left(R_{\alpha\beta}^2 - \frac{1}{3} R^2 \right). \quad (81)$$

This can be done by means of the inverse relations

$$\begin{aligned} R_{\mu\nu\alpha\beta}^2 &= 2C^2 - E_4 + \frac{1}{3} R^2, \\ R_{\alpha\beta}^2 &= \frac{1}{2} C^2 - \frac{1}{2} E_4 + \frac{1}{3} R^2. \end{aligned} \quad (82)$$

After all, the expression for the divergences in an arbitrary parametrization is

$$\Gamma_{\text{div}}^{(1)} = -\frac{\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} \{ g_1 C^2 + g_2 E_4 + g_3 \square R + g_4 R^2 + g_5 \Lambda R + g_6 \Lambda^2 \}, \quad (83)$$

where

$$\begin{aligned}
g_1 &= \frac{7}{20} + x_2^2 + \frac{2c^2}{b_1\gamma_1^2}, & g_2 &= \frac{149}{180} - x_2^2 - \frac{2c^2}{b_1\gamma_1^2}, \\
g_3 &= -\frac{19}{15} + \frac{b_3}{6b_1} - \frac{3}{4}(2x_1 - x_2), \\
g_4 &= \frac{b_3}{6b_1} + \frac{b_3^2}{2b_1^2} + \frac{c^2}{3b_1\gamma_1^2} + \frac{9}{2}(x_1^2 - x_1x_2 + x_1) \\
&\quad - \frac{9}{4}x_2 + \frac{31}{24}x_2^2 + \frac{1}{4}, \\
g_5 &= \frac{b_2(b_1 + 6b_3)}{3b_1^2} + 9[(2x_1 - x_2)(1 + z_1) + z_1 + 1], \\
g_6 &= 18(1 + z_1)^2 + \frac{2b_2^2}{b_1^2}. \tag{84}
\end{aligned}$$

For the sake of completeness, the same coefficients are written in Appendix B in terms of original parameters $\gamma_{1,\dots,6}$, r , and β_3 , describing parametrization ambiguity.

Using Eqs. (59) and (74), we can also evaluate the $\hat{a}_1$ coefficient. The result of this calculation is

$$\begin{aligned}
(\hat{a}_1)_{\text{total}} &= \left[\frac{b_3}{b_1} - \frac{5D^3 - 3D^2 + 12D + 48}{12D} \right. \\
&\quad \left. - \frac{(D-1)(D+2)(Dx_1 - 2x_2)}{2D} \right] R \\
&\quad + \left[\frac{2b_2}{b_1} - (D-1)(D+2)(1 + z_1) \right] \Lambda. \tag{85}
\end{aligned}$$

VI. ANALYSIS OF THE RESULTS: KNOWN LIMITS AND GOING ON SHELL

Let us first consider some special cases of our general answer, Eq. (83). First of all, in the limit

$$\begin{aligned}
x_{1,2} &\rightarrow 0, & z_1 &\rightarrow 0, & c &\rightarrow 0, \\
b_1 &\rightarrow -\frac{1}{16}, & b_2 &\rightarrow \frac{1}{16}, & b_3 &\rightarrow 0, \tag{86}
\end{aligned}$$

one should expect to reproduce the results for GR divergences in the simplest minimal gauge and simplest parametrization. In fact, we get in this limit

$$\begin{aligned}
\Gamma_{\text{div}}^{(1)} &= -\frac{\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} \left\{ \frac{7}{20} C^2 + \frac{149}{180} E_4 - \frac{19}{15} \square R \right. \\
&\quad \left. + \frac{1}{4} R^2 + \frac{26}{3} \Lambda R + 20 \Lambda^2 \right\}. \tag{87}
\end{aligned}$$

Using the relation (81), this expression becomes

$$\begin{aligned}
\Gamma_{\text{div}}^{(1)} &= -\frac{2\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} \left\{ \frac{53}{90} E_4 + \frac{7}{20} R_{\mu\nu}^2 \right. \\
&\quad \left. + \frac{1}{120} R^2 + \frac{13}{3} \Lambda R + 10 \Lambda^2 \right\}, \tag{88}
\end{aligned}$$

which is the famous result of 'tHooft and Veltman [1]. Furthermore, in the limit

$$x_{1,2} \rightarrow 0, \quad z_1 \rightarrow 0, \quad c \rightarrow \beta_3,$$

and

$$\begin{aligned}
b_1 &\rightarrow -\frac{1}{16} - \beta_3 - 4\beta_3^2, \\
b_2 &\rightarrow \frac{1}{16} + 2\beta_3 + 8\beta_3^2, \\
b_3 &\rightarrow \frac{1}{2}\beta_3 + 2\beta_3^2, \tag{89}
\end{aligned}$$

we checked that the result coincides with the one of Peixoto *et al.* [21].

A. On shell analysis near $D=4$

Certainly, the most interesting part is the on shell analysis. The Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} (R + 2\Lambda) = 0 \tag{90}$$

lead to the following relations:

$$\begin{aligned}
R_{\alpha\beta}^2 &= 4\Lambda^2, & R^2 &= 16\Lambda^2, & \square R &= 0, \\
\Lambda R &= -4\Lambda^2, & C^2 &= E_4 - \frac{8\Lambda^2}{3}. \tag{91}
\end{aligned}$$

Using these formulas, the Eq. (83) becomes

$$\begin{aligned}
\Gamma_{\text{div}|_{\text{on shell}}}^{(1)} &= -\frac{\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} \\
&\quad \times \left\{ \frac{53}{45} E_4 + \left[18(2x_1 - x_2 - z_1)^2 - \frac{224}{15} \right. \right. \\
&\quad \left. \left. - \frac{2(b_2 - 2b_3)(2b_1 - 3b_2 + 6b_3)}{3b_1^2} \right] \Lambda^2 \right\}. \tag{92}
\end{aligned}$$

It is not difficult to see that the second term in the integrand vanishes, because

$$2x_1 - x_2 - z_1 = 0. \tag{93}$$

For the last term, we have

$$\begin{aligned}
b_2 - 2b_3 &= \frac{1}{16}(\gamma_1 + 4\gamma_2 + 8r\beta_3)^2, \\
(2b_1 - 3b_2 + 6b_3) &= -\frac{5}{16}(\gamma_1 + 4\gamma_2 + 8r\beta_3)^2 \\
\text{and } b_1^2 &= \frac{1}{16^2}(\gamma_1 + 4\gamma_2 + 8r\beta_3)^4. \quad (94)
\end{aligned}$$

Therefore,

$$-\frac{2(b_2 - 2b_3)(2b_1 - 3b_2 + 6b_3)}{3b_1^2}\Lambda^2 = \frac{10}{3}\Lambda^2 \quad (95)$$

and the expression (92) boils down to

$$\Gamma_{\text{div}}^{(1)}|_{\text{on shell}} = -\frac{\mu^{D-4}}{\epsilon} \int d^4x \sqrt{-g} \left\{ \frac{53}{45} E_4 - \frac{58}{5} \Lambda^2 \right\}. \quad (96)$$

All in all, the one-loop divergences in the on shell limit do not depend of any parametrization or gauge parameters, exactly as it should be, see Eq. (7).

Similarly, for the overall $\hat{a}_1$ coefficient, in the on shell limit, we have

$$(\hat{a}_1)_{\text{total}}|_{\text{on shell}} = \left[\frac{38}{3} + 18(2x_1 - x_2 - z_1) + \frac{2(b_2 - 2b_3)}{b_1} \right] \Lambda. \quad (97)$$

As it was explained before, the second term in the rhs of (97) vanishes. For the third term, one meets

$$\frac{2(b_2 - 2b_3)}{b_1} \Lambda = -2\Lambda, \quad (98)$$

and finally,

$$(\hat{a}_1)_{\text{total}}|_{\text{on shell}} = \frac{32}{3} \Lambda, \quad (99)$$

which does not depend on parametrization or gauge parameters. It is worth to note that the gauge-fixing independence of the same coefficient in $D = 4$ was established before in Ref. [30]

B. D -dimensional on shell analysis

Finally, we can analyze the on shell limit in the Schwinger-DeWitt coefficients for an arbitrary dimension D , where they do not necessary correspond to a divergent part of the effective action. Taking the trace of Einstein's equations, we have

$$R = -\frac{2D}{(D-2)}\Lambda, \quad (100)$$

and consequently, the field equations can be rewritten as

$$R_{\mu\nu} = -\frac{2g_{\mu\nu}}{(D-2)}\Lambda. \quad (101)$$

Using the above equations, we found for the $\hat{a}_1$ coefficient, in the on shell limit, that

$$(\hat{a}_1)_{\text{total}}|_{\text{on shell}} = -\frac{D(D^2 - 3D - 36)}{6(D-2)}\Lambda \quad (102)$$

and

$$\begin{aligned}
(\hat{a}_2)_{\text{total}}|_{\text{on shell}} &= \frac{(D^2 - 33D + 540)}{360} R_{\mu\nu\alpha\beta}^2 \\
&+ \frac{D(5D^3 - 17D^2 - 354D - 720)}{180(D-2)^2} \Lambda^2 \quad (103)
\end{aligned}$$

for the $\hat{a}_2$ coefficient. We can see that both coefficients are gauge and parametrization independent in the on shell limit in general D -dimensional space-time. This feature is a clear sign of the importance of the locality in the gauge-fixing and parametrization independence of the one-loop effective action. The on shell universality holds for an arbitrary D , independent of whether the corresponding term is finite or divergent.

VII. CONCLUSIONS

The universality of beta functions and renormalization group flows in quantum GR is an important issue, due to the applications to asymptotic safety and effective quantum gravity approaches. While the gauge-fixing dependence is controlled by the on shell conditions, the parametrization dependence is not completely covered, especially in the gauge theories. This situation makes interesting the explicit calculations, but such calculations can become incredibly difficult in a nonminimal parametrization of gauge fixing.

By employing the “economic” approach to the one-loop calculations, we verified the on shell universality of the first local coefficients of the Schwinger-DeWitt expansion in an arbitrary dimension D . For the first time, the calculation has been done in the most general parametrization of a quantum metric, while the gauge-fixing parameters were partially constrained to provide the minimal form of the tensor operator of a bilinear form of the total action.

While our calculations were performed only for the first two coefficients of the Schwinger-DeWitt technique, the on shell universality of the result indicated that the parametrization and gauge-fixing independence of the on shell results is due to the locality of these terms in the Schwinger-DeWitt expansion. Therefore, without explicit calculations, one can ensure that further coefficients $\hat{a}_k$ with $k \geq 3$, are also on shell universal.

Indeed, the on shell universality property was always regarded as a useful tool in quantum gravity. As a recent

example, one can mention the gauge-fixing independence of the beta functions in superrenormalizable models of quantum gravity [31], which opens the way for interesting applications, such as the possibility to derive an exact and universal beta function for the Newton constant [32]. Another example is the recent resolution in Ref. [33] of the long-standing discrepancy between the calculations in the phenomenologically interesting tensor-scalar models, which were done in the Einstein [34] and Jordan frames [35]. Our present results indicate that this equivalence can be extended to the finite part of the effective action, at least to the local part and to the nonlocal sectors which can be in principle obtained by the summation of the Schwinger-DeWitt expansions.

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APPENDIX A: ON THE DERIVATION OF THE ACTION OF GHOSTS

Let us expose some details on the derivation of the generator (65). The background field splitting of the metric can be written as

$$g'_{\alpha\beta} = g_{\alpha\beta} + \kappa h_{\alpha\beta}^{(1)} + \kappa^2 h_{\alpha\beta}^{(2)} + \dots, \quad (\text{A1})$$

where

$$h_{\alpha\beta}^{(1)} = \gamma_1 \phi_{\alpha\beta} + \gamma_2 \phi g_{\alpha\beta}, \quad (\text{A2})$$

$$h_{\alpha\beta}^{(2)} = \gamma_3 \phi_{\alpha\rho} \phi_{\beta}^{\rho} + \gamma_4 \phi_{\rho\omega} \phi^{\rho\omega} g_{\alpha\beta} + \gamma_5 \phi \phi_{\alpha\beta} + \gamma_6 \phi^2 g_{\alpha\beta}, \quad (\text{A3})$$

and the dots stand for the σ -dependent terms, which we are not taking into account here. The reason is that, according to Eq. (63), the gauge transformation of these terms must be considered separately. Also, all the terms in (A3) can be safely ignored because they are of the second order in the quantum field. Then, the corresponding part of the gauge generator to (A3) must be proportional to $\phi_{\alpha\beta}$ and, consequently, gives no contribution in the $\phi_{\alpha\beta} \rightarrow 0$ limit. Therefore, the use of Eq. (A2) is sufficient for our purposes, because the σ -dependent terms are relevant only starting from the second loop order. The inverse form of the formula is

$$\phi_{\alpha\beta} = \frac{1}{\gamma_1} \left(\delta_{\alpha\beta}^{\mu\nu} - \frac{\gamma_2}{\gamma_1 + D\gamma_2} g_{\alpha\beta} g^{\mu\nu} \right) h_{\mu\nu}^{(1)}. \quad (\text{A4})$$

Consider the infinitesimal coordinate transformation

$$x^{\mu} \rightarrow x'^{\mu} = x^{\mu} + \xi^{\mu}. \quad (\text{A5})$$

Then,

$$\delta h_{\mu\nu}^{(1)} = -(g_{\mu\rho} \nabla_{\nu} + g_{\nu\rho} \nabla_{\mu}) \xi^{\rho}, \quad (\text{A6})$$

and we finally get

$$\delta \phi_{\alpha\beta} = -\frac{1}{\gamma_1} \left[g_{\alpha\rho} \nabla_{\beta} + g_{\beta\rho} \nabla_{\alpha} - \frac{2\gamma_2}{\gamma_1 + D\gamma_2} g_{\alpha\beta} \nabla_{\rho} \right] \xi^{\rho}, \quad (\text{A7})$$

which directly leads to the formula (65).

APPENDIX B: THE DIVERGENCES IN TERMS OF ORIGINAL PARAMETERS

Our purpose is to write the expressions (84) in terms of the original parameters of parametrization $\gamma_{1,2,\dots,6}$, r and gauge fixing β_3 . In order to avoid repetitions in formulas, let us introduce the notations

$$\begin{aligned} A &= \gamma_3 + 2\gamma_5 + 4r\beta_3\gamma_1, \\ B &= \gamma_1 + 4\gamma_2 + 8r\beta_3, \\ C &= 8r\beta_3(\gamma_1 + 4\gamma_2 + 4r\beta_3) + (\gamma_3 + 4\gamma_4) + 4(\gamma_5 + 4\gamma_6), \\ D &= (\gamma_1 + 4\gamma_2)^2 + 2[(\gamma_3 + 4\gamma_4) + 4(\gamma_5 + 4\gamma_6)] \\ &\quad + 32\beta_3[r(\gamma_1 + 4\gamma_2) + 4r^2\beta_3], \\ E &= \frac{(\gamma_3 + 4\gamma_4)}{\gamma_1^2}. \end{aligned} \quad (\text{B1})$$

Then the coefficients can be cast into the form

$$\begin{aligned} g_1 &= \frac{7}{20} + \frac{4\gamma_3^2}{\gamma_1^4} - \frac{2A^2}{\gamma_1^2 B^2}, \\ g_2 &= \frac{149}{180} - \frac{4\gamma_3^2}{\gamma_1^4} + \frac{2A^2}{\gamma_1^2 B^2}, \\ g_3 &= -\frac{19}{15} + \frac{3E}{2} - \frac{C}{6B^2}, \\ g_4 &= \frac{1}{4} - \frac{9E}{2} + \frac{31\gamma_3^2 + 216\gamma_4(\gamma_3 + 2\gamma_4)}{6\gamma_1^4} - \frac{A^2}{3\gamma_1^2 B^2} - \frac{C}{6B^2} + \frac{C^2}{2B^4}, \\ g_5 &= 9(1 - 2E)^2 - \left(\frac{1}{3} - \frac{2C}{B^2} \right) \frac{D}{B^2}, \\ g_6 &= 18(1 - 2E)^2 + \frac{2D^2}{B^4}. \end{aligned} \quad (\text{B2})$$

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