Seeking a CP -odd Higgs boson via $h \rightarrow \eta_{c,b} \ell^+ \ell^-$

Damir Bečirević,¹ Blaženka Melić,² Monalisa Patra,² and Olcyr Sumensari^{1,3}

¹*Laboratoire de Physique Théorique (Bât. 210) CNRS, Université Paris-Sud,
Université Paris-Saclay, 91405 Orsay, France*

²*Institut Rudjer Bošković, Division of Theoretical Physics, Bijenička 54, HR-10000, Croatia*

³*Instituto de Física, Universidade de São Paulo, C.P. 66.318, 05315-970 São Paulo, Brazil*

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We show that the decay rates of the Higgs boson to a pseudoscalar quarkonium and a pair of leptons, $h \rightarrow P \ell^+ \ell^-$ ($P \in \{\eta_c, \eta_b\}$), can be substantially enhanced in a scenario with two Higgs doublets with a softly broken $\mathbb{Z}_2$ symmetry [two Higgs doublet model (2HDM)] when the CP -odd Higgs A is light, i.e. $m_A \lesssim m_h$. Depending on the type of 2HDM the enhancement of $\mathcal{B}(h \rightarrow \eta_{c,b} \tau^+ \tau^-)$ with respect to its Standard Model value can be an order of magnitude larger, i.e. $\mathcal{O}(10^{-6}-10^{-5})$. The decays $h \rightarrow P \ell^+ \ell^-$ could therefore provide an efficient channel to investigate the presence of a light CP -odd Higgs A and help to disentangle among various 2HDM scenarios.

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I. INTRODUCTION

One of the minimal extensions of the Standard Model consists in enlarging the Higgs sector and, instead of one doublet of scalar fields, introducing an additional Higgs doublet, which comes under the generic name of two Higgs doublet model (2HDM) [1]. With a peculiar choice of Yukawa couplings, 2HDM is embedded in the minimal supersymmetric extension of the Standard Model (MSSM) which made it particularly popular in phenomenological applications of low energy supersymmetry. The fact that the mass of observed Higgs boson was found to be consistent with the Standard Model expectations made MSSM less compelling but the 2HDM remains a convenient framework to study the extensions of the Higgs sector in view of the current experimental searches.

Despite its minimalism 2HDM has a rich structure. Besides the usual Higgs state h , there is an extra CP -even scalar (H) and two additional states of which one is a charged scalar boson ($H^\pm$) and another a CP -odd state (A). The state observed at LHC is identified with h [2], or H [3] which is otherwise expected to be heavier than h . A fact that the measured rates of the tree level weak decays of leptons and mesons are consistent with the Standard Model predictions is a hint that the charged Higgs is heavy too, also confirmed by the direct searches [4]. The CP -odd Higgs boson, instead, remains unconstrained explicitly. It is

often assumed that its mass is larger than that of the observed Higgs, $m_A > m_h$. This, however, is just an assumption which should be tested experimentally. Several proposals to look for a light CP -odd Higgs were made in the past [5–8]. In this paper we propose to study decays of Higgs to the pseudoscalar heavy quarkonia (P), $h \rightarrow P \ell^+ \ell^-$, ($P \in \{\eta_c, \eta_b\}$) processes in which the CP -odd Higgs can contribute at the tree level and make a significant enhancement of various decay rates. The level of such an enhancement is related to the structure of the Yukawa couplings and to the mass of the A state. As we shall see in the following, we find that $\mathcal{B}(h \rightarrow \eta_{c,b} \tau^+ \tau^-)$ can be enhanced by an order of magnitude with respect to its Standard Model value, which is why we find it interesting and worth studying in experiments.

Studies of the Higgs boson to quarkonia attracted quite a bit of attention: Radiative decay $h \rightarrow J/\psi \gamma$ could be used to probe the Yukawa coupling $h c \bar{c}$, a possibility which is compromised in the case of b -quark quarkonia [$\Upsilon(nS)$] due to cancellation of two contributions to the decay amplitude [9]. A possibility to study $h \rightarrow PZ$ and $h \rightarrow VZ$ (where V and P stand for the vector and pseudoscalar quarkonium states, respectively) was elaborated in Refs. [10,11]. Finally, a possibility to search for the effects of lepton flavor violation via $h \rightarrow V \ell_1 \ell_2$ has been proposed in Ref. [12].

The remainder of this paper is organized as follows: In Sec. II we derive the expressions for $\mathcal{B}(h \rightarrow PZ)$ and $\mathcal{B}(h \rightarrow P \ell^+ \ell^-)$ both in the Standard Model and in 2HDM with a light CP -odd Higgs state. In Sec. III we scan the parameter space of 2HDM with a softly broken $\mathbb{Z}_2$ symmetry. The results of our scan are used in Sec. IV where we test the sensitivity of $\mathcal{B}(h \rightarrow \eta_{c,b} \ell^+ \ell^-)$ on the

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FIG. 1. Diagrams contributing $h \rightarrow PZ$ decay.

presence of a light CP -odd Higgs state. We summarize our findings and briefly conclude in Sec. V.

II. EXPRESSIONS FOR $\mathcal{B}(h \rightarrow PZ)$ AND $\mathcal{B}(h \rightarrow P\ell^+\ell^-)$

In this section we provide the expressions for $\mathcal{B}(h \rightarrow PZ)$ and $\mathcal{B}(h \rightarrow P\ell^+\ell^-)$ both in the Standard Model (SM) and in the generic 2HDM scenario. To do so we need to specify the notation. By P we denote the pseudoscalar quarkonium carrying momentum k , while Z flies with momentum p_Z . The dilepton invariant mass squared in the $P\ell^+\ell^-$ final state is considered as $q^2 = (p_\ell + p_{\bar{\ell}})^2$, with $p_{\ell, \bar{\ell}}$ being the momentum of the outgoing leptons.

A. $\mathcal{B}(h \rightarrow PZ)$

In the SM the decay $h \rightarrow PZ$ occurs via the diagrams (a) and (b) shown in Fig. 1. The dominant (“indirect”) contribution comes from $h \rightarrow ZZ^* \rightarrow ZP$ [Fig. 1(b)] which is much larger than the direct one, $h \rightarrow Z\bar{q}q \rightarrow ZP$ [Fig. 1(a)]. In a 2HDM an additional significant contribution arises from $h \rightarrow ZA^* \rightarrow ZP$ [Fig. 1(c)], which can be large if the CP -odd Higgs is light and the corresponding couplings to fermions are non-negligible. The decay amplitude then can be written as

$$\mathcal{M}(h \rightarrow PZ) = \frac{g}{v \cos \theta_W} (k \cdot \varepsilon_Z^*) F^{PZ}, \quad (1)$$

with

$$F^{PZ} \approx \frac{m_Z^2}{m_Z^2 - m_P^2} f_P g_A^q - \frac{f_P v}{m_A^2 - m_P^2 - im_A \Gamma_A} \frac{m_P^2}{2m_q} \frac{m_q}{v} \times C_A^q \cos(\beta - \alpha), \quad (2)$$

where we displayed only the dominant SM contribution and the one arising from the CP -odd Higgs in the 2HDM. In the above formula $g_A^q = T_q^3$, C_A^q is a coupling of the $q\bar{q}$ par to the CP -odd Higgs state

$$\mathcal{L}_Y \supset i \frac{m_f}{v} C_A^f \bar{f} \gamma_5 f A, \quad (3)$$

which will be more explicitly defined below [cf. Eq. (26)]. Furthermore, in Eq. (2) we have also used the standard definition of the decay constant f_P , namely,

$$\langle P(k) | \bar{q} \gamma^\mu \gamma_5 q | 0 \rangle = -i f_P k^\mu, \quad \langle P(k) | \bar{q} \gamma^5 q | 0 \rangle = -i f_P \frac{m_P^2}{2m_q}, \quad (4)$$

where $q = c$ or b , for $P = \eta_c$ or η_b , respectively. We should emphasize that, owing to the fact that it is a flavor singlet, $\eta_{c,b}$ can also couple to $(\alpha_s/4\pi) F_{\mu\nu} \tilde{F}^{\mu\nu}$, with F ($\tilde{F}$) being the (dual) QCD field strength tensor. While such a term is important for the light quark flavor singlets (π^0 , η , η'), it is expected to be very small for the case of heavy quarkonia, which is why it will be neglected in the following.¹ With the above definitions, the decay rate then reads

$$\Gamma(h \rightarrow PZ) = \frac{\lambda^{3/2}(m_h, m_P, m_Z)}{16\pi v^4 m_h^3} |F^{PZ}|^2, \quad (5)$$

where $\lambda(a, b, c) = [a^2 - (b+c)^2][a^2 - (b-c)^2]$. Notice that in the above amplitude we kept the width of the CP -odd Higgs boson different from zero. We emphasize that, for clarity, we disregarded the direct contributions to the amplitude because they are numerically much smaller and, since the corresponding expression is more complicated, we decided to relegate it to the Appendix.² To get the branching fraction in a 2HDM setup, one should be particularly careful with the width of the Higgs boson Γ_h which should not be much larger than its SM value, e.g. $\Gamma_h/\Gamma_h^{\text{SM}} \lesssim 1.4$, a condition that provides a particularly stringent bound on the coupling of h to two light CP -odd Higgses in the situation in which $m_A \leq m_h/2$. To be more specific, from the general 2HDM potential, we can read off the term corresponding to the trilinear interaction, namely,

$$\mathcal{L}_{\text{2HDM}} \supset \frac{v}{2!} \lambda_{hAA} hAA, \quad (6)$$

¹This can be understood by noting that heavy quarkonia are compact hadronic states, $r \sim (m_{c,b} v_P)^{-1}$, where v_P is the velocity of the quarkonia, so that their overlap with soft gluons $\sim \Lambda_{\text{QCD}}^{-1}$ should be very small.

²We checked that the direct contribution is indeed numerically much smaller than the first term in Eq. (2), and that the decay rate becomes larger by no more than 7% when it is included in the calculation.

where $v \approx 246$ GeV, and the coupling

$$\lambda_{hAA} = \frac{1}{2v^2 \sin(2\beta)} \{m_h^2 [\cos(\alpha - 3\beta) + 3 \cos(\alpha + \beta)] + 4m_A^2 \sin(2\beta) \sin(\beta - \alpha) - 4M^2 \cos(\alpha + \beta)\}, \quad (7)$$

with α , β and M^2 explicitly defined in Sec. III. The expression for the decay width reads

$$\Gamma(h \rightarrow AA) = \frac{|\lambda_{hAA}|^2}{32\pi} \frac{v^2}{m_h} \sqrt{1 - \frac{4m_A^2}{m_h^2}}. \quad (8)$$

Besides $h \rightarrow AA$, if opened, the channel $h \rightarrow ZA$ can give significant contribution to the width Γ_h . The decay width of this particular channel is given by

$$\Gamma(h \rightarrow ZA) = \frac{1}{16\pi} \frac{\cos^2(\beta - \alpha)}{m_h^3 v^2} \lambda^{3/2}(m_h, m_Z, m_A). \quad (9)$$

B. $\mathcal{B}(h \rightarrow P\ell^+\ell^-)$

In the Standard Model the situation with $h \rightarrow P\ell^+\ell^-$ is similar to the one discussed in the case of $h \rightarrow PZ$. The dominant contribution comes from the diagram Fig. 2(c) which reads

$$\begin{aligned} \mathcal{M}(h \rightarrow P\ell^+\ell^-)^{2c} = & -\frac{1}{4} \left(\frac{g}{\cos\theta_W} \right)^3 m_Z \frac{g_A^q f_P}{\mathcal{P}_1(q, m_Z, \Gamma_Z) \mathcal{P}_2(k, m_Z)} k^\nu \left(-g_{\nu\alpha} + \frac{k_\nu k_\alpha}{m_Z^2} \right) \\ & \times \left(-g_\mu^\alpha + \frac{q_\mu q^\alpha}{m_Z^2} \right) \bar{u}_\ell \gamma^\mu (g_V^\ell - g_A^\ell \gamma_5) v_\ell, \end{aligned} \quad (10)$$

where $\mathcal{P}_1(a, b, c) = a^2 - b^2 + ibc$ and $\mathcal{P}_2(a, b) = a^2 - b^2$. The full expression, which includes all contributions depicted in Fig. 2, is given in the Appendix of the present paper. We note, however, that the contribution of the diagram in Fig. 2(d) vanishes in the decay rate.

Diagrams arising in the 2HDM setup are shown in Fig. 3, of which the first two are numerically much less significant than the remaining three. The contributions of those latter (dominant) diagrams to the decay amplitude read

$$\begin{aligned} \mathcal{M}(h \rightarrow P\ell^+\ell^-)^{3c} = & -\left(\frac{g}{2\cos\theta_W}\right)^2 \frac{m_q C_A^q m_P^2 f_P}{v} \frac{\cos(\beta-\alpha)}{\mathcal{P}_1(q, m_Z, \Gamma_Z) \mathcal{P}_1(k, m_A, \Gamma_A)} \\ & \times \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_Z^2}\right) (k+p)^\mu \bar{u}_\ell \gamma^\nu (g_V^\ell - g_A^\ell \gamma_5) v_\ell, \end{aligned} \quad (11)$$

$$\begin{aligned} \mathcal{M}(h \rightarrow P\ell^+\ell^-)^{3d} = & \left(\frac{g}{2\cos\theta_W} \right)^2 \frac{m_\ell C_A^\ell}{v} \frac{g_A^q f_P \cos(\beta - \alpha)}{\mathcal{P}_1(q, m_A, \Gamma_A) \mathcal{P}_2(k, m_Z)} \\ & \times \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_Z^2} \right) (q + p)^\mu k^\nu \bar{u}_\ell \gamma_5 v_\ell, \end{aligned} \quad (12)$$

$$\mathcal{M}(h \rightarrow P\ell^+\ell^-)^{3e} = -\lambda_{hAA}v \frac{m_q C_A^q}{v} \frac{m_\ell C_A^\ell}{v} \frac{m_P^2 f_P}{2m_q} \frac{1}{\mathcal{P}_1(q, m_A, \Gamma_A) \mathcal{P}_1(k, m_A, \Gamma_A)} \bar{u}_\ell \gamma_5 v_\ell. \quad (13)$$

In the limit in which $\Gamma_X/m_X \ll 1$, one can work in the narrow width approximation, which amounts to replacing

FIG. 2. Diagrams relevant to $h \rightarrow P\ell^+\ell^-$ decay in the Standard Model. The full dot in the diagram (d) indicates that the vertex is loop induced. Its contribution to the decay rate is nevertheless zero.

FIG. 3. Contributions to the $h \rightarrow P\ell^+\ell^-$ decay amplitude in a 2HDM scenario.

$$\frac{1}{(q^2 - m_X^2)^2 + m_X^2 \Gamma_X^2} \rightarrow \delta(q^2 - m_X^2) \frac{\pi}{m_X \Gamma_X}. \quad (14)$$

This approximation is adopted throughout this paper both for $X = Z$ and $X = A$. To a very good approximation the decay rate can be written as

$$\Gamma(h \rightarrow P\ell^+\ell^-) \simeq \Gamma(h \rightarrow PZ^* \rightarrow P\ell^+\ell^-) + \Gamma(h \rightarrow PA^* \rightarrow P\ell^+\ell^-). \quad (15)$$

We checked that the interference terms are indeed very small and the above formula is useful for the phenomenology we are interested in. For the explicit expressions of the separate rates in Eq. (15) we obtain

$$\begin{aligned} \Gamma(h \rightarrow PZ^* \rightarrow P\ell^+\ell^-) &= \frac{f_P^2 m_Z^3}{384\pi^2 \Gamma_Z m_h^3 v^6} [\cos^2(2\theta_W) + 4\sin^4\theta_W] \\ &\times \left| g_A^q - \frac{C_A^q m_P^2 \cos(\beta - \alpha)}{2(m_A^2 - m_P^2 - im_A \Gamma_A)} \right|^2 \lambda^{3/2}(m_h, m_P, m_Z), \end{aligned} \quad (16)$$

$$\begin{aligned} \Gamma(h \rightarrow PA^* \rightarrow P\ell^+\ell^-) &= \frac{f_P^2 m_A}{512\pi^2 \Gamma_A m_h^3 v^2} \left(\frac{m_\ell C_A^\ell}{v} \right)^2 \left| \lambda_{hAA} \frac{m_P^2}{m_A^2 - m_P^2 - im_A \Gamma_A} \frac{C_A^q}{v} \right. \\ &\left. + 2 \cos(\beta - \alpha) \frac{g_A^q}{v} (m_h^2 - m_A^2) \right|^2 \lambda^{1/2}(m_h, m_P, m_A). \end{aligned} \quad (17)$$

In the above formulas $g_V^f = T_f^3 - 2Q_f \sin^2 \theta_W$, $g_A^f = T_f^3$, and we neglected the additive terms $\propto m_\ell^2/m_Z^2$. We emphasize once again that the above expression for $\Gamma(h \rightarrow P\ell^+\ell^-)$ should be viewed as a very good approximation, whereas the full expressions are provided in the Appendix of the present paper.

Finally, since the above formulas require the knowledge of the width of A , we give that expression too, in which we include $\Gamma_A = \Gamma(A \rightarrow f\bar{f}) + \Gamma(A \rightarrow \gamma\gamma)$, which are given by

$$\begin{aligned} \Gamma(A \rightarrow f\bar{f}) &= \theta(m_A - 2m_f) |C_A^f|^2 \frac{N_c m_f^2}{8\pi v^2} m_A \sqrt{1 - \frac{4m_f^2}{m_A^2}}, \\ \Gamma(A \rightarrow \gamma\gamma) &= \frac{\alpha_{\text{em}}^2}{16\pi^3 v^2} m_A^3 \left| \sum_f C_A^f N_c Q_f^2 x_f F_{\gamma\gamma}(x_f) \right|^2, \end{aligned} \quad (18)$$

where $N_c = 3$ for quarks and $N_c = 1$ for leptons, the electric charge for leptons, up-type and down-type quarks is $Q_\ell = -1$, $Q_u = 2/3$, $Q_d = -1/3$, respectively, and f runs over all available quark and lepton flavors. In the above expression we used the notation $x_f = m_f^2/m_A^2$, and the loop function reads

$$F_{\gamma\gamma}(x) = \begin{cases} \frac{1}{2} \left(i\pi + \log \left[\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}} \right] \right)^2, & \text{for } x < 1/4 \\ -2\text{arcsin}^2\left(\frac{1}{2\sqrt{x}}\right), & \text{otherwise} \end{cases}. \quad (19)$$

Notice that $\Gamma(A \rightarrow \gamma\gamma) \ll \Gamma(A \rightarrow f\bar{f})$ and our conclusion would remain the same even if we neglected $\Gamma(A \rightarrow \gamma\gamma)$. For the same reason we neglect the $\Gamma(A \rightarrow gg)$ contribution to the full width of A .

III. SCANNING THE 2HDM PARAMETER SPACE

We consider a general CP -conserving 2HDM with a softly broken $\mathbb{Z}_2$ symmetry, described by the following potential:

$$\begin{aligned} V(\Phi_1, \Phi_2) = & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 + m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) \\ & + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 \\ & + \frac{\lambda_5}{2} [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2], \end{aligned} \quad (20)$$

where the term proportional to m_{12}^2 breaks the $\mathbb{Z}_2$ symmetry. In the above expression Φ_a ($a = 1, 2$) stand for a scalar doublet

$$\Phi_a(x) = \begin{pmatrix} \phi_a^+(x) \\ \frac{1}{\sqrt{2}}(v_a + \rho_a(x) + i\eta_a(x)) \end{pmatrix}, \quad (21)$$

where $v_{1,2}$ are the vacuum expectation values related to $v^{\text{SM}} = \sqrt{v_1^2 + v_2^2} = 246.22$ GeV [13]. Two of the above fields are Goldstone bosons ($G^0, G^\pm$), two are massive CP -even states (h, H), one CP -odd (A), and one charged Higgs ($H^\pm$). These fields are related to the ones given in Eq. (21) via

$$\begin{aligned} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} &= \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}, & \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} &= \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^0 \\ A \end{pmatrix}, \\ \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} &= \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \end{aligned} \quad (22)$$

where the mixing angles α and β satisfy

$$\tan \beta = \frac{v_2}{v_1}, \quad \tan 2\alpha = \frac{2(-m_{12}^2 + \lambda_{345} v_1 v_2)}{m_{12}^2(v_2/v_1 - v_1/v_2) + \lambda_1 v_1^2 - \lambda_2 v_2^2}, \quad (23)$$

with $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$. The masses of the physical scalars can be written in terms of parameters which appear in the potential as

$$\begin{aligned} m_H^2 &= M^2 \sin^2(\alpha - \beta) + \left(\lambda_1 \cos^2 \alpha \cos^2 \beta + \lambda_2 \sin^2 \alpha \sin^2 \beta + \frac{\lambda_{345}}{2} \sin 2\alpha \sin 2\beta \right) v^2, \\ m_h^2 &= M^2 \cos^2(\alpha - \beta) + \left(\lambda_1 \sin^2 \alpha \cos^2 \beta + \lambda_2 \cos^2 \alpha \sin^2 \beta - \frac{\lambda_{345}}{2} \sin 2\alpha \sin 2\beta \right) v^2, \\ m_A^2 &= M^2 - \lambda_5 v^2, \\ m_{H^\pm}^2 &= M^2 - \frac{\lambda_4 + \lambda_5}{2} v^2, \end{aligned} \quad (24)$$

where for shortness we use $M^2 = m_{12}^2/(\sin \beta \cos \beta)$. As far as the Yukawa sector is concerned, the $\mathbb{Z}_2$ symmetry is imposed to prevent the flavor changing processes to appear at tree level [14] by enforcing each type of the right-handed fermion to couple to a single Higgs doublet. Four choices are then possible and they are called type I, II, X and Z [1]. By writing the Yukawa Lagrangian as

$$\begin{aligned} \mathcal{L}_Y = & -\frac{\sqrt{2}}{v} H^+ \{ \bar{u} [\zeta^d V_{\text{CKM}} m_d P_R - \zeta^u m_u V_{\text{CKM}} P_L] d + \zeta^\ell \bar{\nu} m_\ell P_R \ell \} \\ & - \frac{1}{v} \sum_{f, \phi_i^0 \in \{h, H, A\}} \xi_{\phi_i^0}^f \phi_i^0 [\bar{f} m_f P_R f] + \text{H.c.}, \end{aligned} \quad (25)$$

TABLE I. Couplings ζ^f in various types of 2HDM.

Model	ζ^d	ζ^u	ζ^ℓ
Type I	$\cot \beta$	$\cot \beta$	$\cot \beta$
Type II	$-\tan \beta$	$\cot \beta$	$-\tan \beta$
Type X (lepton specific)	$\cot \beta$	$\cot \beta$	$-\tan \beta$
Type Z (flipped)	$-\tan \beta$	$\cot \beta$	$\cot \beta$

where a specific choice of parameters ζ^f corresponds to the above-mentioned types of 2HDM, cf. Table I. In the expression u , d and ℓ stand for the up-type, down-type quark, and a lepton flavor respectively, and f for a generic fermion. V_{CKM} is the Cabibbo-Kobayashi-Maskawa matrix, lepton mixing is neglected in 2HDM due to the absence of right-handed neutrinos and $P_{L,R} = (1 \mp \gamma_5)/2$. Furthermore, the couplings $\xi_{\varphi_i^0}^f$ are related to ζ^f as

$$\begin{aligned}\xi_h^f &= \sin(\beta - \alpha) + \cos(\beta - \alpha)\zeta^f, \\ \xi_H^f &= \cos(\beta - \alpha) - \sin(\beta - \alpha)\zeta^f, \\ \xi_A^u &= -i\zeta^u, \quad \xi_A^{d,\ell} = i\zeta^{d,\ell}.\end{aligned}\quad (26)$$

The last couplings are related to the one used in Eq. (3) through $C_A^f = -\text{Im}(\xi_A^f)$.

Once we spelled out all the parameters of 2HDM we need to perform a scan of the parameters by taking into account the following general theory constraints:

- (i) *Stability* of the scalar potential is ensured by the requirement that it is bounded from below, which is achieved if the quartic couplings satisfy [15]

$$\begin{aligned}\lambda_{1,2} &> 0, \quad \lambda_3 > -(\lambda_1\lambda_2)^{1/2}, \\ \lambda_3 + \lambda_4 - |\lambda_5| &> -(\lambda_1\lambda_2)^{1/2},\end{aligned}\quad (27)$$

while the stability of the electroweak vacuum requires

$$\begin{aligned}\Delta S^{\text{SM}} &= 0.05 \pm 0.11, \\ \Delta T^{\text{SM}} &= 0.09 \pm 0.13, \\ \Delta U^{\text{SM}} &= 0.01 \pm 0.11,\end{aligned}\quad \text{corr} = \begin{pmatrix} 1 & 0.90 & -0.59 \\ 0.90 & 1 & -0.83 \\ -0.59 & -0.83 & 1 \end{pmatrix},\quad (33)$$

to 99% C.L. Using $X = (\Delta S, \Delta T, \Delta U)$, $\sigma = (0.11, 0.13, 0.11)$, and $\sigma_{ij}^2 \equiv \sigma_i \text{corr}_{ij} \sigma_j$, we build

$$\chi^2 = \sum_{i,j} (X_i - X_i^{\text{SM}})(\sigma^2)_{ij}^{-1}(X_j - X_j^{\text{SM}})\quad (34)$$

and choose the points which agree with the above numbers.

In our scan we identify the lightest CP -even state (h) with the SM-like scalar, observed at the LHC, with mass $m_h = 125.09(24)$ GeV [13]. We assume the branching ratio of the Higgs exotic decay modes,

$$m_{11}^2 + \frac{\lambda_1 v_1^2}{2} + \frac{\lambda_3 v_2^2}{2} = \frac{v_2}{v_1} \left[m_{12}^2 - (\lambda_4 + \lambda_5) \frac{v_1 v_2}{2} \right],\quad (28)$$

$$m_{22}^2 + \frac{\lambda_2 v_2^2}{2} + \frac{\lambda_3 v_1^2}{2} = \frac{v_1}{v_2} \left[m_{12}^2 - (\lambda_4 + \lambda_5) \frac{v_1 v_2}{2} \right].\quad (29)$$

In order for a minimum to be global, the parameters should also satisfy [16]

$$m_{12}^2 \left(m_{11}^2 - m_{22}^2 \sqrt{\lambda_1/\lambda_2} \right) \left(\tan \beta - \sqrt{4\lambda_1/\lambda_2} \right) > 0.\quad (30)$$

- (ii) *Unitarity* of the S -wave of the scalar scattering amplitudes gives rise to the following inequalities [17,18]:

$$|a_\pm|, |b_\pm|, |c_\pm|, |f_\pm|, |e_{1,2}|, |f_1|, |p_1| < 8\pi, \quad (31)$$

where

$$\begin{aligned}a_\pm &= \frac{3}{2}(\lambda_1 + \lambda_2) \pm \sqrt{\frac{9}{4}(\lambda_1 - \lambda_2)^2 + (2\lambda_3 + \lambda_4)^2}, \\ b_\pm &= \frac{1}{2}(\lambda_1 + \lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_4^2}, \\ c_\pm &= \frac{1}{2}(\lambda_1 + \lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2}, \\ e_1 &= \lambda_3 + 2\lambda_4 - 3\lambda_5, \quad e_2 = \lambda_3 - \lambda_5, \\ f_+ &= \lambda_3 + 2\lambda_4 + 3\lambda_5, \quad f_- = \lambda_3 + \lambda_5, \\ f_1 &= \lambda_3 + \lambda_4, \quad p_1 = \lambda_3 - \lambda_4.\end{aligned}\quad (32)$$

- (iii) *Electroweak precision tests* provide important constraint to the 2HDM parameters. We consider the expressions for the parameters S , T and U in 2HDM [19], and the values for ΔS , ΔT , ΔU given in Ref. [20],

FIG. 4. Results of the scan of parameters (35) after imposing constraints as discussed in the text. The blue points satisfy the theoretical constraints and the red points are forbidden by the flavor bounds [22,23]. Notice in particular that the darker (blue, red) points correspond to the *free* scan and the lighter (blue, red) points correspond to the *fine-tuned* scan with $m_H \approx |M|$.

$\mathcal{B}(h \rightarrow \text{exotic}) \leq 0.3$,³ and impose the near-alignment condition, $|\cos(\beta - \alpha)| \leq 0.3$, in order to ensure that g_{hWW} and g_{hZZ} remain consistent with the measured values and in agreement with the SM predictions [21].

The above-mentioned constraints are then imposed onto a set of randomly generated points in the intervals:

$$\begin{aligned} \tan \beta &\in (0.2, 50), \quad \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \quad |M^2| \leq (1 \text{ TeV})^2, \\ m_{H^\pm} &\in (m_W, 1 \text{ TeV}), \quad m_H \in (m_h, 1 \text{ TeV}), \\ m_A &\in (20 \text{ GeV}, m_h). \end{aligned} \quad (35)$$

Due to correlation between m_A and $m_{H^\pm}$, the fact that we choose to work with light CP -odd Higgs implies that the charged Higgs is bounded from above. We find $m_{H^\pm} \lesssim 700 \text{ GeV}$, as shown in Fig. 4, where we show the allowed parameter space for type I and type II models. Similar results are obtained for type X and type Z. The color coding of Fig. 4 is as follows:

³By $\mathcal{B}(h \rightarrow \text{exotic}) \leq 0.3$ we mean the contribution to the width Γ_h arising from the 2HDM alone, namely $h \rightarrow AA, AZ, H^+H^-$. In other words $\mathcal{B}(h \rightarrow \text{exotic}) = (\Gamma_h - \Gamma_h^{\text{SM}})/\Gamma_h$, so that the requirement $\mathcal{B}(h \rightarrow \text{exotic}) \leq 0.3$ implies that $\Gamma_h/\Gamma_h^{\text{SM}} \leq 1.43$.

- (1) A *free* scan of parameters consistent with the constraints listed above favors the moderate and small values of $\tan \beta \in (1, 15]$ shown as darker blue points in Fig. 4.
- (2) Larger values of $\tan \beta$ can be probed around the alignment limit as it can be seen from Eq. (24). For that reason, in addition to the free scan, we also perform a fine-tuned scan, i.e. with $m_H \approx |M|$, shown as lighter blue points in Fig. 4.
- (3) In addition to the above constraints we also impose the bound arising from the comparison between theory predictions and the experimental results concerning the spectrum of the $B \rightarrow X_s \gamma$ decay which for the case of type II and type Z models amounts to a bound $m_{H^\pm} \geq 439 \text{ GeV}$ to 99% C.L. (or $\approx 3\sigma$) [22]. Furthermore, the 3σ bounds coming from the exclusive $b \rightarrow s\mu^+\mu^-$ decay modes, discussed in Ref. [23], have also been included. The points forbidden by the flavor bound in the parameter space of the free (fine-tuned) scan [darker (lighter) blue] are shown in darker (lighter) red.

As it can be seen from Fig. 4, among the remaining points in the parameter space of the type II model, the free scan prefers low $\tan \beta \approx 2$ values, while the large $\tan \beta$ values can only be accessed through the scan in the $m_H \approx |M|$ direction. The same observation applies to the type Z model.

TABLE II. Resulting intervals for the ratios obtained from the scans in various types of 2HDM.

Ratio	$R_{\eta_c}^Z$	$R_{\eta_b}^Z$	$R_{\eta_c}^{\mu\mu}$	$R_{\eta_b}^{\mu\mu}$	$R_{\eta_c}^{\tau\tau}$	$R_{\eta_b}^{\tau\tau}$
Type I	(0.7,1.0)	(0.7,1.0)	(0.7,1.0)	(0.7,1.0)	(0.7,3.3)	(0.7,3.6)
Type II	(0.7,1.0)	(0.3,1.7)	(0.7,1.0)	(0.7,1.7)	(0.8,2.3)	(0.9,130)
Type X	(0.7,1.0)	(0.7,1.0)	(0.7,1.1)	(0.7,1.1)	(0.7,21)	(0.7,23)
Type Z	(0.7,1.0)	(0.3,1.7)	(0.7,1.0)	(0.7,1.7)	(0.7,1.2)	(0.7,1.7)

Notice also that one cannot simultaneously access $m_A < m_h/2$ and have large $\tan\beta$, except for the very fine-tuned solutions. To see that we expand Eq. (7) around the alignment limit, $\delta = \cos(\alpha - \beta) \approx 0$, and obtain

$$\lambda_{hAA} = \frac{m_h^2 + 2m_A^2 - 2M^2}{v^2} + \frac{2(m_h^2 - m_H^2)}{v^2 \tan 2\beta} \delta + \mathcal{O}(\delta^2). \quad (36)$$

To suppress $h \rightarrow \text{exotic}$ in the perfect alignment limit, one needs $2M^2 = m_h^2 + 2m_A^2$, which for the low m_A gives $M^2 \leq 3m_h^2/4 \ll m_H^2$. Therefore, in order to get the large $\tan\beta$ values one needs $M \approx m_H$ which is in contradiction with the previous inequality.

IV. SENSITIVITY OF $\mathcal{B}(h \rightarrow PZ)$ AND $\mathcal{B}(h \rightarrow P\ell^+\ell^-)$ ON THE LIGHT CP -ODD HIGGS STATE

In this section we use the results of the scan discussed above and evaluate

$$\begin{aligned} R_{\eta_{cb}}^Z &= \frac{\mathcal{B}(h \rightarrow \eta_{cb}Z)^{2\text{HDM}}}{\mathcal{B}(h \rightarrow \eta_{cb}Z)^{\text{SM}}}, \\ R_{\eta_{cb}}^{\tau\tau} &= \frac{\mathcal{B}(h \rightarrow \eta_{cb}\tau^+\tau^-)^{2\text{HDM}}}{\mathcal{B}(h \rightarrow \eta_{cb}\tau^+\tau^-)^{\text{SM}}}, \\ R_{\eta_{cb}}^{\mu\mu} &= \frac{\mathcal{B}(h \rightarrow \eta_{cb}\mu^+\mu^-)^{2\text{HDM}}}{\mathcal{B}(h \rightarrow \eta_{cb}\mu^+\mu^-)^{\text{SM}}}, \end{aligned} \quad (37)$$

where η_{cb} means either η_c or η_b . Essential hadronic quantities needed to evaluate $\mathcal{B}(h \rightarrow \eta_{cb}Z)^{\text{SM}}$ and $\mathcal{B}(h \rightarrow \eta_{cb}\ell^+\ell^-)^{\text{SM}}$ are [24,25]

$$f_{\eta_c} = 391 \pm 4 \text{ MeV}, \quad f_{\eta_b} = 667 \pm 7 \text{ MeV}, \quad (38)$$

which together with masses and constants available in Ref. [13], combined with the formulas given in Sec. II, result in

$$\begin{aligned} \mathcal{B}(h \rightarrow \eta_c Z)^{\text{SM}} &= (1.00 \pm 0.01) \times 10^{-5}, \\ \mathcal{B}(h \rightarrow \eta_b Z)^{\text{SM}} &= (2.69 \pm 0.05) \times 10^{-5}, \\ \mathcal{B}(h \rightarrow \eta_c \ell^+\ell^-)^{\text{SM}} &= (3.40 \pm 0.07) \times 10^{-7}, \\ \mathcal{B}(h \rightarrow \eta_b \ell^+\ell^-)^{\text{SM}} &= (8.76 \pm 0.06) \times 10^{-7}, \end{aligned} \quad (39)$$

where we stress again that the additive (numerically insignificant) terms proportional to m_ℓ^2/m_Z^2 have been neglected. After inspection, and by using the result of the scan discussed in the previous section, the range of values for each of the ratios (37), that we obtain by using the results of the scan from the previous section, are summarized in Table II.

Based on the results shown in Table II, one could conclude that the light CP -odd Higgs can modify the decay rates of the processes considered in this paper even in the case in which the muons (or electrons) are in the final state. However, this is mainly due to Γ_h which in 2HDM can be larger than in the Standard Model due to $\Gamma(h \rightarrow \text{exotic}) \approx \Gamma(h \rightarrow AA) + \Gamma(h \rightarrow AZ)$. This width enhancement can be as large as $0.4 \times \Gamma_h^{\text{SM}}$, which is why the ratios (37) can be reduced by about 30%, cf. Table II. The ratios of decay widths alone

$$\frac{\Gamma(h \rightarrow \eta_{cb}Z)^{2\text{HDM}}}{\Gamma(h \rightarrow \eta_{cb}Z)^{\text{SM}}} \quad \text{and} \quad \frac{\Gamma(h \rightarrow \eta_{cb}\mu^+\mu^-)^{2\text{HDM}}}{\Gamma(h \rightarrow \eta_{cb}\mu^+\mu^-)^{\text{SM}}}, \quad (40)$$

remain either insensitive to 2HDM scenarios or only slightly enhanced, up to about 10% with respect to their Standard Model values. In other words, the contribution arising from the diagram shown in Fig. 3(c) is indeed small. In order to exacerbate the sensitivity to the CP -odd Higgs, one should consider τ -leptons in the final state. This is because the second part in Eq. (15) becomes important, $\Gamma(h \rightarrow PA^* \rightarrow P\ell^+\ell^-) \propto m_\ell^2$, which can also be seen by using an approximate relation, $\Gamma(h \rightarrow \eta_{cb}A^* \rightarrow \eta_{cb}\tau\tau) \approx \Gamma(h \rightarrow \eta_{cb}A)\mathcal{B}(A \rightarrow \tau\tau)$. Indeed, on the basis of the results presented in Table II we see that the ratios $R_{\eta_c}^{\tau\tau}$ and $R_{\eta_b}^{\tau\tau}$ depend much more on the light CP -odd Higgs than the ones with the light leptons in the final state. This is particularly true for the type I, II and X models, the results highlighted in Table II, and illustrated in Fig. 5 for type II and X. We already stressed that the type II model is far more constrained than type X because of the constraint coming from $B \rightarrow X_s\gamma$. Yet the results for $R_{\eta_b}^{\tau\tau}$ exhibit a similar enhancement in both models, which can be traced back to $\Gamma(h \rightarrow Z^*A^* \rightarrow P\ell^+\ell^-) \propto m_\ell^2 \tan^2\beta$, a common feature of both models. The enhancement in the case of type II model can be even larger due to the contribution $\Gamma(h \rightarrow A^*A^* \rightarrow P\ell^+\ell^-) \propto m_\ell^2 \tan^4\beta$, which contributes in the type X model only as $\Gamma(h \rightarrow A^*A^* \rightarrow P\ell^+\ell^-) \propto m_\ell^2$.

FIG. 5. Results of the scan of parameters (35) after imposing constraints discussed in the text. The darker blue points correspond to the *free* scan and the lighter blue points correspond to the fine-tuned scan with $m_H \approx |M|$.

Notice, however, that for larger values of m_A , the value of $\mathcal{B}(h \rightarrow \eta_{cb} \tau^+ \tau^-)$ rapidly approaches its Standard Model result, which is why we focus on these decay modes as possible probes of the light CP -odd Higgs ($m_A \lesssim m_h$).

Finally, it is worth mentioning a correlation between $R_{\eta_c}^{\tau\tau}$ and $R_{\eta_b}^{\tau\tau}$ in type I and type X models, which is shown in Fig. 6. It is easy to understand its origin once one realizes that $\Gamma(h \rightarrow PA^* \rightarrow P\ell^+\ell^-)$ dominates the full decay rate (15), and since the couplings to charm and to bottom quarks are equal in both models, $|\xi_A^c| = |\xi_A^b| = 1/\tan\beta$, the correlation becomes quite obvious. Similar reasoning, but this time with respect to ξ_A^τ , can be used to explain why the enhancement in type X model ($|\xi_A^\tau| = \tan\beta$) is much more pronounced than the one in type I model ($|\xi_A^\tau| = 1/\tan\beta$).

A. Impact on $R_{\eta_{c,b}}$ from the direct searches of extra Higgs states

Beside the constraints discussed in previous section, several bounds coming from direct searches of the extra Higgs states at the LHC and LEP experiments result in important constraints on the parameter space. Since most of the direct searches are made in the context of the Type II 2HDM, we focus on that model in this part of our paper. Other types of 2HDM are more difficult to study in the direct searches, when focusing on the $b\bar{b}\tau\tau$ final state, because of the lack of enhancement due to $\tan\beta > 1$.

The CMS Collaboration has excluded a light pseudo-scalar Higgs boson of mass in the intervals $m_A \in (25, 80)$ GeV and $m_A \in (25, 60)$ GeV for large $\tan\beta$, based on nonobservation of $\sigma(pp \rightarrow b\bar{b}A)\mathcal{B}(A \rightarrow \tau\tau)$

FIG. 6. Correlation of the ratios $R_{\eta_c}^{\tau\tau}$ and $R_{\eta_b}^{\tau\tau}$ in type I and type X models arises from the fact that the Yukawa couplings of the charm and bottom quarks to the CP -odd Higgs are equal in these two models.

FIG. 7. Same as in Fig. 4 but after including the constraints coming from direct searches at LEP and LHC, discussed in the text.

FIG. 8. Same as in Fig. 5 for the type II 2HDM, but after including the bounds on the Higgs states deduced from the direct experimental searches (see text).

[26] and $\sigma(pp \rightarrow b\bar{b}A)\mathcal{B}(A \rightarrow \mu\mu)$ [27], respectively. This production channel is favored compared to the gluon-gluon fusion at low m_A due to a higher signal over background ratio, the significant background being from the Z peak.⁴

The search for the heavy neutral Higgs bosons in the mass range $m_H \in (0.1, 1)$ TeV has been carried out by both the ATLAS ($\sqrt{s} = 8$ TeV) and the CMS ($\sqrt{s} = 7$ and 8 TeV) Collaborations in the $\tau\tau$ final state through [a] gluon fusion and [b] b -associated production [30,31], and exclusion limits are obtained for the production cross section times branching fraction of a scalar particle as a function of its mass.

A nonobservation of the $e^+e^- \rightarrow H^+H^-$ signal at the LEP [32] lead to a lower bound $m_{H^\pm} \gtrsim 80$ GeV, while the searches at the LHC first excluded $m_{H^\pm} \in (80, 160)$ GeV through the top-quark decays ($t \rightarrow H^\pm b$, $H^\pm \rightarrow \tau^\pm \nu_\tau$) [33], and then those $m_{H^\pm} \in (200, 1000)$ GeV, through the direct production $pp \rightarrow t(b)H^\pm$ with $H^\pm \rightarrow \tau^\pm \nu_\tau$ [33,34], $H^\pm \rightarrow tb$ [35] or $H^\pm \rightarrow W^\pm Z$ [36].

⁴Previous limits on $m_A + m_h$ deduced from the searches of $e^+e^- \rightarrow Ah$ at LEP through various final states ($b\bar{b}b\bar{b}$, $\tau^+\tau^-b\bar{b}$ [28], and $b\bar{b}\gamma\gamma$ [29]) will not be considered here because the bounds reported in these papers are very model dependent. We note in passing that the Ah production cross section is proportional to $\cos^2(\beta - \alpha)$ and will therefore be very small in the near-alignment limit we are considering here.

All of the above-mentioned constraints are combined with the ones discussed in Sec. III for the Type II 2HDM, and the resulting plots are shown in Fig. 7 in which we clearly see that a large fraction of parameter space is forbidden. After including the results of the direct searches the maximal possible enhancement of $\Gamma(h \rightarrow \eta_{c,b}\tau\tau)$ becomes lower than the limits presented in Table II. We obtain

$$\begin{aligned} R_{\eta_c}^Z &\in (0.7, 1.0), & R_{\eta_b}^Z &\in (0.8, 1.1), \\ R_{\eta_c}^{\mu\mu} &\in (0.7, 1.0), & R_{\eta_b}^{\mu\mu} &\in (0.7, 1.1), \\ R_{\eta_c}^{\tau\tau} &\in (1.0, 2.3), & R_{\eta_b}^{\tau\tau} &\in (0.9, 9.0). \end{aligned} \quad (41)$$

For the reader's convenience, in Fig. 8 we also present the plots similar to those shown in Fig. 5 in which the bounds on the Higgs states obtained from the direct experimental searches have been included.

V. SUMMARY AND CONCLUSION

In this paper we elaborated on a possibility to search the signal of a light CP -odd Higgs state through the decays $h \rightarrow PZ$ and $h \rightarrow P\ell^+\ell^-$, with P being either η_c or η_b . We derived the relevant expressions, identified the dominant parts of the decay amplitudes and then focused on various types of the 2HDM. In our scan of the 2HDM parameter

space we fixed $m_h = 125.09(24)$ GeV and varied other parameters in the ranges indicated in Eq. (35), and then selected them as acceptable if consistent with a number of general theoretical constraints to which we also added those coming from the exclusive $b \rightarrow s\mu^+\mu^-$ modes [23] as well as the one arising from the inclusive $b \rightarrow s\gamma$ decay. That latter mode provides a substantial shift of the lower bound on $m_{H^\pm}$ in type II and type Z models [22].

We find that most of the branching fractions are either consistent with their Standard Model values or can get lowered (increased) by about 30% (10%). Notable exceptions are the models of type I, II and X, in which the enhancement can be as large as a factor of ~ 3 , and 2 orders of magnitude, as highlighted in Table II where we summarized the numerical results of our study. The origin of that enhancement is due to $m_A \lesssim m_h$ and it is related to the Yukawa couplings to the CP -odd Higgs, which explains why the effect is so pronounced in the case of τ -leptons in the final state (in contrast to the case of muons or electrons).

The large enhancement of $\mathcal{B}(h \rightarrow \eta_b \tau^+ \tau^-)$ in type II and type X models with respect to its Standard Model value is also possible in the case of $\mathcal{B}(h \rightarrow \eta_c \tau^+ \tau^-)$, but only in the type X model, which is a consequence of the structure of Yukawa couplings and the amplitude mediated by $h \rightarrow AA$ vertex in the decay. Inclusion of the direct experimental searches regarding the Higgs states H , A and $H^\pm$ reduce the possible enhancement of the mentioned decay rate in the type II 2HDM. The enhancement of the $h \rightarrow \eta_b \tau^+ \tau^-$ decay width can, however, be as large as a factor of 9 for $m_A \approx m_h/2$ and becomes lower for larger m_A . While in this paper we focused on the decay of the SM Higgs boson, one could also consider a production of Higgs associated with the quarkonium state, along the lines similar to what has been discussed in Ref. [37].

Concerning the sensitivity of the LHC searches in observing the Higgs decay channels considered here, we should mention the result of Ref. [38] in which the upper limits on $\mathcal{B}(h \rightarrow J/\psi\gamma)$ and $\mathcal{B}(h \rightarrow Y(1S, 2S, 3S)\gamma)$, through the dimuon decay channel of the vector quarkonia were set to be $\mathcal{O}(10^{-3})$. The pseudoscalar quarkonia dominantly decay to a hadronic final state and will therefore have a large QCD background. Since the outgoing $\eta_{c,b}$ will be highly boosted their decay products will be highly collimated. In that situation one can employ the tools of Ref. [39] to increase the sensitivity of the signal over the background. The observation of the branching ratio will point to models with additional CP -odd scalars contributing to the $Z\eta_{c,b}$ final state.

We should also add that in this paper we focused on the lowest lying pseudoscalar quarkonia, but that our discussion could be trivially extended to the excited pseudoscalar quarkonia. Strategies for their detections have been discussed in Ref. [40] and references therein.

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APPENDIX: EXPRESSIONS FOR THE AMPLITUDES CONSIDERED

We show here expressions for the direct contributions from diagrams in Figs. 1(a), 2(a) and 3(a), and the indirect contributions involving leptons in Figs. 1(b), 2(b).

The amplitude for the diagrams in Figs. 1(a), 2(a) and 3(a) where the Higgs couples directly to the quark and antiquark pair from which the meson is formed can be calculated in the QCD factorization approach, with the bound state effect of the highly energetic hadrons in the final state accounted for in terms of light-cone distribution amplitudes (LCDA) of these hadrons. The light-cone projector for a pseudoscalar meson P in momentum space, defined in terms of the matrix elements of the nonlocal quark and gluon current, up to twist-3 can be written as [41,42]

$$M_P(k, x, \mu) = \frac{if_P}{4} \left\{ k\gamma_5 \phi_P(x, \mu) - \mu_P(\mu) \gamma_5 \right. \\ \times \left[\phi_P(x, \mu) - i\sigma_{\mu\nu} \frac{k^\mu \bar{n}^\nu}{k \cdot \bar{n}} \frac{\phi'_\sigma(x, \mu)}{6} \right. \\ \left. \left. + i\sigma_{\mu\nu} k^\mu \frac{\phi_\sigma(x, \mu)}{6} \frac{\partial}{\partial k_{\perp\nu}} \right] \right. \\ \left. + 3\text{-particle LCDAs} \right\}, \quad (\text{A1})$$

where x is the longitudinal momentum fraction of meson carried by one of the valence quarks q , $\mu \sim m_h$ is the factorization scale, $\mu_P(\mu) = m_P^2/(2m_q(\mu))$, ϕ_P is the twist-2 LCDA, $\phi_{p,\sigma}$ are the two-particle twist-3 LCDAs, while the three-particle ones are neglected which is often referred to as Wandzura-Wilczek approximation [43], which amounts to

$$\phi_P(x, \mu) = 1, \quad \phi_\sigma(x, \mu) = 6x(1-x). \quad (\text{A2})$$

The lightlike vector $\bar{n}$ in the above expression is aligned into the opposite direction of $\vec{k}$.

We have used the following definition in the calculation of amplitudes for the diagrams in Figs. 1(a), 2(a) and 3(a) which involve the integrals over the LCDA of the pseudoscalar meson:

$$\begin{aligned}
I_1(x, y) &= \frac{1}{\mathcal{P}(x, 1-x, y)} + \frac{1}{\mathcal{P}(1-x, x, y)}, & I_3(x, y) &= \frac{1}{\mathcal{P}(x, 1-x, y)^2} + \frac{1}{\mathcal{P}(1-x, x, y)^2}, \\
I_2(x, y) &= \frac{x}{\mathcal{P}(x, 1-x, y)} + \frac{1-x}{\mathcal{P}(1-x, x, y)}, & I_4(x, y) &= \frac{x}{\mathcal{P}(x, 1-x, y)^2} + \frac{1-x}{\mathcal{P}(1-x, x, y)^2}, \\
\bar{I}_2(x, y) &= \frac{1-x}{\mathcal{P}(x, 1-x, y)} + \frac{x}{\mathcal{P}(1-x, x, y)}, & \bar{I}_4(x, y) &= \frac{1-x}{\mathcal{P}(x, 1-x, y)^2} + \frac{x}{\mathcal{P}(1-x, x, y)^2},
\end{aligned} \tag{A3}$$

where $\mathcal{P}(a, b, y) = (am_h^2 + by^2 - abm_p^2 - m_q^2)/m_h^2$. The expressions below are listed for the asymptotic form of the leading-twist LCDA, $\phi_p(x, \mu) = 6x(1-x)$.

The direct contribution to the $h \rightarrow PZ$ decay amplitude, including the light-cone projector at leading and subleading twist is

$$\begin{aligned}
F_D^{PZ} &= -f_P \frac{m_q}{2m_h^2} g_A^q \int_0^1 dx \left[m_q \phi_p(x, \mu) I_1(x, m_Z) - \mu_P(\mu) \left\{ \phi_p(x, \mu) I_2(x, m_Z) \right. \right. \\
&\quad \left. \left. + \frac{\phi_\sigma(x, \mu)}{6} \left(3I_1(x, m_Z) - 2r_Z I_3(x, m_Z) - \frac{2}{m_h^2} (k \cdot p_Z) I_4(x, m_Z) \right) \right\} \right] \\
&= f_P g_A^q \frac{m_q}{m_h^2} [2\mu_P(\mu) - 3m_q] \frac{1 - r_Z^2 + 2r_Z \ln r_Z}{(1 - r_Z)^3},
\end{aligned} \tag{A4}$$

where g_A^q is the axial-vector coupling of the Z boson to the constituent quark of the meson, $k \cdot p_Z = m_h^2(1 - r_Z - r_P)/2$ and $r_X = m_X^2/m_h^2$, $X = Z, P, q$. This direct contribution is completely negligible as it is suppressed relative to the leading term in Eq. (2) by a factor of m_p^2/m_h^2 or m_q^2/m_h^2 . We have also checked that the effect of the CP -violating coupling of the quarks to h if present cannot be seen in these two body decays.

We next list the direct contribution to the three body Higgs decays Figs. 2(a) and 3(a), along with the indirect contribution where the Higgs couples to the leptons Figs. 2(b) and 3(b).

$$\begin{aligned}
\mathcal{M}(h \rightarrow P\ell^+\ell^-)^{2a} &= f_P \frac{m_q}{vm_h^2} \left(\frac{g}{2\cos\theta_W} \right)^2 \frac{g_A^q}{(q^2 - m_Z^2 + im_Z\Gamma_Z)} \\
&\quad \times \left[(C_1 k^\mu + C_2 p^\mu) \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_Z^2} \right) (\bar{u}_\ell \gamma^\nu (g_V^\ell - g_A^\ell \gamma_5) v_\ell) \right],
\end{aligned} \tag{A5}$$

$$\begin{aligned}
\mathcal{M}(h \rightarrow P\ell^+\ell^-)^{2b} &= f_P \frac{m_\ell}{v} \left(\frac{g}{2\cos\theta_W} \right)^2 \frac{g_A^q}{(k^2 - m_Z^2 + im_Z\Gamma_Z)} \left[k^\mu \left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_Z^2} \right) \right. \\
&\quad \times \left\{ \frac{1}{(k + p_\ell)^2} (\bar{u}_\ell \gamma^\nu (g_V^\ell - g_A^\ell \gamma_5) (\not{k} + \not{p}_\ell) v_\ell) \right. \\
&\quad \left. \left. + \frac{1}{(k + p_{\bar{\ell}})^2} (\bar{u}_\ell (\not{k} + \not{p}_{\bar{\ell}}) \gamma^\nu (g_V^\ell - g_A^\ell \gamma_5) v_\ell) \right\} \right],
\end{aligned} \tag{A6}$$

$$\mathcal{M}(h \rightarrow P\ell^+\ell^-)^{3a} = -f_P \frac{m_\ell m_q^2}{v^3 m_h^2} \frac{\xi_A^\ell \xi_A^q}{(q^2 - m_A^2 + im_A\Gamma_A)} \mathcal{F}_A^{P\ell^+\ell^-} (\bar{u}_\ell \gamma_5 v_\ell), \tag{A7}$$

$$\begin{aligned}
\mathcal{M}(h \rightarrow P\ell^+\ell^-)^{3b} &= f_P \mu_P(\mu) \frac{m_\ell^2 m_q}{v^3} \frac{\xi_A^\ell \xi_A^q}{(k^2 - m_A^2 + im_A\Gamma_A)} \\
&\quad \times \left\{ \frac{1}{(k + p_\ell)^2} [\bar{u}_\ell \gamma_5 (\not{k} + \not{p}_\ell) v_\ell] + \frac{1}{(k + p_{\bar{\ell}})^2} [\bar{u}_\ell (\not{k} + \not{p}_{\bar{\ell}}) \gamma_5 v_\ell] \right\},
\end{aligned} \tag{A8}$$

where we have

$$\begin{aligned}
\mathcal{C}_1 &= f_P \frac{m_q}{2m_h^2} g_A^q \int_0^1 dx \left[m_q \phi_P(x, \mu) I_1(x, q) - \mu_P(\mu) \left\{ -\phi_P(x, \mu) \bar{I}_2(x, q) \right. \right. \\
&\quad \left. \left. + \frac{\phi_\sigma(x, \mu)}{6} \left(3I_1(x, q) - 2I_3(x, q) + 2 \frac{(k \cdot p)}{m_h^2} \bar{I}_4(x, q) \right) \right\} \right] \\
&= \frac{1}{m_h(1-r_q)^4} \{ (r_q - 1) [6m_q m_h (r_q^2 - 1) - \mu_P(\mu) (2E_P(r_q + 5) + m_h(r_q - 1)(r_q + 13))] \\
&\quad + 2 \ln r_q [2E_P \mu_P(\mu)(2r_q + 1) + m_h(r_q - 1)(\mu_P(\mu)(4r_q + 3) - 6m_q r_q)] \}, \\
\mathcal{C}_2 &= f_P \frac{m_q}{2m_h^2} g_A^q \int_0^1 dx \left[-\mu_P(\mu) \left\{ \phi_P(x, \mu) I_1(x, q) \right. \right. \\
&\quad \left. \left. + \frac{\phi_\sigma(x, \mu)}{6} \left(2 \frac{(p_P \cdot p_h)}{m_h^2} I_3(x, q) - 2r_P \bar{I}_4(x, q) \right) \right\} \right] \\
&= \frac{2\mu_P(\mu)}{m_h(1-r_q)^3} [2E_P(1+r_q) + m_h(1-r_q)^2 + 4E_P(1-r_q)], \tag{A9}
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_A^{P\ell^+\ell^-} &= \int_0^1 dx \{ \phi_P(x, \mu) [m_h^2 r_P I_2(x, q) + (k \cdot q) I_1(x, q)] - \mu_P(\mu) \phi_P(x, \mu) m_q I_1(x, q) \} \\
&= -\frac{2}{(1-r_q)^3} [3((1-r_q^2) + 2r_q \ln r_q)(k \cdot q) + \mu_P(\mu) m_q (1-r_q^2) \ln r_q], \tag{A10}
\end{aligned}$$

and $k \cdot q = 1/2(m_h^2 - m_P^2 - q^2)$, $k \cdot p = m_h E_P$, with $E_P = 1/(2m_h)(m_h^2 + m_P^2 - q^2)$. The amplitudes for Fig. 2(a) [Fig. 2(b)] listed in Eq. (A5) [(A6)] being proportional to m_q (m_ℓ) can be safely neglected compared to Fig. 2(c), which is proportional to m_Z , Eq. (10). The CP -violating coupling of the fermions to h if present will contribute to $P\ell^+\ell^-$ final state, through the trilinear Higgs (hhh) coupling. We find that the $h \rightarrow P\ell^+\ell^-$ is insensitive to the CP -odd coupling, the reason being it enters quadratically and is proportional to the mass of the final state fermions.

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