Addendum: New approach to the resummation of logarithms in Higgs-boson decays to a vector quarkonium plus a photon [Phys. Rev. D **95**, 054018 (2017)]

Geoffrey T. Bodwin,^{1,*} Hee Sok Chung,^{2,†} June-Haak Ee,^{3,‡} and Jungil Lee^{3,§}

¹*High Energy Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

²*Theory Department, CERN, 1211 Geneva 23, Switzerland*

³*Department of Physics, Korea University, Seoul 02841, Korea*

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In this addendum to Phys. Rev. D **95**, 054018 (2017) we recompute the rates for the decays of the Higgs boson to a vector quarkonium plus a photon, where the vector quarkonium is J/ψ , $\Upsilon(1S)$, $\Upsilon(2S)$, or $\Upsilon(3S)$. We correct an error in the Abel-Padé summation formula that was used to carry out the evolution of the quarkonium light-cone distribution amplitude in Phys. Rev. D **95**, 054018 (2017). We also correct an error in the scale of quarkonium wave function at the origin in Phys. Rev. D **95**, 054018 (2017) and introduce several additional refinements in the calculation.

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In Ref. [1], rates for the decays of the Higgs boson to a vector heavy quarkonium plus a photon ($H \rightarrow V + \gamma$) were computed by making use of the light-cone amplitude for the direct decay process. In the direct process, the Higgs boson decays into a heavy-quark-antiquark pair ($Q\bar{Q}$), which radiates a gluon and evolves into a vector heavy quarkonium. In this addendum to Ref. [1], we make several improvements to the calculation of the direct amplitude in Ref. [1] and also correct an error in the scale of the heavy-quarkonium wave function at the origin and an error in the application of the Abel-Padé method to the evolution of the light-cone distribution amplitude (LCDA).

In Ref. [1], $\phi_V(x)$, the heavy-quarkonium light-cone distribution amplitude at light-cone momentum fraction x , was evolved from the initial scale μ_0 , which is of order the heavy-quark mass, to the final scale μ , which is of order the Higgs-boson mass m_H , by making use of the Abel-Padé method.

As was explained in Ref. [1], the Abel summation in the Abel-Padé method defines generalized functions (distributions) in the nonrelativistic QCD (NRQCD) expansion of the LCDA at the initial scale μ_0 as limits of sequences of ordinary functions. In Eq. (31) of Ref. [1], the Abel summation was erroneously applied to $\phi_V(\mu)$, rather than to $\phi_V(\mu_0)$. Equation (31) of Ref. [1] should be corrected to the following:

*gtb@anl.gov
 †hee.sok.chung@cern.ch
 ‡chodigi@gmail.com
 §jungil@korea.ac.kr

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$$\mathcal{M} = \lim_{z \rightarrow 1} \sum_{n,m=0}^{\infty} T_m(\mu) U_{mn}(\mu, \mu_0) z^n \phi_n^\perp(\mu_0). \quad (1)$$

Here, $\mathcal{M}$ is the light-cone amplitude, $\phi_n^\perp(\mu_0)$ is the n th Gegenbauer moment of $\phi_V^\perp(x, \mu_0)$ [Eq. (19b) of Ref. [1]], $U_{mn}(\mu, \mu_0)$ is the evolution matrix for the LCDA [Eq. (B1) of Ref. [1]], and T_m is the m th Gegenbauer moment of the hard-scattering amplitude [Eq. (20b) of Ref. [1]]. The essential change in Eq. (1) is to replace z^m with z^n .

Because the evolution matrix in leading logarithmic (LL) order is diagonal, this change affects the expression for the light-cone amplitude only at next-to-leading logarithmic (NLL) order and higher. In Ref. [1], only the LCDA at leading order in α_s and v was evolved at NLL order. (Here, α_s is the QCD running coupling and v is the velocity of the Q or $\bar{Q}$ in the quarkonium rest frame.) Because the sums in Eq. (1) are not absolutely convergent, it is not obvious that the correction in Eq. (1) would not affect the results. However, it turns out that the correction in Eq. (1) does not shift the numerical results for the direct amplitudes significantly from those that were reported in Ref. [1], although it does improve the stability of the convergence of the Abel-Padé procedure.

The light-cone amplitude for the direct process can be written [2], by virtue of NRQCD factorization [3], as a sum of products of short-distance coefficients with NRQCD long-distance matrix elements (LDMEs) [4]. We denote the NRQCD LDME of leading order in v by $\langle \mathcal{O}_1^V \rangle$. The quantity $\langle \mathcal{O}_1^V \rangle$ can be written in terms of the quarkonium wave function at the origin $\Psi_V(0)$ (Ref. [3]):

$$\langle \mathcal{O}_1^V \rangle = \frac{1}{2N_c} |\Psi_V(0)|^2, \quad (2)$$

where $N_c = C_A = 3$ for SU(3) color.

For the computations in Ref. [1], which involve J/ψ and $\Upsilon(nS)$ final states, with $n = 1, 2$, or 3 , values for $\Psi_V(0)$ were inferred from values of $\langle \mathcal{O}_1^V \rangle$ that were given in Refs. [5,6]. In those papers, $\langle \mathcal{O}_1^V \rangle$ was computed by making use of the NRQCD expansion of the electronic decay rate $\Gamma(V \rightarrow e^+e^-)$ through orders $\alpha_s(\mu)$ and v^2 .¹ That expansion can be obtained from Eq. (2), the relation

$$\Gamma(V \rightarrow e^+e^-) = \frac{4\pi}{3m_V} \alpha^2(m_V) e_Q^2 f_V^{\parallel 2}, \quad (3)$$

and the NRQCD expansion

$$f_V^{\parallel} = \frac{\sqrt{2N_c} \sqrt{2m_V}}{m_V} \Psi_V(0) \times \left[1 - \frac{1}{6} \langle v^2 \rangle_V - 8 \frac{\alpha_s(\mu) C_F}{4\pi} + \mathcal{O}(\alpha_s^2, \alpha_s v^2, v^4) \right]. \quad (4)$$

Here, $f_V^{\parallel}$ is the decay constant for a longitudinally polarized quarkonium, $\alpha(\mu)$ is the electromagnetic fine-structure constant at the scale μ , e_Q is the heavy-quark charge, m_V is the quarkonium mass, $C_F = (N_c^2 - 1)/(2N_c)$, and $\langle v^2 \rangle_V$ is the ratio of the NRQCD decay LDME of relative order v^2 to the NRQCD decay LDME of leading order in v ($\langle \mathcal{O}_1^V \rangle$). (See Eq. (6) of Ref. [1].) In Refs. [5,6], the scale μ in Eq. (4) was taken to be m_V . Hence, the values of $\langle \mathcal{O}_1^V \rangle_V$ that are given in those papers are at the scale m_V .

The decay rate for the process $H \rightarrow V + \gamma$ depends on the decay constant for a transversely polarized quarkonium $f_V^{\perp}(\mu)$, which has the NRQCD expansion [4,7]

$$f_V^{\perp}(\mu) = \frac{\sqrt{2N_c} \sqrt{2m_V}}{2m_Q} \Psi_V(0) \left[1 - \frac{5}{6} \langle v^2 \rangle_V + \left(-\log \frac{\mu^2}{m_Q^2} - 8 \right) \frac{\alpha_s(\mu) C_F}{4\pi} + \mathcal{O}(\alpha_s^2, \alpha_s v^2, v^4) \right], \quad (5)$$

where m_Q is the heavy-quark pole mass.

In Ref. [1], $f_V^{\perp}(\mu)$ was assumed to be at the scale $\mu = m_Q$. However, the values of $\Psi_V(0)$ that were used in Ref. [1] were inferred from the values of $\langle \mathcal{O}_1^V \rangle$ in

Refs. [5,6]. Because those values of $\langle \mathcal{O}_1^V \rangle$ are at the scale $\mu = m_V$, it is necessary to correct for the difference in scales. One can do this straightforwardly by making the following replacement in the expression for $f_V^{\perp}$ in Eq. (5):

$$\Psi_V(0) \rightarrow \frac{1 - \frac{1}{6} \langle v^2 \rangle_V - 8 \frac{C_F \alpha_s(m_V)}{4\pi}}{1 - \frac{1}{6} \langle v^2 \rangle_V - 8 \frac{C_F \alpha_s(m_Q)}{4\pi}} \Psi_V(0). \quad (6)$$

Instead, we follow a procedure that was suggested in Ref. [8]. We compute $f_V^{\parallel}$ from the measured quarkonium electronic decay rates, using Eq. (3). Then, we use Eq. (4) to eliminate $\Psi_V(0)$ from the amplitude and use Eqs. (4) and (5) to express $f_V^{\perp}$ in terms of $f_V^{\parallel}$:

$$f_V^{\perp}(\mu) = \frac{m_V}{2m_Q} \left[1 - \frac{2}{3} \langle v^2 \rangle_V - \frac{\alpha_s(\mu) C_F}{4\pi} \log \frac{\mu^2}{m_Q^2} + \mathcal{O}(\alpha_s^2, \alpha_s v^2, v^4) \right] f_V^{\parallel} \quad (7a)$$

$$= \frac{m_V}{2\bar{m}_Q} \left[1 - \frac{2}{3} \langle v^2 \rangle_V + \frac{\alpha_s(\mu) C_F}{4\pi} \left(-\log \frac{\mu^2}{\bar{m}_Q^2} - 4 \right) + \mathcal{O}(\alpha_s^2, \alpha_s v^2, v^4) \right] f_V^{\parallel}. \quad (7b)$$

Because the pole mass is subject to renormalon ambiguities and, hence, is ill defined, we have used the one-loop relation between the $\overline{\text{MS}}$ mass $\bar{m}_Q$ at the scale $\bar{m}_Q$ and the pole mass [9], namely,

$$m_Q = \bar{m}_Q \left[1 + C_F \frac{\alpha_s(\bar{m}_Q)}{\pi} + \mathcal{O}(\alpha_s^2) \right], \quad (8)$$

to express $f_V^{\perp}/f_V^{\parallel}$ in terms of the $\overline{\text{MS}}$ mass in Eq. (7b). As can be seen, if μ is approximately equal either to m_Q or to $\bar{m}_Q$, then the apparent rates of convergence of both the velocity expansions and the α_s expansions in Eqs. (7a) and (7b) are better than the corresponding rates of convergence in Eq. (5). Furthermore, the removal of $\Psi_V(0)$ from the amplitude reduces some of the theoretical uncertainties. The resulting expression for the direct amplitude, which corresponds to Eq. (37) in Ref. [1], is

$$\begin{aligned} i\mathcal{M}_{\text{dir}}^{\text{LC}}[H \rightarrow V + \gamma] &= \frac{i}{2} e e_Q \kappa_Q \bar{m}_Q(\mu) (\sqrt{2} G_F)^{1/2} \left(-\epsilon_V^* \cdot \epsilon_\gamma^* + \frac{\epsilon_V^* \cdot p_\gamma p \cdot \epsilon_\gamma^*}{p_\gamma \cdot p} \right) \frac{f_V^{\perp}(\mu)}{f_V^{\perp}(\mu_0)} f_V^{\parallel} \frac{m_V}{2\bar{m}_Q} \\ &\times \left[1 - \frac{2}{3} \langle v^2 \rangle_V + \frac{\alpha_s(\mu_0) C_F}{4\pi} \left(-\log \frac{\mu_0^2}{\bar{m}_Q^2} - 4 \right) \right] \\ &\times \left[\mathcal{M}^{(0,0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} \mathcal{M}^{(1,0)}(\mu) + \frac{\alpha_s(\mu_0)}{4\pi} \mathcal{M}^{(0,1)}(\mu) + \langle v^2 \rangle_V \mathcal{M}^{(0,v^2)}(\mu) \right]. \end{aligned} \quad (9)$$

¹The calculations in Refs. [5,6] also included a class of corrections of order v^4 and higher. However, there are additional corrections of order v^4 that were not considered in those papers. In order to streamline the discussion, we show explicitly only contributions through order v^2 in the present paper.

Here, $e(>0)$ is the electric charge at momentum scale zero, κ_Q is a factor that accounts for deviations of the $HQ\bar{Q}$ coupling from the standard model (SM) value ($\kappa_Q = 1$ is the SM value), G_F is the Fermi constant, p and ϵ_V^* are the quarkonium momentum and polarization, respectively, and p_γ and ϵ_γ^* are the photon momentum and polarization, respectively. Each of the quantities $\mathcal{M}^{(i,j)}$ is a convolution of a component of the light-cone hard-scattering kernel at the scale μ with a component of the LCDA of the quarkonium, evolved from the scale μ_0 to the scale μ . The detailed definitions of the $\mathcal{M}^{(i,j)}$ are given in Ref. [1]. As in that work, we carry out the evolution of the LCDAs at next-to-leading-logarithmic accuracy, using the Abel-Padé method that is presented in that paper to sum the resulting series expansions in the eigenfunctions (Gegenbauer polynomials) of the leading-order evolution operator. Our uncertainties are also computed as outlined in Ref. [1].

Note that, because we include the NRQCD expansion for $f_V^\perp/f_V^\parallel$ as an overall factor in the direct amplitude in Eq. (9), this expression contains cross terms of orders α_s^2 and $\alpha_s v^2$ that are beyond the accuracy of the present calculation. We structure the amplitude in this way because of the possibility that the perturbation expansion of $f_V^\perp/f_V^\parallel$ may be better behaved in higher orders in α_s than the perturbation expansion of $f_V^\parallel$ [10–12]. We use Eq. (9) to compute the direct amplitude, taking $\mu = m_H$, as in Ref. [1], but taking $\mu_0 = \bar{m}_Q$, rather than $\mu_0 = m_Q$ because, as we have already remarked, the pole mass is ill defined.

As was pointed out in Ref. [13], there is also a contribution to the decay process from an indirect amplitude, which arises from the decay of the Higgs boson through a W -boson loop or a quark loop into a photon plus a virtual photon, with the virtual photon decaying into a quarkonium. We compute the indirect amplitude as in Ref. [1].

We obtain the results that are shown in Table I for the direct and indirect amplitudes. The results in Table I for the direct amplitude are shifted by about 5% for $H \rightarrow J/\psi + \gamma$ and by about 2% for $H \rightarrow \Upsilon(nS) + \gamma$, relative to the results that were given in Ref. [1]. The correction factor in Eq. (6) would shift the direct amplitude by about 13% for $H \rightarrow J/\psi + \gamma$ and by about 4–5% for $H \rightarrow \Upsilon(nS) + \gamma$. Apparently, the additional refinements that we have introduced in this calculation work to reduce those shifts in the direct amplitude.

Our results for the decay rates and branching fractions are shown in Table II. The results in Table II for the rates are shifted by about –1% for $H \rightarrow J/\psi + \gamma$, by about –28% for $H \rightarrow \Upsilon(1S) + \gamma$, by about –42% for $H \rightarrow \Upsilon(2S) + \gamma$, and by about –49% for $H \rightarrow \Upsilon(3S) + \gamma$, relative to the results that were given in Ref. [1]. The strong sensitivity of the rates for $H \rightarrow \Upsilon(nS) + \gamma$ to small changes in the direct

TABLE I. Values of the parameters α_V and β_V in $\Gamma(H \rightarrow V + \gamma) = |\alpha_V - \beta_V \kappa_Q|^2 \times 10^{-10}$ GeV for $V = J/\psi$ and $\Upsilon(nS)$.

V	α_V	β_V
J/ψ	11.71 ± 0.17	$(0.659_{-0.085}^{+0.085}) + (0.073_{-0.035}^{+0.035})i$
$\Upsilon(1S)$	3.283 ± 0.036	$(2.962_{-0.078}^{+0.078}) + (0.332_{-0.079}^{+0.079})i$
$\Upsilon(2S)$	2.155 ± 0.028	$(2.072_{-0.056}^{+0.056}) + (0.227_{-0.054}^{+0.054})i$
$\Upsilon(3S)$	1.803 ± 0.024	$(1.782_{-0.050}^{+0.050}) + (0.192_{-0.046}^{+0.046})i$

TABLE II. SM values of $\Gamma(H \rightarrow V + \gamma)$ in units of GeV and $\text{Br}(H \rightarrow V + \gamma)$ for $V = J/\psi$ and $\Upsilon(nS)$.

V	$\Gamma(H \rightarrow V + \gamma)$ (GeV)	$\text{Br}(H \rightarrow V + \gamma)$
J/ψ	$1.221_{-0.041}^{+0.042} \times 10^{-8}$	$2.99_{-0.15}^{+0.16} \times 10^{-6}$
$\Upsilon(1S)$	$2.13_{-0.69}^{+0.82} \times 10^{-11}$	$5.22_{-1.70}^{+2.02} \times 10^{-9}$
$\Upsilon(2S)$	$5.82_{-2.34}^{+2.93} \times 10^{-12}$	$1.42_{-0.57}^{+0.72} \times 10^{-9}$
$\Upsilon(3S)$	$3.72_{-1.55}^{+1.97} \times 10^{-12}$	$0.91_{-0.38}^{+0.48} \times 10^{-9}$

amplitude occurs because of the near cancellation of the direct and indirect amplitudes in these processes.

Given the corrections and refinements to the calculation of the rates for $H \rightarrow V + \gamma$ that we have outlined above, we believe that the numerical results that we have presented in this paper are more accurate and reliable than those in Ref. [1] and should be used in preference to the results in Ref. [1] in future comparisons with experimental measurements.

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