Erratum: Threshold expansion of the three-particle quantization condition [Phys. Rev. D **93**, 096006 (2016)]

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We have found an error in Eq. (126) of our paper, which leads to a change in the coefficient of the geometrical constant $\mathcal{K}$ in the final result for the threshold energy shift.

The left-hand side of Eq. (126) involves factors of

$$\tilde{G}_{0p} = \tilde{G}_{p0} = \frac{1}{8L^3 m \omega_p^2} \frac{1}{E - m - 2\omega_p}, \quad (1)$$

evaluated at $E = 3m + \Delta E_{\text{th}}$. The missed term occurs in the region where $pL \sim L^0$, for which we have

$$\tilde{G}_{0p} = -\frac{1}{8m^2 L} \left[\frac{1}{p^2 L^2} + \frac{x}{L} \frac{1}{p^4 L^4} + \mathcal{O}(L^{-2}) \right], \quad (2)$$

where $x = \Delta E_{\text{th}} L^3 m^2 \sim L^0$. The term proportional to x in this equation was erroneously dropped in our paper. Including this term, the left-hand side (lhs) of Eq. (126) is augmented by the following term,

$$9\tilde{\mathcal{K}}_{2,00}^2 \frac{2\pi a}{m^3} \frac{\mathcal{K}x}{(2\pi)^6}, \quad [\text{add to lhs of Eqs. (126), (127) and (135)}], \quad (3)$$

where $\mathcal{K} = \sum_{\vec{n} \neq 0} 1/(\vec{n}^2)^3$ is a numerical constant. This correction then propagates into the left-hand sides of Eqs. (127) and (135). This, in turn, leads to the following additional contribution to the right-hand side (rhs) of Eq. (135),

$$\frac{576}{\pi^2} a^4 m^2 24\mathcal{K}, \quad [\text{add to rhs of Eq. (135)}]. \quad (4)$$

This changes the coefficient of $\mathcal{K}$ on the rhs of Eq. (135) from -9 to $+15$, and this change carries over to the final result of our paper, Eq. (136), which gives the coefficient of the L^{-6} contribution to the threshold energy shift for three particles in a finite, cubic box. The corrected form of this equation reads

$$\begin{aligned} \frac{a_6(L)}{a_3} = & \left(\frac{a}{\pi} \right)^3 \left[-\mathcal{I}^3 + \mathcal{I}\mathcal{J} + 15\mathcal{K} + \frac{16\pi^3}{3} (3\sqrt{3} - 4\pi) \log\left(\frac{mL}{2\pi}\right) + \mathcal{C}_F + \mathcal{C}_4 + \mathcal{C}_5 \right] \\ & + \frac{64\pi^2 a^2}{m} \mathcal{C}_3 + \frac{3\pi a}{m^2} + 6\pi r a^2 - \frac{\mathcal{M}_{3,\text{thr}}}{48m^3 a_3}. \end{aligned} \quad (5)$$

We note that, with this change, the coefficient of $\mathcal{K}$ agrees with that obtained using nonrelativistic quantum mechanics (NRQM) in Ref. [1]. We had not previously been concerned about having different coefficients for $\mathcal{K}$ because, as shown in our paper, relativistic effects start to enter in $a_6(L)$. Furthermore, the two works used different ultraviolet regularization schemes, which could lead to differences in the finite terms. Nevertheless, it is interesting that the coefficient of $\mathcal{K}$ (like those of $\mathcal{I}^3$ and $\mathcal{I}\mathcal{J}$) do agree between the relativistic and NR calculations.

This error was discovered when performing a check of the result for $a_6(L)$ in $\lambda\phi^4$ theory. Previously, we had checked the contributions of third order in λ (equivalent to third order in a) [2]; the check has now been extended to fourth order [3], verifying the corrected form in Eq. (5).

[1] S. R. Beane, W. Detmold, and M. J. Savage, *Phys. Rev. D* **76**, 074507 (2007).

[2] M. T. Hansen and S. R. Sharpe, *Phys. Rev. D* **93**, 014506 (2016).

[3] S. R. Sharpe, [arXiv:1707.04279](https://arxiv.org/abs/1707.04279).