Comment on “Self-gravitating spherically symmetric solutions in scalar-torsion theories”

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We find a crucial miscalculation in [G. Kofinas, E. Papantonopoulos, and E. N. Saridakis, Self-gravitating spherically symmetric solutions in scalar-torsion theories, *Phys. Rev. D* **91**, 104034 (2015)] which leads to the wrong master equation. This follows that there is no wormhole-like solution for hyperbolic scalar potential and the solution at large distances differs from that of [G. Kofinas, E. Papantonopoulos, and E. N. Saridakis, Self-gravitating spherically symmetric solutions in scalar-torsion theories, *Phys. Rev. D* **91**, 104034 (2015)].

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I. MASTER EQUATION

Let us start our discussion in Sec. III in [1]. After redefining some new variables (x, y, z) , the field equations transform into

$$\begin{aligned} \dot{y} + \frac{3}{2}y^2 - \frac{e^{-2x}}{2\nu^2} + \frac{\kappa^2(\varepsilon\nu^2 + 2V)}{4\nu^2(1 - 2\xi\kappa^2\nu^2)} &= 0, \\ yz = \frac{\kappa^2}{\nu^2(1 - 6\xi\nu^2\kappa^2)} \left(\frac{(1 - 2\xi\nu^2\kappa^2)}{\kappa^2} e^{-2x} + \frac{\nu^2}{2} - V \right) &\equiv F, \\ \dot{z} + \frac{1}{2}z^2 + \frac{e^{-2x}}{2\nu^2} + \frac{3\kappa^2(\nu^2 + 2V)}{4\nu^2(1 - 2\xi\kappa^2\nu^2)} &= 0, \end{aligned} \quad (1.1)$$

where $\varepsilon = \pm 1$. For the case at hand we choose $\varepsilon = +1$ since we have a nonphantom scalar field [1]. Next, we introduce again a set of new variables

$$\begin{aligned} Y &\equiv y^2, \\ Z &\equiv z^2, \end{aligned} \quad (1.2)$$

such that we could have

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$$\begin{aligned} \frac{1}{2} \frac{dY}{dx} &= \dot{y}, \\ \frac{1}{2} \varsigma \sqrt{\frac{Y}{Z}} \frac{dZ}{dx} &= \dot{z}, \end{aligned} \quad (1.3)$$

where $\varsigma = \pm 1$. Then, (1.1) can be rewritten as

$$\begin{aligned} \frac{dY}{dx} + 3Y - \frac{e^{-2x}}{\nu^2} + \frac{\kappa^2(\nu^2 + 2V)}{2\nu^2(1 - 2\xi\kappa^2\nu^2)} &= 0, \\ YZ &= F^2, \\ \varsigma \sqrt{\frac{Y}{Z}} \frac{dZ}{dx} + Z + \frac{e^{-2x}}{\nu^2} + \frac{3\kappa^2(\nu^2 + 2V)}{2\nu^2(1 - 2\xi\kappa^2\nu^2)} &= 0. \end{aligned} \quad (1.4)$$

The third equation in (1.4) differs from that of Eq. (A5) in [1]. The crucial miscalculation is the missing prefactor $\varsigma \sqrt{\frac{Y}{Z}}$ in the second equation of (1.3). Thus, they obtained the wrong master equation as we will see below. After some computations using all equations in (1.4) we get the correct master equation

$$\begin{aligned} 2\varsigma \frac{d^2Y}{dx^2} + \frac{1}{Y} \left(\frac{\tilde{\eta}}{2\eta} - \varsigma \right) \left(\frac{dY}{dx} \right)^2 \\ + \left(\frac{2\nu^2(\tilde{\eta} - \varsigma\eta)}{Y} + \frac{3\tilde{\eta}}{\eta} + 3\varsigma - \frac{6\eta}{\tilde{\eta}} \right) \frac{dY}{dx} \\ + \left(\frac{9\tilde{\eta}}{2\eta} - \frac{18\eta}{\tilde{\eta}} \right) Y + 6\tilde{\eta}\nu^2 + \frac{2\eta\tilde{\eta}\nu^4}{Y} + \frac{8\eta}{\tilde{\eta}\nu^2} e^{-2x} &= 0, \end{aligned} \quad (1.5)$$

where η and $\tilde{\eta}$ are constant defined as

$$\eta \equiv \frac{\kappa^2}{2\nu^2(1-2\xi\nu^2\kappa^2)},$$

$$\tilde{\eta} \equiv \frac{\kappa^2}{\nu^2(1-6\xi\nu^2\kappa^2)}. \quad (1.6)$$

We also have

$$\left(\frac{dx}{d\phi}\right)^2 = Y(x),$$

$$\left[\frac{d \ln(RN^2)}{dx}\right]^2 = \frac{Z}{Y}, \quad (1.7)$$

such that the static metric (3.1) in [1] can be written down as

$$ds^2 = -N^2 dt^2 + \frac{dR^2}{\nu^2 R^2 Y} + R^2 d\Omega^2, \quad (1.8)$$

where $d\Omega^2$ is the 2-sphere metric.

II. SOLUTIONS

A. Solutions at large distances

In the asymptotic limit, namely $x \rightarrow +\infty$ the function $Y(x)$ can be written as $Y(x) = Y_0 + Y_1 + \dots$ such that $|Y_0| \gg |Y_1(x)|$. Such a setup simplifies (1.5) into a linearized differential equation. At the zeroth order, we have

$$Y_0 = \frac{2\eta\nu^2}{3(2\sigma-1)}, \quad (2.1)$$

where

$$\sigma \equiv \frac{\varepsilon\eta}{\tilde{\eta}}, \quad (2.2)$$

with $\varepsilon = \pm 1$. The function Y_1 satisfies the following linear equation

$$\frac{d^2 Y_1}{dx^2} + \frac{3}{2}\varepsilon\left(\frac{1+\varepsilon\tilde{\sigma}-2\sigma^2}{\sigma}\right)\frac{dY_1}{dx} + \frac{9}{4}\varepsilon\left(\frac{1-4\sigma^2}{\sigma}\right)Y_1 + \varepsilon\frac{4\sigma}{\nu^2}e^{-2x} = 0. \quad (2.3)$$

The general solution of (2.3) is given by

$$Y_1 = C_1 e^{\lambda_1 x} + D_1 e^{\lambda_2 x} - \frac{4\varepsilon\sigma e^{-2x}}{\nu^2(\lambda_1+2)(\lambda_2+2)}, \quad (2.4)$$

where $C_1, D_1 \in \mathbb{R}$ with

$$\lambda_{1,2} = -\frac{3\varepsilon}{4\sigma}(1+\varepsilon\sigma-2\sigma^2 \pm [(1+\varepsilon\sigma-2\sigma^2)^2 + 4\varepsilon\sigma(1-4\sigma^2)]^{1/2}), \quad (2.5)$$

The values of λ_1 or λ_2 should be negative, since $\lim_{x \rightarrow +\infty} Y_1(x) \rightarrow 0$. So, the solution (2.3) belongs to the following two cases. First, for $\varepsilon = 1$ we have $0 < \sigma < \frac{1}{2}$ and then, $-\frac{3}{2} < \lambda_1 < 0$ and $\lambda_2 < -2$. Second, in the case of $\varepsilon = -1$ we have $-\frac{1}{2} < \sigma < 0$ which follows $\lambda_1 < -2$ and $-\frac{3}{2} < \lambda_2 < 0$.

Solving the second equation in (1.7), we obtain the first order lapse function f

$$f(R) = A_0 Y_0^{\varepsilon/2\sigma} R^{3\varepsilon-1} \left(1 + \frac{\varepsilon Y_1}{\sigma Y_0}\right). \quad (2.6)$$

while the first equation gives us

$$\phi(R) = \phi_0 + \int \frac{dx}{\sqrt{Y_0 + Y_1}}$$

$$\approx \phi_0 + Y_0^{-1/2} \ln R - \frac{1}{2} Y_0^{-3/2} \left(\frac{C_1}{\lambda_1} R^{\lambda_1} + \frac{D_1}{\lambda_2} R^{\lambda_2} + \frac{2\varepsilon\sigma}{\nu^2(\lambda_1+2)(\lambda_2+2)} R^{-2} \right). \quad (2.7)$$

The first order scalar potential can be obtained from the first equation in (1.4), namely

$$V = -\frac{\nu^2(2\sigma+7)}{2(2\sigma-1)} - \frac{2}{\eta} (C_1(\lambda_1+3)R^{\lambda_1} + D_1(\lambda_2+3)R^{\lambda_2}) + \left(\frac{4\varepsilon\sigma}{\nu^2(\lambda_1+2)(\lambda_2+2)} - \frac{1}{\nu^2} \right) R^{-2}. \quad (2.8)$$

Looking back at the lapse function (2.6), its lowest order leads to spaces of constant scalar curvature whose Ricci scalar has the form

$$R = -(9+3\varepsilon)\nu^2 Y_0. \quad (2.9)$$

For $\varepsilon = 1$ we have asymptotically AdS spacetime, while in the case of $\varepsilon = -1$ the asymptotic spacetime is not Einstein.

B. No Wormhole-like Solutions

Now, let us consider a case with scalar potential

$$V(x) = \alpha e^{-2x} + \beta, \quad (2.10)$$

where α and β are real constants. As $x \rightarrow +\infty$, we should have

$$\beta = -\frac{\nu^2(2\sigma + 7)}{2(2\sigma - 1)}. \quad (2.11)$$

Finally, inserting (2.12) into the master Eq. (1.5) we conclude that for $c_1 = 0$ there is no solution for α real.

Inserting (2.10) into the first equation in (1.4) we then have

$$Y = c_1 e^{-3x} + \frac{e^{-2x}}{\nu^2} - \frac{\eta \nu^2}{2} - 2\eta \alpha e^{-2x} - \frac{2}{3}\eta \beta. \quad (2.12)$$

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- [1] G. Kofinas, E. Papantonopoulos, and E. N. Saridakis, Self-gravitating spherically symmetric solutions in scalar-torsion theories, *Phys. Rev. D* **91**, 104034 (2015).