**Erratum: Momentum transport in strongly coupled anisotropic plasmas
in the presence of strong magnetic fields
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In Appendix B of the paper (FCRN), where we derived Kubo formulas for the shear and bulk viscosities in the presence of an external magnetic field using gravitational fluctuations, we stated that the convention adopted for the viscosity coefficients was the same used in Ref. [1] (HRS). This is not actually the case because $\eta_2^{(\text{FCRN})} = -\eta_2^{(\text{HSR})}$ and $\eta_3^{(\text{FCRN})} = -2\eta_3^{(\text{HSR})}$. Moreover, there are also some inconsistencies in the derivations presented for some of the Kubo formulas for the shear viscosity coefficients, which we correct below.

In Appendix B of the paper, all equations are correct up to Eq. (B9). Below, we repeat the steps between Eqs. (B9) and (B33), correcting the expressions where appropriate,

$$\delta(i) = -\eta_0 \left(\partial_i h_{ij} - \frac{1}{3} \delta_{ij} \partial_t h_k^k \right), \quad (1)$$

$$\delta(ii) = -\frac{1}{4} \eta_1 \left[-(\delta_{ij} - 3b_i b_j) \partial_t h_k^k + \frac{3}{2} (\delta^{kl} - b^k b^l) (\delta_{ij} - 3b_i b_j) \partial_t h_{kl} \right], \quad (2)$$

$$\delta(iii) = \eta_2 [b_i b^k \partial_t h_{jk} + b_j b^k \partial_t h_{ik} - 2b_i b_j b^k b^l \partial_t h_{kl}], \quad (3)$$

$$\delta(iv) = \frac{1}{2} \eta_3 \partial_t h_{kl} (\delta_i^k \epsilon_j^{lz} + \delta^{jl} \epsilon_i^{kz} - b_i b^k \epsilon_j^{lz} - b_j b^k \epsilon_i^{lz}), \quad (4)$$

$$\delta(v) = -\eta_4 (b_{ik} b_j b_l + b_{jk} b_i b_l) \partial_t h^{kl}, \quad (5)$$

$$\delta(vi) = -\frac{3}{4} \zeta_{\perp} (\delta_{ij} - b_i b_j) (\partial_t h_k^k + \partial_t h_{zz}), \quad (6)$$

$$\delta(vii) = -\frac{3}{2} \zeta_{\parallel} b_i b_j \partial_t h_{zz}. \quad (7)$$

The next step is to write the variations above in Fourier space using a plane-wave ansatz for the perturbations, which gives the following expressions,

$$\delta(i) = \frac{i\omega}{2} h_{kl}(\omega) \left[\eta_0 \left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k - \frac{2}{3} \delta_{ij} \delta^{kl} \right) \right], \quad (8)$$

$$\delta(ii) = \frac{i\omega}{2} h_{kl}(\omega) \frac{1}{4} \eta_1 [-2\delta^{kl} (\delta_{ij} - 3b_i b_j) + 3(\delta^{kl} - b^k b^l) (\delta_{ij} - 3b_i b_j)], \quad (9)$$

$$\delta(iii) = -\frac{i\omega}{2} h_{kl}(\omega) \eta_2 (b_i b^k \delta_j^l + b_i b^l \delta_j^k + b_j b^k \delta_i^l + b_j b^l \delta_i^k - 4b_i b_j b^k b^l), \quad (10)$$

$$\delta(iv) = -\frac{i\omega}{2} h_{kl}(\omega) \eta_3 (\delta_i^k \epsilon_j^{lz} + \delta^{jl} \epsilon_i^{kz} - b_i b^k \epsilon_j^{lz} - b_j b^k \epsilon_i^{lz}), \quad (11)$$

$$\delta(iv) = \frac{i\omega}{2} h_{kl}(\omega) \eta_4 (b_i^k b_j b^l + b_i^l b_j b^k + b_j^k b_i b^l + b_j^l b_i b^k), \quad (12)$$

$$\delta(v) = \frac{i\omega}{2} h_{kl}(\omega) \left[\frac{3}{2} \zeta_{\perp} (\delta_{ij} \delta^{kl} + \delta_{ij} \delta_z^k \delta_z^l - b_i b_j \delta^{kl} - b_i b_j \delta_z^k \delta_z^l) \right], \quad (13)$$

$$\delta(vii) = \frac{i\omega}{2} h_{kl}(\omega) [3\zeta_{\parallel} b_i b_j \delta_z^k \delta_z^l]. \quad (14)$$

Collecting all the variations in Fourier space, we write

$$\begin{aligned} \delta\Pi_{ij}(\omega) = \frac{i\omega}{2} h_{kl}(\omega) & \left[\eta_0 \left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k - \frac{2}{3} \delta_{ij} \delta^{kl} \right) + \frac{1}{4} \eta_1 [-2\delta^{kl} (\delta_{ij} - 3b_i b_j) + 3(\delta^{kl} - b^k b^l) (\delta_{ij} - 3b_i b_j)] \right. \\ & - \eta_2 (b_i b^k \delta_j^l + b_i b^l \delta_j^k + b_j b^k \delta_i^l + b_j b^l \delta_i^k - 4b_i b_j b^k b^l) - \eta_3 (\delta_i^k \epsilon_j^{lz} + \delta_i^l \epsilon_j^{kz} - b_i b^k \epsilon_j^{lz} - b_j b^k \epsilon_i^{lz}) \\ & \left. + \eta_4 (b_i^k b_j b^l + b_i^l b_j b^k + b_j^k b_i b^l + b_j^l b_i b^k) + \frac{3}{2} \zeta_{\perp} (\delta_{ij} \delta^{kl} + \delta_{ij} \delta_z^k \delta_z^l - b_i b_j \delta^{kl} - b_i b_j \delta_z^k \delta_z^l) + 3\zeta_{\parallel} b_i b_j \delta_z^k \delta_z^l \right], \end{aligned} \quad (15)$$

which allows us to express the retarded Green's function in terms of the viscosities,

$$\begin{aligned} -\lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_{ij}^{R,kl}(\omega) = \eta_0 & \left(\delta_i^k \delta_j^l + \delta_i^l \delta_j^k - \frac{2}{3} \delta_{ij} \delta^{kl} \right) + \frac{1}{4} \eta_1 [-2\delta^{kl} (\delta_{ij} - 3b_i b_j) + 3(\delta^{kl} - b^k b^l) (\delta_{ij} - 3b_i b_j)] \\ & - \eta_2 (b_i b^k \delta_j^l + b_i b^l \delta_j^k + b_j b^k \delta_i^l + b_j b^l \delta_i^k - 4b_i b_j b^k b^l) - \eta_3 (\delta_i^k \epsilon_j^{lz} + \delta_i^l \epsilon_j^{kz} - b_i b^k \epsilon_j^{lz} - b_j b^k \epsilon_i^{lz}) \\ & + \eta_4 (b_i^k b_j b^l + b_i^l b_j b^k + b_j^k b_i b^l + b_j^l b_i b^k) + \frac{3}{2} \zeta_{\perp} (\delta_{ij} \delta^{kl} + \delta_{ij} \delta_z^k \delta_z^l - b_i b_j \delta^{kl} - b_i b_j \delta_z^k \delta_z^l) \\ & + 3\zeta_{\parallel} b_i b_j \delta_z^k \delta_z^l. \end{aligned} \quad (16)$$

The final step is to isolate the viscosities and obtain the corresponding Kubo formulas. For such a task, we only need to select specific components of $G_{ij,kl}^R$. For instance, if we take $i = k = x$ and $j = l = y$ in (16), we have

$$\eta_0 = -\lim_{\omega \rightarrow 0} \frac{1}{\omega} G_{T_{xy} T_{xy}}^R(\omega), \quad (17)$$

and so forth. Finally, we obtain the following Kubo formulas,

$$\eta_0 = -\lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_{T_{xy} T_{xy}}^R(\omega), \quad (18)$$

$$\eta_1 = -\frac{4}{3} \eta_0 + 4 \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_{P_{\parallel} P_{\perp}}^R(\omega), \quad (19)$$

$$\eta_2 = \eta_0 + \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_{T_{xz} T_{xz}}^R(\omega), \quad (20)$$

$$\eta_3 = \lim_{\omega \rightarrow 0} \frac{1}{\omega} G_{P_{\perp} T_{12}}^R(\omega), \quad (21)$$

$$\eta_4 = -\lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_{T_{xz} T_{yz}}^R(\omega), \quad (22)$$

$$\zeta_{\perp} = -\frac{2}{3} \lim_{\omega \rightarrow 0} \frac{1}{\omega} [\text{Im} G_{P_{\perp} P_{\perp}}^R(\omega) + \text{Im} G_{P_{\parallel} P_{\perp}}^R(\omega)], \quad (23)$$

$$\zeta_{\parallel} = -\frac{4}{3} \lim_{\omega \rightarrow 0} \frac{1}{\omega} [\text{Im} G_{P_{\perp} P_{\parallel}}^R(\omega) + \text{Im} G_{P_{\parallel} P_{\parallel}}^R(\omega)], \quad (24)$$

where

$$P_{\perp} \equiv \frac{1}{2} T^a_a = \frac{1}{2} (T^x_x + T^y_y), \quad P_{\parallel} \equiv \frac{1}{2} T^z_z. \quad (25)$$

The results in Eqs. (18)–(24) now agree with the ones obtained in Ref. [2]. Moreover, we now define,¹

$$\eta_{\parallel} \equiv \eta_0 - \eta_2. \quad (26)$$

In summary, the final Kubo formulas obtained above for η_1 , η_2 , η_3 , and η_4 , previously derived in the paper, have now been corrected while η_0 , $\zeta_{\perp}$, and $\zeta_{\parallel}$ were left unchanged. We note that η_1 , η_3 , and η_4 vanish for the backgrounds considered in the paper (as already mentioned there), while with the new definition given in Eq. (26) above for $\eta_{\parallel}$, which replaces the right-hand side of Eq. (B37), all the results in the main text of the paper are left unchanged.

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¹Due to the different sign conventions for η_2 , this definition differs from the one adopted in Ref. [3].