Tau properties in $B \rightarrow D\tau\nu$ from visible final-state kinematicsRodrigo Alonso,¹ Jorge Martin Camalich,¹ and Susanne Westhoff²¹*CERN, Theoretical Physics Department, CH-1211 Geneva 23, Switzerland*²*Institute for Theoretical Physics, Heidelberg University, D-69120 Heidelberg, Germany*

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In semileptonic B decays with a tau lepton, features of the production process are imprinted on the tau helicity states. Since the tau momentum cannot be fully reconstructed experimentally, the available information on the tau properties is encoded in its visible decay products. Focusing on the process $B \rightarrow D\tau\nu$, we find explicit relations between the tau properties and the kinematics of the charged particles in the decays $\tau \rightarrow \pi\nu$, $\tau \rightarrow \rho\nu$, and $\tau \rightarrow \ell\nu\bar{\nu}$. In particular, we show that the perpendicular polarization $P_\perp$ and the forward-backward asymmetry A_τ of the tau lepton can simultaneously be extracted from an angular asymmetry of the charged particle against the D meson. For the most sensitive decay channel, $\tau \rightarrow \pi\nu$, we expect a relative statistical precision of about 10% for $P_\perp$ and A_τ in a measurement based on 50 ab^{-1} of data at Belle II.

DOI: [10.1103/PhysRevD.95.093006](https://doi.org/10.1103/PhysRevD.95.093006)**I. INTRODUCTION**

The tau leptons in $B \rightarrow D\tau\nu$ and $B \rightarrow D^*\tau\nu$ decays serve as a wide test ground for physics in and beyond the standard model (SM). Within the SM, these decays differ from the decays $B \rightarrow D^{(*)}\ell\nu$ with light leptons $\ell = e, \mu$ by the larger mass of the tau lepton, which facilitates its production through a longitudinally polarized virtual W boson [1]. The ratios of decay rates into taus and light leptons, R_D and R_{D^*} , can be predicted very precisely using model-independent calculations of form factors and experimental input from the spectra of $B \rightarrow D\ell\nu$ decays [2–10]. On the experimental side, increasingly precise measurements of total decay rates and single differential distributions have been achieved by BABAR [11,12], Belle [13–15], and LHCb [16] (see Ref. [17] for a review). The large amount of data expected in the near future from Belle II and LHCb will enable us to probe $B \rightarrow D^{(*)}\tau\nu$ decays in great detail. Exploring the properties of B decays with tau leptons is thus a key target at future B physics experiments [18].

The precise SM predictions of $B \rightarrow D^{(*)}\tau\nu$ observables are also key to searches for physics beyond the SM. Two decades ago, semileptonic B decays with taus had already been proposed as sensitive probes of new charged scalars [19,20] and for a model-independent analysis of new physics [21]. More recently, an observed discrepancy between SM predictions and measurements of R_D and R_{D^*} [17] has triggered a major effort to scrutinize possible explanations

in terms of new physics (see, e.g., Refs. [22–28] for an overview of models and model-independent analyses). This unsolved puzzle calls for a careful investigation of other observables, such as those related to tau polarizations [19], angular distributions of the tau [29,30], or other decay modes induced by the same elementary $b \rightarrow c\tau\nu$ transition [31,32].

Nonetheless, an important experimental challenge of B decays with tau leptons is their fast decay. The final state of the tau lepton necessarily involves one or more neutrinos, which escape the detector and hinder a full reconstruction of the tau kinematics.¹ The maximal accessible information on the $b \rightarrow c\tau\nu$ transition is encoded in the visible decay products of the τ lepton, where the three dominant decay modes $\tau \rightarrow \ell\nu\bar{\nu}$, $\tau \rightarrow \rho\nu$, and $\tau \rightarrow \pi\nu$ make up a branching ratio of more than 70%. It is therefore suitable to construct observables directly from final-state kinematics of the visible decay particle $d = \{\ell, \rho, \pi\}$, without relying on the reconstruction of the tau momentum. This approach has been considered [20] and later pioneered [33] in the context of searches for charged scalars in $B \rightarrow D\tau\nu$. Subsequently, a variety of observables—partially or fully based on different visible final states—have been proposed to measure tau polarizations [34,35], angular asymmetries [36], CP violation [37], to study the impact of leptonic tau decay on light lepton distributions [38], or to facilitate a comprehensive analysis of new physics [39,40].

In this work, we find analytic relations between properties of tau leptons produced via $B \rightarrow D\tau\nu$ decays and corresponding observables obtained from the kinematics of visible final-state particles from the leptonic and two-body hadronic decays of the tau. This is achieved by expressing the decay rates in terms of helicity amplitudes and by

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¹A detection of the displaced vertex of the tau decay could add this missing piece of information [19].

analytically solving the phase-space integrals related to the kinematics of the virtual tau and final-state neutrinos. In particular, we show that the perpendicular polarization $P_\perp$ [19] and the forward-backward asymmetry A_τ [29] of the tau lepton can be directly extracted from an angular asymmetry of the visible decay products of the tau. This article is organized as follows. In Sec. II, we introduce the tau properties in $B \rightarrow D\tau\nu$ at the production level and discuss their kinematic features. Subsequently, in Sec. III we show how to extract the tau polarizations and forward-backward asymmetry from final-state kinematics, focusing on the full decay chains $B \rightarrow D\nu[\tau \rightarrow \{\pi\nu, \rho\nu, \ell\nu\}]$. In Sec. IV, we compare the sensitivity of the different tau decay modes to the tau properties and discuss the measurement prospects at Belle II. We conclude in Sec. V.

II. PROPERTIES OF THE TAU LEPTON PRODUCED VIA $B \rightarrow D\tau\nu$ DECAYS

The differential decay rate for $B \rightarrow D^{(*)}\tau\nu$, $d\Gamma$, as well as the projections onto the three polarization states of the tau lepton are defined in terms of helicity matrix elements $\mathcal{M}_\pm$ as

$$\begin{aligned} d\Gamma &= \frac{(2\pi)^4 d\Phi_3}{2m_B} (|\mathcal{M}_+|^2 + |\mathcal{M}_-|^2), \\ d\Gamma_L &= \frac{(2\pi)^4 d\Phi_3}{2m_B} (|\mathcal{M}_+|^2 - |\mathcal{M}_-|^2), \\ d\Gamma_\perp &= \frac{(2\pi)^4 d\Phi_3}{2m_B} 2\text{Re}[\mathcal{M}_+ \mathcal{M}_-^\dagger], \\ d\Gamma_T &= \frac{(2\pi)^4 d\Phi_3}{2m_B} 2\text{Im}[\mathcal{M}_+ \mathcal{M}_-^\dagger], \end{aligned} \quad (1)$$

where $d\Phi_3$ is the corresponding three-body phase-space element. We define the longitudinal, perpendicular, and transversal tau polarizations at a specific point in phase space by

$$dP_L = \frac{d\Gamma_L}{d\Gamma}, \quad dP_\perp = \frac{d\Gamma_\perp}{d\Gamma}, \quad dP_T = \frac{d\Gamma_T}{d\Gamma}. \quad (2)$$

Notice that dP_L and $dP_\perp$ depend on the frame in which the helicities of the tau lepton are defined. In turn, dP_T points perpendicular to the $D - \tau$ decay plane and is thus invariant under boosts contained in this plane. In particular, it is invariant under boosts that connect the B rest frame, the τ rest frame, and the q rest frame, where q is the four-momentum of the $\tau - \nu$ pair. In the absence of strong phases, a nonzero P_T polarization signals violation of time-reversal symmetry.

In this work, we define the polarizations in the tau rest frame. The matrix element $\mathcal{M}_\lambda$ in Eq. (2) thus corresponds with the production of a tau lepton of helicity $\lambda = \pm 1/2$ in this frame. The decay rate for a τ lepton polarized along a direction $\hat{s}$ is then given by [19]

FIG. 1. Kinematics of the chain decay $B \rightarrow D\nu[\tau \rightarrow d\nu(\bar{\nu})]$, where $d = \{\pi, \rho, \ell\}$. The momenta of the D meson and the tau lepton span the decay plane in $B \rightarrow D\tau\nu$ (in green). The momenta of the tau lepton and its visible decay product d span the plane of the subsequent decay $\tau \rightarrow d\nu(\bar{\nu})$ (in blue).

$$d\Gamma(\hat{s}) = \frac{1}{2} [d\Gamma + (d\Gamma_L \hat{e}_\tau + d\Gamma_\perp \hat{e}_\perp + d\Gamma_T \hat{e}_T) \cdot \hat{s}]. \quad (3)$$

We choose our coordinate system $\{\hat{e}_\tau, \hat{e}_\perp, \hat{e}_T\}$ as

$$\hat{e}_\tau = \frac{\vec{p}_\tau}{|\vec{p}_\tau|}, \quad \hat{e}_T = \frac{\vec{p}_D \times \vec{p}_\tau}{|\vec{p}_D \times \vec{p}_\tau|}, \quad \hat{e}_\perp = \hat{e}_T \times \hat{e}_\tau, \quad (4)$$

where $\vec{p}_\tau$ and $\vec{p}_D$ are the momenta of the τ lepton and the D meson defined in the q rest frame. The decay kinematics are illustrated in Fig. 1.

The unpolarized differential rate depends on two kinematic variables that can be taken to be q^2 and the angle that the τ lepton forms with the recoil against the direction of the D in the q rest frame, $\cos \theta_\tau = -\vec{p}_D \cdot \vec{p}_\tau / |\vec{p}_D| |\vec{p}_\tau|$. We define the forward-backward asymmetry associated to this angle as

$$\begin{aligned} \frac{d\Gamma}{dq^2} A_\tau(q^2) &= \int_0^1 d\cos \theta_\tau \frac{d^2\Gamma}{dq^2 d\cos \theta_\tau} \\ &\quad - \int_{-1}^0 d\cos \theta_\tau \frac{d^2\Gamma}{dq^2 d\cos \theta_\tau}. \end{aligned} \quad (5)$$

The differential rate $d\Gamma/dq^2$, normalized to the total decay rate for $B^- \rightarrow D^0 \tau^- \bar{\nu}_\tau$, denoted as Γ , is shown in the left panel of Fig. 2. The differential tau polarizations $P_L(q^2)$ and $P_\perp(q^2)$ are obtained by partially integrating dP_L and $dP_\perp$ in Eq. (2) over the respective phase space. In the right panel of Fig. 2, we show $P_L(q^2)$, $P_\perp(q^2)$, and $A_\tau(q^2)$ in $B^- \rightarrow D^0 \tau^- \bar{\nu}_\tau$ for the kinematic range $m_\tau^2 \leq q^2 \leq (m_B - m_D)^2$. All three quantities are sizable over most of the q^2 spectrum, which will be beneficial for a measurement. Near the end point $q^2 = (m_B - m_D)^2$, the tau lepton recoils back to back against the right-handed antineutrino in the B rest frame. Therefore, the τ^- is purely longitudinally polarized with $P_L = +1$, as can be observed in the figure. The average tau polarizations and asymmetry in the full sample of $B \rightarrow D\tau\nu$ events are given by

FIG. 2. Left: Normalized differential decay rate $(d\Gamma/dq^2)/\Gamma$ [GeV^{-2}] (black) and forward-backward asymmetry $A_d(q^2)$ from Eq. (19) for the pion (green), ρ meson (orange), and charged lepton ℓ (blue) in $B^- \rightarrow D^0 \bar{\nu}_\tau [\tau^- \rightarrow \pi^- \nu_\tau]$. Right: Longitudinal polarization $P_L(q^2)$ (dotted), perpendicular polarization $P_\perp(q^2)$ (plain), and τ forward-backward asymmetry $A_\tau(q^2)$ (dashed) from Eq. (5).

$$\begin{aligned}
 P_L &= \frac{1}{\Gamma} \int dq^2 \frac{d\Gamma}{dq^2} P_L(q^2), \\
 P_\perp &= \frac{1}{\Gamma} \int dq^2 \frac{d\Gamma}{dq^2} P_\perp(q^2), \\
 A_\tau &= \frac{1}{\Gamma} \int dq^2 \frac{d\Gamma}{dq^2} A_\tau(q^2).
 \end{aligned} \quad (6)$$

Numerically, in the SM these average tau properties in $B^- \rightarrow D^0 \tau^- \bar{\nu}_\tau$ amount to²

$$\begin{aligned}
 P_L &= 0.34(3), \\
 P_\perp &= -0.839(7), \\
 A_\tau &= -0.359(3).
 \end{aligned} \quad (7)$$

It is worth noting that the uncertainties for $P_\perp$ and A_τ are much smaller than for P_L . This is mainly due to the fact that in the SM prediction for $B \rightarrow D\tau\nu$, P_L is the result of a strong cancellation between the helicity-favored ($\lambda = -1/2$) and helicity-suppressed ($\lambda = +1/2$) contributions to the rate. Only the latter depend on the ratio of form factors $(f_0(q^2)/f_+(q^2))^2$, which causes a larger overall uncertainty than in the case of $P_\perp$ and A_τ .

By inspecting Eqs. (1) and (2), it is apparent that the longitudinal tau polarization $P_L(q^2)$ is independent from the differential rate $d\Gamma/dq^2$, so that more information is needed to determine it unambiguously. Similarly, the perpendicular polarization $P_\perp(q^2)$, which intrinsically

contains information on the interference between the two tau helicity states, cannot be obtained from $d\Gamma/dq^2$ either. The asymmetry $A_\tau(q^2)$ probes the interference between the longitudinal and time-like components in the production of the $\tau - \nu$ pair and projects on tau leptons with positive helicity, $\lambda = +1/2$ (see, for instance, Ref. [39]). Since the angle θ_τ cannot be reconstructed from the τ decay products, A_τ is not a direct observable either. In what follows, we will show that $P_L(q^2)$, $P_\perp(q^2)$, and $A_\tau(q^2)$ can be extracted with good sensitivity from kinematic distributions of the τ decay products beyond the differential decay rate $d\Gamma/dq^2$ in $B \rightarrow D\tau\nu$.

III. OBSERVABLES FROM FINAL-STATE KINEMATICS

In order to extract the τ polarizations directly from final-state kinematics, we need to consider the full production and decay chain of the τ lepton, i.e., $B \rightarrow D\nu[\tau \rightarrow \{\pi\nu, \rho\nu, \ell\nu\}]$. The two-body decays are particularly promising in this regard, because the meson carries more information on the τ kinematics than the lepton from the three-body decay.

The visible kinematics of the decay chain $B \rightarrow D\nu[\tau \rightarrow d\nu(\bar{\nu})]$ can be described in terms of three variables. We choose them as q^2 , the energy of the charged particle d in the τ decay, E_d , and the angle $\cos\theta_d = -\vec{p}_d \cdot \vec{p}_D / (|\vec{p}_d||\vec{p}_D|)$, the latter two being defined in the q rest frame. The fully differential decay rate can then be expressed as

$$\begin{aligned}
 \frac{d^3\Gamma_d}{dq^2 dE_d d\cos\theta_d} &= \mathcal{B}_d \frac{\mathcal{N}}{2m_\tau} [I_0(q^2, E_d) + I_1(q^2, E_d) \cos\theta_d \\
 &\quad + I_2(q^2, E_d) \cos^2\theta_d],
 \end{aligned} \quad (8)$$

where $\mathcal{B}_d$ denotes the branching ratio of the respective decay channel $\tau \rightarrow d\nu(\bar{\nu})$. Analytical formulas of the angular coefficient functions $I_{0,1,2}(q^2, E_d)$ and the normalization $\mathcal{N}$ can be found for $\tau \rightarrow \ell\nu\bar{\nu}$ in Ref. [39]. We have

²The errors quoted for these predictions are due to form factor uncertainties. Form factors have been implemented as described in Ref. [39]; $f_+(q^2)$ is obtained from fits to the measured $B \rightarrow D\ell\nu$ spectra by the Heavy Flavor Averaging Group in Ref. [17], whereas for the scalar form factor $f_0(q^2)$ we use the lattice QCD calculation in Ref. [9]. Our predictions confirm results found in earlier studies. In particular, we find agreement with P_L in Ref. [34], $P_\perp$ in Ref. [41], and A_τ in Ref. [36]. Sign differences are due to the different choices of reference directions made in these articles.

calculated the corresponding functions for $\tau \rightarrow \pi\nu$ and $\tau \rightarrow \rho\nu$ using the same methods.³ By integrating over $\cos\theta_d$, we find the double-differential rate

$$\begin{aligned} \frac{d^2\Gamma_d}{dq^2 dE_d} &= \int_{-1}^1 d\cos\theta_d d^3\Gamma_d \\ &= \mathcal{B}_d \frac{\mathcal{N}}{m_\tau} \left[I_0(q^2, E_d) + \frac{1}{3} I_2(q^2, E_d) \right]. \end{aligned} \quad (9)$$

Complementary to the decay rate, we define the forward-backward asymmetry of the pion with respect to the D meson,

$$\begin{aligned} \mathcal{B}_d \frac{d\Gamma}{dq^2} \frac{dA_d}{dE_d} &= \int_0^1 d\cos\theta_d d^3\Gamma_d - \int_{-1}^0 d\cos\theta_d d^3\Gamma_d \\ &= \mathcal{B}_d \frac{\mathcal{N}}{2m_\tau} I_1(q^2, E_d). \end{aligned} \quad (10)$$

Hence, $d^2\Gamma_d/dq^2 dE_d$ probes the angular coefficients I_0 and I_2 , whereas dA_d/dE_d is sensitive to I_1 . The asymmetry dA_d/dE_d is purely induced by the interference of longitudinal and time-like intermediate states of the $\tau - \nu$ pair.

Let us now focus on the dependence of these double-differential distributions [Eqs. (9) and (10)] on the tau forward-backward asymmetry $A_\tau(q^2)$ [defined in Eq. (5)] and the differential tau polarizations $P_L(q^2)$ and $P_\perp(q^2)$. We introduce the dimensionless variables

$$s_d = \frac{E_d}{\sqrt{q^2}}, \quad r_d = \frac{m_d}{\sqrt{q^2}}, \quad r_\tau = \frac{m_\tau}{\sqrt{q^2}}. \quad (11)$$

For the sake of simplicity, in what follows we neglect the mass effects r_π and r_ℓ in the decays $\tau \rightarrow \pi\nu$ and

$\tau \rightarrow \ell\nu\bar{\nu}$.⁴ The differential decay rate can then be expressed in terms of $d\Gamma/dq^2$ and $P_L(q^2)$ as [34]

$$\frac{d^2\Gamma_d}{dq^2 ds_d} = \mathcal{B}_d \frac{d\Gamma}{dq^2} [f_0^d + f_L^d P_L(q^2)], \quad (12)$$

where the integrated coefficient functions satisfy

$$\begin{aligned} \int_{s_d^{\min}}^{s_d^{\max}} ds_d f_0^d &= 1 \quad \text{and} \quad \int_{s_d^{\min}}^{s_d^{\max}} ds_d f_L^d = 0, \\ \text{with} \quad s_d^{\min} &= \frac{r_\tau^2}{2} \left(1 + \frac{r_d^2}{r_\tau^2} \right), \\ s_d^{\max} &= \frac{1}{2} (1 + r_d^2). \end{aligned} \quad (13)$$

For the pion and rho decay channels, the coefficients are given by

$$\begin{aligned} f_0^\pi &= \frac{2}{1 - r_\tau^2}, \\ f_L^\pi &= -\frac{2(1 - 4s_\pi + r_\tau^2)}{(1 - r_\tau^2)^2}, \\ f_0^\rho &= \frac{2r_\tau^2}{(1 - r_\tau^2)(r_\tau^2 - r_\rho^2)}, \\ f_L^\rho &= -\frac{2r_\tau^2(r_\tau^2 - 2r_\rho^2)((r_\tau^2 + 1)(r_\rho^2 + r_\tau^2) - 4r_\tau^2 s_\rho)}{(1 - r_\tau^2)^2(r_\tau^2 - r_\rho^2)^2(2r_\rho^2 + r_\tau^2)}. \end{aligned} \quad (14)$$

For the lepton mode, the coefficients are defined as piecewise functions, depending on the region of phase space [39]:

$$\begin{aligned} s_\ell \in \left(0, \frac{r_\tau^2}{2}\right): \quad f_0^\ell &= \frac{8s_\ell^2(9r_\tau^4 + 9r_\tau^2 - 8s_\ell r_\tau^4 - 8s_\ell r_\tau^2 - 8s_\ell)}{3r_\tau^6}, \\ f_L^\ell &= \frac{8s_\ell^2(1 - r_\tau^2)(8s_\ell r_\tau^2 + 8s_\ell - 3r_\tau^2)}{3r_\tau^6}, \\ s_\ell \in \left(\frac{r_\tau^2}{2}, \frac{1}{2}\right): \quad f_0^\ell &= \frac{2(1 - 2s_\ell)(5 + 10s_\ell - 16s_\ell^2)}{3(1 - r_\tau^2)}, \\ f_L^\ell &= \frac{2(1 - 2s_\ell)(16s_\ell^2 r_\tau^2 + 2s_\ell r_\tau^2 - 32s_\ell^2 + 2s_\ell + r_\tau^2 + 1)}{3(1 - r_\tau^2)^2}. \end{aligned} \quad (15)$$

The energy of the visible decay particle d , or equivalently the variable s_d , is essential to extract $P_L(q^2)$ from $d^2\Gamma_d/dq^2 ds_d$. Similarly, the differential forward-backward asymmetry dA_d/ds_d can be expressed in terms of $A_\tau(q^2)$ and $P_\perp(q^2)$ as

$$\frac{dA_d}{ds_d} = f_A^d A_\tau(q^2) + f_\perp^d P_\perp(q^2), \quad (16)$$

³As the fully differential expressions are not necessary for the observables considered in this article, we refrain from listing them here. Results are available from the authors upon request.

⁴The numerical effect of this approximation on our observables is at the per mille level.

where the coefficients for the pion and rho decay modes are given by

$$\begin{aligned}
 f_A^\pi &= -\frac{4(s_\pi - r_\tau^2 + s_\pi r_\tau^2)(r_\tau^2 - 2s_\pi)}{s_\pi(1 - r_\tau^2)^3}, \\
 f_\perp^\pi &= -\frac{8r_\tau(1 - 2s_\pi)(r_\tau^2 - 2s_\pi)}{\pi s_\pi(1 - r_\tau^2)^3}, \\
 f_A^\rho &= -\frac{4r_\tau^4(2r_\rho^4 + r_\rho^2(1 - 4s_\rho) - r_\tau^4 + 2r_\tau^2 s_\rho)(r_\rho^2 - (r_\tau^2 + 1)s_\rho + r_\tau^2)}{(1 - r_\tau^2)^3(r_\tau^2 - r_\rho^2)^2(2r_\rho^2 + r_\tau^2)\sqrt{s_\rho^2 - r_\rho^2}}, \\
 f_\perp^\rho &= -\frac{8r_\tau^3(r_\tau^2 - 2r_\rho^2)(r_\rho^2 - 2s_\rho + 1)(r_\rho^2 + r_\tau^4 - 2r_\tau^2 s_\rho)}{\pi(1 - r_\tau^2)^3(r_\tau^2 - r_\rho^2)^2(2r_\rho^2 + r_\tau^2)\sqrt{s_\rho^2 - r_\rho^2}},
 \end{aligned} \tag{17}$$

and the coefficients for the lepton mode read

$$\begin{aligned}
 s_\ell \in \left(0, \frac{r_\tau^2}{2}\right): f_A^\ell &= \frac{16s_\ell^2((2 - 4s_\ell)r_\tau^2 - 1)}{3r_\tau^4}, \\
 f_\perp^\ell &= \frac{64s_\ell^2(r_\tau^2 - 2s_\ell(r_\tau^2 + 1))}{3\pi r_\tau^5}, \\
 s_\ell \in \left(\frac{r_\tau^2}{2}, \frac{1}{2}\right): f_A^\ell &= \frac{4(1 - 2s_\ell)^2(4s_\ell^2(r_\tau^4 - 3r_\tau^2 + 3) + s_\ell(2r_\tau^4 - 5r_\tau^2 + 3) + r_\tau^2(r_\tau^2 - 2))}{3s_\ell(1 - r_\tau^2)^3}, \\
 f_\perp^\ell &= \frac{8(1 - 2s_\ell)^2 r_\tau(4s_\ell^2(r_\tau^2 - 2) + 2s_\ell(r_\tau^2 - 1) + r_\tau^2)}{3\pi s_\ell(1 - r_\tau^2)^3}.
 \end{aligned} \tag{18}$$

Since A_τ probes purely longitudinally polarized tau leptons (see Sec. II), $A_\tau(q^2)$ and $P_\perp(q^2)$ are clearly independent quantities. They can be extracted from a two-dimensional fit to the energy distribution of the forward-backward asymmetry dA_d/ds_d . For later convenience, we also define the s_d -integrated asymmetries,

$$A_d(q^2) = F_A^d A_\tau(q^2) + F_\perp^d P_\perp(q^2), \tag{19}$$

with the integrated coefficient functions

$$F_A^d = \int_{s_d^{\min}}^{s_d^{\max}} ds_d f_A^d, \quad F_\perp^d = \int_{s_d^{\min}}^{s_d^{\max}} ds_d f_\perp^d. \tag{20}$$

The distributions $A_d(q^2)$ are shown in the left panel of Fig. 2. At the end point, where the D meson is produced at rest, $A_d(q^2)$ tends to zero, since both $A_\tau(q^2)$ and $P_\perp(q^2)$ vanish (see right panel of Fig. 2). Otherwise, the asymmetries for the pion and rho modes are sizable over the remaining q^2 range. The total asymmetries are given by⁵

⁵The inclusive pion and rho asymmetries, A_π and A_ρ , have previously been suggested as discriminators between various contributions of new physics to $B \rightarrow D^{(*)}\tau\nu$ decays [36]. Our results agree with those from Ref. [36], with the sign difference being due to different definitions of the angle θ_d .

$$\begin{aligned}
 A_\pi &= -0.54, \\
 A_\rho &= -0.32, \\
 A_\ell &= +0.06.
 \end{aligned} \tag{21}$$

The magnitude of A_π and A_ρ suggests that they will be sensitive observables of $P_\perp$ and A_τ . We will quantify this fact in what follows.

IV. PHENOMENOLOGY AND OBSERVATION PROSPECTS AT BELLE II

After having determined the analytic relations between visible final-state kinematics and the τ properties in $B \rightarrow D\tau\nu$ decays, we now seek to quantify the sensitivity of the differential rate $d^2\Gamma_d/dq^2 ds_d$ to $P_L(q^2)$, and of the angular asymmetry dA_d/ds_d to $P_\perp(q^2)$ and $A_\tau(q^2)$. The energy of the visible decay particle $\bar{d}$, s_d , will serve as a τ polarimeter for the observables. For the longitudinal polarization, a similar analysis has previously been performed in Ref. [34] for the decays $\tau \rightarrow \pi\nu$ and $\tau \rightarrow \ell\nu\bar{\nu}$. Here we extend the analysis of $d^2\Gamma_d/dq^2 ds_d$ by the decay $\tau \rightarrow \rho\nu$ and present new results for the asymmetry dA_d/ds_d in all three decay modes $\tau \rightarrow \pi\nu$, $\tau \rightarrow \rho\nu$, and $\tau \rightarrow \ell\nu\bar{\nu}$.

Let us first consider the longitudinal polarization of the tau, P_L . Assuming an ideal experiment with unlimited resolution in q^2 and E_d , we define the statistical uncertainty

FIG. 3. Relative statistical uncertainties on the longitudinal polarization $P_L(q^2)$ measured in the differential rate $d^2\Gamma_d/dq^2 ds_d$ (left), and on the perpendicular polarization $P_\perp(q^2)$ (plain) and angular asymmetry $A_\tau(q^2)$ (dashed) measured in the asymmetry dA_d/ds_d (right). Shown are the three decay channels $\tau \rightarrow \pi \nu$ (green), $\tau \rightarrow \rho \nu$ (orange), and $\tau \rightarrow \ell \nu \bar{\nu}$ (blue). Projections are for Belle II with a total of $N = 3000$ events per channel (50 ab^{-1}) of data.

$\delta P_L(q^2)$ of measuring the longitudinal polarization in the differential rate $d^2\Gamma_d/dq^2 ds_d$ as

$$\delta P_L(q^2) = \frac{1}{\sqrt{N(q^2)} S_L(q^2)}. \quad (22)$$

Here $N(q^2)$ is the number of events $i = 1, \dots, N(q^2)$ with energy s_d^i for a fixed momentum q^2 . For a large data sample $N(q^2)$, the sensitivity $S_L(q^2)$ is given by (cf. Refs. [34,42])

$$S_L^2(q^2) = \int ds_d \frac{f_L(r_\tau, s_d)^2}{f_0(r_\tau) + f_L(r_\tau, s_d) P_L(q^2)}. \quad (23)$$

In the left panel of Fig. 3, we show the relative statistical uncertainty $\delta P_L(q^2)/P_L(q^2)$ in $d^2\Gamma_d/dq^2 ds_d$ from $B^- \rightarrow D^0 \tau^- \bar{\nu}_\tau$ for the three decay modes $\{\tau^- \rightarrow \pi^- \nu_\tau, \tau^- \rightarrow \rho^- \nu_\tau, \tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau\}$ as expected at Belle II. Assuming the same detector performance as for Belle, the expected total number of events is roughly the same in each decay channel. For a luminosity of $50(5) \text{ ab}^{-1}$, we expect $N \approx 3000(300)$ events per decay mode, based on the predictions made in Ref. [18]. In all three channels, the statistical sensitivity reaches its maximum near the kinematic end point at large q^2 . A precise measurement of the energy of the visible decay product in this region will thus facilitate

the extraction of the longitudinal tau polarization to a good accuracy. By comparing the different tau decay modes, it is apparent that the pion in $\tau \rightarrow \pi \nu$ (green) has the best analyzing power, since the pion kinematics translate directly into the polarization of the tau lepton. In $\tau \rightarrow \rho \nu$ (orange), the sensitivity is reduced due to the additional decay into transversely polarized rho mesons. In $\tau \rightarrow \ell \nu \bar{\nu}$ (blue), the relation between the final-state lepton and the tau polarization is washed out by the second invisible neutrino. From the theory point of view, the decay $\tau \rightarrow \pi \nu$ is therefore the preferred channel to observe the longitudinal tau polarization.

In the full sample of N events for $B^- \rightarrow D^0 \tau^- \bar{\nu}_\tau$, the statistical uncertainty on the average longitudinal polarization P_L from Eq. (6) is given by

$$\delta P_L = \frac{1}{\sqrt{N} S_L}, \quad \text{with} \quad S_L^{-2} = \frac{1}{\Gamma} \int dq^2 \frac{d\Gamma}{dq^2} S_L^{-2}(q^2). \quad (24)$$

In Table I, we compare the relative statistical uncertainty $\delta P_L/P_L$ in $B^- \rightarrow D^0 \tau^- \bar{\nu}_\tau$ for the decays $\tau \rightarrow \pi \nu$, $\tau \rightarrow \rho \nu$, and $\tau \rightarrow \ell \nu \bar{\nu}$. As expected from the left panel of Fig. 3, the decay mode $\tau \rightarrow \pi \nu$ has the best overall sensitivity to the longitudinal tau polarization. Already with the complete

TABLE I. Relative statistical uncertainties on the τ polarizations P_L and $P_\perp$ and angular asymmetry A_τ in $B^- \rightarrow D^0 \tau^- \bar{\nu}_\tau$ for different τ decays $\{\tau \rightarrow \pi \nu, \tau \rightarrow \rho \nu, \tau \rightarrow \ell \nu \bar{\nu} | \text{comb}\}$ and their combination (“comb”). Predictions are given for the full data set from Belle and projections for Belle II.

	Belle [total]	Belle II [1 year]	Belle II [total]
$\mathcal{L}[\text{ab}^{-1}]/N$ [events]	1/60	5/300	50/3000
$\delta P_L/P_L$	{0.21, 0.49, 0.62 0.19}	{0.10, 0.22, 0.28 0.08}	{0.03, 0.07, 0.09 0.03}
$\delta P_\perp/P_\perp$	{0.62, 1.8, 4.0 0.58}	{0.28, 0.81, 1.8 0.26}	{0.09, 0.25, 0.57 0.08}
$\delta A_\tau/A_\tau$	{0.74, 0.69, 2.8 0.50}	{0.33, 0.31, 1.3 0.22}	{0.11, 0.10, 0.40 0.07}

data set collected at Belle, P_L can be measured in this channel up to a statistical uncertainty of $\delta P_L/P_L = 21\%$, which will be reduced to the 3% level by the end of Belle II. In the long term, $\tau \rightarrow \rho\nu$ and $\tau \rightarrow \ell\nu\bar{\nu}$ will also be promising decay modes to observe P_L with less than 10% statistical uncertainty. To estimate the sensitivity expected from a combination of all three decay channels, we also give the combined statistical uncertainty $\delta P_L^{\text{comb}}/P_L$, with

$$\delta P_L^{\text{comb}} = \frac{1}{\sqrt{\sum_i (\delta P_L^i)^{-2}}},$$

$$i = \{\tau \rightarrow \pi\nu, \tau \rightarrow \rho\nu, \tau \rightarrow \ell\nu\bar{\nu}\}, \quad (25)$$

assuming independent measurements. From this combination, an ultimate statistical accuracy of P_L at the subpercent level is expected at Belle II.

A first measurement of the longitudinal τ polarization in $B \rightarrow D^*\tau\nu$, with hadronic decays $\tau \rightarrow \pi\nu$ and $\tau \rightarrow \rho\nu$, has

recently been performed by the Belle Collaboration [43]. As in our approach, Belle measures the quantities q^2 and E_d , which determine the pion or rho scattering angle against the τ direction in the q^2 frame, $\cos\theta_{\tau d}$. The helicity angle $\cos\theta_{\text{hel}}$, which is sensitive to the polarization in the τ rest frame P_L , is then obtained by boosting the event to a pseudo τ rest frame on a cone around the d direction. The sensitivity to P_L obtained through this procedure is the same as in our distribution $d^2\Gamma_d/dq^2 dE_d$ from Eq. (8). We therefore suggest to directly extract P_L from the energy distribution of the visible decay particle d , as was pointed out earlier in Ref. [34].⁶

To extract $P_\perp(q^2)$ and $A_\tau(q^2)$ from the forward-backward asymmetry A_d , we propose an unbinned maximum likelihood fit to the energy distribution dA_d/ds_d from Eq. (16). We define the probabilities $\mathcal{P}_+(q^2, s_d)$ and $\mathcal{P}_-(q^2, s_d)$ to find an event with decay particle energy s_d and $\cos\theta_d > 0$ or $\cos\theta_d < 0$, respectively, for a given q^2 (bin) as

$$\mathcal{P}_+(q^2, s_d) = \frac{f_0(r_\tau) + f_L(r_\tau, s_d)P_L(q^2) + f_A(r_\tau, s_d)A_\tau(q^2) + f_\perp(r_\tau, s_d)P_\perp(q^2)}{1 + F_A(r_\tau)A_\tau(q^2) + F_\perp(r_\tau)P_\perp(q^2)}, \quad (26)$$

$$\mathcal{P}_-(q^2, s_d) = \frac{f_0(r_\tau) + f_L(r_\tau, s_d)P_L(q^2) - f_A(r_\tau, s_d)A_\tau(q^2) - f_\perp(r_\tau, s_d)P_\perp(q^2)}{1 - F_A(r_\tau)A_\tau(q^2) - F_\perp(r_\tau)P_\perp(q^2)}. \quad (27)$$

For a data set of $N(q^2)$ events from the decay $B \rightarrow D\nu[\tau \rightarrow d\nu(\bar{\nu})]$, the log likelihood function of the variable s_d with the parameters $P_\perp(q^2)$ and $A_\tau(q^2)$ is given by

$$\ln \mathcal{L}(s_d | \{P_\perp(q^2), A_\tau(q^2)\}) = \sum_{i=1}^{N_+(q^2)} \ln(\mathcal{P}_+(q^2, s_d^i)) + \sum_{j=1}^{N_-(q^2)} \ln(\mathcal{P}_-(q^2, s_d^j)),$$

$$\text{where } N_\pm(q^2) = N(q^2) \frac{1 \pm A_d(q^2)}{2}. \quad (28)$$

For large $N(q^2)$, the likelihood function is Gaussian distributed around the estimators $\hat{P}_\perp(q^2)$ and $\hat{A}_\tau(q^2)$ which maximize $\ln \mathcal{L}$. The covariance matrix is then obtained from the second derivatives of the log likelihood,

⁶The approach taken in Ref. [35] is the same as in the Belle measurement; the angle θ_d in the former corresponds to the angle θ_{hel} in the latter.

$$\text{cov}(\hat{P}_\perp(q^2), \hat{A}_\tau(q^2)) = \begin{pmatrix} \sigma_\perp^2 & \rho\sigma_\perp\sigma_A \\ \rho\sigma_\perp\sigma_A & \sigma_A^2 \end{pmatrix}, \quad \text{with}$$

$$\text{cov}^{-1}(\hat{V}_1, \hat{V}_2)_{ij} = -\frac{\partial^2 \ln \mathcal{L}}{\partial V_i \partial V_j} \Big|_{V_1=\hat{V}_1, V_2=\hat{V}_2}. \quad (29)$$

Here, $\sigma_\perp$ and σ_A are the standard deviations of the maximum likelihood estimators in a data set of $N(q^2)$ events. The correlation coefficient for $P_\perp(q^2)$ and $A_\tau(q^2)$ is denoted as ρ . The parameter pairs $\{P_\perp(q^2), A_\tau(q^2)\}$, which are n standard deviations away from the estimators, lie on an ellipse defined by

$$\ln \mathcal{L}(s_d | \{\hat{P}_\perp \pm n\sigma_\perp, \hat{A}_\tau \pm n\sigma_A\}) - \ln \mathcal{L}(s_d | \{\hat{P}_\perp, \hat{A}_\tau\}) = -\frac{n^2}{2}. \quad (30)$$

In the left panel of Fig. 4, we display this ellipse for the decay mode $\tau \rightarrow \pi\nu$. We choose $q^2 = 7 \text{ GeV}^2$, where $d\Gamma/dq^2$ and $A_\pi(q^2)$ reach their maximum (see the left panel of Fig. 2). The data sample in our example comprises $N(q^2 = 7 \text{ GeV}^2) = 3000 \times d\Gamma(q^2 = 7 \text{ GeV}^2)/\Gamma = 540$ events, as roughly expected in a bin of size $\Delta q^2 = 1 \text{ GeV}^2$ around $q^2 = 7 \text{ GeV}^2$ from a total data set of $N = 3000$

FIG. 4. Left: Statistical uncertainty ellipse expected from an unbinned maximum likelihood fit of $P_{\perp}(q^2)$ and $A_{\tau}(q^2)$ in the asymmetry dA_{π}/ds_{π} of $B^- \rightarrow D^0 \bar{\nu}_{\tau} [\tau^- \rightarrow \pi^- \nu_{\tau}]$ to $N(q^2) = 540$ events at $q^2 = 7 \text{ GeV}^2$. The long dotted vertical and horizontal lines indicate the values of the estimators $\hat{P}_{\perp}(q^2)$ and $\hat{A}_{\tau}(q^2)$, which are equal to their standard-model predictions. The contours of the plain and dashed green ellipses lie one and two standard deviations away from the estimators, respectively. Right: Correlation $\rho(q^2)$ between $\hat{P}_{\perp}(q^2)$ and $\hat{A}_{\tau}(q^2)$ in dA_d/ds_d for the three decay modes $\tau \rightarrow \pi\nu$ (green), $\tau \rightarrow \rho\nu$ (orange), and $\tau \rightarrow \ell\nu\bar{\nu}$ (blue).

events at Belle II with 50 ab^{-1} of data luminosity. We have assumed that the estimators are equal to their standard-model expectations, $\hat{P}_{\perp}(7) = -0.86$ and $\hat{A}_{\tau}(7) = -0.37$. Shown are the contours $n = 1$ (plain) and $n = 2$ (dashed), corresponding to one and two standard deviations from the estimators. Since the standard deviations for $A_{\tau}(7)$ and $P_{\perp}(7)$ are comparable in magnitude, we expect that A_{τ} and P_L can be extracted with similar precision from a given data set. The tilt of the ellipse indicates the correlation $\rho(7) = 0.38$ between $P_{\perp}(7)$ and $A_{\tau}(7)$. In the right panel of Fig. 4, the correlation coefficient $\rho(q^2)$ between $\hat{P}_{\perp}(q^2)$ and $\hat{A}_{\tau}(q^2)$ is shown for all three tau decay modes. In the hadronic decays, the correlation is moderate at low to intermediate q^2 and in particular around $q^2 = 7$, where most of the events are expected. In this region, E_d is thus a good discriminator between $P_{\perp}$ and A_{τ} . In the leptonic modes, the correlation between $P_{\perp}$ and A_{τ} is different, which could, in principle, allow for an independent extraction of these quantities by combining results from different tau decay modes.

To estimate the sensitivity of the angular asymmetry to $P_{\perp}(q^2)$ and $A_{\tau}(q^2)$ individually, we proceed as for $P_L(q^2)$. The statistical uncertainties are given by the standard deviations as

$$\begin{aligned} \delta P_{\perp}(q^2) &= \frac{1}{\sqrt{N(q^2)S_{\perp}(q^2)}} = \sigma_{\perp}(q^2), \\ \delta A_{\tau}(q^2) &= \frac{1}{\sqrt{N(q^2)S_A(q^2)}} = \sigma_A(q^2). \end{aligned} \quad (31)$$

In the right panel of Fig. 3, we show the relative statistical uncertainties for $P_{\perp}(q^2)$ (plain) and $A_{\tau}(q^2)$ (dashed) as expected from a maximum likelihood fit of dA_d/ds_d to Belle II data corresponding to 50 ab^{-1} luminosity. In each of the three tau decay modes, the uncertainties are smallest in the region of intermediate q^2 , where the decay rate is

high. As in the case of the longitudinal polarization, the pion from the decay $\tau \rightarrow \pi\nu$ has the highest analyzing power. In this decay mode, $P_{\perp}(q^2)$ and $A_{\tau}(q^2)$ can be extracted from dA_{π}/ds_{π} with a precision that is similar to $P_L(q^2)$ extracted from $d^2\Gamma_{\pi}/dq^2 ds_{\pi}$ over the range of intermediate q^2 (cf. left panel of Fig. 3). Interestingly, the decay mode $\tau \rightarrow \rho\nu$ can compete with $\tau \rightarrow \pi\nu$ in its sensitivity to A_{τ} (green and orange dashed lines in the right panel of Fig. 3). Since A_{τ} probes the tau polarization along the tau momentum, only the longitudinal component of the rho meson contributes. This component has the same analyzing power as the pion (see also Ref. [35]). The small difference in sensitivity between $\tau \rightarrow \rho\nu$ and $\tau \rightarrow \pi\nu$ is due to the meson mass effects. The decay $\tau \rightarrow \ell\nu\bar{\nu}$ is much less sensitive to $P_{\perp}$ and A_{τ} and a significant increase in statistics, such as the one that could be provided by LHCb, would be necessary to make this mode competitive with the hadronic ones.

The statistical uncertainties on the average perpendicular polarization $\delta P_{\perp}$ and tau asymmetry δA_{τ} are defined as for the longitudinal polarization in Eq. (24), with P_L replaced by $P_{\perp}$ or A_{τ} , respectively. The expected accuracy for a measurement of $P_{\perp}$ and A_{τ} at Belle is shown in Table I. For $\tau \rightarrow \pi\nu$, the sensitivities to $P_{\perp}$ and A_{τ} are comparable, while for $\tau \rightarrow \rho\nu$ and $\tau \rightarrow \ell\nu\bar{\nu}$ the sensitivity to A_{τ} is higher than for $P_{\perp}$. While the current statistical sensitivity of Belle is limited, it will improve significantly with the larger data set expected at Belle II. In the decay mode $\tau \rightarrow \pi\nu$, $P_{\perp}$ and A_{τ} are expected to be accessible with a precision of 9% and 11%, respectively. Remarkably, the decay $\tau \rightarrow \rho\nu$ serves as an alternative channel to observe A_{τ} at the 10% level. From the combination of the three tau decay channels, at Belle II we expect an ultimate statistical accuracy of measuring $P_{\perp}$ and A_{τ} at the percent level. While the statistical sensitivity of the asymmetry A_d to $P_{\perp}$ and A_{τ} is somewhat lower than for P_L , the differential decay rate and the

decay particle's asymmetry are complementary observables of the tau properties in $B \rightarrow D\tau\nu$.

V. CONCLUSIONS

In this work, we have established explicit analytical relations between the tau properties in $B \rightarrow D\tau\nu$ decays and the kinematics of visible final-state particles. For the three dominant decay modes $\tau \rightarrow \pi\nu$, $\tau \rightarrow \rho\nu$, and $\tau \rightarrow \ell\nu\bar{\nu}$, it was shown how the longitudinal tau polarization P_L can be obtained from the energy distribution of the charged decay particle d in the full decay rate. Complementary to the decay rate, the angular asymmetry of d against the D meson direction allows us to extract the perpendicular polarization $P_\perp$ and forward-backward asymmetry A_τ of the tau lepton. These results provide a sound framework to gain the maximal available information on the production mechanism of the tau lepton directly from its visible decay products. The benefit of this approach is that the partial reconstruction of the tau rest frame can be avoided, so that the interpretation of the final state in terms of tau properties is immediate and transparent.

To quantify our results, we have performed a numerical statistics analysis for the Belle and Belle II experiments. Among the three considered decay modes, $\tau \rightarrow \pi\nu$ is shown by our analysis to be the most sensitive channel to measure the tau polarizations P_L and $P_\perp$, because the kinematics of the scalar pion directly reflect the tau helicity state. The decay $\tau \rightarrow \rho\nu$ has a comparable sensitivity to the asymmetry A_τ , which imprints itself only on the longitudinal component of the vector-meson rho. We therefore strongly encourage experimentalists to continue and intensify the investigation of two-body hadronic tau decays. In the leptonic decay $\tau \rightarrow \ell\nu\bar{\nu}$, the access to the tau properties is washed out by the presence of the second neutrino, making them much less sensitive. Nonetheless,

this could be compensated by sheer statistics if the kinematic distributions were measured at LHCb. Alternative decay modes, such as the three-prong tau decay into three pions, might be similarly sensitive to the tau properties.

With the full data set obtained at Belle, we expect a relative statistical precision of $\delta P_L/P_L = 21\%$, $\delta P_\perp/|P_\perp| = 62\%$, and $\delta A_\tau/|A_\tau| = 74\%$ from the decay channel $\tau \rightarrow \pi\nu$. In combination with $\tau \rightarrow \rho\nu$ and $\tau \rightarrow \ell\nu\bar{\nu}$, the statistical accuracy of the longitudinal polarization and the tau asymmetry can be improved to $\delta P_L^{\text{comb}}/P_L = 19\%$ and $\delta A_\tau^{\text{comb}}/|A_\tau| = 50\%$, respectively. At Belle II, with 50 ab^{-1} of data, the sensitivity is significantly improved, yielding an ultimate combined statistical precision of $\delta P_L^{\text{comb}}/P_L = 3\%$, $\delta P_\perp^{\text{comb}}/|P_\perp| = 8\%$, and $\delta A_\tau^{\text{comb}}/|A_\tau| = 7\%$.

The goal of our paper was to outline the strategy to observe the tau properties P_L , $P_\perp$, and A_τ from final-state kinematics with the application to $B \rightarrow D\tau\nu$ decays. A similar analysis for $B \rightarrow D^*\tau\nu$ is an interesting extension of this framework, which we leave for future work. Beyond the standard model, the investigation of the tau properties along the same lines will provide us with valuable information on a possible modification of the production process. The results can shed light on the apparent discrepancy between the SM predictions and measurements of semileptonic B decays with taus.

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