**Erratum: Kaon-nucleon scattering to one-loop order
in heavy baryon chiral perturbation theory
[Phys. Rev. D **92**, 114033 (2015)]**

Bo-Lin Huang and Yun-De Li

(Received 20 December 2016; published 17 January 2017)

DOI: [10.1103/PhysRevD.95.019903](https://doi.org/10.1103/PhysRevD.95.019903)

There were a number of errors in our paper. Equation (10) should take the following form:

$$T_{KN,\bar{K}N}^{(I)} = \left(\frac{E_N + M_N}{2M_N} \right) \{ V_{KN,\bar{K}N}^{(I)}(q) + i\boldsymbol{\sigma} \cdot (\mathbf{q}' \times \mathbf{q}) W_{KN,\bar{K}N}^{(I)}(q) \}, \quad (1)$$

then, Eqs. (12), (14), (20), (22), (29), and (31) should have their signs changed. The C_1 and C_2 of Eq. (36) should also have their signs changed. The C_3 and C_6 of Eq. (36) are replaced by $C_3 = 4b_{10} + b_{11}$ and $C_6 = -4b_9 - b_{11}$, respectively.

In Appendix A, Eq. (A2) should have its sign changed to be $Q^2 = 2q^2(z-1)$. Equations (A5), (A7), (A14), (A16), (A31), (A33), (A72), and (A74) should also have their signs changed. α_K^{DF} , α_η^{DF} , β_K^{DF} , β_η^{DF} , δ_K^{DF} , and δ_η^{DF} of Eq. (A12) should be given by

$$\begin{aligned} \alpha_K^{DF} &= D^4 - 30D^2F^2 + 45F^4, \\ \alpha_\eta^{DF} &= -D^4 + 9D^3F - 15D^2F^2 - 9DF^3, \\ \beta_K^{DF} &= -7D^4 + 18D^2F^2 - 27F^4, \\ \beta_\eta^{DF} &= -5D^4 + 21D^3F - 27D^2F^2 + 27DF^3, \\ \delta_K^{DF} &= 5D^4 - 54D^2F^2 + 81F^4, \\ \delta_\eta^{DF} &= D^4 + 3D^3F - 9D^2F^2 - 27DF^3. \end{aligned} \quad (2)$$

The terms $w^3 \ln \frac{m_K}{\lambda}$, $w^3 \ln \frac{m_\pi}{\lambda}$, and $w^3 \ln \frac{m_\eta}{\lambda}$ should be multiplied by 2 in Eqs. (A21), (A23), (A25), and (A27). The terms $+(6wm_\pi^2 - 2w^3) \ln \frac{m_\pi}{\lambda}$, $+(6wm_K^2 - 2w^3) \ln \frac{m_K}{\lambda}$, and $+(6wm_\eta^2 - 2w^3) \ln \frac{m_\eta}{\lambda}$ of Eqs. (A21), (A23), (A25), and (A27) should be replaced by $-2w(m_\pi^2 + w^2 \ln \frac{m_\pi}{\lambda})$, $-2w(m_K^2 + w^2 \ln \frac{m_K}{\lambda})$, and $-2w(m_\eta^2 + w^2 \ln \frac{m_\eta}{\lambda})$, respectively. Equation (A22) should be given by

$$\begin{aligned} W_{KN}^{(1)} &= \frac{1}{288\pi^2 w^2 f_K^4} \left\{ -\alpha_\pi^{DF} \left[w^3 + \pi m_\pi^3 - 2w \left(m_\pi^2 + w^2 \ln \frac{m_\pi}{\lambda} \right) - 2(w^2 - m_\pi^2)^{3/2} \ln \frac{w + \sqrt{w^2 - m_\pi^2}}{m_\pi} \right] \right. \\ &\quad - \alpha_{K,1}^{DF} \left[w^3 + \pi m_K^3 - 2w \left(m_K^2 + w^2 \ln \frac{m_K}{\lambda} \right) - 2(w^2 - m_K^2)^{3/2} \ln \frac{w + \sqrt{w^2 - m_K^2}}{m_K} \right] \\ &\quad + \alpha_{K,2}^{DF} \left[\pi m_K^3 + 2wm_K^2 - w^3 + 2w^3 \ln \frac{m_K}{\lambda} - 2(w^2 - m_K^2)^{3/2} \left(i\pi - \ln \frac{w + \sqrt{w^2 - m_K^2}}{m_K} \right) \right] \\ &\quad - \alpha_\eta^{DF} \left[w^3 + \pi m_\eta^3 - 2w \left(m_\eta^2 + w^2 \ln \frac{m_\eta}{\lambda} \right) - 2(m_\eta^2 - w^2)^{3/2} \left(\arccos \frac{w}{m_\eta} \right) \theta(m_\eta^2 - w^2) \right. \\ &\quad \left. \left. - 2(w^2 - m_\eta^2)^{3/2} \left(\ln \frac{w + \sqrt{w^2 - m_\eta^2}}{m_\eta} \right) \theta(w^2 - m_\eta^2) \right] \right\}. \end{aligned} \quad (3)$$

Equation (A24) should be given by

$$\begin{aligned}
 W_{KN}^{(0)} = \frac{1}{288\pi^2 w^2 f_K^4} & \left\{ -\beta_\pi^{DF} \left[w^3 + \pi m_\pi^3 - 2w \left(m_\pi^2 + w^2 \ln \frac{m_\pi}{\lambda} \right) - 2(w^2 - m_\pi^2)^{3/2} \ln \frac{w + \sqrt{w^2 - m_\pi^2}}{m_\pi} \right] \right. \\
 & - \beta_{K,1}^{DF} \left[w^3 + \pi m_K^3 - 2w \left(m_K^2 + w^2 \ln \frac{m_K}{\lambda} \right) - 2(w^2 - m_K^2)^{3/2} \ln \frac{w + \sqrt{w^2 - m_K^2}}{m_K} \right] \\
 & + \beta_{K,2}^{DF} \left[\pi m_K^3 + 2w m_K^2 - w^3 + 2w^3 \ln \frac{m_K}{\lambda} - 2(w^2 - m_K^2)^{3/2} \left(i\pi - \ln \frac{w + \sqrt{w^2 - m_K^2}}{m_K} \right) \right] \\
 & - \beta_\eta^{DF} \left[w^3 + \pi m_\eta^3 - 2w \left(m_\eta^2 + w^2 \ln \frac{m_\eta}{\lambda} \right) - 2(m_\eta^2 - w^2)^{3/2} \left(\arccos \frac{w}{m_\eta} \right) \theta(m_\eta^2 - w^2) \right. \\
 & \left. \left. - 2(w^2 - m_\eta^2)^{3/2} \left(\ln \frac{w + \sqrt{w^2 - m_\eta^2}}{m_\eta} \right) \theta(w^2 - m_\eta^2) \right] \right\}. \tag{4}
 \end{aligned}$$

Equation (A26) should be given by

$$\begin{aligned}
 W_{KN}^{(1)} = -\frac{1}{576\pi^2 w^2 f_K^4} & \left\{ \gamma_\pi^{DF} \left[\pi m_\pi^3 + 2w m_\pi^2 - w^3 + 2w^3 \ln \frac{m_\pi}{\lambda} - 2(w^2 - m_\pi^2)^{3/2} \left(i\pi - \ln \frac{w + \sqrt{w^2 - m_\pi^2}}{m_\pi} \right) \right] \right. \\
 & + \gamma_{K,1}^{DF} \left[\pi m_K^3 + 2w m_K^2 - w^3 + 2w^3 \ln \frac{m_K}{\lambda} - 2(w^2 - m_K^2)^{3/2} \left(i\pi - \ln \frac{w + \sqrt{w^2 - m_K^2}}{m_K} \right) \right] \\
 & - \gamma_{K,2}^{DF} \left[w^3 + \pi m_K^3 - 2w \left(m_K^2 + w^2 \ln \frac{m_K}{\lambda} \right) - 2(w^2 - m_K^2)^{3/2} \ln \frac{w + \sqrt{w^2 - m_K^2}}{m_K} \right] \\
 & + \gamma_\eta^{DF} \left[\pi m_\eta^3 + 2w m_\eta^2 - w^3 + 2w^3 \ln \frac{m_\eta}{\lambda} - 2(m_\eta^2 - w^2)^{3/2} \left(\arccos \frac{-w}{m_\eta} \right) \theta(m_\eta^2 - w^2) \right. \\
 & \left. \left. - 2(w^2 - m_\eta^2)^{3/2} \left(i\pi - \ln \frac{w + \sqrt{w^2 - m_\eta^2}}{m_\eta} \right) \theta(w^2 - m_\eta^2) \right] \right\}. \tag{5}
 \end{aligned}$$

Equation (A28) should be given by

FIG. 1. Fits and predictions for the SP92 phase shifts versus the kaon laboratory momentum q_K in KN scattering. The fits in the S01, P01, and P03 waves are the data between 100 and 140 MeV (solid dots), while the S11, P11, and P13 waves are between 50 and 90 MeV. For higher and lower energies, the phase shifts are predicted.

$$\begin{aligned}
 W_{\bar{K}N}^{(0)} = & -\frac{1}{576\pi^2 w^2 f_K^4} \left\{ \delta_\pi^{DF} \left[\pi m_\pi^3 + 2wm_\pi^2 - w^3 + 2w^3 \ln \frac{m_\pi}{\lambda} - 2(w^2 - m_\pi^2)^{3/2} \left(i\pi - \ln \frac{w + \sqrt{w^2 - m_\pi^2}}{m_\pi} \right) \right] \right. \\
 & + \delta_{K,1}^{DF} \left[\pi m_K^3 + 2wm_K^2 - w^3 + 2w^3 \ln \frac{m_K}{\lambda} - 2(w^2 - m_K^2)^{3/2} \left(i\pi - \ln \frac{w + \sqrt{w^2 - m_K^2}}{m_K} \right) \right] \\
 & - \delta_{K,2}^{DF} \left[w^3 + \pi m_K^3 - 2w \left(m_K^2 + w^2 \ln \frac{m_K}{\lambda} \right) - 2(w^2 - m_K^2)^{3/2} \ln \frac{w + \sqrt{w^2 - m_K^2}}{m_K} \right] \\
 & + \delta_\eta^{DF} \left[\pi m_\eta^3 + 2wm_\eta^2 - w^3 + 2w^3 \ln \frac{m_\eta}{\lambda} - 2(m_\eta^2 - w^2)^{3/2} \left(\arccos \frac{-w}{m_\eta} \right) \theta(m_\eta^2 - w^2) \right. \\
 & \left. \left. - 2(w^2 - m_\eta^2)^{3/2} \left(i\pi - \ln \frac{w + \sqrt{w^2 - m_\eta^2}}{m_\eta} \right) \theta(w^2 - m_\eta^2) \right] \right\}. \quad (6)
 \end{aligned}$$

α_π^{DF} , α_η^{DF} , β_π^{DF} , $\beta_{K,1}^{DF}$, and β_η^{DF} of Eq. (A29) should have their signs changed. $\alpha_{K,1}^{DF}$ and δ_π^{DF} of Eq. (A29) should be given by

$$\begin{aligned}
 \alpha_{K,1}^{DF} &= -13D^4 + 42D^2F^2 - 45F^4, \\
 \delta_\pi^{DF} &= 27D^4 - 54D^2F^2 + 27F^4. \quad (7)
 \end{aligned}$$

FIG. 2. The antikaon-nucleon ($\bar{K}N$) phase shifts versus the kaon laboratory momentum q_K . The solid lines denote our predictions through the LECs determined by the KN phase shifts and the corresponding data.

α_K^{DF} , α_η^{DF} , β_K^{DF} , β_η^{DF} , δ_π^{DF} , δ_K^{DF} , and δ_η^{DF} of Eq. (A38) should be given by

$$\begin{aligned}
 \alpha_K^{DF} &= 40D^4 - 48D^3F + 144D^2F^2 + 144DF^3 + 360F^4, \\
 \alpha_\eta^{DF} &= 16D^4 + 48D^2F^2, \\
 \beta_K^{DF} &= 104D^4 - 240D^3F + 144D^2F^2 - 432DF^3 - 216F^4, \\
 \beta_\eta^{DF} &= 32D^4 - 96D^3F, \\
 \delta_\pi^{DF} &= 12D^4 + 72D^3F + 108D^2F^2, \\
 \delta_K^{DF} &= 4D^4 + 24D^3F + 72D^2F^2 + 216DF^3 + 324F^4, \\
 \delta_\eta^{DF} &= 4D^4 + 24D^3F + 36D^2F^2.
 \end{aligned} \tag{8}$$

The terms $[2r(m_\pi)(\arctan \frac{1}{r(m_\pi)})\theta(4m_\pi^2 - Q^2) + (r(m_\pi) \ln |\frac{1+r(m_\pi)}{1-r(m_\pi)}| - i\pi r(m_\pi))\theta(Q^2 - 4m_\pi^2)]$ and $r(m_K) \arctan \frac{1}{r(m_K)}$ should be replaced by $r(m_\pi) \ln |\frac{1+r(m_\pi)}{1-r(m_\pi)}|$ and $\frac{1}{2} r(m_K) \ln |\frac{1+r(m_K)}{1-r(m_K)}|$ in Eqs. (A58), (A59), (A60), and (A61), respectively. The terms $-12\pi m_\pi^2 Q^2 K_0(m_\pi)$, $-4\pi m_K^2 Q^2 K_0(m_K)$, $(8m_K^2 + 3Q^2)$, $(15m_\pi^2 + 21m_\eta^2 - 18m_\pi m_\eta + 3Q^2)$, $(12m_\eta^4 - 60m_\pi^2 m_\eta^2 - 18m_\pi^2 Q^2 - 6m_\eta^2 Q^2 + 3Q^4)$, and $(12m_\pi^4 - 36m_\pi^2 m_\eta^2 - 18m_\eta^2 Q^2 - 6m_\pi^2 Q^2 + 3Q^4 - 24m_\eta^4)$ of Eqs. (A62), (A63), (A64), and (A65) should be replaced by $+36\pi m_\pi^2 Q^2 K_0(m_\pi)$, $+12\pi m_K^2 Q^2 K_0(m_K)$, $(8m_K^2 - 9Q^2)$, $(15m_\pi^2 + 21m_\eta^2 - 18m_\pi m_\eta - 9Q^2)$, $-(12m_\eta^4 - 60m_\pi^2 m_\eta^2 + 30m_\pi^2 Q^2 - 6m_\eta^2 Q^2 + 3Q^4)$, and $-(12m_\pi^4 - 36m_\pi^2 m_\eta^2 + 30m_\eta^2 Q^2 - 6m_\pi^2 Q^2 + 3Q^4 - 24m_\eta^4)$, respectively. The remaining Q^2 's of Eqs. (A62), (A63), (A64), and (A65) should have their signs changed. Equations (A66), (A67), (A68), and (A69) should be given by

FIG. 3. Convergence properties as exemplified by our predictions for the KN phase shifts. The dashed, dotted, and dashed-dotted lines denote the first, second, and third order, respectively. The solid lines give the sum of the first-, second-, and third-order contributions.

$$K_0(m) = -\frac{1}{8\pi\sqrt{-Q^2}} \arctan \frac{\sqrt{-Q^2}}{2m}, \quad (9)$$

$$H_0(m) = -m^2 + \frac{Q^2}{6} - 5m^2 \ln \frac{m}{\lambda} + \left(2m^2 + \frac{Q^2}{4}\right) \left[1 - 2 \ln \frac{m}{\lambda} - r(m) \ln \left| \frac{1+r(m)}{1-r(m)} \right| \right], \quad (10)$$

$$H(m_1, m_2) = \frac{1}{32\pi^2\sqrt{-Q^2}} [F_1(y_1) + F_1(y_2) + F_2(x_1) + F_2(x_2) - F_3(z_1) - F_3(z_2)], \quad (11)$$

and

$$\begin{aligned} F_1(x) &= \text{Li}_2\left(\frac{z_0 + 2Q^2}{z_0 - 2\sqrt{-Q^2}x}\right) - \text{Li}_2\left(\frac{z_0}{z_0 - 2\sqrt{-Q^2}x}\right), \\ F_2(x) &= -\text{Li}_2\left(\frac{z_0 + 2Q^2}{z_0 - 2\sqrt{-Q^2}x}\right) - \frac{1}{2} \ln^2\left(\frac{z_0 - 2\sqrt{-Q^2}x}{\lambda^2}\right), \\ F_3(x) &= -\text{Li}_2\left(\frac{z_0}{z_0 - 2\sqrt{-Q^2}x}\right) - \frac{1}{2} \ln^2\left(\frac{z_0 - 2\sqrt{-Q^2}x}{\lambda^2}\right), \\ z_0 &= m_2^2 - m_1^2 - Q^2, \quad x_{1,2} = \sqrt{-Q^2} \pm im_2, \quad z_{1,2} = \pm im_1, \\ y_{1,2} &= -\frac{1}{2\sqrt{-Q^2}} \left(m_1^2 - m_2^2 + Q^2 \pm \sqrt{[Q^2 - (m_1 + m_2)^2][Q^2 - (m_1 - m_2)^2]} \right), \end{aligned} \quad (12)$$

respectively.

In the results and discussion, the discussion about the low-energy constant C_2 (in the second paragraph of the sixth page) should be deleted, and Fig. 4 should also be deleted. The statements “and successfully reduce one parameter” in the abstract and “and find a constraint among the LECs” in the summary should be deleted. Equations (42) and (43) about the constants should be given by

$$\begin{aligned} C_1 &= 1.99 \pm 0.11 \text{ GeV}^{-1}, \\ C_2 &= -0.45 \pm 0.11 \text{ GeV}^{-1}, \\ C_3 &= 6.36 \pm 0.09 \text{ GeV}^{-1}, \end{aligned} \tag{13}$$

with $\chi^2/\text{d.o.f.} \sim 0.91$ and

$$\begin{aligned} C_4 &= 3.01 \pm 0.21 \text{ GeV}^{-1}, \\ C_5 &= -5.10 \pm 0.21 \text{ GeV}^{-1}, \\ C_6 &= -5.13 \pm 0.12 \text{ GeV}^{-1}, \end{aligned} \tag{14}$$

with $\chi^2/\text{d.o.f.} \sim 2.49$. The statement “One can find that the uncertainty of the parameter C_2 is very large when compared to the C_2 value. For this, a more detailed discussion is given below.” in the last paragraph of the fifth page should be deleted. Figure 3 on the sixth page should be replaced by the following Fig. 1. On the sixth page the first sentence below Eq. (44) should be replaced by “The LEC combination $C_1 + C_2$ determined by the phase shifts from Eq. (13) is 1.54 GeV^{-1} , while the $C_4 + C_5$ from Eq. (14) is -2.09 GeV^{-1} .” Figure 5 on the seventh page should be replaced by the following Fig. 2. $a_{KN}^{(0)}$, $a_{\bar{K}N}^{(1)}$, and $a_{\bar{K}N}^{(0)}$ should be -0.01 , 0.41 , and 1.63 in “Prediction A” of Table I, respectively. Figure 6 on the eighth page should be replaced by the following Fig. 3. The changed formulas result in these changed constants, figures, and statements; the other main statements are not changed. These corrections are all fully incorporated into the paper arXiv:1612.06163.