Applicability of approximations used in calculations of the spectrum of dark matter particles produced in particle decays

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For the warm dark matter (WDM) candidates the momentum distribution of particles becomes important, since it can be probed with observations of Lyman-α forest structures and confronted with coarse grained phase space density in galaxy clusters. We recall the calculation [M. Kaplinghat, Phys. Rev. D **72**, 063510 (2005)] of the spectrum in the case of dark matter nonthermal production in decays of heavy particles emphasizing the inherent applicability conditions, which are rather restrictive and sometimes ignored in literature. The cold part of the spectrum requires special care when WDM is considered.

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I. INTRODUCTION

One of the major puzzles in physics as we know it at present—dark matter (DM) phenomenon—requires new massive electrically neutral collisionless particles stable at the cosmological time scale [1]. They must be produced in the early Universe before the plasma temperature T drops below 1 eV, since later cosmological stages definitely need the DM [2].

While the Universe expansion schedule is sensitive only to the total energy density associated with the new nonrelativistic particles (and hence to their number density at a given particle mass), the evolution of spatial *inhomogeneities* of matter is also sensitive to the velocity distribution of the DM particles. Indeed, free streaming of the DM particles smooths out all the inhomogeneities smaller than the so-called free-streaming length $l_{\mathrm{f.s.}}$. The latter is the typical distance traveled by a DM particle, which is of order $l_{\mathrm{f.s.}} \sim v \times l_H$, where v is the DM average velocity and l_H is the Hubble horizon size at a given time. In order for successful generation of the smallest observed primordial structures—dwarf galaxies—one needs $v \lesssim 10^{-3}$ at the epoch of radiation-matter energy density equality, $T \sim 1$ eV. This requirement defines the border line between faster and slower candidates named as hot and cold DM.

The candidates *right at the border* are called warm DM, and the question about velocity distribution is especially relevant for them. In fact, the hot DM is disfavored by structure formation and may be only a small fraction of DM (precise amount depends on the velocity distribution). On the other extreme, the cold DM candidates, like weakly interacting massive particles, are typically very slow at equality, and allow for formation of structures much smaller (and lighter) than the dwarf galaxies. These structures are expected to be starless and empty of baryons after reionization epoch and (partially) destroyed during subsequent formation of heavier structures. Yet if some of them remained, searches for gravitational lensing events in galaxies may (in principle) detect them and determine the structure abundance (the DM velocity distribution defines the size of smallest structures). Therefore, both hot and cold DM components suggest potential observables sensitive to the velocity distribution. However, this is the intermediate case of warm DM, where the corresponding observables provide the most nontrivial constraints on the DM models, and they have been actively exploited in the literature.

The most promising observable for this task is the small structures in the Lyman-α forest [3,4]. Their studies have already allowed one to rule out some warm dark matter (WDM) models, e.g. (keV scale) sterile neutrino DM [5] produced nonthermally by nonresonant oscillations of the active neutrino in primordial plasma [6]. However, many other candidates are still valid (e.g. light gravitino [7], axino [8], etc.), which are mostly nonthermally produced, see [9] for a review. Moreover, even in the case of sterile neutrino DM various other production mechanisms were proposed, such as resonant production [10], thermal production with subsequent dilution [11], production in decays of scalar particles [12–15], all providing some ways to evade the present Lyman-α constraints [16,17].

To use the observations of Lyman-α forest structure in a particular model, one must know the velocity distribution of the DM particles. In this paper we focus our attention on

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the DM produced in the early Universe in decays of some particle, which we, following [12], denote as DDM. Several regimes are possible, corresponding to the DDM particle being in or out of thermal equilibrium. The case of DDM in thermal equilibrium due to annihilations in the SM particles, while also having a small decay branching ratio into the DM, is analyzed in [13,14]. The production happens mostly at a temperature of about the DDM mass, $T \sim M$, leading to the distribution with average momentum slightly below the thermal one. After production the spectrum can be cooled further due to the decrease of degrees of freedom of the relativistic plasma in the expanding Universe [14]. This mechanism leads to the lower bound on the DM mass in the model, $m_{\mathrm{DM}} > 7.8$ keV where we used the recent Lyman-α analysis from [17]. Another situation corresponds to the case of DDM decaying while being *out of thermal equilibrium* [12] (in particular, this may correspond to the DDM itself produced in a nonthermal way). It was argued that, for sufficiently long living DDM, the majority of its decays happen when it is significantly nonrelativistic, leading to a peculiar momentum distribution, which can be strongly shifted towards low momenta. In this paper we show that the approximation of nonrelativistic decay is actually valid only for the high energy part of the DM spectrum, while the low energy part is produced at earlier stages, when DDM particles still have non-negligible velocities.

The results of the paper show that for the proper description of the cold parts of DM it is important to analyze exactly the decays of the DDM at early times, when it is close to being relativistic (no matter whether it is in or out of thermal equilibrium). The detailed analysis of several possibilities of this type is present in [13–15,18,19].

In Sec. II we introduce the generic formalism, and review the nonrelativistic approximation used to obtain the DM spectrum in Sec. III. Regions of applicability of this approximation and comparison with exact numerical results are presented in Sec. IV.

II. GENERAL FORMALISM

In the model we have two sets of particles—the decaying DDM of mass M and the dark matter DM, which is stable with mass m_{DM}. DM is produced in two-body decay of the initial DDM particle. Distribution of the particles over momentum $f(p)$ are normalized to the physical particle number density n in the expanding Universe with scale factor a as

$$n = \int \frac{d^3p}{(2\pi)^3} f(pa) = \int \frac{d^3k}{(2\pi)^3 a^3} f(k) = \int_0^\infty \frac{k^2 dk}{2\pi^2 a^3} f(k), \tag{1}$$

where p and $k \equiv pa$ are physical and conformal 3-momenta, respectively. We allow to be sloppy in writing the conformal or physical momentum as an argument to f, as far as they can always be mapped to another. One should just be careful in the solution of the kinetic equations, where the conformal momentum must be always used. Also we often drop the explicit time dependence where it does not lead to ambiguities. We use conformal η and physical t times (related by $dt = a d\eta$) interchangeably. The normalization used corresponds to $f(k)$ remaining constant in time in the absence of interactions, and (physical) number density decreasing as $n \propto 1/a^3$.

The kinetic equation for the DDM evolution is [cf. Eq. (1) of [12]]

$$\frac{df_{\mathrm{DDM}}(k_{\mathrm{DDM}}, \eta)}{d\eta} = -\frac{aM}{\tau E_{\mathrm{DDM}}} f_{\mathrm{DDM}}(k_{\mathrm{DDM}}, \eta), \tag{2}$$

where τ is the DDM lifetime. The equation for the DM is

$$\begin{aligned} \frac{df_{\mathrm{DM}}(k_{\mathrm{DM}}, \eta)}{d\eta} &= \frac{aM^2}{\tau E_{\mathrm{DM}} p_{\mathrm{DM}} p_{\mathrm{CM}}} \int_{E_1}^{E_2} f_{\mathrm{DDM}}(ap) dE \\ &\overset{m_{\mathrm{DM}} \to 0}{=} \frac{a^3 2M}{\tau k_{\mathrm{DM}}^2} \int_{p_{\mathrm{DM}} + \frac{M^2}{4p_{\mathrm{DM}}}}^{\infty} f_{\mathrm{DDM}}(ap) dE, \end{aligned} \tag{3}$$

where $p_{\mathrm{DM}} \equiv k_{\mathrm{DM}}/a$, $p \equiv \sqrt{E^2 - M^2}$, and p_{CM} is the DM momentum in the center of mass frame of DDM, $E_{1,2}$ are given by (8). There is overall coefficient 2 as compared to (2) of [12], which assumed that only one of the two-body decay products is the DM. In the present paper we assume that both DDM decay products are DM.

III. ANALYTICAL SOLUTIONS TO THE EQUATIONS IN THE CASE OF RADIATION DOMINATION AND CONSTANT g_*

The solution to Eq. (2) at the radiation dominated stage with scale factor $a \equiv c\eta$ is (cf. Ref. [15])

$$\begin{aligned} f_{\mathrm{DDM}}(k, \eta) = f_{\mathrm{DDM}}(k, \eta_i) &\left(\frac{\eta + \sqrt{\eta^2 + \frac{k^2}{M^2 c^2}}}{\eta_i + \sqrt{\eta_i^2 + \frac{k^2}{M^2 c^2}}} \right)^{\frac{k^2}{2\tau c M^2}} \\ &\times e^{-\frac{c}{2\tau}\left(\eta \sqrt{\eta^2 + \frac{k^2}{M^2 c^2}} - \eta_i \sqrt{\eta_i^2 + \frac{k^2}{M^2 c^2}} \right)}. \end{aligned} \tag{4}$$

Hereafter the subscript i refers to the moment when the DDM particles freeze out or appear in the Universe through another mechanism, leading to some fixed spectrum $f_{\mathrm{DDM}}(k, \eta_i)$. The analytical solution (4) assumes a constant number of relativistic degrees of freedom in plasma from the DDM production η_i until its decay. The solution (4) can be rewritten through the physical momenta $p \equiv k/a$ and the Hubble parameter given by

$$H \equiv \frac{da/d\eta}{a^2} = \frac{1}{c\eta^2}. \tag{5}$$

In the limit of very nonrelativistic particles one obtains approximately from (4)

$$f_{\mathrm{DDM}}(k,\eta) = f_{\mathrm{DDM}}(k,\eta_i) \times \mathrm{e}^{-\frac{1}{2\tau}\left(\frac{1}{H}-\frac{1}{H_i}\right)}, \tag{6}$$

and at the next-to-leading order both for the exponent (which remains the same) and the prefactor

$$f_{\mathrm{DDM}}(k,\eta) = f_{\mathrm{DDM}}(k,\eta_i)\left(1+\frac{k^2}{a^2M^2}\frac{1}{4\tau H}\log\frac{H_i}{H}\right)\mathrm{e}^{-\frac{1}{2\tau}\left(\frac{1}{H}-\frac{1}{H_i}\right)}. \tag{7}$$

To solve (3) one must evaluate the upper E_2 and lower E_1 limits of the integration in the rhs. One gets approximately (in the relativistic limit, $p_{\mathrm{DM}} \gg m_{\mathrm{DM}}$)

$$E_2 = p_{\mathrm{DM}}\frac{M^2}{m_{\mathrm{DM}}^2} - p_{\mathrm{DM}} + \frac{M^2}{4p_{\mathrm{DM}}} \to \infty,$$
$$E_1 = p_{\mathrm{DM}} + \frac{M^2}{4p_{\mathrm{DM}}}. \tag{8}$$

In the nonrelativistic limit (6), when all decaying particles DDM are (almost) at rest, it is reasonable to assume that their distribution function is

$$f_{\mathrm{DDM}}(k,\eta) = F\frac{2\pi^2\delta(k)}{k^2}, \tag{9}$$

where the normalization is fixed by (1), and

$$F(\eta) = F_i \times \mathrm{e}^{-\frac{1}{2\tau}\left(\frac{1}{H}-\frac{1}{H_i}\right)} \equiv \tilde{F}_i \times \mathrm{e}^{-\frac{1}{2\tau H}}. \tag{10}$$

To avoid singularities at $E = E_1$ in (3), it is convenient to regularize (9) as

$$f_{\mathrm{DDM}}(k) = F\frac{2\pi^2\delta(k-\kappa)}{k^2}. \tag{11}$$

This regularization has physical meaning, as it assumes that the DDM particles are not exactly at rest, but move with some small conformal momentum κ. Note that the normalization (10) means that at the moment η_i of DDM freeze-out its concentration is given by $n_{\mathrm{DDM}}(\eta_i) = F_i/a_i^3$. In this approximation the collision integral in (3) can be taken easily:

$$\int_{p_{\mathrm{DM}}+\frac{M^2}{4p_{\mathrm{DM}}}}^{\infty} f_{\mathrm{DDM}}(ap)dE = \frac{2\pi^2 F}{a^3 M}\delta\left(p_{\mathrm{DM}}-\frac{M}{2}\right). \tag{12}$$

Using Eq. (12), Eq. (3) can be reduced to

$$\frac{df_{\mathrm{DM}}(k)}{d\eta} = \frac{4\pi^2 F}{\tau k^2}\delta\left(\frac{k}{a}-\frac{M}{2}\right) = \frac{16\pi^2 F}{\tau a^2 M^2}\delta\left(\frac{k}{a}-\frac{M}{2}\right) \tag{13}$$

and can be directly integrated for each k individually. The δ-function in (13) gives the moment, $\eta = \eta_*$, when the particle with properly rescaled 3-momentum

$$p = \frac{k}{a} = \frac{a_*}{a}\frac{k}{a_*} = \frac{M}{2}\frac{a_*}{a} \tag{14}$$

was created, so the integration leads to

$$f_{\mathrm{DM}}(k) = \frac{16\pi^2 F(\eta_*)}{\tau a_*^2 M^2}. \tag{15}$$

The conformal time (for some given number of degrees of freedom encoded in c_*) is $\eta_* = a_*/c_*$ and (14) implies $\eta_* = 2k/(c_* M)$. Then for the Hubble parameter one obtains from (5)

$$H_* = \frac{1}{c_*\eta_*^2} = \frac{c_*}{a_*^2} = \frac{c_* M^2}{4k^2} = \frac{c_*}{c}\frac{HM^2}{4p^2}. \tag{16}$$

Using (10) we get

$$f_{\mathrm{DM}}(k) = \frac{32\pi^2\tilde{F}_i}{\tau M^3}\frac{1}{a_*^3 H_*} \times \mathrm{e}^{-\frac{1}{2\tau H_*}}$$
$$= \frac{1}{a^3}\frac{16\pi^2}{\tau M^2}\tilde{F}_i\frac{c}{c_* H}\frac{1}{p} \times \mathrm{e}^{-\frac{c}{c_*}\frac{2p^2}{\tau H M^2}}. \tag{17}$$

Finally, introducing the effective number of degrees of freedom in the plasma $g_*(T)$, photon temperature T and

$$M_{\mathrm{Pl}}^*(T) \equiv \frac{M_{\mathrm{Pl}}}{1.66 \times \sqrt{g_*(T)}}, \qquad \text{so that } H = \frac{T^2}{M_{\mathrm{Pl}}^*},$$

we find for the DM distribution in physical momentum $p = k/a$ the following expression:

$$f_{\mathrm{DM}}(p) = \frac{16\pi^2}{\tau M^2}\tilde{F}_i\left(\frac{g_*(T_*)}{g_*(T)}\right)^{2/3}\frac{M_{\mathrm{Pl}}^*}{T^3}$$
$$\times \frac{T}{p} \times \mathrm{e}^{-\left(\frac{g_*(T_*)}{g_*(T)}\right)^{2/3}\frac{p^2}{T^2}\frac{2M_{\mathrm{Pl}}^*}{\tau M^2}}, \tag{18}$$

which precisely coincides with the results from [12].

IV. APPLICABILITY OF THE NONRELATIVISTIC APPROXIMATION

The final spectrum (18) is valid as far as the nonrelativistic approximation (12) for the DDM distribution was applicable. This puts two bounds on the allowed values of the momenta p of the DM particle. The *upper* bound is rather trivial and is irrelevant for most considerations, as far as it kicks in the region where the spectrum is anyway exponentially suppressed. Above the value

$$p_{\max} = M/2, \qquad f(p_{\mathrm{DM}} > p_{\max}) = 0, \tag{19}$$

the result is completely cut off due to absence (or exponential suppression in precise calculation) of high momenta DDM particles at the present time. The more interesting bound modifies the spectrum for the *low* values of momenta.[1] This is not a hard cutoff, but a suppression of the spectrum. This suppression is not grasped by (18) as it is obtained in the assumption that at the moment η_* the DDM particle was already nonrelativistic, and hence the DM momentum at the moment of production is

$$p_{\mathrm{DM}}|_{\eta=\eta_*} = \frac{M}{2} \gg \langle p_{\mathrm{DDM}}\rangle|_{\eta=\eta_*}. \tag{20}$$

Later the Universe expands and the DM momentum gets redshifted, so that at temperature T (today) it equals

$$p_{\mathrm{DM}} = \left(\frac{g_*(T)}{g_*(T_*)}\right)^{1/3} \frac{T}{T_*}\frac{M}{2}.$$

Smaller momenta correspond thus to higher Universe temperature T_* at the time of production. As far as the typical momentum of a particle in the plasma is $p \sim 3T$, the DDM particles are nonrelativistic only at $T_* \ll M/3$, and the spectrum (18) is valid only for high momenta,

$$p_{\mathrm{DM}} \gg \frac{3}{2}\left(\frac{g_*(T)}{g_*(T_*)}\right)^{1/3} T. \tag{21}$$

So the cold part of the distribution is not described by (18), and analysis beyond the nonrelativistic approximation (12) is required. This constraint alone is also not always an important correction for the spectrum (18), as far as the spectrum is again suppressed at low momenta, at least for large τ, see e.g. Fig. 1 and the average momentum (23).

There are two more constraints referring to the model parameters, which leave the spectrum (18) valid. One is very relevant in practice. It follows from the analysis of the assumption of decaying particle being (almost) at rest and implies the lower bound on the DDM lifetime,

$$1 \ll \tau H(T = M/3) = \frac{2\tau M^2}{18 M^*_{\mathrm{Pl}}(T = M/3)} \equiv \frac{1}{18}\frac{1}{\Lambda}, \tag{22}$$

which limits significantly the exponential factor $\propto \Lambda$ in (18). Note that the limit corresponds to explosion of the expansion in the preexponent in (7). Basically, the DDM particles with short lifetime decay mostly before becoming nonrelativistic. So spectrum (18) is justified only for $\Lambda \ll 0.05$, otherwise we come out of the applicability region.

[1]We do not discuss here the issue of Pauli blocking at low momenta which might be relevant for any mechanism with light fermionic dark matter.

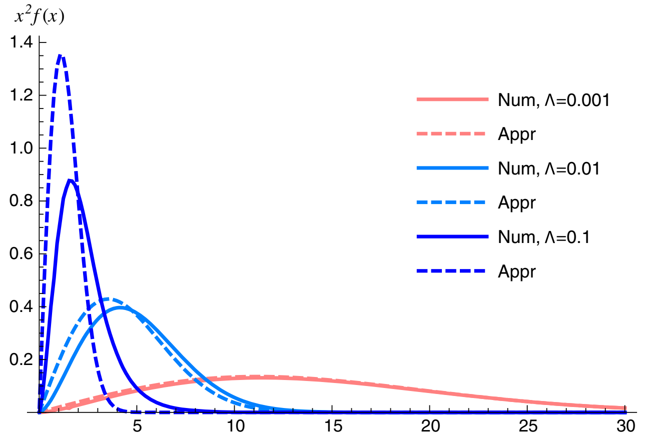

FIG. 1. DM spectra (arbitrary units) obtained by exploiting the approximate formula (18) (dashed curves) and by integrating the equations numerically (solid curves), through inserting (4) into (3). Parameter Λ (22) takes three different values: the smaller the value is, the more shallow the curves. The horizontal axis is $x = (g_*(T_*)/g_*(T))^{1/3} p/T$.

On the contrary, for very long lifetime τ of the DDM particle (small Λ) and large enough initial abundance the DDM may start to dominate the Unverse expansion and lead to a temporary matter dominated stage. This means that for large τ formula (18), which is derived assuming radiation domination stage, also cannot be applied.

We illustrate the statements above with Fig. 1. Here, for several values of Λ we plot DM spectra (18) and the exact numerical solution of Eq. (3), with DDM spectra given by (4) (see [15]). The maximum of the distribution moves towards larger momenta when Λ diminishes. The smaller the latter, the more accurate the approximation (22) becomes. For larger values of Λ pronounced deviation develops between the approximate formula (18) and the exact numerical spectrum. At low momenta, $p/T \lesssim 3/2(g_*(T)/g_*(T_*))^{1/3}$ the approximate spectrum (18) is always incorrect, since this region is beyond the border (21).

Account for these limits impacts the calculations of the average velocity,

$$\langle v_{\mathrm{DM}}\rangle = \frac{\langle p_{\mathrm{DM}}\rangle}{T}\frac{T}{m_{\mathrm{DM}}}.$$

When integration is performed using distribution (17) over all momenta [i.e. violating the bounds (19) and (21)], and/or ignoring the constraint (22) (e.g. [20–23]) the obtained average velocity is not correct. Recall that the average velocity is usually adopted in estimates of the free-streaming length important for the small scale structure formation and tested with Lyman-α forest data. The *coldest* component of

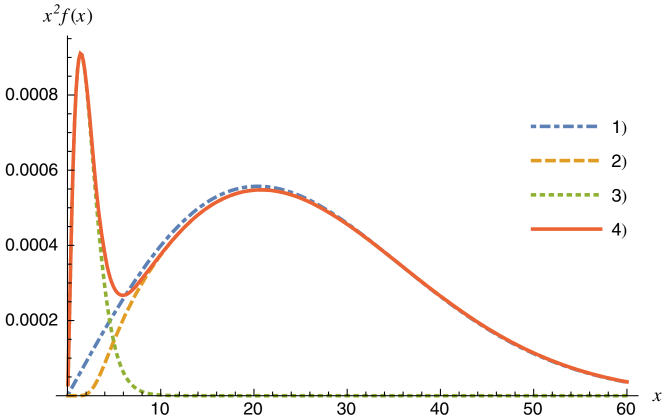

FIG. 2. DM spectra obtained (the same variable x as in Fig. 1): 1) by exploiting the approximate formula (18); 2) by integrating the equations numerically, through inserting (4) into (3); 3) from decays of a particle while it is still in thermal equilibrium (here it was assumed to freeze out at $T \sim M/7.5$), see e.g. [13]; 4) by summing contributions (2) and (3), total answer, see e.g. [15]. We choose $\Lambda = 3 \times 10^{-4}$, which obeys (22).

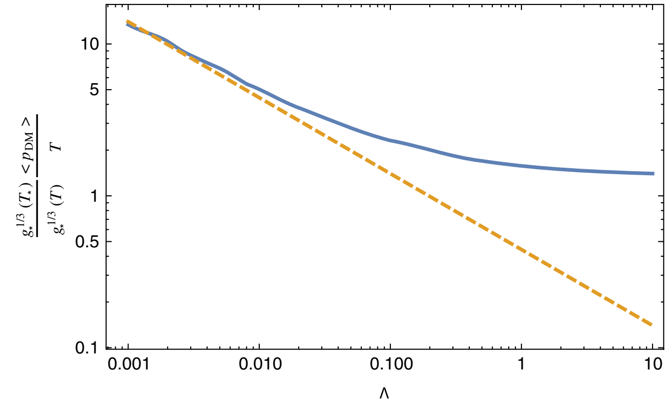

FIG. 3. Average velocity of the DM calculated with formula (23) derived for the approximated spectrum (18) (dashed line); numerical integration with spectrum obtained through inserting (4) into (3) (solid curve).

the dark matter may be obtained for decays of DDM which just start to be nonrelativistic. To grasp this part of the spectrum the exact solution of (3) is required instead of the approximation (18), see [15] for a detailed solution. An even more interesting situation happens if the DDM particle stays in thermal equilibrium long enough, and decays (partially) into the DM while still in thermal equilibrium with the rest of the Universe, as analyzed in [13,14]. This situation corresponds to using the purely thermal DDM distribution instead of (4) for early production times [or, equivalently, restoring the full collision integral in the rhs of (2), including all production and destruction terms], and the relevant contribution to the DDM can be read of [14]. Then, an additional peak in the spectrum arises at low momenta, leading to an interesting two-component DDM spectrum with very pronounced two maxima, see Fig. 2. This situation was also analyzed in detail numerically in [19] for the case of DDM decaying only to DM particles.

The average velocity for different spectra can differ significantly. As we show below, it takes place even in the one-component case of Fig. 1. For the approximate solution (18) the average momentum corresponds to $p/T \simeq (g_*(T)/g_*(T_*))^{1/3}/\sqrt{\Lambda}$ and should always be much larger than one, if the applicability condition (22) is satisfied. Therefore, the relative contribution of low momenta $p/T \lesssim (g_*(T)/g_*(T_*))^{1/3}$ is small, and one can neglect the bound (21) and obtain the approximation for the average momentum using the distribution (18):

$$\frac{\langle p_{\rm DM}\rangle}{T} = \frac{\sqrt{\pi}}{2\sqrt{\Lambda}}\left(\frac{g_*(T)}{g_*(T_*)}\right)^{1/3} = \frac{\sqrt{\pi\tau}M}{\sqrt{2M_{\rm Pl}^*}}\left(\frac{g_*(T)}{g_*(T_*)}\right)^{1/3}, \qquad (23)$$

where presently $g_*(T = T_0) = 43/11 \approx 3.9$ (and not 3.36 as in [15,20–23]). This value must be compared to the exact numerical estimate obtained by averaging over the spectra derived by inserting (4) into (3). Both results depend on the parameter Λ, and coincide when it is sufficiently small, $\Lambda \ll 0.05$, so that the approximation (22) is valid. It happens when the model parameters, DDM mass M and lifetime τ, obey the inequality

$$1 \ll \left(\frac{M}{1\ \rm TeV}\right)^2 \left(\frac{\tau}{4\times 10^{-12}\ \rm s}\right) \approx \frac{0.05}{\Lambda}.$$

When Λ increases, the numerical estimate reaches the finite asymptote $\langle p_{\rm DM}\rangle/T \sim 1.3 \times (g_*(T)/g_*(T_*))^{1/3}$, while the approximate formula (18) yields steadily decreasing to zero average velocity (23), which would imply colder and colder DM, see Fig. 3. Therefore, the decay of the DDM particles after they leave thermal equilibrium can lead to DM slightly colder than a thermally produced one with $\langle p\rangle/T \sim 3.15 \times (g_*(T)/g_*(T_*))^{1/3}$.

To summarize we conclude that the process of nonthermal generation of the DM in decays of another particle has many nontrivial features, and should be approached with care. The situations that can be tracked analytically are the case of in-equilibrium decay, leading to the colder component of the DM, and decay of relatively long-lived particles that decay out of equilibrium when they became nonrelativistic, leading to a relatively hot DM component. Intermediate situations of short DDM lifetime should be analyzed exactly numerically, as in [15,18,19].

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