**Erratum: Soft gluon emission process in the color-octet
model for heavy quarkonium production
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There is a typographical error in Eq. (76) of Ref. [1] which gives the Gamow factor $K_L(\eta)$ for the Coulomb interaction $V(r) = \alpha/r$, for a state with an angular momentum L . We would like to write down the correct expression and give the derivation of the Gamow factor in Eq. (12) below.

The regular Coulomb wave function for the Coulomb potential $V(r) = \alpha/r$, as given by Eq. (14.1.3) of Ref. [2], is

$$u_L(r) = F_L(\eta, \rho) = C_L(\eta) \rho^{L+1} e^{-i\rho} M(L+1-i\eta, 2L+2, 2i\rho), \quad (1)$$

where $\eta = \alpha/v$ and $\rho = kr$. We would like to represent the above wave function by the amplitude $A_L(r)$ and phase shift $\delta_L(r)$ in terms of free-potential Riccati-Bessel wave functions $\hat{j}_L(kr)$ and $\hat{n}_L(kr)$ as in Calogero [3],

$$u_L(r) = \frac{A_L(r)}{A_L(\infty)} [\cos \delta_L(r) \hat{j}_L(kr) - \sin \delta_L(r) \hat{n}_L(kr)], \quad (2)$$

where

$$\hat{j}_L(\rho) = \left(\frac{\pi\rho}{2}\right)^{1/2} J_{L+1/2}(\rho), \quad (3)$$

$$\hat{n}_L(\rho) = (-1)^{L+1} \left(\frac{\pi\rho}{2}\right)^{1/2} J_{-(L+1/2)}(\rho). \quad (4)$$

The functions $A_L(r)$ and $\delta_L(r)$ need to be solved as a function of r as formulated by Calogero [1,3,4], starting with $\delta_L(0) = 0$, to yield the proper phase shift at $r \rightarrow \infty$ appropriate for the Coulomb potential.

The Gamow factor (or K factor) expresses the effects of the Coulomb interaction and is defined as the ratio of the absolute square of the amplitude at the origin and at infinity [1,4],

$$K_L = \left| \frac{A_L(0)}{A_L(\infty)} \right|^2. \quad (5)$$

We can get K_L by matching the wave functions of Eqs. (1) and (2). Near the origin with $\delta_L(0) = 0$, Eq. (2) gives

$$\lim_{r \rightarrow 0} u_L(r) = \frac{A_L(0)}{A_L(\infty)} \hat{j}_L(\rho) = \frac{A_L(0)}{A_L(\infty)} \frac{\rho^{L+1}}{(2L+1)!!}. \quad (6)$$

On the other hand, in Eq. (1) as $M(r \rightarrow 0) = 1$, the wave function u_L near the origin is

$$\lim_{r \rightarrow 0} u_L(r) = C_L(\eta) \rho^{L+1}. \quad (7)$$

Matching Eq. (6) with Eq. (7), we obtain

$$\frac{A(0)}{A(\infty)} = (2L+1)!! C_L(\eta). \quad (8)$$

Therefore, the Gamow factor is related to $C_L(\eta)$ by

$$K_L(\eta) = |(2L+1)!!C_L(\eta)|^2. \quad (9)$$

From Eq. (14.1.7) of [2], the Gamow factor for $L = 0$ is

$$K_0(\eta) = |C_0(\eta)|^2 = \frac{2\pi\eta}{\exp\{2\pi\eta\} - 1}. \quad (10)$$

The coefficient $C_L(\eta)$ satisfies the recurrence relation [2],

$$C_L(\eta) = \frac{(L + \eta^2)^{1/2}}{L(2L+1)} C_{L-1}(\eta), \quad (11)$$

and thus

$$|C_L(\eta)|^2 = \frac{(L^2 + \eta^2)[(L-1)^2 + \eta^2] \dots (1 + \eta^2)}{[L!(2L+1)!!]^2} \frac{2\pi\eta}{\exp\{2\pi\eta\} - 1}.$$

The Gamow factor for the Coulomb interaction $V(r) = \alpha/r$, for a state with angular momentum $L \geq 1$, is

$$K_L(\eta) = \frac{(L^2 + \eta^2)[(L-1)^2 + \eta^2] \dots (1 + \eta^2)}{[L!]^2} \frac{2\pi\eta}{\exp\{2\pi\eta\} - 1}. \quad (12)$$

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