Erratum: Studying the precision of ray tracing techniques with Szekeres models [Phys. Rev. D **92**, 023532 (2015)]

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This erratum serves to give corrections of two errors made in Koksang and Hannestad [Phys. Rev. D, **92**, 023532 (2015)]. One error consists of having used the expression for the Doppler convergence for a flat background to study the convergence on curved backgrounds. The other error which was made, is a typo in the numerical code used to study the convergence in onion models with curved backgrounds. After correcting this typo, the results of Sec. VI A in Koksang and Hannestad [Phys. Rev. D, **92**, 023532 (2015)] were recomputed. Contrary to the original results, the new results show that the ray-tracing scheme studied in Koksang and Hannestad [Phys. Rev. D, **92**, 023532 (2015)] can reproduce the exact results in LTB onion models very well. The corrections and new results are described more elaborately below.

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I. RECOMPUTED RESULTS FOR SEC. VI A

The main result of Sec. VI A was that ray tracing through onion models leads to significantly less accurate results than ray tracing through swiss cheese models. In order to draw this conclusion, the mapping between spacetimes and the ray-tracing scheme were adapted to curved backgrounds so that the ray tracing could be done on different Friedmann-Lemaître-Robertson-Walker (FLRW) backgrounds. An important point with this adaption is that the age of the background and the exact model should be the same. As explained in Appendix C of Ref. [1], this requires that the Hubble constant is changed appropriately when the curvature of the background is changed. Due to a typo, this was not done consistently throughout the code used to study onion models. The results obtained after correcting this typo are presented below.

FIG. 1 (color online). Density along exact and ray-traced radial rays in the onion model $O1_{lb}$ of Ref. [1]. The ray tracing has been done on different backgrounds. The average density of model $O1_{lb}$ along the ray corresponds approximately to $\Omega_{m,0} = 0.8$. (a) Density along radial ray in model $O1_{lb}$ with $\Omega_{m,0} = 0.65$, (b) Density along radial ray in model $O1_{lb}$ with $\Omega_{m,0} = 0.8$, (c) Density along radial ray in model $O1_{lb}$ with $\Omega_{m,0} = 0.83$, and (d) Density along radial ray in model $O1_{lb}$ with $\Omega_{m,0} = 0.95$.

FIG. 2 (color online). Convergence along exact and ray-traced radial rays in the onion model $O1_{\text{ltb}}$ of Ref. [1]. The exact convergence is shown together with the sum of the Doppler and gravitational convergences. The gravitational convergence is also shown alone. The average density of model $O1_{\text{ltb}}$ along the ray corresponds approximately to $\Omega_{m,0} = 0.8$. (a) Convergence, model $O1_{\text{ltb}}$ with $\Omega_{m,0} = 0.65$, (b) Convergence, model $O1_{\text{ltb}}$ with $\Omega_{m,0} = 0.8$, (c) Convergence, model $O1_{\text{ltb}}$ with $\Omega_{m,0} = 0.83$, and (d) Convergence, model $O1_{\text{ltb}}$ with $\Omega_{m,0} = 0.95$.

Figure 1 shows the density along a single, radial ray in the model $O1_{\text{ltb}}$. The density as a function of the redshift according to the exact model is compared to the same relation obtained using the ray-tracing scheme with different backgrounds. Changing the background induces a change in the peculiar velocity field. This velocity field is used when computing the observable redshift. Hence, changing the velocity field leads to shifts in the density field along the rays when plotted against the redshift. As seen in Fig. 1, the effect is quite small, at least as long as there are only small or moderate changes in the curvature.

In Fig. 2, the convergence along the radial ray is shown. As seen, the accuracy of the reproduction of the convergence depends on the background. The best reproduction is obtained when using a background defined by a present time density parameter $\Omega_{m,0} := \frac{\rho(t_0)}{\rho_{\text{crit},0}} \approx 0.83$. The average density along the ray corresponds to $\Omega_{m,0} \approx 0.8$. The reproduction is not quite as good as along rays in swiss cheese and single void models. It seems likely that this is simply a result of the density fluctuations in the onion model oscillating about a “mean” that changes in the radial direction. In particular, the mean starts below, but converges towards, the critical density. This feature is not particularly realistic and it seems likely that the reproduction of the convergence and redshift would be even better in more realistic models or if the studies of the model used here were restricted to redshift intervals where the oscillations occur approximately about the same mean. For instance, Fig. 1 shows that at redshifts below approximately 0.15, the change in this mean is very small. In accordance with this, the convergence in the same redshift interval is well described by ray tracing with a constant background corresponding to $\Omega_{m,0} = 0.8$.

II. ERROR IN THE EXPRESSION FOR THE DOPPLER CONVERGENCE

The expression for the Doppler convergence used in Ref. [1] is only valid for flat FLRW backgrounds. As explained below, this has not had any significant consequences for the results obtained when ray tracing on curved backgrounds. Nonetheless, the correct expression for the Doppler convergence on a curved background is given below for completeness.

A general FLRW line element can be written as

$$ds^2 = -c^2 dt^2 + a^2 \left(\frac{dr^2}{1 - Kr^2/R_0^2} + \frac{r^2}{E^2} (dp^2 + dq^2) \right). \quad (1)$$

$K \in \{-1, 0, 1\}$ gives the sign of the spatial curvature while $R_0 = \frac{c}{H_0 \sqrt{|\Omega_{m,0}-1|}} = \frac{c}{H_0 \sqrt{|\Omega_{k,0}|}}$ is the curvature radius.

The function $\tilde{E}$ is given by $\tilde{E} = \frac{1}{2}(p^2 + q^2 + 1)$.

For the observer placed at the origin and with the definition $r = r(z_{\text{obs}})$, the expression for the Doppler convergence can be written as

$$\kappa_v c = \left(1 - \frac{c}{a_{,t} r} \sqrt{1 - Kr^2/R_0^2}\right) \mathbf{n} \cdot (\mathbf{v} - \mathbf{v}_0) + \mathbf{n} \cdot \mathbf{v}_0. \quad (2)$$

The factor $\sqrt{1 - Kr^2/R_0^2}$ is often written as $\cosh(\sqrt{\Omega_{k,0}} d_c/d_H)$, where $d_c = c \int_0^z \frac{dz'}{H(z')}$ and $d_H = \frac{c}{H_0}$.

In Ref. [1], the factor $\sqrt{1 - Kr^2/R_0^2}$ was not included. Including the factor has no significant effects on the results since $r \ll R_0$ in the studies of Ref. [1].

III. SUMMARY

The main point with the onion model results shown in Sec. VI A of Ref. [1] was that they indicated that there could be issues regarding the use of ray-tracing techniques in spacetimes that do not have FLRW backgrounds explicitly visible in the models' density fields in the form of homogeneous density patches. This result was due to an error in the numerical code. The results presented here show that using a background corresponding roughly to the average density along a light ray in an LTB onion model leads to a good reproduction of the exact results. The reproductions are not quite as precise as in the case of swiss cheese models. The reproductions are accurate and precise enough for it to seem unlikely that the lack of an explicit background has any significant consequences for using perturbation theory to interpret real observations and to study ray tracing through N -body simulations.

The overall conclusion of Ref. [1] is thus that even the simple ray-tracing scheme studied in Ref. [1] yields quite accurate and precise reproductions of the exact redshift and convergence in Szekeres swiss cheese and onion models. The only caveat is that the good reproduction requires that the ray-traced rays move along spacetime paths equivalent to the exact paths. This is not generally the case which may compromise results, especially if they involve either 1) nonlocal observables such as the gravitational convergence which is computed as an integral along ray paths, or 2) observables which are studied on maps of the sky, such as maps of cosmic microwave background temperature fluctuations.

- [1] S. M. Koksang and S. Hannestad, Studying the precision of ray tracing techniques with Szekeres models, [Phys. Rev. D **92**, 023532 \(2015\)](#).