**Erratum: Bottomonium hyperfine splittings from lattice nonrelativistic
QCD including radiative and relativistic corrections
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A numerical error in two previous theoretical papers [1,2] means that the coefficients d_1 and d_2 of four-quark operator terms used in this paper are changed. The details of this error are given in the corresponding errata to [1,2]. For clarity we reproduce here the whole of the Table I, but only the entries in columns 7 and 8 are changed, giving respectively the coefficients $d_1\alpha_s^2$ and $d_2\alpha_s^2$.

TABLE I. Parameters used for the valence quarks. am_b is the bare b quark mass in lattice units, u_{0L} is the Landau link. The c_i, d_i are coefficients of terms in the NRQCD action [see Eq. (1)]. c_4, d_1 and d_2 use α_s in the V scheme at scale π/a . The other coefficients use different scales as discussed in Ref. [21].

Set	am_b	u_{0L}	c_1, c_6	c_5	c_4	$d_1\alpha_s^2$	$d_2\alpha_s^2$	$\alpha_s(\pi/a)$
1	3.31	0.8195	1.36	1.21	1.23	-0.1021	0.0306	0.275
2	2.73	0.834	1.31	1.16	1.19	-0.058	0.016	0.255
3	1.95	0.8525	1.21	1.12	1.18	-0.026	0.006	0.225

TABLE III. Fit results in lattice units for the $1S$ and $2S$ energies of the Υ and η_b for the $\mathcal{O}(v^6)$ action with and without four-quark interactions.

Set	$aE_\Upsilon(1S)$	$aE_\Upsilon(2S)$	$aE_{\eta_b}(1S)$	$aE_{\eta_b}(2S)$
$\mathcal{O}(v^6)$ action				
1	0.27547(6)	0.6917(14)	0.22950(5)	0.6748(12)
2	0.28786(3)	0.6332(9)	0.25148(3)	0.6180(6)
3	0.30269(3)	0.5573(10)	0.27484(2)	0.5461(9)
$\mathcal{O}(v^6)$ action + four-quark interactions				
1	0.27699(6)	0.6997(13)	0.22460(5)	0.6755(13)
2	0.28850(3)	0.6340(10)	0.24897(3)	0.6166(9)
3	0.30271(2)	0.5564(6)	0.27378(2)	0.5437(5)

We have redone the calculation for the bottomonium hyperfine splitting using the correct coefficients and we present the revised results in Table III for the $1S$ and $2S$ energies of the Υ and η_b in the case where the four-quark interactions are included. We reproduce the whole of the revised Table III for clarity but only the last three rows are changed. We also give here the revised figures for Figs. 1–3. Table IV gives the revised error budget. This has changed very little, but a typographical error in the row marked “EM effects” is corrected.

In the abstract the results given in the last two sentences should read, respectively, $M_\Upsilon(1S) - M_{\eta_b}(1S) = 60.0(6.4)$ MeV and $(M_\Upsilon(2S) - M_{\eta_b}(2S))/(M_\Upsilon(1S) - M_{\eta_b}(1S)) = 0.445(28)$.

On page 1, second column, the sentence starting “Including the ...” should read “Including the corrections gave a hyperfine splitting of 65(9) MeV where ...”.¹ In the text below Eq. (4) the sentence beginning “We take Gaussian priors”

¹The hyperfine splitting using the uncorrected values of d_1, d_2 was found to be 70(9) MeV in [21]. We estimate the correction to the hyperfine splitting using a potential model analysis, as done in [21], but using the corrected values of these coefficients. We find the corrected hyperfine splitting to be 65(9) MeV, and it is this value that is plotted in Fig. 3.

FIG. 1 (color online). The fit result for the 1S hyperfine splitting from the v^6 action with spin-dependent four-quark operators. The two error bars are as follows: the smaller band includes statistics and scale setting, and the larger band includes correlated errors from missing radiative corrections to c_4, d_1, d_2 and quark mass tuning errors. The grey band is our final result, including all systematic errors. Also shown are the pure v^6 results, which are not included in the fit, and the PDG average [2].

FIG. 2 (color online). The fit result for the hyperfine ratio from the v^6 action with spin-dependent four-quark operators. Also shown are the pure v^6 results, which are not included in the fit, and the results from Belle and Dobbs *et al.* [5, 6].

should read “We take Gaussian priors of mean 0 and width 1 on all the coefficients in the fit to the 1S hyperfine splitting. We tighten the prior on k_1 to 0.0(3) in fitting R_H since the action includes radiatively improved a^2 lattice spacing corrections and we also expect cancellation of discretization errors in the ratio.”

Equation (5) becomes

$$\begin{aligned}\Delta_{\text{hyp}}(1S) &= 60.0(6.4) \text{ MeV} \\ \Delta_{\text{hyp}}(2S)/\Delta_{\text{hyp}}(1S) &= 0.445(28).\end{aligned}\tag{5}$$

On page 5, in Sec. IV, the sentence starting “Our result for the 1S splitting of ...” should be removed. Equation (6) should read

$$\Delta_{\text{hyp}}(2S) = 27.7(1.7)_{\text{latt}}(1.4)_{\text{exp}} \text{ MeV}.\tag{6}$$

FIG. 3 (color online). Comparison of our result (top) with other lattice calculations [18, 20, 21, 23, 35] and experimental [1–6] results for the hyperfine splittings. The $2S$ hyperfine splitting is on the left and the $1S$ on the right. The $1S$ value plotted for the earlier $\mathcal{O}(\alpha_s v^4)$ HPQCD result has been corrected for the d_1, d_2 error and the $2S$ value uses the 2013 PDG average for $\Delta_{\text{hyp}}(1S)$ to extract $\Delta_{\text{hyp}}(2S)$ from the ratio given in that paper. Meinel quotes two results for $\Delta_{\text{hyp}}(2S)$, normalizing the result using either the $1S$ hyperfine or the $1P$ tensor splitting. The results are taken as given in Ref. [23].

TABLE IV. Full error budget for the $1S$ hyperfine splitting and the $2S$ to $1S$ ratio. All errors are in percent.

Error %	$\Delta_{\text{hyp}}(1S)$	R_H
Stats/fitting	0.2	4.1
Uncertainty in a	2.2	0.0
Scale dependence	1.3	2.5
NRQCD am_b dependence	3.6	< 0.0
NRQCD radiative $\alpha_s v^6$	3.7	0.0
NRQCD radiative $\alpha_s^2 v^4$ in c_4	7.1	< 0.1
Statistical error in $c_4^{(1)}$	3.2	1.4
NRQCD relativistic spin v^8	1.0	0.5
NRQCD radiative four-quark $\alpha_s^3 v^3$	3.0	1.3
m_b tuning	0.7	< 0.1
$m_{1,\text{sea}}$ dependence	< 0.1	3.5
EM effects	0.3	< 0.1
Total	10.8	6.3

[1] T. C. Hammant, A. G. Hart, G. M. von Hippel, R. R. Horgan, and C. J. Monahan, *Phys. Rev. Lett.* **107**, 112002 (2011).

[2] T. C. Hammant, A. G. Hart, G. M. von Hippel, R. R. Horgan, and C. J. Monahan, *Phys. Rev. D* **88**, 014505 (2013).