Bajc-Melfo vacua enable Yukawon ultraminimal grand unified theories

Charanjit S. Aulakh*

*Indian Institute of Science Education and Research Mohali,
Sector 81, S. A. S. Nagar, Manauli 140306, India
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Bajc-Melfo (BM) two-field (S, ϕ) superpotentials define metastable F-term supersymmetry-breaking vacua suitable as hidden sectors for calculable and realistic family and grand unification models. The undetermined vacuum expectation value (VEV) $\langle S_s \rangle$ of the Polonyi field that breaks supersymmetry can be fixed either by coupling to $N = 1$ supergravity or by radiative corrections. BM hidden sectors extend to symmetric multiplets $(S, \phi)_{ab}$ of a gauged $O(N_g)$ family symmetry, broken at the GUT scale, so that the $O(N_g)$ charged component VEVs $\langle \hat{S}_{ab} \rangle$ are also undetermined before accounting for the $O(N_g)$ D-terms, which fix them by cancellation against D-term contributions from the visible sector. This facilitates Yukawon ultraminimal GUTs (YUMGUTs) proposed in [C. S. Aulakh and C. K. Khosa, Phys. Rev. D **90**, 045008 (2014)] by relieving the visible sector from the need to give null D-terms for the family symmetry $O(N_g)$. We analyze symmetry breaking and spectra of the hidden-sector fields in the supergravity resolved case when $N_g = 1, 2, 3$. Besides the Polonyi field S_s , most of the superfields $\hat{S}_{ab}$ remain light, with fermions getting masses only from loop corrections. Such modes may yield novel dark matter lighter than 100 GeV. Possible Polonyi and moduli problems associated with the fields S_{ab} call for detailed investigation of loop effects due to the Yukawa and gauge interactions in the hidden sector and of postinflationary field relaxation dynamics.

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I. INTRODUCTION

Supersymmetry (SUSY) imposes remarkable restrictions on its own spontaneous violation and thus severely constrains models of physics beyond the standard model. Traditionally [1–5] gravity mediation has been used to generate phenomenologically acceptable soft SUSY-breaking terms for globally supersymmetric GUTs. This requires supersymmetry breaking in a hidden sector with no superpotential couplings to the “visible” sector in which we live. The GUT Minkowski space vacuum is metastable [4] in the sense of there being a SUSY-preserving vacuum with lower (negative) energy, but it is thought that it cannot decay to the lower vacuum by tunneling, or at any rate the time scale for doing so is much larger than the age of the Universe. The vacuum in the gravity mediated scenario is defined as the perturbation of a global SUSY-preserving minimum by a hidden sector with a *global* minimum which breaks supersymmetry spontaneously due to specialized superpotentials (Polonyi, O’Raifeartaigh [6,7], etc.) or due to strong dynamics (gaugino condensation, etc.). After long struggles to tame the hard problem of finding

phenomenologically usable SUSY-breaking global minima, it was reemphasized [8,9] that metastable, i.e. local, minima could serve just as well as global minima, since the unifying theory would anyway have vacua at lower energy [4] than the phenomenologically relevant Minkowski vacuum on which the MSSM lived. This realization was liberating for phenomenologists, because models defined on supersymmetry-breaking vacua which are only local minima offer a much larger range of candidate vacua. Various strongly coupled supersymmetric gauge theories [9,10] were studied to provide the SUSY-breaking dynamics, albeit now considering also metastable vacua. Unfortunately, even the metastable dynamical SUSY-breaking models seem so arbitrarily elaborate as to rival the complexity of the phenomenological theory (GUT, MSSM, etc.) for which they were supposed to provide the service of supersymmetry breaking, making testability of both a moot issue.

Recently Bajc and Melfo (BM) [11] suggested that, from a “calculable unification” viewpoint, it would be more interesting to consider SUSY-breaking metastable vacua of simple two-field hidden-sector superpotentials. BM models are related to the classic O’Raifeartaigh SUSY-breaking superpotentials and are variants of a class of models with SUSY-breaking local minima studied systematically earlier in Ref. [12]. Such superpotentials typically have flat directions, and there is a longstanding vision whereby the determination of such flat directions (running out of SUSY-breaking minima) by radiative corrections [13] or

* aulakh@pu.ac.in; aulakh@iisermohali.ac.in; <http://14.139.227.202/Faculty/aulakh/>

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supergravity [14,15] might profitably fix the undetermined vacuum expectation values (VEVs) at a high unification scale. Bajc and Melfo took up this challenge and formulated models where the undetermined VEVs were those of **24** or **75** irreps of an $SU(5)$ unified theory and the flat directions were determined by radiative corrections. The use of $N = 1$ supergravity potentials to lift the flat directions rolling out of the Bajc-Melfo SUSY-breaking local minimum has not been analyzed.

From a Grand Unified viewpoint, the only credible sign of flavor unification so far is third-family Yukawa unification at large $\tan\beta$ in $SO(10)$ at scales of order $M_X > 10^{16}$ GeV. On the other hand, minimal supersymmetric (MS) $SO(10)$ GUTs [16–23] have been shown to be completely realistic and even to suppress $d = 5$ proton decay [24,25] by a novel and generic mechanism based on the necessary emergence of a pair of light doublet Higgs in the effective MSSM. Motivated by these two clues, we exploited the defining (i.e. **126**) representation of MSGUTs (which gets large VEVs yet couples to matter fermions) to design a *renormalizable* Grand Unified Yukawa/familion model [26] in which we generate hierarchical matter fermion Yukawas from flavor-bland parameters of the MSGUT extended by an $O(N_g)$ family symmetry. Family gauge symmetry is broken at the unification scale, and the visible-sector GUT Higgs fields effloresce into symmetric (**210**, **126**, **126**, **10**) or antisymmetric (**120**) irreps of the family group [as dictated by the properties of bilinears in the **16**-plet matter irreps in $SO(10)$ GUTs], while the matter **16**-plet Yukawa couplings become flavor bland. The neat and exactly soluble [18] gauge symmetry breaking [$SO(10) \rightarrow \text{MSSM}$] of the family singlet MSGUT Higgs system then generalizes straightforwardly for the family symmetry variant MSGUT Higgs multiplets. The supersymmetric breaking of family symmetry will be discussed in detail in this paper. Extraction of the light MSSM Higgs from the plethora of MSSM doublets present in the $O(N_g) \times SO(10)$ GUT via calculation of the “Higgs fractions” [18,20,23,27] (which specify the makeup of the MSSM Higgs) determined by flavor-blind GUT parameters generates candidate flavor structures.

In implementing this flavor-generation scenario an obstacle arises. Although the solution of the F- and D-term conditions for the GUT multiplets in the $SO(10)$ sector with flavor nondiagonal VEVs is straightforward, it is very hard or impossible to ensure that the same configurations simultaneously cancel the family symmetry D-terms. It thus becomes necessary to introduce additional fields to soak up the contribution of the GUT VEVs to the $O(N_g)$ D-terms. Since minimality, simplicity and solubility of the GUT Higgs sector symmetry breaking is a central virtue of MSGUTs, it is natural to ask whether the additional fields might not be located in a hidden sector with its own flat directions arising from supersymmetry breaking in a way which did not interfere with the already

accomplished MSGUT symmetry breaking [16–18]. Besides making a completely unexpected connection between family symmetry and supersymmetry breaking, this idea is straightforward to implement in a context where the BM flat directions are lifted by a combination of supergravity and $O(N_g)$ gauge D-term minimization effects.

A notable phenomenological implication of our model is the presence of weakly coupled light fields (“moduli”). In particular, in the supergravity mediated scenario presented here, there is a Polonyi (scalar) field (Planck-scale VEV, weak-scale mass and intermediate-scale SUSY-breaking F-term). In addition, there are other light fermions and scalars associated with the flat directions of the family nonsinglet BM fields. These modes may be interesting light (< 50 GeV) SUSY WIMP dark matter of the sort perhaps indicated by the DAMA/LIBRA experiment [28] and so far completely missing from MSGUT spectra. On the other hand, this clutch of modes may also pose the typical problems associated with “moduli” fields found in other fundamental theories [29]. Recall that the Polonyi problem arises due to late decay of the oscillations of the supersymmetry-breaking Polonyi field while it settles into the zero-temperature potential’s minimum because of the suppression of its couplings by the Planck scale. The dark matter candidates’ role in cosmogony also needs to be checked for consistency. In view of the new Yukawa and gauge interactions of the hidden sector, the cosmological implications require a separate detailed study and thus should not prematurely prejudice the novel flavor-generation aspects of our model. In this paper we give the relevant details for the symmetry breaking, VEVs and masses in the BM sector for direct use in our “Yukawonification” [26] models. We will return to a study of cosmological constraints and alternative models elsewhere.

II. THE BAJC MELFO VACUA AND SUPERGRAVITY

The Bajc-Melfo superpotential [11] is a quadratic variant of the O’Raifeartaigh superpotential and its relatives [7,12] which breaks supersymmetry at a *metastable* minimum

$$W_{\text{BM}}(S_s, \phi_s) = W_0 + S_s(\mu_B \phi_s + \lambda_B \phi_s^2). \quad (1)$$

We have added a constant term W_0 for later convenience. It is then easy to see [11,12] that besides the SUSY-preserving global minima at $S_s = \phi_s = 0$ and $S_s = 0, \phi_s = -\mu_B/\lambda_B$ with zero vacuum energy, there is also a local minimum at $\langle \phi_s \rangle = -\mu_B/2\lambda_B$ with $\langle S_s \rangle$ undetermined provided

$$|\langle S_s \rangle| \geq \left| \frac{\langle \phi_s \rangle}{\sqrt{2}} \right|. \quad (2)$$

This local minimum breaks supersymmetry, since $\langle F_S \rangle = -\mu_B^2/4\lambda_B \equiv \theta$. Thus, the chiral fermion in the multiplet S_s provides the Goldstino field, while its partner scalar, whose VEV remains undetermined, is massless. When supersymmetry is made local by coupling the theory to gravity, the Goldstino can be gauged away by a local supersymmetry transformation, leaving a massive gravitino while the scalar picks up the gravitino mass common to light scalars in gravity mediation. For clarity we first review the effect of coupling to supergravity on extrema of globally supersymmetric Lagrangians [4].

The generic $N = 1$ supergravity potential [1,3,5] for scalar fields z_I interacting through a superpotential $W(z_I)$ [with canonical Kahler potential $K(z, \bar{z})$ and gauge kinetic function $f_{\alpha\beta}(z) = \delta_{\alpha\beta}$, κ is the Planck length] is

$$V = E \left(\sum_I |\mathcal{F}^I|^2 - 3\kappa^2 |W|^2 \right) + \sum_\alpha \frac{g_\alpha^2}{4} \left[\frac{(\mathcal{F}^I (T^\alpha)_I^J z_J)^2}{\kappa^4 W^2} + \text{H.c.} \right],$$

$$\mathcal{F}^I = \frac{\partial W}{\partial z_I} + z^{*I} \kappa^2 W; \quad E \equiv e^{\kappa^2 \sum_I |z_I|^2}. \quad (3)$$

As is well known, the constant term in the superpotential W , which is irrelevant in global supersymmetry, can be used to tune the vacuum energy in *one* of its extrema to zero after coupling to supergravity. This means that for a supersymmetric minimum with VEVs $\langle z_I \rangle = \bar{z}_I$ derived from

$$\langle F^I \rangle = \langle \partial W / \partial z_I \rangle = 0; \quad \langle D^\alpha \rangle = \langle z^{*I} T_I^{\alpha J} z_J \rangle = 0 \quad (4)$$

there is no shift in the global VEVs $\langle \bar{z}_I \rangle$ at all: since by tuning $\langle W \rangle = \bar{W} = 0$, we ensure that the VEV $\bar{\mathcal{F}}^I = \bar{F}^I = 0$, the vacuum energy at the old minimum vanishes, and so do terms in the extremization condition which come from the variations of the fields in the exponential factor (since they multiply a separately vanishing factor). It is, however, amusing that the D-term VEV in (3) above is then indeterminate, seemingly indicating an incompatibility between supergravity and preserved supersymmetry with $\langle W \rangle = 0$.

On the other hand, if the global superpotential breaks supersymmetry spontaneously (as, for example, the Polonyi or O’Raifeartaigh superpotentials at a global minimum or the BM superpotential at a local minimum), then the nonvanishing of F at the minimum means that the total superpotential VEV must be nonzero to cancel the global contribution to the vacuum energy in Eq. (3). The VEVs of the supersymmetry-preserving visible-sector fields may also shift from their globally determined values by small terms of order $O(m_{3/2})$ and maintain zero value for their local F-terms. Special behavior is seen only for

the components of the sliding multiplet S which breaks supersymmetry.

Using the potential (3) in the pure BM case, we see that the field S_s whose value was *undetermined* at the local/metastable minimum of the global superpotential described above now picks up a potential which fixes its value to be of the order of the Planck scale. The other BM field ϕ_s , which already had a large VEV [think of it as $\bar{\phi}_s \sim 10^{12}$ GeV on phenomenological grounds in the context of the gravity-mediated (SUGRA) models we are developing], will only experience a tiny shift in its VEV. To leading order we can determine the VEV $\langle S_s \rangle \equiv \bar{S}_s$ by fixing ϕ_s at its globally determined value $\bar{\phi}_s = -\mu_B/2\lambda_B$. The potential to be minimized is then

$$V(S_s) = e^{\kappa^2 |S_s|^2 + \delta} \left\{ (\delta \kappa^2 |W_0 + S_s \theta|^2 + |\theta + \kappa^2 S_s^* (W_0 + S_s \theta)|^2 - 3\kappa^2 |W_0 + S_s \theta|^2) \right\}. \quad (5)$$

Here $\delta = \kappa^2 (|\bar{\phi}_s|^2)$ is a tiny background fixed *parameter* ($\sim 10^{-12}$) for the purposes of the extremization with respect to the globally undetermined VEV of S_s and has essentially no effect on the value of S_s at the extremum found. Defining dimensionless variables

$$x = \kappa \frac{\hat{W}_0}{\theta}; \quad y = \kappa S_s, \\ \varphi_x = \text{Arg}[x]; \quad \varphi_y = \text{Arg}[y], \quad (6)$$

the potential becomes

$$\tilde{V} \equiv \frac{V}{|\theta|^2} = \{ (|x|^2 + |y|^2)(\delta - 3) + (1 + |y|^2)^2 + |x|^2 |y|^2 + 2 \cos(\varphi_y - \varphi_x) |x| |y| (|y|^2 + \delta - 2) \}. \quad (7)$$

We emphasize that x (which is the dimensionless form of the constant term in the superpotential) is not a dynamical variable but a parameter to be tuned to maintain the Minkowski vacuum energy at zero. Similarly, δ is a parameter fixed by the globally determined background to the S_s extremization at a lower order of the expansion in powers of κ and not a variable.

The only stable minimum of (7) with respect to y (i.e. S_s) with zero energy (i.e., for the following, where field subscripts denote partial derivatives):

$$V = V_{|y|} = V_{\theta_y} = 0 = V_{|y|, \theta_y}; \quad V_{|y|, |y|}, V_{\theta_y, \theta_y} > 0 \quad (8)$$

is achieved as

$$\varphi_y = \varphi_x; \quad x = 2 - \sqrt{3 - \delta}; \quad y = y_0 = \sqrt{3 - \delta} - 1, \\ \frac{\partial_{|y|} \partial_{|y|} \tilde{V}}{\partial_{\varphi_y} \partial_{\varphi_y} \tilde{V}} = 4\sqrt{3 - \delta}; \\ \frac{\partial_{\varphi_y} \partial_{\varphi_y} \tilde{V}}{\partial_{\varphi_y} \partial_{\varphi_y} \tilde{V}} = 4\delta\sqrt{3 - \delta} - 16\delta - 32\sqrt{3 - \delta} + 56. \quad (9)$$

It is clear that the condition for a local minimum (2) is satisfied for

$$\delta < 3 - \left(1 + \sqrt{\left|\frac{\kappa^2 \theta}{2\lambda_B}\right|}\right)^2 \simeq 2. \quad (10)$$

As mentioned, δ is tiny, and this condition is trivially satisfied. Thus, we have fixed the leading contribution $\tilde{S}_s$ at the Planck scale by the combination of a gravitino-scale ($m_{3/2} = \kappa^2 \langle W \rangle$) mass and a tiny $[\sim (m_{3/2}/M_{\text{Planck}})^2]$ quartic coupling for the sliding field S_s . The leading-order VEVs $\tilde{\phi}_s, \tilde{S}_s$ specify a Minkowski vacuum, with the degeneracy in S_s lifted, to leading order in $\kappa \tilde{\phi}_s \sim 10^{-13}$. There is no immediate reason to compute next-order shifts by perturbing around these VEVs, but it is straightforward to do so iteratively.

We now wish to build upon these complementary simple cases to first analyze a model in which a hidden-sector BM superpotential $W_{\text{BM}}(S_s, \phi_s) = W_H$ with gauge singlets is coupled to a visible (GUT) sector (with fields z_i) which has a globally supersymmetric minimum associated with superpotential $W_O(z_i)$ and is invariant under a grand unifying gauge group broken spontaneously by the VEVs $\tilde{z}_i$ at a superheavy scale ($M_G \sim 10^{16}$ GeV) while preserving SUSY [20,23,26]. There is no coupling between the fields of the two sectors in the superpotential, so their global extrema and the nondetermination of $\tilde{S}_s$ are unchanged. It is eminently reasonable to pursue the determination of the leading correction by setting fields ϕ, z_i already determined at the global level to those values ($\tilde{\phi}, \tilde{z}_i$) and to determine the undetermined field (i.e. S_s) by minimizing (3) with respect to S_s , tuning W_0 to maintain zero energy at the vacuum. In this case, the entire discussion goes through verbatim with two superficial modifications:

- (1) The constant term in the superpotential now also receives a contribution from the VEV of the visible sector, i.e. $W_O(\tilde{z}_i)$.
- (2) The tiny background-determined parameter δ now also contains the visible-sector values

$$\delta = \kappa^2 \left(|\tilde{\phi}_s|^2 + \sum_i |\tilde{z}_i|^2 \right). \quad (11)$$

Thus, the determination of S_s is unchanged. The value of δ is still very small, since $M_{\text{GUT}} \ll M_{\text{Planck}}$, $\delta < 10^{-6}$. Condition (10) is still obviously trivially satisfied.

Finally, we arrive at the novel construction proposed in Ref. [26], where a (family) gauge symmetry links the BM hidden sector to the, via D terms, visible sector even at the global level, but there is still no link at the level of the superpotential. As we will see in the following sections, besides the field S_s , also the gauge nonsinglet parts $\hat{S}$ of the sliding field remain undetermined by the global F-term

conditions. *However*, the visible-sector contributions to the family gauge group D-terms *will not* [26] vanish. In the BM sector, VEVs of the family nonsinglet part of ϕ field have zero VEV, so that ϕ makes no contribution to the family D-terms, while the undetermined $\hat{S}$ nonsinglet BM VEVs will contribute to the *positive definite* family group D-terms. Since the visible contribution is $\sim M_{\text{GUT}}^4$, it is obviously highly favorable for the $\hat{S}$ fields to take VEVs and cancel this positive definite contribution to the global SUSY potential. For definiteness, let the family gauge group be the $O(N_g)$, $N_g = 2, 3$ actually used in Ref. [26] (for cogent structural and phenomenological reasons). Then an $O(N_g)$ gauge transformation can always align the family D-term VEV with any one generator of the gauge group and thus requires (in that aligned basis) only one $\hat{S}$ component to get a VEV. It is clear that there will be a pseudo-Goldstone degeneracy among the remaining $\hat{S}$ fields which may be lifted by supergravity effects. Since field $\hat{S}$ gets a VEV $\tilde{\hat{S}} \ll M_{\text{Planck}}$, it too affects the S_s VEV (like $\tilde{\phi}_s, \tilde{z}_i$) only very weakly through δ , which now takes the value

$$\delta = \kappa^2 \left(|\tilde{\phi}_s|^2 + \sum_i |\tilde{z}_i|^2 + \text{Tr} \tilde{\hat{S}}^\dagger \tilde{\hat{S}} \right), \quad (12)$$

and the rest of the determination of the BM-Polonyi field S_s goes through unchanged.

As usual in gravity-mediated scenarios, the chiral fermion in the local supersymmetry-breaking S multiplet is the global SUSY-breaking Goldstino that furnishes the longitudinal mode of the gravitino, which gets the mass

$$m_{\frac{3}{2}} = \kappa^2 |\sqrt{E}(W_0 + W_O(\tilde{z}_i) + \overline{W_H})|. \quad (13)$$

Since this mass should be $\sim 10^3$ – 10^5 GeV in typical gravity-mediated scenarios and typically $\overline{W}_{\text{GUT}} \sim M_X^3 > 10^{48}$ GeV, it is clear that W_0 must be used to cancel $\overline{W}_{\text{GUT}}$ so that

$$|\langle W \rangle| \simeq M_p |\theta| \simeq 10^{39} \text{--} 10^{41} \text{ GeV}^3. \quad (14)$$

The scale of supersymmetry breaking is fixed by the BM superpotential parameters:

$$\sqrt{|F^S|} = \sqrt{|\theta|} = \left| \frac{\mu_B}{2\sqrt{\lambda_B}} \right| \sim 10^{10.5} \text{--} 10^{11.5} \text{ GeV}. \quad (15)$$

Provided $|\lambda_B| > 10^{-14}$, one has $|\lambda_B S| \gg |\mu_B|$.

Having discussed the case $N_g = 1$ in this section and indicated the general structure and solution strategy for our model [26], where family gauge symmetry—*but not* superpotential couplings—ties the visible and hidden sectors together, we will discuss the details of the global superpotentials and family representations in the next

section for $N_g = 2, 3$ while the moduli mass spectra associated with the $\hat{S}$ “familion moduli” are discussed in Sec. IV.

A slightly subtle point that follows from the role of $\mathcal{F}$ as a local supersymmetry order parameter is that z_i, ϕ_s get small corrections of order $m_{3/2}$ so as to maintain $\mathcal{F}_{z_i, \phi_s} \equiv 0$, and similarly the family gauge nonsinglet BM fields $\hat{\phi}$ remain zero (see the next section) so that $\mathcal{F}_{\hat{\phi}} \equiv 0$. However since $\bar{S}_s \sim M_{\text{Planck}}$, $\bar{\hat{S}} \sim M_{\text{GUT}}$ while $\bar{W} \neq 0$, it is clear that $\mathcal{F}_{S_s, \hat{S}}$ are not zero but $\mathcal{F}_{\hat{S}}/\mathcal{F}_{S_s} \sim M_{\text{GUT}}/M_{\text{Planck}} \ll 1$, so that S_s still dominates the supersymmetry breaking.

III. GAUGED $O(N_g)$ AND BM VACUA

Assuming that the visible-sector supersymmetry conditions are satisfied, the vanishing of the *family symmetry* D-terms for the GUT sector VEVs ($\bar{z}_i$) alone is by no means guaranteed. In global SUSY, the $O(N_g)$ D-terms have the form

$$D_{O(N_g)}^a = \text{Tr}(\hat{\phi}^\dagger [T^a, \hat{\phi}] + \hat{S}^\dagger [T^a, \hat{S}]) + \bar{D}_X^a, \quad (16)$$

$$\bar{D}_X^a = \sum_i \bar{z}_i^\dagger T^a z_i,$$

where $\bar{D}_X^a$ is the contribution of the visible-sector fields, and $T^a, \bar{T}^a$ are the generators of $O(N_g)$ in the fundamental and generic representations.

Since we wish to use a flat direction of the hidden-sector superpotential to cancel the contribution $\bar{D}_X^a$ of the visible sector, we consider the situation where ϕ_s, S_s become the trace modes of $O(N_g)$ symmetric representations ϕ_{ab}, S_{ab} so that the BM superpotential becomes

$$W_H = \text{Tr} S(\mu_B \phi + \sqrt{N_g} \lambda_B \phi^2). \quad (17)$$

Generically,

$$S = \hat{S} + \frac{1}{\sqrt{N_g}} S_s \mathcal{I}_{N_g}; \quad \text{Tr} \hat{S} = 0. \quad (18)$$

$\mathcal{I}_{N_g}$ is the N_g -dimensional unit matrix.

Before performing the case-wise analysis, consider the general features of the minimization of the Bajc-Melfo potential arising from the hidden-sector potential W_H alone when $N_g > 1$. The BM local minimum we find is characterized by an indeterminate VEV of all the fields $S_s, \hat{S}$ in the symmetric multiplet S , while the VEV of ϕ is fixed. The arguments given above and the evaluations in the next section show that the VEV of the family singlet in S is determined by supergravity effects, while the others are fixed by minimizing the family D-terms [which are of $O(M_{\text{GUT}}^4)$] together with the leading corrections to them, namely the supergravity induced soft mass terms

$[O(m_{3/2}^2 M_{\text{GUT}}^2)]$, all of which starts from considering the BM local supersymmetry-breaking minimum and its flat direction for $N_g > 1$ together with the globally supersymmetric VEVs $\bar{z}_i$ as the background in which the determination of the sliding multiplet VEVs $S_s, \hat{S}$ are determined. We built up the argument for this fixation stepwise in Sec. II, and the novelty is now only the determination of the $\hat{S}$ VEVs.

The F-terms arising from N_g are

$$F_\phi = \mu_B S + \lambda \sqrt{N_g} (S \cdot \phi + \phi \cdot S),$$

$$F_S = \mu_B \phi + \lambda \sqrt{N_g} \phi \cdot \phi. \quad (19)$$

Clearly the associated global superpotential

$$V(\phi, S) = \text{Tr}(F_S^\dagger F_S + F_\phi^\dagger F_\phi) \quad (20)$$

is semipositive definite. We search for the generalization of the BM local minimum by the ansatz

$$\langle \phi \rangle = \frac{1}{\sqrt{N_g}} \bar{\phi}_s \mathcal{I}_{N_g}. \quad (21)$$

We emphasize that this implies that the gauge nonsinglet VEV $\hat{\phi} \equiv 0$ for all N_g . In fact, this feature implies that after the introduction of supergravity corrections $\mathcal{F}_{\hat{\phi}} = 0$ [unlike $\mathcal{F}_{\hat{S}}$, which develops a value $O(m_{3/2} M_{\text{GUT}})$ and thus makes a $M_{\text{GUT}}/M_{\text{Planck}}$ suppressed contribution to the Goldstino]. With the ansatz (21), then,

$$\langle F_\phi \rangle = \langle S \rangle (\mu_B + 2\lambda \bar{\phi}_s),$$

$$\langle F_S \rangle = \langle \bar{\phi}_s \rangle (\mu_B + \lambda \bar{\phi}_s) \mathcal{I}_{N_g}, \quad (22)$$

so that $\phi_s = -\mu_B/(2\lambda)$ still ensures that $\langle F_\phi \rangle, \langle F_{\hat{S}} \rangle$ vanish provided $\langle \phi \rangle$ has the form in (21). Moreover, the fluctuation $\tilde{\phi}$ around this VEV receives a positive definite mass squared

$$V = 4|\lambda \langle S_s \rangle|^2 \text{Tr} \tilde{\phi}^\dagger \tilde{\phi} + \dots \quad (23)$$

from the VEV of the singlet in S_s , which is undetermined by the BM potential but fixed by supergravity corrections to be of order M_p as shown above. These contributions dominate all others. On the other hand,

$$\langle F_S \rangle = -\frac{\mu_B^2}{4\lambda} \mathcal{I}_{N_g} \quad (24)$$

breaks supersymmetry and causes corrections of order $\pm \mu_B^2$ to the scalar masses of the ϕ supermultiplet. We again emphasize that the F-term of $\hat{S}$ vanishes for the BM

minimum for all N_g . The mass spectrum is discussed in detail in Section IV.

A. Determination of $S_\pm$ when $N_g = 2$

For $N_g = 2$ it is convenient to use the isomorphism $O(2) \simeq U(1)$ and define

$$S = \frac{1}{2} \begin{pmatrix} \sqrt{2}S_s + S_+ + S_- & i(S_- - S_+) \\ i(S_- - S_+) & \sqrt{2}S_s - (S_+ + S_-) \end{pmatrix}, \quad (25)$$

and similarly for $\phi = [\phi_{ab}]$, where $S_\pm, S_s$ are properly normalized fields so that $\text{Tr} S^\dagger S = S_+^\dagger S_+ + S_-^\dagger S_- + S_s^\dagger S_s$ and $\text{Tr} S\phi = S_s\phi_s + S_{(+}\phi_{-})$. In this notation an $O(2)$ vector ψ_a has charges $\pm 1/2$: $\psi_{\pm 1/2} = (\psi_1 \pm i\psi_2)/\sqrt{2}$. The superpotential becomes

$$W_H = \mu_B(S_s\phi_s + S_{(+}\phi_{-})) + \lambda_B(S_s\phi_s^2 + 2S_s\phi_+\phi_- + 2\phi_s S_{(+}\phi_{-})), \quad (26)$$

and it is easy to check that $\bar{\phi}_s = -\mu_B/2\lambda_B$, $\bar{\phi}_\pm = 0$ solves the F-term conditions for W_H (all F-terms vanish except that $\bar{\partial}_{S_s} \bar{W} = \theta$ as before). This verifies that the general form of the VEVs of ϕ , F_S is indeed that given in (21) and (22). The fields $S_{s,\pm}$ remain undetermined at this local minimum, and the superpotential reduces to the one for $N_g = 1$ when the fields that *are* determined are equated to their values at the local minimum. Notice that the VEV $\bar{\phi}$ makes the coefficient of $S_{(+}\phi_{-})$ vanish so that there is no Dirac mass mixing the chiral fermions in $S_\pm, \phi_\mp$.

The calculation for the singlet goes through unchanged fixing $S_s \sim M_p$ as in Eqs. (21) and (22). With this VEV, the $O(2)$ charged fermionic components of $\hat{\phi}_\pm$ get masses $\sim \lambda_B \bar{S}_s$. Along with the GUT-scale breaking of the family symmetry, the large mass of the modes in ϕ_{ab} is responsible for quelling the percolation of the supersymmetry breaking

coded in F_{S_s} and ensuring the hidden sector is actually hidden. Since $\hat{\phi}_\pm$ have zero VEVs at the SUSY-breaking minimum, they do not mix with the family symmetry gaugino. Since the charged S fields can have VEVs of at most the order of the GUT scale $M_{\text{GUT}} \ll M_p$, the potential for their determination to leading order in $m_{3/2}$ is simply the common scalar mass term generated by the supergravity potential and the D-term of the family symmetry. In the $O(2)$ case, the leading-order potential for the flat family charged S directions is

$$V(S_+, S_-) = m_{3/2}^2(|S_+|^2 + |S_-|^2) + \frac{g_f^2}{2}(|S_+|^2 - |S_-|^2 + \bar{D}_X)^2; \quad (27)$$

here $\bar{D}_X = \sum q_i |\bar{z}_i|^2$, where $q_i, \bar{z}_i$ are the family symmetry charges of the global supersymmetric VEVs in the visible sector. Clearly, if $x = \text{Sign}[\bar{D}_X]$, the minimum will occur when

$$S_{-x} = \sqrt{|\bar{D}_X| - x \frac{m_{3/2}^2}{g^2}}, \quad \bar{S}_x = 0. \quad (28)$$

Notice that the shift in the S_{-x} VEV relative to $\sqrt{\bar{D}_X}$ is tiny, but the gravity induced mass term does enforce the choice of which component of $\hat{S}$ gets a VEV. The vacuum energy after minimization is $\sim m_{3/2}^2 |\bar{D}_X|$ and can be tuned to zero by a shift $\delta \hat{W}_0 \sim m_{3/2} |\bar{D}_X| < m_{3/2} M_p^2$. Note that since $\bar{F}_{\hat{S}} = 0$ from (26) (and $\bar{F}_{z_i} = 0$ by our scenario), the supergravity corrections to the global D-terms shown in (3) are zero.

B. Fixation of S VEVs for $N_g = 3$

For $N_g = 3$ we define the components of S_{ab} in terms of T_3 eigenfields to be

$$\begin{pmatrix} \frac{1}{6}(\sqrt{6}S_0 + 3iS_{-2} - 3iS_{+2} + 2\sqrt{3}S_s) & \frac{1}{2}(S_{+2} + S_{-2}) & \frac{1}{2}(S_+ + S_-) \\ \frac{1}{2}(S_{-2} + S_{+2}) & \frac{1}{6}(\sqrt{6}S_0 - 3iS_{-2} + 3iS_{+2} + 2\sqrt{3}S_s) & \frac{1}{2}i(S_+ - S_-) \\ \frac{1}{2}(S_+ + S_-) & \frac{1}{2}i(S_+ - S_-) & \frac{1}{\sqrt{3}}(S_s - \sqrt{2}S_0) \end{pmatrix},$$

and similarly for $\phi = [\phi_{ab}]$. $S_{s,0,\pm,\pm 2}$ are properly normalized fields, so that

$$\begin{aligned} \text{Tr} S^\dagger S &= S_{+2}^\dagger S_{+2} + S_{-2}^\dagger S_{-2} + S_+^\dagger S_+ + S_-^\dagger S_- + S_s^\dagger S_s, \\ \text{Tr} S\phi &= S_s\phi_s + S_{(+}\phi_{-}) + S_{(+2}\phi_{-2}). \end{aligned} \quad (29)$$

In this case the explicit form of the BM superpotential is [where (square) round brackets denote (anti-)symmetrization]

$$\begin{aligned}
W_{\text{BM}} = & \mu_B(S_s\phi_s + S_0\phi_0 + S_{(+}\phi_{-}) + S_{(+2}\phi_{-2})) + \lambda_B \left[(S_s(\phi_s^2 + \phi_0^2 + 2\phi_{-}\phi_{+} + 2\phi_{-2}\phi_{+2}) \right. \\
& + S_0(-\phi_0^2/\sqrt{2} + 2\phi_s\phi_0 - \sqrt{2}\phi_{-}\phi_{+} + \sqrt{2}\phi_{-2}\phi_{+2}) + 2S_{(+}\phi_{-})\phi_s - \frac{1}{\sqrt{2}}S_{(+}\phi_{-})\phi_0 + i\sqrt{3}(S_{+}\phi_{-2}\phi_{+} - S_{-}\phi_{-}\phi_{+2}) \\
& \left. + \sqrt{2}S_{(+2}\phi_{-2})\phi_0 + 2S_{(+2}\phi_{-2})\phi_s + \frac{i}{2}S_{[+2}\phi_{-2}^2] \right]. \quad (30)
\end{aligned}$$

Using this one may again verify that the VEVs are of the form given in Eqs. (21) and (22).

When $N_g = 3$, the D-terms form a vector of $O(3)$, and it is advantageous to change to a basis (denoted by primes) where $\bar{D}'_X = \delta_3^a |\bar{D}_X|$. This is easily achieved by a rotation in the 12 plane to rotate the $\bar{D}_X^a$ vector into the 23 plane, and then a 23 plane rotation to make it point in the 3 direction:

$$\begin{aligned}
O &= R_{23}[\theta_X] \cdot R_{12} \left[\frac{\pi}{2} - \varphi_X \right], \\
\theta_X &= \text{ArcTan} \left[\sqrt{\frac{V_1^2 + V_2^2}{V_3^2}} \right]; \quad \varphi_X = \text{ArcTan} \left[\frac{V_2}{V_1} \right], \quad (31)
\end{aligned}$$

where $V^a = \bar{D}_X^a$. The potential for the flat directions from $\hat{S}'$ is now

$$\begin{aligned}
V[\hat{S}'] &= m_{3/2}^2 (|S'_0|^2 + |S'_+|^2 + |S'_{-1}|^2 + |S'_{+2}|^2 + |S'_{-2}|^2) \\
&+ \frac{g_f^2}{2} \{ (|S'_+|^2 + 2|S'_{+2}|^2 - |S'_-|^2 - 2|S'_{-2}|^2 + (\bar{D}_X^3)^2) \\
&+ 2\text{Tr}(S'^{\dagger}[T_+, S'])\text{Tr}(S'^{\dagger}[T_-, S']) \}. \quad (32)
\end{aligned}$$

This has a solution along the same lines as for the $N_g = 2$ case. Dropping primes, one gets

$$\begin{aligned}
|\bar{S}_{-2}| &= \sqrt{\frac{|\bar{D}_X|}{2} - \frac{m_{3/2}^2}{4g_f^2}}, \\
\bar{S}_{-,+,+2} &= 0. \quad (33)
\end{aligned}$$

IV. MASS SPECTRUM

The spectrum from the visible-sector gauge interactions and superpotential is sensible by assumption and worked out for realistic cases in Ref. [26]. Our interest here is in the BM fields and—where relevant—their mixing with visible-sector fields through gauge or supergravity terms. We shall confine ourselves to working out the leading contributions and ignore the effects of small shifts ($\bar{z}_i \rightarrow \bar{z}_i + \delta_i$, where δ_i of order $m_{3/2}$) in visible-sector VEVs which we assume [via the usual shift $\delta_i = -(W'')_{ij}^{-1} \bar{z}^j \kappa^2 \bar{W}$] maintain $\mathcal{F}_{z_i} = 0$.

We first consider the mass terms that arise in the globally supersymmetric Lagrangian when the fields $\phi_s, z_i, S_s, \hat{S}$ are replaced by their VEVs. The supergravity corrections are necessarily suppressed by the Planck length and are therefore important only if a field does not obtain a mass at one of the large scales present, $\{M_P, M_{\text{GUT}}, \sqrt{M_P M_{3/2}} \gg m_{3/2}\}$. At the global level, VEVs of F-terms do not couple to fermion fields at all, so that any SUSY-breaking contributions will be visible only in the scalar sector. Since only $F_{S_s} \neq 0$, it follows that it makes a SUSY-breaking mass contribution of order $\bar{F}_{S_s} \sim m_{3/2} M_P$ to the scalars that it couples to in the cubic terms of the superpotential.

In the family gauge terms (we ignore the visible gauge sector under which the BM fields are singlets) the (global SUSY-preserving) $\hat{S}$ VEVs mix the chiral fermion in the $\hat{S}$ multiplet with the visible-sector chiral fermion combination that couples to the family gauginos to form a Dirac multiplet. Since $\hat{S}$ terms otherwise lack mass terms, this leads to a residual massless fermion from the orthogonal mode. This is explained more explicitly below.

In the supergravity Lagrangian, only fermion bilinears coupling with nonvanishing $\mathcal{F}$ terms can possibly obtain mass terms sensitive to supersymmetry breaking. Only $\mathcal{F}_{S_s} \sim m_{3/2} M_{\text{Planck}}$ can be nonzero at leading order, although the $\mathcal{F}_{z_i, \hat{S}}$ will get smaller contributions of order $m_{3/2} M_{\text{GUT}}$.

A. Mass spectrum for $N_g = 1$

In this case only gravity couples the visible and hidden sectors. The corrections to the visible sector are the familiar soft supersymmetry-breaking supergravity corrections which are insensitive to the detailed form of the hidden-sector superpotential or whether its SUSY-breaking minimum is local or global. The VEV $\bar{\phi}_s = -\mu_B/2\lambda_B$ implies that the field S_s (which has no mass term $W = m_s S_s^2 + \dots$) does not get a mass by mixing with ϕ_s and provides the Goldstino as expected, even though the supersymmetry is broken by a local rather than a global minimum. The fermion masses consist of the ϕ_s mode with mass $2\lambda_B \bar{S}_s$ and the gravitino with longitudinal mode from the Goldstino $\tilde{S}_s$.

In the BM sector, the *only* fermion mass terms at the global level are

$$-\mathcal{L} = \lambda_B \bar{S}_s \tilde{\phi}_s \tilde{\phi}_s = \frac{m_{\tilde{\phi}_s}}{2} \tilde{\phi}_s \tilde{\phi}_s. \quad (34)$$

Mass terms due to the VEV $\tilde{\phi}_s$ cancel. The scalar mass spectrum consists of two real scalar modes from ϕ_s with masses squared $|2\lambda_B \bar{S}|^2 \pm |\mu_B|^2/2$ (notice the supersymmetry-breaking splitting of the scalars from the chiral fermion's Majorana mass $2\lambda_B \bar{S}_s$) and two real scalars from S_s with masses squared $m_{3/2}^2((7 + 4\sqrt{3}) \pm (5 + 3\sqrt{3}))$. This completes the discussion of the $N_g = 1$ case apart from explicitly working out the effective theory, which will be of the usual supergravity type (global SUSY GUT-derived MSSM plus soft SUSY-breaking terms).

B. Mass spectrum for $N_g = 2$

Consider the masses of the superfields $\hat{\phi}_{ab}$. The fermions do not couple directly to the SUSY-violating F-term. Inserting the VEVs in the fermion bilinear terms obtained from the superpotential in Eq. (26), we get the following fermion mass terms *only* (tildes here denote fermionic components):

$$-\mathcal{L} = \lambda_B \bar{S}_s (\tilde{\phi}_s \phi_s + 2\tilde{\phi}_+ \tilde{\phi}_-) + 2\lambda_B \tilde{\phi}_s \tilde{\phi}_- (\bar{S}_+). \quad (35)$$

Since $\bar{S}_s \gg \bar{S}_\pm$, it is clear that the $\tilde{\phi}_s \tilde{\phi}_\pm$ mixing is far subdominant to the diagonal Majorana and Dirac mass terms $\tilde{\phi}_s^2, \tilde{\phi}_+ \tilde{\phi}_-$. Since there is no residual gauge group, the chiral mass matrix is just $\partial_i \partial_j \bar{W}$. We have a symmetric 3×3 chiral mass matrix with rows and columns labeled by $\{\phi_s, \phi_+, \phi_-\}$:

$$m_{\phi\phi} = \lambda_B \bar{S}_s \begin{pmatrix} 1 & \epsilon & 0 \\ \epsilon & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (36)$$

where $\epsilon = \bar{S}_{-x}/\bar{S}_s \sim M_X/M_p$. This can easily be diagonalized and gives large masses in the GUT-scale range for reasonable values $\lambda_B \sim 10^{-3}$ to 10^{-1} . In the scalar sector the ϕ fields also receive mass-squared contributions from $\lambda_B F_{S_s} \sim \mu_B^2$ besides the contributions shown for the fermions. However, since $|\mu_B| \ll M_X \ll M_p$, these will lead to small splitting of the dominant contributions shared with the fermions. All this is a straightforward extension of the behavior of the BM ϕ for $N_g = 1$.

With the VEVs given in Sec. III for $N_g = 2$, and due to the lack of any other mass term for $S_\pm$, it is easy to check that the fermionic component of S_{-x} mixes with the visible-sector Higgsino field combination that combines to form a Dirac multiplet with the single family gaugino of the $O(2)$ gauge symmetry, because the visible-sector Higgs fields carry symmetric representation of the family group. Thus, the Dirac partner Λ for the $U(1)_f$ gaugino λ_f with Dirac mass

$$m_{\lambda_B} = \sqrt{2}g_f \sqrt{|\tilde{S}_{-s}|^2 + \sum_i q_i^2 |\tilde{z}_i|^2} \quad (37)$$

is defined by

$$\begin{aligned} \mathcal{L} &= -m_\lambda \Lambda \lambda_g + \text{H.c.}, \\ \Lambda &= (\cos[\theta_S] \tilde{Z} - \sin[\theta_S] \tilde{S}_{-x}) \frac{\sqrt{2}g_f}{m_{\lambda_B}}, \\ \tilde{Z} &= \frac{\sum_i q_i \tilde{z}_i^* \tilde{z}_i}{\sqrt{\sum_i q_i^2 |\tilde{z}_i|^2}}, \\ \tan \theta_S &= \frac{x |\tilde{S}_{-x}|}{\sqrt{\sum_i q_i^2 |\tilde{z}_i|^2}}. \end{aligned} \quad (38)$$

However, due to the absence of other mixing terms for $S_{\pm s}$, the orthogonal combination $\tilde{\chi}$ and S_x remain massless at leading order. Here

$$\tilde{\chi} = \cos[\theta_S] \tilde{S}_{-x} + \sin[\theta_S] \tilde{Z} \quad (39)$$

does not mix with the remaining massive modes, since it is made of (SM neutral) zero modes $\tilde{S}_{-x}, \Lambda$ of the chiral mass matrices in the hidden and visible sectors, respectively. Shifts due to supergravity are too small to shift the masses $m_{S_+ \phi_-} = m_{\phi_+ S_-} = \mu_B + 2\lambda \tilde{\phi}_s = 0$ appreciably. So the leading contributions come from loop effects (see Fig. 1) (analogous to the one-loop corrections to neutrino masses induced by soft SUSY breaking [30]) and have value

FIG. 1. Diagram for fermionic mass $m_{\tilde{S}}$.

$$m_S^{1\text{-loop}} \simeq \frac{|\lambda_B|^2 \bar{F}_S}{16\pi^2 \bar{S}_s} \simeq \frac{|\lambda_B|^2}{16\pi^2} \frac{m_{3/2}}{\sqrt{3}-1}. \quad (40)$$

Thus, these masses may be as small as a few GeV even for a large gravitino mass. Due to their weak interactions with light modes, they may be suitable candidates for light cold dark matter. The scalar partners have masses

$$(m^2)_{S_s} = 2m_{3/2}^2; \quad m_{\text{Re}(S_{-x})}^2 = 2g_f^2 |D_X| - 2m_{3/2}^2. \quad (41)$$

Like $\tilde{S}_{-x}$, the hidden-sector Goldstone scalar component $\text{Im}[S_{-x}]$ will again lead to a nearly massless mode, since it too will mix with the $O(2)$ Goldstone contributions from the visible sector, and hence the orthogonal mode (partner of the pseudo-Goldstino $\tilde{\chi}$) will be massless before loop effects. The scalar pseudo-Goldstone mode χ which remains massless even after symmetry breaking due to pseudo-doubling of Goldstone modes (i.e. independent breaking of the family symmetry in the potentials from the unlinked hidden- and visible-sector superpotentials) can also obtain loop induced mass from the partner of the graph shown for the fermions, but it will remain light. This is because the supersymmetry-breaking F-term couples to other particles through ϕ_{ab} propagators whose large masses $\lambda_B \langle S_s \rangle \sim M_X$ ensure that the effect of one-loop corrections is a power of $F/M_X \sim m_{3/2} M_p/M_X$. It is clear that since all the other modes have large masses much greater than the gravitino mass, and we have already taken into account the dominant contributions to the fields that participate in the SUSY breaking, the supergravity corrections to their masses are subdominant and of no immediate significance to warrant their presentation here.

C. Mass spectrum for $N_g = 3$

The analysis for this case proceeds in a manner closely parallel to the case of two generations. Once again, all contributions from the VEV of ϕ_s cancel, leaving for the BM fermion mass terms *only*

$$-\mathcal{L} = \lambda_B \tilde{S}_s (\tilde{\phi}_s \tilde{\phi}_s + \tilde{\phi}_0 \tilde{\phi}_0 + 2\tilde{\phi}_+ \tilde{\phi}_- + 2\tilde{\phi}_{+2} \tilde{\phi}_{-2}) + 2\lambda_B \tilde{\phi}_s \tilde{\phi}_{+2} \tilde{S}_{-2}. \quad (42)$$

Once again we have diagonal Majorana/Dirac masses $2\lambda_B \tilde{S}_s$ for all the components of ϕ and a much smaller off-diagonal contribution from the VEV of the nonsinglet $\hat{S}$. These will be accompanied by scalar masses of the same magnitude up to small splitting $O(m_{3/2})$. Similarly, the scalars from the S_s multiplet will obtain masses of the order of the gravitino mass.

As for the $N_g = 2$ case, fermionic partners of the $\hat{S}$ particles (except S_{-2} , but even so, the analog of $\tilde{\chi}$ will remain light via the mechanism explained for the $N_g = 2$ case) do not get mass to leading order. The Dirac partners of

the family symmetry gauginos in the primed basis, where $\bar{D}^a \sim \delta_s^a$, are as follows:

$$\Lambda^3 = \frac{-2\bar{S}'_{-2} \tilde{S}'_{-2} + \sum_i q_i \bar{z}'_i \tilde{z}'_i}{(4\bar{S}'_{-2} \tilde{S}'_{-2} + \sum_i \bar{q}_i^2 \bar{z}'_i \tilde{z}'_i)^{1/2}},$$

$$\Lambda^\pm = \frac{\sum_i \bar{z}'_i T^\pm \tilde{z}'_i}{(\sum_i \bar{z}'_i T^\pm \tilde{z}'_i)^{1/2}}. \quad (43)$$

As before, the fermion mode orthogonal to Λ^3 will remain massless along with $\tilde{S}_{0,\pm,2}$.

The scalar masses are positive (apart from the Goldstone boson)

$$M_{S_0}^2 = m_{3/2}^2; \quad M_{S_-}^2 = \frac{1}{2} m_{3/2}^2,$$

$$M_{S_+}^2 = \frac{3}{2} m_{3/2}^2; \quad M_{S_{+2}}^2 = 2m_{3/2}^2,$$

$$M_{R_{-2}}^2 = 4g_f^2 \left| \frac{\bar{D}_X}{2} \right| - 2m_{3/2}^2; \quad M_{T_{-2}}^2 = 0. \quad (44)$$

In the above, since we are working in the primed basis, we have exhibited only the Goldstone longitudinal component of the $O(3)$ gauge boson in the 3 direction which comes from the field S_{-2} : of course, all three $O(N_g)$ massive gauge bosons will get longitudinal mode contributions from the visible sector (which we assume breaks the family symmetry completely) which we have not discussed explicitly here [see, however, Ref. [26] for more details in a concrete model based on a realistic $SO(10)$ GUT]. Since only a linear combination of I_{-2} and a visible-sector massless mode will be eaten, we would again expect the orthogonal combination to remain massless.

Now we will have five light fermionic and one scalar degree of freedom lighter than the gravitino mass and with very weak couplings to ordinary matter. Again they will pick up mass at one loop due to supersymmetry breaking and will remain as the fossils of flavor and supersymmetry breaking. Thus, they are either candidates for multi-component relic dark matter or must be removed somehow.

From the above discussion, it is clear that the BM SUSY-breaking Minkowski vacuum is characterized by the presence of several light fields coming from the supersymmetry-breaking (S_s) and from the $O(N_g)$ variant $\hat{S}$ fields over and above the usual MSSM light fields in the visible sector. The superfields (ϕ_{ab}) are superheavy, with masses of order $\lambda_B \tilde{S}_s \sim M_X$. The effective low-energy theory must be written in terms of both sets of light fields by equating the heavy fields to their VEVs and separating the light fields out via an expansion in $m_{3/2}/M_p$. This calculation proceeds largely like that in Ref. [5], but care must be taken due to the additional gauge invariance active in both hidden and gauge sectors, the main effects of which the present analysis sought to clarify.

It bears mention that radiative effects in such systems may be quite significant and have an important bearing on cosmological behavior [31]. The loop-corrected Kahler potential is available up to two loops [32]. We will return to the study of the vacuum and cosmological predictions in the theory with both gravity and radiative effects in a sequel.

V. DISCUSSION

In this paper we have shown how gravity mediation of Bajc-Melfo-type calculable supersymmetry breaking at metastable vacua in a hidden sector containing fields variant under a family gauge symmetry ensures the viability of GUT and family symmetry breaking in the visible sector of a “grand Yukawonification” [26] model. In such models, one aims at generating the observed hierarchical fermion flavor structure from a gauged family symmetry model with only generation blind couplings. The special role of the BM supersymmetry breaking is that it provides flat directions in both the $O(N_g)$ singlet and the gauge variant parts of a symmetric chiral supermultiplet $S_{ab} = S_{ba}$. Since it is very difficult to make the contribution of the visible-sector GUT fields to the $O(N_g)$ D-terms vanish, the $\hat{S}$ flat direction performs the invaluable function of canceling this contribution *without disturbing the symmetry breaking in the visible sector*. The determination of the viability of the “grand Yukawonification” model then becomes a matter of searching the relatively small remaining parameter space for viable parameter sets that fit the fermion data at M_X while taking account of threshold corrections at low and high scales and while respecting constraints on crucial quantities like the proton lifetime [20,23,24,25]. Note that in this approach, not only are the hard parameters of the visible-sector superpotential reduced by replacement of the flavored parameters by bland family symmetric ones, but also the soft supersymmetry-breaking parameters are

determined by the two parameters of the hidden-sector superpotential and the Planck scale.

The structure used entails yet further stringent constraints, since the masslessness of the moduli multiplets $\hat{S}_{ab}$ before supersymmetry breaking implies the existence of $N_g(N_g + 1)/2 - 1$ SM singlet fermions generically lighter than the gravitino mass scale and possibly as light as a few GeV. In addition, the Polonyi mode S_s may also lead to difficulties in the cosmological scenario. Thus, such modes can be both a boon and a curse for familion GUT models. A boon because generic SUSY GUT models are hard put if asked to provide SUSY WIMPs of mass below 100 GeV as CDM candidates, as suggested by the DAMA/LIBRA experiment [28]. A curse because there are strong constraints on the existence of such light moduli which normally demand that their mass be rather large (> 10 TeV) due to the robust cosmological (“Polonyi”) problems arising from decoupled modes with Planck scale VEVs [29]. In contrast to the simple Polonyi model and String moduli, the BM moduli have explicit couplings to light fields through family D-term mixing and loops. Moreover, the MSGUT scenario favors [23,25] large gravitino masses $> 5\text{--}50$ TeV. Thus, the Polonyi and moduli problems may be evaded. In any case, the cosmology need be considered seriously only after we have shown that the MSSM fermion spectrum is indeed generated by the “Yukawonified” NMSGUT [26].

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