Invariant-mass spectrum of $\Lambda\bar{\Lambda}$ pair in the process $e^+e^- \rightarrow \phi\Lambda\bar{\Lambda}$

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We show that the final-state interaction perfectly explains the recent experimental data of the BESIII Collaboration on the near-threshold invariant-mass spectrum of $\Lambda\bar{\Lambda}$ in the process $e^+e^- \rightarrow \phi\Lambda\bar{\Lambda}$. The position of the peak in the invariant-mass spectrum is above the threshold, though the interaction of Λ and $\bar{\Lambda}$ is due to an attractive potential. This potential has a simple form and depends only on two parameters. We show that the invariant-mass spectrum is consistent with the quantum numbers $J^{PC} = 1^{++}$ and 2^{++} of $\Lambda\bar{\Lambda}$ pair but the quantum numbers $J^{PC} = 2^{-+}$ are less likely.

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Nowadays, many resonances have been observed with masses only slightly above the threshold of the production of particles into which these resonances mainly decay. Although there are a lot of experimental data, the discussion about the nature of these resonances still continues. Very recently, the BESIII Collaboration published the experimental results [1] on annihilation $e^+e^- \rightarrow \phi\Lambda\bar{\Lambda}$. These results clearly demonstrate a strong enhancement of $\Lambda\bar{\Lambda}$ pair production in the near-threshold mass region. A study of the angular distributions carried out in Ref. [1] shows that the possible quantum numbers of the $\Lambda\bar{\Lambda}$ pair are $J^{PC} = 1^{++}, 2^{++}, 2^{-+}$. In the first two cases, the relative angular momentum l of the $\Lambda\bar{\Lambda}$ pair, its total spin S , and the relative angular momentum L of a pair and ϕ meson are $l = 1, S = 1$, and $L = 0$. In the third case, the corresponding quantum numbers are $l = 2, S = 0$, and $L = 1$.

The invariant-mass spectrum of $\Lambda\bar{\Lambda}$ is similar to that observed in $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ and $\Upsilon(4S) \rightarrow B^+B^-$ decays [2–4]. These decays were discussed within various theoretical approaches [5–11] with completely different qualitative predictions. In our recent work [12], a simple exactly solvable model has been proposed for describing $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ and $\Upsilon(4S) \rightarrow B^+B^-$ decays. This model is based on the account for the final-state interaction of B and $\bar{B}$ mesons. The results of Ref. [12] are in good agreement with the available experimental data. The method used in Ref. [12] is similar to that developed earlier in Refs. [13–15] for calculating the cross section of e^+e^-

annihilation into proton-antiproton and neutron-antineutron pairs near the pair production threshold. The effects of final-state interaction in the hadronic systems were also discussed in Refs. [16–19].

The approach of Ref. [12] can also be applied for calculation of $\Lambda\bar{\Lambda}$ invariant-mass spectrum. In fact, in our case, it is even simpler because there is no Coulomb interaction and there is only isoscalar exchange between Λ and $\bar{\Lambda}$. Of course, at present, not much is known about the corresponding potential $V(r)$ of $\Lambda\bar{\Lambda}$ interaction. However, for our purposes, we can use any simple phenomenological potential with some specific properties (see below). Our approach ensures agreement between the theoretical predictions and the experimental data. Besides, it clarifies the physics of near-threshold resonances. This is the main goal of our work.

Following Ref. [12], consider the radial wave function $\Psi(r)$ of the $\Lambda\bar{\Lambda}$ pair. The function $\psi(r) = kr\Psi(r)$ satisfies the equation

$$\left[-\frac{1}{M} \frac{\partial^2}{\partial r^2} + \frac{l(l+1)}{Mr^2} + V(r) - E \right] \psi(r) = 0, \quad (1)$$

where M is the mass of Λ , $k = \sqrt{ME}$, and E is the energy of the pair, counted from $M_{\text{th}} = 2M$, $\hbar = c = 1$. It is necessary to find a solutions $\psi(r)$ of (1) with asymptotics at large distances

$$\begin{aligned} \psi(r) &= \frac{1}{2i} (S\chi^+ - \chi^-), \\ \chi^\pm &= \exp[\pm i(kr - l\pi/2)], \end{aligned} \quad (2)$$

where S is some function of energy.

There are two stages in the process under consideration. There is production of the hadronic system at small distances followed by its transformation into $\Lambda\bar{\Lambda}\phi$ at large

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distances. The amplitude at small distances is almost energy independent near the $\Lambda\bar{\Lambda}$ pair production threshold. Therefore, the strong dependence of the cross section on the energy of the pair is determined by the phase space of $\Lambda\bar{\Lambda}$ and by their final-state interaction due to the small relative velocity of Λ and $\bar{\Lambda}$. As is known (see, e.g., Refs. [13–15]), the final-state interaction can be taken into account by the wave function of a pair at small distances, similar to the well-known Gamow-Sakharov-Sommerfeld factor. Therefore, near the threshold, the cross sections of the process are given by the functions W_l ,

$$\begin{aligned} W_1 &= N_1 k \left| \frac{\partial}{\partial r} \Psi(0) \right|^2 \quad \text{for } l = 1, \\ W_2 &= N_2 k \left| \frac{\partial^2}{\partial r^2} \Psi(0) \right|^2 \quad \text{for } l = 2, \end{aligned} \quad (3)$$

where $N_{1,2}$ are some constants. In fact, there is some smooth dependence of $N_{1,2}$ on E via the momentum of ϕ meson and its interaction with $\Lambda\bar{\Lambda}$ pair. However, the relative velocity between a pair and ϕ meson is not small, in contrast to the relative velocity between Λ and $\bar{\Lambda}$. Therefore, the effect of rescattering between the ϕ meson and $\Lambda\bar{\Lambda}$ pair on the energy dependence of the cross section in the vicinity of the pair production threshold can be neglected.

For a model, we choose a simplest potential $V(r) = -V_0\theta(a-r)$, where $V_0 > 0$ and a are some parameters and $\theta(x)$ is the Heaviside function. These parameters can be chosen so that the corresponding results describe the available experimental data well. For this potential, the analytical solution has a simple form [12],

$$\begin{aligned} W_1 &= b_1 \frac{kq^2}{M^3} \left| \frac{q}{kh'_1(ka)f_1(qa) - qh_1(ka)f'_1(qa)} \right|^2, \\ W_2 &= b_2 \frac{kq^4}{M^5} \left| \frac{q}{kh'_2(ka)f_2(qa) - qh_2(ka)f'_2(qa)} \right|^2, \\ h_1(x) &= \left(\frac{1}{x} - i \right) e^{ix}, \quad f_1(x) = \frac{\sin x}{x} - \cos x, \\ h_2(x) &= \left(\frac{3}{x^2} - \frac{3i}{x} - 1 \right) e^{ix}, \\ f_2(x) &= \left(\frac{3}{x^2} - 1 \right) \sin x - 3 \frac{\cos x}{x}, \end{aligned} \quad (4)$$

where $q = \sqrt{M(E+V_0)}$, $b_{1,2}$ are some energy-independent constants and $Z'(x) \equiv \partial Z(x)/\partial x$. As should be, $W_1 \propto k^3$ and $W_2 \propto k^5$ at $E \rightarrow 0$. However, these asymptotics are valid only for very small energy E .

The observed peak in the invariant-mass spectrum appears due to a strong reflection from the border of the potential, so that $q_R a = \sqrt{M(V_0 + E_R)} a = n\pi$, where $n = 1, 2, \dots$ and E_R is a parameter close to the value of the peak position, $E_R \approx 100$ MeV. Thus,

FIG. 1. Energy dependence of the functions W_1 (solid curve) and W_2 (dashed curve) normalized to the experimental data [events/(5 MeV)]. The experimental data are from Ref. [1] (BESIII).

$$V_0 = \frac{(n\pi)^2}{Ma^2} - E_R. \quad (5)$$

In the case of $e^+e^- \rightarrow N\bar{N}$ annihilation, a nucleon-antinucleon pair is produced in the superposition of $l=0$ and $l=2$ waves. As a result, it is necessary to use an optical potential with the negative imaginary part, which accounts for annihilation of $N\bar{N}$ pair into mesons. In our cases, $l=1$ or $l=2$; hence, annihilation of $\Lambda\bar{\Lambda}$ into mesons is not important, so we use a real value of V_0 . The dependence of the functions W_1 and W_2 on E (4) is shown in Fig. 1, and data are taken from Ref. [1] (BESIII). Values V_0 and a of the potential and the external factors b_l in functions W_l are chosen to minimize χ^2 . The best $\chi^2/N_{df} = 1.3$ for $l=1$ is obtained at $a = 1.2$ fm and $V_0 = 836$ MeV. Our predictions for $l=1$ are given in Fig. 1 by the solid curve. The values of the parameters a and V_0 correspond to $n=2$ and $E_R = 106$ MeV; see Eq. (5). The relation $E_R \ll V_0$ corresponds to strong reflection from the border of the potential. So, we have good agreement between our predictions and the experimental data for $l=1$.

For $l=2$, the best fit of experimental data is obtained at $a = 1.9$ fm and $V_0 = 979$ MeV. These values give $\chi^2/N_{df} = 2.4$, which is essentially worse than for $l=1$. The corresponding predictions for $l=2$ are given in Fig. 1 by the dashed curve.

Note that the experimental data shown in Fig. 1 were obtained in Ref. [1] by summing up the data samples taken at different values of e^+ and e^- beam energies. However, we have checked that our predictions (scaled to the number of events) are in good agreement with the data from each sample given in the supplementary materials of Ref. [1]. Therefore, strong dependence of the cross section of $e^+e^- \rightarrow \phi\Lambda\bar{\Lambda}$ annihilation on the invariant mass of $\Lambda\bar{\Lambda}$ pair close to the threshold is given solely by the functions W_l .

To understand the dependence of the theoretical predictions on the shape of the potential, we obtained numerically the predictions for $l = 1$ and $l = 2$ using the potential of the Woods-Saxon type $V(r) = -V_0/(1 + \exp[(r - a)/b])$. We found that good agreement with the experimental data can be achieved for $l = 1$ only for relatively small $b \lesssim 0.2$ fm. This is an argument in favor of strong attraction at small distances $r < 1$ fm. Therefore, additional contribution to the potential corresponding to the pion exchange has no influence on the shape of W_l . This statement was checked numerically. For $l = 2$, increasing of the value of b in the potential of the Woods-Saxon type also results in the increasing of χ^2 .

In the nonrelativistic approximation, spin-orbit interaction is not taken into account, and the function W_1 is the same for $J^{PC} = 1^{++}$ and $J^{PC} = 2^{++}$ states, since in both cases $l = 1$. In the case $l = 2$, our attempts to achieve a small χ^2 for reasonable values of V_0 and a were not successful. This allows us to claim that, within our model, the case $l = 2$ ($J^{PC} = 2^{-+}$) does not agree with the experimental data.

To conclude, it is shown that our predictions for the invariant-mass spectrum of $\Lambda\bar{\Lambda}$ in $e^+e^- \rightarrow \phi\Lambda\bar{\Lambda}$

annihilation are in good agreement with the recent experimental data of BESIII Collaboration. These predictions are based on the account for the final-state interaction within a simple model. Of course, the real potential differs from that considered in our work. However, the real potential is currently unknown. In fact, its specific form is not very important, since for many parametrizations one can find the values of parameters which provide a given shape and position of the above-threshold resonance. Our parametrization is convenient because it allows an analytical solution of the problem. We argue that $\Lambda\bar{\Lambda}$ pairs are produced mainly in the states with quantum numbers $J^{PC} = 1^{++}$ or 2^{++} but the state $J^{PC} = 2^{-+}$ is less likely. Our results for $\Lambda\bar{\Lambda}$ invariant-mass spectrum strongly support the statement that the origin of near-threshold peaks in various processes is related to the final-state interaction of the produced hadrons.

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