Spin and polarization analysis of the Z_{cs} state

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A polarization analysis is performed for the recent observation of Z_{cs} exotic state in the e^+e^- annihilation experiment with motivation for measuring its spin quantum number in the future. Starting with the unpolarized electron and positron beam, the polarization transfer to the Z_{cs} state and its decay angular distribution patterns are investigated. Some observables are suggested for determination of the spin parity quantum numbers. An ensemble of Monte-Carlo events is used to show some moment distributions special for manifestation of the different Z_{cs} spin scenarios.

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I. INTRODUCTION

Search for exotic hadronic states, such as tetraquark or pentaquark states, has received much more attention both in experimental and theoretical physics community in recent years [1–3]. Some exciting observations in experiment have been reported from the e^+e^- collider or hadron collider experiments. Most impressive among these are the charged or neutral $Z_c(3900)$ and $Z_c(4020)$ states, reported by the BESIII, Belle, and CLEO collaborations [4–13]. These observations open a new era for study of exotic hadronic spectroscopy.

Very recently, a new structure, dubbed $Z_{cs}(3985)^-$, is observed in the threshold enhancement of $D_s^- D^{*0}$ or $D_s^{*-} D^0$ invariant mass with significance of 5.3 standard deviation [14]. The pole position is determined to be $(3982.5^{+1.8}_{-2.6} \pm 2.1) - \frac{i}{2}(12.8^{+5.3}_{-4.4} \pm 3.0)$ MeV. This structure is suggested as a candidate of $c\bar{c}s\bar{u}$ tetraquark state, as predicted by the theoretical models [15].

The spin and parity of $Z_{cs}(3985)^-$ is not determined experimentally due to limited events observed in the BESIII data. It was conjured as a $J^P = 1^+$ state in the experimental determination of its detection efficiency. Theoretically, it was investigated as $J^P = 0^+, 1^+$ or 2^+ state in the molecular and tetraquark state scenarios [16]. In this work, we perform the polarization analysis with a motivation to

suggest some spin observables for the spin and parity measurement for this structure.

II. POLARIZATION ANALYSIS

We consider an exact experiment of Z_{cs} production as observed in the unpolarized e^+e^- collisions. The center-of-mass (CM) energy of e^+e^- beams is set at about $\sqrt{s} = 4.7$ GeV, so the Z_{cs} production is dominated by the electric magnetic process, from which the e^+e^- annihilate into a virtual photon (γ^*), then it couples the $K^+ Z_{cs}^-$ final states. Without loss of generality, we assume that we detect the Z_{cs} state with a decay mode $Z_{cs}^- \rightarrow D_s^- D^{*0}$.

It is convenient for us to perform the Z_{cs} spin analysis and investigate the polarization transfer using the method of helicity amplitude. The Z_{cs} production and decay are described with helicity angles and amplitude as defined in Table I. The angles θ_1 and ϕ_1 are defined as the polar and azimuthal angles of Z_{cs} momentum in the e^+e^- CM system, with the z axis taken along the positron moving direction. While the angle θ_2 is spanned by the three momentum of Z_{cs} and D^{*0} ; here, D^{*0} is boosted to the Z_{cs} rest frame, and ϕ_2 is the angle spanned by the Z_{cs} production and decay planes, as shown in Fig. 1.

We perform the Z_{cs} spin analysis by calculating its spin density matrix (SDM), which encodes the complete polarization information and allow us to access it by studying the angular distribution of its final states $D^{*0} D_s^-$. Starting with the process of e^+e^- annihilation into a virtual photon, one has the γ^* SDM as $\rho(\gamma^*) = \text{diag}\{1, 0, 1\}$, since this electric magnetic process conserves the parity and helicity. One can see that the γ^* SDM deviates from the unit matrix, which indicates that the virtual photon has some degree of tensor polarization, while it is not longitudinally polarized.

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TABLE I. Definition of helicity angles and amplitudes for the Z_{cs} production and decay; here, λ_i ($i = 1, 2$) denotes the helicity values of corresponding particle.

Decay	Angles	Amplitudes
$e^+e^- \rightarrow \gamma^* \rightarrow K^+ Z_{cs}^-(\lambda_1)$	$\Omega_1(\theta_1, \phi_1)$	A_{λ_1}
$Z_{cs}^- \rightarrow D_s^- D^{*0}(\lambda_2)$	$\Omega_2(\theta_2, \phi_2)$	B_{λ_2}

The polarization transfer to the Z_{cs} can be estimated by calculation of the SDM in a straightforward manner with the method of helicity amplitude, under an assumption that Z_{cs} has the spin and parity $J^P = 0^-, 1^\pm, 2^\pm$, or $3^\pm$. Note that the 0^+ assignment is not allowed due to the fact that it violates the spin and parity in the process $e^+e^- \rightarrow \gamma^* \rightarrow K^+ Z_{cs}^-$. In the space of Z_{cs} spin configuration, calculation of Z_{cs} SDM, $\rho(Z_{cs})$, can be visualized as the linear transformation with the decay matrix M . It reads [17]

$$\rho(Z_{cs}) = M \cdot \rho(\gamma^*) \cdot M^\dagger, \quad (1)$$

with $M_{\lambda_1, m} = D_{m, \lambda_1}^1(\phi_1, \theta_1, 0)A_{\lambda_1}$; here, D_{m, λ_1}^J is an element of Wigner D -matrix. The number of independent helicity amplitude A_{λ_1} is constrained by the helicity conservation law. It yields the relations,

$$A_{-\lambda_1} = P(-1)^{J-1}A_{\lambda_1}, \quad (2)$$

for the J^P assignment to the Z_{cs} state. Due to the constraint imposed by the D_{m, λ_1}^1 matrix, the helicity amplitudes will be vanishing if $\lambda_1 > 1$. This means that we are only left with two nonvanishing components, A_0 and A_1 . For the assignments of $1^-, 2^+$, and 3^- , the above parity conserving relation will further yield $A_0 = 0$.

Information on a particle polarization is often characterized with a set of multipole parameters t_M^L , which are used to define its SDM in terms of the spherical tensor operators T_M^L . Due to the property of non-Hermitian T_M^L and the complex numbers t_M^L , it is not closely related the t_M^L parameters to the experimental measurements. Here, we choose an alternative Hermitian bases, Q_M^L , and real multipole parameters r_M^L to form the SDM for a spin- J particle [18]. It reads

FIG. 1. Orientation of helicity axes describing the Z_{cs} production and decay.

$$\rho(Z_{cs}) = \frac{r_0^0}{2J+1} \left(I + 2J \sum_{L=1}^{2J} \sum_{M=-L}^L r_M^L Q_M^L \right), \quad (3)$$

where I is a unit matrix with $(2J+1) \times (2J+1)$ dimensions, and r_0^0 is a unpolarization cross section, with $r_0^0 = \text{Tr}\rho(Z_{cs})$.

To form a SDM for the spin- J particle, one needs at the most the $J(J+2)$ real parameters, r_M^L , with the highest rank $L = 2J$. The number of independent parameters r_M^L will be greatly reduced if one imposes the parity conservation to the Z_{cs} production process and the rank condition to the calculation of Z_{cs} SDM. The parity conservation yields the symmetry relation,

$$\rho_{-m, -m'} = (-1)^{m-m'} \rho_{m, m'}. \quad (4)$$

The rank condition requires that the rank of Z_{cs} SDM should be less or equal to that of virtual photon, namely

$$\text{Rank}(\rho(Z_{cs})) \leq 2.$$

The lists of nonvanishing real parameters r_M^L are given in Table II, and their expressions in terms of helicity amplitudes are given in Appendices A–G.

The degree of Z_{cs} polarization measures the departure of its SDM from isotropy. We take the definition for the spin- J particle as [19]

$$d = \frac{1}{\sqrt{2J}} [(2J+1)\text{Tr}\tilde{\rho}^2 - 1]^{1/2}, \quad (5)$$

where $\tilde{\rho}$ is the normalized SDM, and its angular dependence has been integrated out. With the obtained SDM, the degree of polarization for different J^P assignment to Z_{cs} are calculated to be

TABLE II. List of real multipole parameters (r_M^L) and analyzing powers ($\mathcal{A}_M^L$) for different J^P assignments to the Z_{cs} particle.

J^P	1^+	1^-	2^+	2^-	3^+	3^-
r_M^L	$r_{0,1,2}^2$	$r_{0,2}^2$	$r_{0,2}^2$ $r_{0,2}^4$	$r_{0,1,2}^2$ $r_{0,1,2}^4$	$r_{0,1,2}^2$ $r_{0,1,2}^4$ $r_{0,1,2}^6$	$r_{0,2}^2$ $r_{0,2}^4$ $r_{0,2}^6$
$\mathcal{A}_M^L$	$\mathcal{A}_{0,1,2}^2$	$\mathcal{A}_{0,2}^2$	$\mathcal{A}_{0,2}^2$ $\mathcal{A}_{0,2}^4$	$\mathcal{A}_{0,1,2}^2$ $\mathcal{A}_{0,1,2}^4$	$\mathcal{A}_{0,1,2}^2$ $\mathcal{A}_{0,1,2}^4$ $\mathcal{A}_{0,1,2}^6$	$\mathcal{A}_{0,2}^2$ $\mathcal{A}_{0,2}^4$ $\mathcal{A}_{0,2}^6$

$$d = \begin{cases} \frac{\sqrt{7}}{4} & \text{for } 1^-, \\ \frac{1}{2} \sqrt{\frac{4+r(7r-8)}{(1+2r)^2}} & \text{for } 1^+, \\ \frac{1}{4} \sqrt{\frac{17}{2}} & \text{for } 2^+, \\ \frac{1}{2\sqrt{2}} \sqrt{\frac{8-8r+17r^2}{(1+2r)^2}} & \text{for } 2^-, \\ \frac{1}{2\sqrt{3}} \sqrt{\frac{12+r(27r-8)}{(1+2r)^2}} & \text{for } 3^+, \\ \frac{3}{4} & \text{for } 3^-, \end{cases} \quad (6)$$

where $r = |A_1/A_0|$. One can see that the Z_{cs} has a degree of polarization larger than 60% for the assignment of $J^P = 1^-, 2^+$, and 3^- . The Z_{cs} polarization originates from the polarization transfer of the transverse polarization of virtual photon.

To access the Z_{cs} polarization information, one needs to study the implication of the D^{*0} angular distribution in its decaying final states. Thus, the decay $Z_{cs}^- \rightarrow D_s^- D^{*0}$ serves as a polarimetry. Likewise, we investigate the polarization transfer from the Z_{cs} to D^{*0} by calculation of the SDM for the D^{*0} particle. It reads

$$\rho(D^{*0}) = N \cdot \rho(Z_{cs}) \cdot N^\dagger, \quad (7)$$

with decay matrix $N_{\lambda_2, \lambda_1} = B_{\lambda_2} D_{\lambda_1, \lambda_2}^J(\phi_2, \theta_2, 0)$. Then the joint angular distribution for the two sequential

decay can be calculated by taking trace of $\rho(D^{*0})$; thus, one has

$$\mathcal{I}(\Omega_1, \Omega_2) = \mathcal{I}_0 \left(1 + \sum_{L=1}^{2J} \sum_{M=-L}^L r_M^L \mathcal{A}_M^L \right), \quad (8)$$

with

$$\mathcal{I}_0 = \frac{r_0^0}{2J+1} \text{Tr} N_f N_f^\dagger, \quad (9)$$

$$\mathcal{A}_M^L = \frac{2J \text{Tr} N_f Q_M^L N_f^\dagger}{\text{Tr} N_f N_f^\dagger}, \quad (10)$$

$$\mathcal{A}_0^0 = \text{Tr} N_f N_f^\dagger, \quad (11)$$

where $\mathcal{I}_0$ is the cross section corresponding to the unpolarization case, and $\mathcal{A}_M^L$ is always known as the analyzing power. The list of nonvanishing $\mathcal{A}_M^L$ is given in Table II, and their expressions in terms of amplitudes B_{λ_2} and angles Ω_2 are given in Appendices A–G.

To figure out the Z_{cs} spin and parity, an intuitive way is to check the joint angular distribution in respect of $\cos \theta_1$ and $\cos \theta_2$. It is easy to get it by integrating out the ϕ_2 in the $\mathcal{I}(\Omega_1, \Omega_2)$ distribution. Then one has

$$\frac{dN}{dx_1 dx_2} \propto \begin{cases} (3+5r_2) + (3+r_2)x_1^2 + (r_2-1)(5x_1^2+1)x_2^2, & \text{for } 1^+ \\ (1+x_1^2)(1+x_2^2), & \text{for } 1^- \\ (1+x_1^2)(1-3x_2^2+4x_2^4), & \text{for } 2^+ \\ 2r_1(x_1^2+1)[(4r_2-3)x_2^4-3(r_2-1)x_2^2+r_2] \\ + (x_1^2-1)[3(4r_2-3)x_2^4+(6-12r_2)x_2^2-1], & \text{for } 2^- \\ 4(x_1^2-1)[3r_2(1-5x_2^2)^2(x_2^2-1)-2(5x_2^3-3x_2)^2] \\ + r_1(x_1^2+1)[r_2(225x_2^6-305x_2^4+111x_2^2+1)-6(1-5x_2^2)^2(x_2^2-1)], & \text{for } 3^+ \\ (1+x_1^2)(1+111x_2^2-305x_2^4+225x_2^6), & \text{for } 3^- \end{cases}$$

where $x_1 = \cos \theta_1, x_2 = \cos \theta_2, r_1 = |A_1/A_0|^2, r_2 = |B_1/B_0|^2$.

For the cases of $1^-, 2^+$, and 3^- assignments to the Z_{cs} state, one can see that the angular distributions are factored out from the undetermined helicity amplitudes in the above equations, and their patterns are ambiguously determined according to the Z_{cs} spin and parity quantum numbers. Due to the factor that the virtual photon is produced from the e^+e^- annihilation, and the vector coupling to the e^-/e^+ Dirac spinor conserves their helicities, the component of helicity zero is negligible. This leads to deviation of the SDM of virtual photon from the unit matrix. This is equivalent to the description of tensor polarization for the

vector particles, which is further manifested in the nontrivial pattern of the Z_{cs} angular distributions. Especially, the polarization transfer to the Z_{cs} state leads to it acquiring some degree of longitudinal polarizations, and this gives rise to the unflat profile of angular distribution for the decayed particles. In experiment, one can get knowledge about these quantum numbers by interpreting the implication of angular distribution $\frac{dN}{dx_2}$, which was shown in Fig. 2 for the $J^P = 1^-, 2^+$, and 3^- cases.

For the cases of $J^P = 1^+, 2^-$, and 3^+ assignments, the angular distribution is dependent on the amplitudes ratios of r_1 and r_2 , which are related to the study of polarization

FIG. 2. Angular distribution of $\frac{dN}{d\cos\theta_2}$ for the spin and parity assignments $J^P = 1^-, 2^+, \text{ and } 3^-$.

transfer to the Z_{cs} state. However, the polarization information is inaccessible directly in the modern electromagnetic spectrometers, since they are often designed with a general purpose to detect the charged particle and neutral showers for high energy experiments. In the considered e^+e^- collision experiment, we express the spin transfer to the Z_{cs} states with a set of real multipole parameters, r_M^L , as we list in Table II. Nonetheless, one can study the implication of analyzing power for the subsequent decay, $Z_{cs} \rightarrow D_s D^{*0}$, to get the polarization information. In experiments, one can require the mass of $D_s D^{*0}$ falling into the Z_{cs} mass region and look at the distribution of an analyzing power with respect of the Z_{cs} polar angle. This is equivalent to looking at the distribution of first moments $\langle a_M^L \rangle \sim \cos\theta_1$; here, a_M^L is the angular-dependent factor of the analyzing power $\mathcal{A}_M^L$.

Using the joint angular distribution $\mathcal{I}(\Omega_1, \Omega_2)$, we get

$$\langle a_M^L \rangle \sim \int a_M^L(\Omega_2) \mathcal{I}(\Omega_1, \Omega_2) d\Omega_2 \sim r_M^L(x), \quad (12)$$

with $x = \cos\theta_1$. Explicitly, we list the first moments distribution $\langle a_M^L \rangle$ versus x in Table III. For the components

$M = 0$, one has the first moments $\langle a_0^L \rangle = 1 + \alpha_L x$ with $L = 2, 4, 6$; here, α_L is an angular distribution parameter, determined by the helicity amplitudes A_{λ_1} . They are given in the Appendix H.

III. APPLICATIONS

We consider the discovery process of $e^+e^- \rightarrow K^+ Z_{cs}^-, Z_{cs}^- \rightarrow D_s^- D^{*0}$, in which the $Z_{cs}(3985)^-$ state was observed for the first time in the recoiling mass spectrum of $RM(K^+)$ [14]. The mass and width of the Z_{cs} state was measured based on the 85 observed signal events. Analysis on the angular distributions needs a large size of data events. In the present, the hints on the spin and parity numbers can only be investigated from the mass spectrum based on the amplitude model, using the dependence of the line shape of Z_{cs} state on the spin and parity numbers.

A. Amplitude model

We construct the amplitude model with the method of the covariant tensor formalism, in which we consider the hypotheses that the Z_{cs} state has the spin and parity numbers as $J^P = 0^-, 1^\pm, 2^\pm$, and $3^\pm$, respectively. The possibility of other higher spin assignments should be depressed by the factor of the centrifugal barrier. We take the 0^- hypothesis as an example to illustrate the construction of amplitude model. For the first decay, $\gamma^*(p_1) \rightarrow Z_{cs}^-(p_2) K^+(p_3)$; here, p_1, p_2 , and p_3 denote the momenta of the corresponding particle, and only one partial wave, $L = 1$, is allowed due to the decay conserves the spin and parity. The form of coupling vertex is taken as $f_2 p_3^\mu$ with a complex coupling constant f_2 . For the second decay, $Z_{cs}^-(p_2) \rightarrow D_s^-(p_4) D^{*0}(p_5)$, the coupling vertex can be written as $f_5 p_5^\nu$ with a complex parameter f_5 . Then the joint amplitude reads

$$\mathcal{M}_{0^-} = \frac{f_2 f_5 (\epsilon^* \cdot p_3) (\epsilon_{D^*} \cdot p_5)}{m_{D_s D^*}^2 - M^2 - iM\Gamma}, \quad (13)$$

TABLE III. The analyzing power a_M^L and its first moments distribution $\langle a_M^L \rangle$ in terms of $x = \cos\theta_1$. The symbols $\checkmark$ below the J^P assignment indicate that it has the corresponding analyzing power.

a_M^L	$\langle a_M^L \rangle$	$J^P = 1^+$	1^-	2^+	2^-	3^+	3^-
$a_2^2 = \sin^2 \theta_2 \cos(2\phi_2)$	$1 - x^2$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$a_1^2 = \sin(2\theta_2) \cos(\phi_2)$	$x\sqrt{1-x^2}$	$\checkmark$			$\checkmark$	$\checkmark$	
$a_0^2 = 3 \cos(2\theta_2)$	$1 + \alpha_2 x^2$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$a_2^4 = \sin^2 \theta_2 [7 \cos(2\theta_2) + 5] \cos(2\phi_2)$	$1 - x^2$			$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$a_1^4 = \sin \theta_2 [9 \cos(\theta_2) + 7 \cos(3\theta_2)] \cos \phi_2$	$x\sqrt{1-x^2}$				$\checkmark$	$\checkmark$	
$a_0^4 = 20 \cos(2\theta_2) + 35 \cos(4\theta_2)$	$1 + \alpha_4 x^2$			$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$
$a_2^6 = \sin^2 \theta_2 [60 \cos(2\theta_2) + 33 \cos(4\theta_2) + 35] \cos(2\phi_2)$	$1 - x^2$					$\checkmark$	$\checkmark$
$a_1^6 = [5 \sin(2\theta_2) + 12 \sin(4\theta_2) + 33 \sin(6\theta_2)] \cos(\phi_2)$	$x\sqrt{1-x^2}$					$\checkmark$	
$a_0^6 = 105 \cos(2\theta_2) + 126 \cos(4\theta_2) + 231 \cos(6\theta_2)$	$1 + \alpha_6 x^2$					$\checkmark$	$\checkmark$

TABLE IV. The tensor forms of coupling vertexes for the two decays. Here, $p_i (i = 1, \dots, 5)$ are the momenta of corresponding particles. The symbols f_i denote the coupling constants, and they are taken as complex numbers. The Greek letters are the Lorentz indexes.

Decay	$\gamma^*(p_1) \rightarrow Z_{cs}^-(p_2)K^+(p_3)$	$Z_{cs}^-(p_2) \rightarrow D_s^-(p_4)D^{*0}(p_5)$
0^-	$f_2 p_3^\mu$	$f_5 p_5^\nu$
1^-	$-f_{10} \epsilon_{\alpha\beta\mu\nu} p_2^\alpha p_3^\beta$	$-f_{11} \epsilon_{\alpha\beta\mu'\nu'} p_2^\alpha p_4^\beta$
1^+	$p_{3\mu} p_{3\nu} f_6 + g_{\mu\nu} f_7$	$p_{4\mu'} p_{4\nu'} f_8 + g_{\mu'\nu'} f_9$
2^-	$p_{3\mu} p_{3\nu} p_{3\alpha} f_{18} + p_{3\mu} g_{\nu\alpha} f_{19}$	$p_{4\mu'} p_{4\nu'} p_{4\alpha'} f_{20} + p_{4\mu'} g_{\nu'\alpha'} f_{21}$
2^+	$-\epsilon_{\lambda\beta\mu\alpha} p_{1\lambda} p_{2\beta} p_{3\nu} f_{16}$	$-\epsilon_{\lambda\beta\mu'\alpha'} p_{2\lambda} p_{4\beta} p_{4\nu'} f_{17}$
3^-	$-\epsilon_{\lambda\eta\mu\beta} p_{1\lambda} p_{3\eta} p_{3\nu} p_{3\alpha} f_{28}$	$-\epsilon_{\lambda\eta\mu'\beta'} p_{2\lambda} p_{4\eta} p_{4\nu'} p_{4\alpha'} f_{29}$
3^+	$p_{3\mu} p_{3\nu} p_{3\alpha} p_{3\beta} f_{24} + p_{3\mu} p_{3\nu} g_{\alpha\beta} f_{25}$	$p_{4\mu'} p_{4\nu'} p_{4\alpha'} p_{4\beta'} f_{26} + p_{4\mu'} p_{4\nu'} g_{\alpha'\beta'} f_{27}$

where $\epsilon(\epsilon_{D^*})$ is the polarization vector for the virtual photon (D^{*0}).

The tensor forms of coupling vertexes for other hypotheses, $J^P = 1^\pm, 2^\pm$, and $3^\pm$, are summarized in Table IV, which are constructed with the Levi-Civita tensor $\epsilon_{\alpha\beta\mu\nu}$ and p^μ , respecting the CP conservation. The joint amplitude for the two-step sequential decay is constructed by multiplying the propagator to contract the Lorentz indexes of the Z_{cs} resonance. The propagators are parametrized with function of relativistic Breit-Wigner, and they are given in Appendix I. It is worthy noting that the structure of tensor $\tilde{g}_{\mu\nu}$ in the Breit-Wigner is dependent on the spin assignment to the Z_{cs} state; this leads to the line shape of Z_{cs} state dependent on the spin quantum numbers. This is the idea we adopt to investigate the spin hints from the mass spectrum, but the conclusive spin and polarization analysis is needed to directly perform the angular analysis if the large size of data events is available further.

One feature in the joint amplitude is that the coupling constants, f_i , can be factored out from the part of tensor amplitude for hypotheses, $J^P = 0^-, 1^-, 2^+$, and 3^- . This is equivalent to the separation of the angular distribution as discussed in the polarization analysis with the helicity amplitude. This leads to the fact that the spin analysis for these four assignments is model independent. However, to distinguish them from other spin assignments, i.e., $J = 1^+, 2^-$, and 3^+ , one needs to do amplitude analysis to determine the coupling constants by fitting the mass spectrum of data events.

B. Fit to data events

We determine the coupling constant for the spin assignments, $1^+, 2^-$, and 3^+ , using the mass spectrum as reported by the BESIII collaboration [14]. The Z_{cs} state was searched in the four datasets within the e^+e^- center-of-mass energy from $\sqrt{s} = 4.681\text{--}4.698$ GeV. Only the significant Z_{cs} state was observed at the first dataset.

We take the recoiling mass spectrum of K^+ at this energy point to do the amplitude analysis. The combinatory background shape is modeled by the probability density distribution,

$$M_i^{\text{bg}} = g_{\text{bg}} \exp[-a \cdot m_i - b \cdot m_i^2], \quad (14)$$

where m_i is the observed recoil mass of K^+ in the i -th bin, and the parameters $g_{\text{bg}} = 1.07 \pm 0.11$ GeV $^{-1/2}$, $a = -6.44 \pm 1.08$ GeV $^{-1}$, and $b = 1.37 \pm 0.34$ GeV $^{-2}$ are determined by fitting the line shape of combinatory background events as reported in the experiment. Then the observed events in the i -th bin can be calculated with

$$N_i^{\text{th}} = c_0 \left(M_i^{\text{bg}} + \int \frac{1}{(2\pi)^5} \frac{1}{16s} |\overline{\mathcal{M}_{J^P}}|^2 |\vec{p}_{D_s^-}^*| |\vec{p}_{K^+}| \times d\Omega_{D_s^-}^* d\Omega_{K^+} \right), \quad (15)$$

where c_0 is an overall parameter, and $\mathcal{M}_{J^P}$ is the amplitude corresponding to the spin and parity assignments to the Z_{cs} state. With s being the center-of-mass energy, $|\vec{p}_{D_s^-}^*|$ and $\Omega_{D_s^-}^*$ are the momentum and solid angle of D_s^- in the rest frame of $D^*D_s^-$ system, respectively. $|\vec{p}_{K^+}|$ and Ω_{K^+} are the momentum and solid angle of K^+ , respectively, and the overline means the spin average over the initial γ^* and the spin sum over the final D^* .

We determine the coupling constants, f_i , using the least square method. The object function to be minimized is defined as

$$\chi^2 = \sum_i \frac{(N_i^{\text{dt}} - N_i^{\text{th}})^2}{N_i^{\text{dt}}}, \quad (16)$$

where N_i^{dt} is the number of observed events in the i -bin, and the uncertainty of observed events follows the Poisson

FIG. 3. Fit results of the recoil mass $RM(K)$ for the different hypotheses of Z_{cs} spin and parity, (a) 1^+ , (b) 2^- , and (c) 3^+ . The data events and background shape are quoted from Ref. [14].

distribution as reported in experiment, and the sum runs over all bins in the fit range.

Since the Z_{cs} state was observed at the mass threshold of $D_s^- D^{*0}/D^{*-} D^0$, and the reported line shape above 4.02 GeV can be explained with the combinatory background events, we limit the fit range to $RM(K^+) < 4.02$ GeV. We determine the coupling constants f_i for the each spin assignment 1^+ , 2^- , or 3^+ , in which we have four parameters in the joint amplitude. Due to the fact that the number of observed events is defined to be proportional to the density of amplitude, it will introduce a non-physical parameter in the Eq. (15). This implies that only the relative values of coupling constants can be chosen as the physics parameters in the fit. From Table IV, we chose the parameters f_6 , f_8 , f_{18} , f_{20} , f_{24} , and f_{26} as reference parameters by fixing the value at 1 in the fit. In this way, the other determined parameters are understood as the relative values to the reference parameters.

In the fit, the mass and width of the Z_{cs} state are fixed to the reported values for each spin and parity assignment, namely $M = 3982.5$ MeV and $\Gamma = 12.8$ MeV. The parameters are obtained by minimizing the χ^2 object function, and they are determined to be $f_7 \approx 0.186 \pm 0.035$, $f_9 \approx -0.074 \pm 0.018$ for the hypothesis $J^P = 1^+$, and $f_{19} = 0.299 \pm 0.036$, $f_{21} = -0.278 \pm 0.067$ for the 2^- case, and $f_{25} = 0.012 \pm 0.002$, $f_{27} = 0.297 \pm 0.047$ for the 2^- case. Here, we take the coupling parameters as real numbers. In general, they are able to be taken as complex numbers, but this choice will introduce some redundant parameters, relative to the limited data events available.

The fit results are displayed in Fig. 3, and they are in good agreement with the data events for the 1^+ and 2^- hypotheses. The fit goodness for these two cases is almost the same, but it gets worse for the 3^- hypothesis. The line shape for the 3^- hypothesis gets flat above the Z_{cs} mass, since the momentum dependence in the Breit-Wigner function introduces more high power terms in the line shape, and this leads to the enhancement with the mass increase. In view of fit goodness, the possibility of 3^- assignment can be ruled out.

C. Monte-Carlo simulations

We generate an ensemble of Monte-Carlo events using the amplitude model, with the coupling constant fixed to the determined values. Figure 4 shows the helicity angular distribution $\cos \theta_2$ for the assignments of $J^P = 0^-, 1^-, 2^+$, and 3^- . One can see that these distributions are distinguishable from each other, and they are consistent with that shown in Fig. 2. To compare with the pattern of observed angular distribution, one can figure out the spin and parity assignment of Z_{cs} state. The advantage using these distributions is that they are model independent and are easily checked in experiment.

However, to distinguish the hypotheses of spin and parity $1^+, 2^-$, and 3^+ , one needs an analysis of data events, especially the angular distribution. With the coupling constants obtained from the fit to the data spectrum, we present some distributions with Monte-Carlo events as shown in Fig. 5, serving as the footprint mark. If we check the moment a_2^0 distribution, the distributions are almost indistinguishable for these three J^P assignments. One needs further to check the distribution of high moments, such as a_4^0 and a_6^0 , and the 2^- case can be figured out. Then we can further distinguish the 1^+ from 3^+ hypothesis by checking the $\cos \theta_2$ distribution as shown Fig. 5(d).

FIG. 4. Angular distribution of generated events for the spin and parity assignments $J^P = 0^-, 1^-, 2^+$, and 3^- .

FIG. 5. Distributions of the first moments a_M^L versus $\cos \theta_1$ for the different spin and parity (J^P) assignments. The plots are filled with a set of Monte-Carlo events as described in the text.

IV. SUMMARY

We carry out an analysis on the Z_{cs} polarization for the motivation of measuring the Z_{cs} spin and parity in the future experiment. We consider the production process $e^+e^- \rightarrow \gamma^* \rightarrow K^\pm Z_{cs}^\mp, Z_{cs}^\mp \rightarrow D_s^- D^{*0}$ or $D_s^{*-} D^0$ at the e^+e^- annihilation experiment. As the spin and parity conservation is concerned for the Z_{cs} production, the quantum numbers of $J^P = 0^+$ are not allowed for the virtual photon coupled to the $K^\pm Z_{cs}^\mp$ states. Hence, the analysis is performed for the scenarios of $J^P = 0^-, 1^\pm$, and $2^\pm$. Due to fact that the virtual photon from the e^+e^- annihilation is of tensor polarization, the electromagnetic coupling to the Z_{cs} state will transfer some degree of polarization to the Z_{cs} of nonzero spin state. The subsequent Z_{cs} decay can be used as polarimeter to manifest its spin and parity by studying the angular distribution of decayed particles. The spin assignments $0^-, 1^-, 2^+$, and 3^- can be distinguishable by checking the angular distribution of the Z_{cs} decayed particle, which are model independent, while to figure out the $1^+, 2^-,$ and 3^+ hypotheses, one needs to perform the amplitude analysis. With a few data events available, we perform a fit to the mass spectrum and get hints for these spin hypotheses. Using obtained parameters in the amplitude model, an ensemble of toy MC events is generated, and we present some distributions as a

footprint mark to figure out these spin hypotheses. Some moments distributions are formulated and suggested as polarization observables to manifest the Z_{cs} spin and parity in the experiment if a large size dataset is available in the future.

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APPENDIX A: $J^P = 0^-$

Multipole parameter r_0^0 is expressed in terms of helicity amplitude A_0 for the Z_{cs} $J^P = 0^-$ hypothesis,

$$r_0^0 = |A_0|^2 \sin^2 \theta_1.$$

APPENDIX B: $J^P = 1^+$

There are four multipole parameters (r_M^L) and analyzing powers (A_M^L) for the Z_{cs} $J^P = 1^+$ hypothesis. The definitions are given in Table II and Eq. (8). A_{λ_1} and B_{λ_2} are helicity amplitudes as defined in Table I.

$$\begin{aligned}
r_0^0 &= \frac{1}{4} \{2|A_0|^2 \sin^2 \theta_1 + |A_1|^2 [\cos(2\theta_1) + 3]\} \\
r_0^0 r_2^2 &= \frac{\sqrt{3}}{4} |A_1|^2 \sin^2 \theta_1 \\
r_0^0 r_1^2 &= -\frac{\sqrt{3}}{4} \text{Re}(A_1 A_0^*) \sin(2\theta_1) \\
r_0^0 r_0^2 &= \frac{1}{8} \{|A_1|^2 [\cos(2\theta_1) + 3] - 4|A_0|^2 \sin^2 \theta_1\}.
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_0^0 &= 2|B_1|^2 + |B_0|^2, \\
\mathcal{A}_0^0 \mathcal{A}_2^2 &= \sqrt{3}(|B_1|^2 - |B_0|^2) \sin^2 \theta_2 \cos(2\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_1^2 &= \sqrt{3}(|B_1|^2 - |B_0|^2) \sin(2\theta_2) \cos(\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_0^2 &= \frac{1}{2} (|B_1|^2 - |B_0|^2) [3 \cos(2\theta_2) + 1].
\end{aligned}$$

APPENDIX C: $J^P = 1^-$

There are three multipole parameters (r_M^L) and analyzing powers ($\mathcal{A}_M^L$) for the Z_{cs} $J^P = 1^-$ hypothesis. The definitions are referred to in Table II and Eq. (8). A_{λ_1} and B_{λ_2} are helicity amplitudes as defined in Table I.

$$\begin{aligned}
r_0^0 &= \frac{1}{4} |A_1|^2 [\cos(2\theta_1) + 3], \\
r_0^0 r_2^2 &= -\frac{1}{4} \sqrt{3} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_0^2 &= \frac{1}{8} |A_1|^2 [\cos(2\theta_1) + 3],
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_0^0 &= 2|B_1|^2, \\
\mathcal{A}_0^0 \mathcal{A}_2^2 &= \sqrt{3} |B_1|^2 \sin^2 \theta_2 \cos(2\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_0^2 &= \frac{1}{2} |B_1|^2 [3 \cos(2\theta_2) + 1].
\end{aligned}$$

APPENDIX D: $J^P = 2^+$

There are five multipole parameters (r_M^L) and analyzing powers ($\mathcal{A}_M^L$) for the Z_{cs} $J^P = 2^+$ hypothesis. The definitions are referred to in Table II and Eq. (8). A_{λ_1} and B_{λ_2} are helicity amplitudes as defined in Table I.

$$\begin{aligned}
r_0^0 &= \frac{1}{4} |A_1|^2 [\cos(2\theta_1) + 3], \\
r_0^0 r_2^4 &= \frac{1}{2} \sqrt{\frac{5}{14}} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_0^4 &= -\frac{1}{2\sqrt{14}} |A_1|^2 [\cos(2\theta_1) + 3], \\
r_0^0 r_2^2 &= -\frac{1}{4} \sqrt{\frac{15}{14}} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_0^2 &= -\frac{1}{8} \sqrt{\frac{5}{14}} |A_1|^2 [\cos(2\theta_1) + 3].
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_0^0 &= 2|B_1|^2, \\
\mathcal{A}_0^0 \mathcal{A}_2^4 &= -\sqrt{\frac{10}{7}} |B_1|^2 \sin^2 \theta_2 [7 \cos(2\theta_2) + 5] \cos(2\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_0^4 &= -\frac{1}{4\sqrt{14}} |B_1|^2 [20 \cos(2\theta_2) + 35 \cos(4\theta_2) + 9], \\
\mathcal{A}_0^0 \mathcal{A}_2^2 &= -\sqrt{\frac{30}{7}} |B_1|^2 \sin^2 \theta_2 \cos(2\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_0^2 &= -\sqrt{\frac{5}{14}} |B_1|^2 [3 \cos(2\theta_2) + 1].
\end{aligned}$$

APPENDIX E: $J^P = 2^-$

There are seven multipole parameters (r_M^L) and analyzing powers ($\mathcal{A}_M^L$) for the Z_{cs} $J^P = 2^-$ hypothesis. The definitions are referred to in Table II and Eq. (8). A_{λ_1} and B_{λ_2} are helicity amplitudes as defined in Table I.

$$\begin{aligned}
r_0^0 &= \frac{1}{4} \{2|A_0|^2 \sin^2 \theta_1 + |A_1|^2 [\cos(2\theta_1) + 3]\}, \\
r_0^0 r_2^4 &= -\frac{1}{2} \sqrt{\frac{5}{14}} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_1^4 &= \frac{1}{4} \sqrt{\frac{15}{7}} \text{Re}(A_1 A_0^*) \sin(2\theta_1), \\
r_0^0 r_0^4 &= \frac{3|A_0|^2 \sin^2 \theta_1 - |A_1|^2 [\cos(2\theta_1) + 3]}{2\sqrt{14}},
\end{aligned}$$

$$\begin{aligned}
r_0^0 r_2^2 &= \frac{1}{4} \sqrt{\frac{15}{14}} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_1^2 &= -\frac{1}{4} \sqrt{\frac{5}{14}} \text{Re}(A_1 A_0^*) \sin(2\theta_1), \\
r_0^0 r_0^2 &= -\frac{1}{8} \sqrt{\frac{5}{14}} \{4|A_0|^2 \sin^2 \theta_1 + |A_1|^2 [\cos(2\theta_1) + 3]\}.
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_0^0 &= 2|B_1|^2 + |B_0|^2, \\
\mathcal{A}_0^0 \mathcal{A}_2^4 &= \frac{1}{2} \sqrt{\frac{5}{14}} (3|B_0|^2 - 4|B_1|^2) \sin^2 \theta_2 \cos(2\phi_2) \\
&\quad \times [7 \cos(2\theta_2) + 5], \\
\mathcal{A}_0^0 \mathcal{A}_1^4 &= \frac{1}{4} \sqrt{\frac{5}{7}} (3|B_0|^2 - 4|B_1|^2) \sin \theta_2 \cos \phi_2 \\
&\quad \times [9 \cos(\theta_2) + 7 \cos(3\theta_2)], \\
\mathcal{A}_0^0 \mathcal{A}_0^4 &= \frac{(3|B_0|^2 - 4|B_1|^2)[20 \cos(2\theta_2) + 35 \cos(4\theta_2) + 9]}{16\sqrt{14}}, \\
\mathcal{A}_0^0 \mathcal{A}_2^2 &= -\sqrt{\frac{30}{7}} (|B_0|^2 + |B_1|^2) \sin^2 \theta_2 \cos(2\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_1^2 &= -\sqrt{\frac{30}{7}} (|B_0|^2 + |B_1|^2) \sin(2\theta_2) \cos \phi_2, \\
\mathcal{A}_0^0 \mathcal{A}_0^2 &= -\sqrt{\frac{5}{14}} (|B_0|^2 + |B_1|^2) [3 \cos(2\theta_2) + 1].
\end{aligned}$$

APPENDIX F: $J^P = 3^+$

There are 10 multipole parameters (r_M^L) and analyzing powers ($\mathcal{A}_M^L$) for the Z_{cs} $J^P = 3^+$ hypothesis. The definitions are referred to in Table II and Eq. (8). A_{λ_1} and B_{λ_2} are helicity amplitudes as defined in Table I.

$$\begin{aligned}
r_0^0 &= \frac{1}{4} \{2|A_0|^2 \sin^2 \theta_1 + |A_1|^2 [\cos(2\theta_1) + 3]\}, \\
r_0^0 r_2^6 &= \frac{1}{4} \sqrt{\frac{35}{33}} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_1^6 &= -\frac{5}{12} \sqrt{\frac{7}{11}} \text{Re}(A_1 A_0^*) \sin(2\theta_1), \\
r_0^0 r_0^6 &= \frac{5\{3|A_1|^2 [\cos(2\theta_1) + 3] - 8|A_0|^2 \sin^2(\theta_1)\}}{24\sqrt{22}}, \\
r_0^0 r_2^4 &= -\frac{1}{2} \sqrt{\frac{5}{33}} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_1^4 &= \frac{1}{4} \sqrt{\frac{5}{11}} \text{Re}(A_1 A_0^*) \sin(2\theta_1), \\
r_0^0 r_0^4 &= \frac{12|A_0|^2 \sin^2(\theta_1) + |A_1|^2 (\cos(2\theta_1) + 3)}{8\sqrt{33}}, \\
r_0^0 r_2^2 &= \frac{1}{2\sqrt{6}} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_1^2 &= -\frac{1}{12} \text{Re}(A_1 A_0^*) \sin(2\theta_1), \\
r_0^0 r_0^2 &= \frac{-8|A_0|^2 \sin^2(\theta_1) - 3|A_1|^2 [\cos(2\theta_1) + 3]}{24\sqrt{2}}.
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_0^0 &= 2|B_1|^2 + |B_0|^2, \\
\mathcal{A}_0^0 \mathcal{A}_2^6 &= -\frac{5}{128} \sqrt{\frac{105}{11}} (2|B_0|^2 - 3|B_1|^2) \sin^2 \theta_2 \{60 \cos(2\theta_2) \\
&\quad + 33 \cos(4\theta_2) + 35\} \cos(2\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_1^6 &= -\frac{5}{128} \sqrt{\frac{21}{22}} (2|B_0|^2 - 3|B_1|^2) \{5 \sin(2\theta_2) \\
&\quad + 12 \sin(4\theta_2) + 33 \sin(6\theta_2)\} \cos(\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_0^6 &= -\frac{5}{256\sqrt{22}} (2|B_0|^2 - 3|B_1|^2) \{105 \cos(2\theta_2) \\
&\quad + 126 \cos(4\theta_2) + 231 \cos(6\theta_2) + 50\}, \\
\mathcal{A}_0^0 \mathcal{A}_2^4 &= \frac{1}{4} \sqrt{\frac{15}{11}} (3|B_0|^2 + |B_1|^2) \sin^2 \theta_2 [7 \cos(2\theta_2) + 5] \\
&\quad \times \cos(2\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_1^4 &= \frac{1}{8} \sqrt{\frac{15}{22}} (3|B_0|^2 + |B_1|^2) [2 \sin(2\theta_2) \\
&\quad + 7 \sin(4\theta_2)] \cos(\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_0^4 &= \frac{1}{32} \sqrt{\frac{3}{11}} (3|B_0|^2 + |B_1|^2) [20 \cos(2\theta_2) \\
&\quad + 35 \cos(4\theta_2) + 9], \\
\mathcal{A}_0^0 \mathcal{A}_2^2 &= -\sqrt{\frac{3}{2}} (2|B_0|^2 + 3|B_1|^2) \sin(\theta_2)^2 \cos(2\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_1^2 &= -\sqrt{6} (2|B_0|^2 + 3|B_1|^2) \sin(\theta_2) \cos(\theta_2) \cos(\phi_2), \\
\mathcal{A}_0^0 \mathcal{A}_0^2 &= -\frac{(2|B_0|^2 + 3|B_1|^2)(3 \cos(2\theta_2) + 1)}{2\sqrt{2}}.
\end{aligned}$$

APPENDIX G: $J^P = 3^-$

There are seven multipole parameters (r_M^L) and analyzing powers ($\mathcal{A}_M^L$) for the Z_{cs} $J^P = 3^-$ hypothesis. The definitions are referred to in Table II and Eq. (8). A_{λ_1} and B_{λ_2} are helicity amplitudes as defined in Table I.

$$\begin{aligned}
r_0^0 &= \frac{1}{4} |A_1|^2 [\cos(2\theta_1) + 3], \\
r_0^0 r_2^6 &= -\frac{1}{4} \sqrt{\frac{35}{33}} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_0^6 &= \frac{5|A_1|^2 [\cos(2\theta_1) + 3]}{8\sqrt{22}}, \\
r_0^0 r_2^4 &= \frac{1}{2} \sqrt{\frac{5}{33}} |A_1|^2 \sin^2 \theta_1, \\
r_0^0 r_0^4 &= \frac{|A_1|^2 (\cos(2\theta_1) + 3)}{8\sqrt{33}}, \\
r_0^0 r_2^2 &= -\frac{|A_1|^2 \sin^2 \theta_1}{2\sqrt{6}},
\end{aligned}$$

APPENDIX H: ANGULAR DISTRIBUTION PARAMETERS

Here are the lists of $\alpha_L (L = 2, 4, 6)$ values defined in Table III:

$$r_0^0 r_0^2 = -\frac{|A_1|^2 (\cos(2\theta_1) + 3)}{8\sqrt{2}}.$$

$$\begin{aligned} \mathcal{A}_0^0 &= 2|B_1|^2, \\ \mathcal{A}_0^0 \mathcal{A}_2^6 &= \frac{15}{128} \sqrt{\frac{105}{11}} |B_1|^2 \sin^2 \theta_2 \{60 \cos(2\theta_2) \\ &\quad + 33 \cos(4\theta_2) + 35\} \cos(2\phi_2), \end{aligned}$$

$$\begin{aligned} \mathcal{A}_0^0 \mathcal{A}_0^6 &= \frac{15}{256\sqrt{22}} |B_1|^2 \{105 \cos(2\theta_2) \\ &\quad + 126 \cos(4\theta_2) + 231 \cos(6\theta_2) + 50\}, \\ \mathcal{A}_0^0 \mathcal{A}_2^4 &= \frac{1}{4} \sqrt{\frac{15}{11}} |B_1|^2 \sin^2 \theta_2 [7 \cos(2\theta_2) + 5] \cos(2\phi_2), \\ \mathcal{A}_0^0 \mathcal{A}_0^4 &= \frac{1}{32} \sqrt{\frac{3}{11}} |B_1|^2 [20 \cos(2\theta_2) + 35 \cos(4\theta_2) + 9], \\ \mathcal{A}_0^0 \mathcal{A}_2^2 &= -3 \sqrt{\frac{3}{2}} |B_1|^2 \sin^2 \theta_2 \cos(2\phi_2), \\ \mathcal{A}_0^0 \mathcal{A}_0^2 &= -\frac{3|B_1|^2 [3 \cos(2\theta_2) + 1]}{2\sqrt{2}}. \end{aligned}$$

$$\alpha_2 = \begin{cases} \frac{|A_1|^2 - 2|A_0|^2}{|A_1|^2 + 2|A_0|^2}, & \text{for } 1^+, \\ \frac{|A_1|^2 - 2|A_0|^2}{|A_1|^2 + 2|A_0|^2}, & \text{for } 2^-, \\ \frac{3|A_1|^2 - 4|A_0|^2}{3|A_1|^2 + 4|A_0|^2}, & \text{for } 3^+, \\ 1 & \text{for } 1^-, 2^+ \text{ and } 3^-. \end{cases}$$

$$\alpha_4 = \begin{cases} \frac{2|A_1|^2 + 3|A_0|^2}{2|A_1|^2 - 3|A_0|^2}, & \text{for } 2^-, \\ \frac{|A_1|^2 - 6|A_0|^2}{|A_1|^2 + 6|A_0|^2}, & \text{for } 3^-, \\ \frac{|A_1|^2 - 6|A_0|^2}{|A_1|^2 + 6|A_0|^2}, & \text{for } 3^+, \\ 1 & \text{for } 2^+. \end{cases}$$

$$\alpha_6 = \begin{cases} \frac{3|A_1|^2 + 4|A_0|^2}{3|A_1|^2 - 4|A_0|^2}, & \text{for } 3^+, \\ 1 & \text{for } 3^-. \end{cases}$$

APPENDIX I: PROPAGATOR FOR DIFFERENT J^P HYPOTHESES

We give the propagators for the spin $J = 1, 2, 3$ here:

$$\begin{aligned} BW_1(k, \mu, \nu) &= \frac{\tilde{g}_{\mu\nu}(k)}{k^2 - M^2 - iM\Gamma}, \quad \text{with } \tilde{g}_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}, \quad \text{for } J = 1, \\ BW_2(k, \mu, \nu, \alpha, \beta) &= \frac{i}{k^2 - M^2 - iM\Gamma} \left[\frac{1}{2} (\tilde{g}_{\nu\alpha}(k) \tilde{g}_{\mu\beta}(k) + \tilde{g}_{\nu\beta}(k) \tilde{g}_{\mu\alpha}(k)) - \frac{1}{3} (\tilde{g}_{\alpha\beta}(k) \tilde{g}_{\mu\nu}(k)) \right], \quad \text{for } J = 2, \\ BW_3(k, \mu, \nu, \alpha, \beta, \gamma, \eta) &= \frac{i}{k^2 - M^2 - iM\Gamma} \left[\frac{1}{6} (\tilde{g}_{\alpha\beta}(k) \tilde{g}_{\nu\gamma}(k) \tilde{g}_{\mu\eta}(k) + \tilde{g}_{\alpha\gamma}(k) \tilde{g}_{\nu\beta}(k) \tilde{g}_{\mu\eta}(k) + \tilde{g}_{\alpha\beta}(k) \tilde{g}_{\nu\eta}(k) \tilde{g}_{\mu\gamma}(k) \right. \\ &\quad + \tilde{g}_{\alpha\eta}(k) \tilde{g}_{\nu\beta}(k) \tilde{g}_{\mu\gamma}(k) + \tilde{g}_{\alpha\gamma}(k) \tilde{g}_{\nu\eta}(k) \tilde{g}_{\mu\beta}(k) + \tilde{g}_{\alpha\eta}(k) \tilde{g}_{\nu\gamma}(k) \tilde{g}_{\mu\beta}(k)) \\ &\quad + \frac{-1}{15} (\tilde{g}_{\mu\beta}(k) \tilde{g}_{\gamma\eta}(k) \tilde{g}_{\nu\alpha}(k) + \tilde{g}_{\mu,\gamma}(k) \tilde{g}_{\beta\eta}(k) \tilde{g}_{\nu\alpha}(k) + \tilde{g}_{\mu,\eta}(k) \tilde{g}_{\beta\gamma}(k) \tilde{g}_{\nu\alpha}(k) + \tilde{g}_{\nu,\beta}(k) \tilde{g}_{\gamma\eta}(k) \tilde{g}_{\mu\alpha}(k) \\ &\quad + \tilde{g}_{\nu,\gamma}(k) \tilde{g}_{\beta\eta}(k) \tilde{g}_{\mu,\alpha}(k) + \tilde{g}_{\nu,\eta}(k) \tilde{g}_{\beta\gamma}(k) \tilde{g}_{\mu\alpha}(k) + \tilde{g}_{\alpha\beta}(k) \tilde{g}_{\gamma\eta}(k) \tilde{g}_{\mu\nu}(k) + \tilde{g}_{\alpha\gamma}(k) \tilde{g}_{\beta\eta}(k) \tilde{g}_{\mu\nu}(k) \\ &\quad \left. + \tilde{g}_{\alpha\eta}(k) \tilde{g}_{\beta\gamma}(k) \tilde{g}_{\mu\nu}(k)) \right], \quad \text{for } J = 3, \end{aligned}$$

where k is the momentum of $D_s D^*$ system, and $M(\Gamma)$ is the mass (width) of Z_{cs} .

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