Leptoquark and vectorlike quark extended models as the explanation of the muon $g - 2$ anomaly

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Leptoquark (LQ) models can be the solution to the $(g - 2)_\mu$ anomaly because of their chiral enhancements. In this work, we consider the models extended by the LQ and vectorlike quark (VLQ) simultaneously, which can be used to explain the anomaly because of the top quark and top partner (denoted as T) mixing. For the one-LQ and one-VLQ extended models, there are contributions from the top and T quarks. In the minimal LQ models, only the R_2 and S_1 representations can lead to the chiral enhancements. Here, we find one new S_3 solution to the anomaly in the presence of a $(X, T, B)_{L,R}$ triplet. We also consider the one-LQ and two-VLQ extended models, which can solve the anomaly even in the absence of top interactions and mixings. Then, we propose new LQ search channels under the constraints of $(g - 2)_\mu$. Besides the traditional $t\mu$ decay channel, the LQ can also decay into $T\mu$ final states, which will lead to the characteristic multitop and multimueon signals at hadron colliders.

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I. INTRODUCTION

Lepton magnetic moments play an important role in the development of the standard model (SM) of elementary particle physics. One interesting related topic is the muon anomalous magnetic dipole moment, which is labeled as $a_\mu \equiv (g - 2)_\mu/2$. Its deviation from the SM prediction can give us some clues to the new physics [1,2]. The so-called $(g - 2)_\mu$ anomaly was first reported by the E821 experiment at BNL [3]. Recently, the FNAL muon $g - 2$ experiment has increased the tension between the SM prediction and experiment data [4]. The average experimental result is $a_\mu^{\text{Exp}} = 116592061(41) \times 10^{-11}$ after combining the BNL and FNAL data. The SM prediction is $a_\mu^{\text{SM}} = 116591810(43) \times 10^{-11}$ [5–25], which results in a 4.2σ discrepancy with $\Delta a_\mu \equiv a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (251 \pm 59) \times 10^{-11}$. This excess may be pinned down with more data at FNAL and J-PARC [26].

Although this anomaly could be caused by the experimental and theoretical uncertainties [27], we are more interested in the new physics interpretations. Many new physics models are motivated to explain this anomaly [1,28–31]: for example, supersymmetric models [32–36], scalar mediator models [37–43], new chiral lepton

models [44,45], vectorlike lepton (VLL) models [46–54], and so on. Leptoquarks (LQs) are well motivated in the grand unified theories [55–57], and they can also be an alternative choice for the $(g - 2)_\mu$ anomaly [58–64] because of the significant chiral enhancement m_t/m_μ . Although they can also be used to explain the anomalies in B physics [60,65–69], we will not discuss those here. On the other hand, vectorlike fermions will occur in many new physics models [70–73]. However, most studies focus on the VLL extended models, and the vectorlike quark (VLQ) models usually draw little attention. In this work, we will consider the simultaneous scalar LQ and VLQ extended models.

II. RESULTS IN THE LQ EXTENDED MODELS

A. Minimal LQ models

First of all, let us consider the minimal extension of SM with only one scalar LQ [29,58,60,74,75]. There is a fermion number $F = 3B + L$ for the LQ interactions, in which B is the baryon number and L is the lepton number. In fact, there are two types of interactions, $\bar{\psi}_R \psi_L$ and $(\bar{\psi}_L)^c \psi_L$, depending on the fermion number carried by the LQ. We name the first type “ $F = 0$ ” (or “A”) and the second type “ $F = 2$ ” (or “B”). Then, the Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_{F=0} &\supset \bar{\mu}(y_L^{S_A \mu q_A} \omega_- + y_R^{S_A \mu q_A} \omega_+) q_A S_A + \text{H.c.}, \\ \mathcal{L}_{F=2} &\supset \bar{\mu}(y_L^{S_B \mu q_B} \omega_- + y_R^{S_B \mu q_B} \omega_+) q_B^C S_B + \text{H.c.}\end{aligned}\quad (1)$$

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TABLE I. Coupling patterns of the Yukawa couplings between muons and top quarks in the minimal LQ models for different representations.

$SU(3)_C \times SU(2)_L \times U(1)_Y$ representation	Label	F	$y_L^{\text{LQ}\mu t}$	$y_R^{\text{LQ}\mu t}$
$(\bar{3}, 3, 1/3)$	S_3	-2	0	$\mathcal{O}(y)$
$(3, 2, 7/6)$	R_2	0	$\mathcal{O}(y)$	$\mathcal{O}(y)$
$(3, 2, 1/6)$	$\bar{R}_2$	0	0	0
$(\bar{3}, 1, 4/3)$	$\tilde{S}_1$	-2	0	0
$(\bar{3}, 1, 1/3)$	S_1	-2	$\mathcal{O}(y)$	$\mathcal{O}(y)$

In the above, q_B^C is the charge conjugate of the q_B quark, which means that $q_B^C = C\bar{q}_B^T$, with¹ $C \equiv i\gamma^2\gamma^0$. In Table I, we list different scalar LQs and the coupling patterns between muons and up-type quarks [60].

When involving the LQ, we need to consider two types of Feynman diagrams at the same time, shown in Fig. 1. The contributions to $(g-2)_\mu$ are slightly different for the $F=0$ -type and $F=2$ -type interactions [75]. The one-loop results have been calculated in previous studies [60,76], and we note that there should be a color factor $N_C=3$. Here, we consider the muon coupled to the LQ and top quark. Thus, $m_\mu \ll m_q, m_S$ is a good

FIG. 1. The typical Feynman diagrams contributing to $(g-2)_\mu$.

approximation. For convenience, we define the following four integrals:

$$\begin{aligned}
 f_{LL}^q(x) &= \frac{2 + 3x - 6x^2 + x^3 + 6x \log x}{12(1-x)^4}, \\
 f_{LR}^q(x) &= -\frac{3 - 4x + x^2 + 2 \log x}{4(1-x)^3}, \\
 f_{LL}^S(x) &= -\frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x}{12(1-x)^4}, \\
 f_{LR}^S(x) &= -\frac{1 - x^2 + 2x \log x}{4(1-x)^3}.
 \end{aligned} \tag{2}$$

For $F=0$ or A-type interactions, the contributions to $(g-2)_\mu$ are given as

$$\begin{aligned}
 \Delta a_\mu^{F=0} &= -\frac{N_C m_\mu^2}{8\pi^2 m_{S_A}^2} \left[(|y_L^{S_A \mu q_A}|^2 + |y_R^{S_A \mu q_A}|^2) (\mathcal{Q}_{q_A} f_{LL}^q(x_A) + \mathcal{Q}_{S_A} f_{LL}^S(x_A)) \right. \\
 &\quad \left. + \frac{m_{q_A}}{m_\mu} (y_L^{S_A \mu q_A} (y_R^{S_A \mu q_A})^* + y_R^{S_A \mu q_A} (y_L^{S_A \mu q_A})^*) (\mathcal{Q}_{q_A} f_{LR}^q(x_A) + \mathcal{Q}_{S_A} f_{LR}^S(x_A)) \right],
 \end{aligned} \tag{3}$$

where x_A is defined as $m_{q_A}^2/m_{S_A}^2$, $\mathcal{Q}_{q_A}$ is the electric charge of quark q_A , and $\mathcal{Q}_{S_A}$ is the electric charge of scalar S_A in Eq. (1). For $F=2$ or B-type interactions, we need to perform the transformations $y_{L,R}^{S_A \mu q_A} \rightarrow y_{L,R}^{S_B \mu q_B}$, $\mathcal{Q}_{q_A} \rightarrow -\mathcal{Q}_{q_B}$, $\mathcal{Q}_{S_A} \rightarrow \mathcal{Q}_{S_B}$, $m_{q_A} \rightarrow m_{q_B}$, and $x_A \rightarrow x_B$ in Eq. (3). Because the quark field is q_B^C in the type-B interactions, it should be expressed as² $\mathcal{Q}_{q_B^C} = -\mathcal{Q}_{q_B}$. More specifically, the contributions to $(g-2)_\mu$ are given as

$$\begin{aligned}
 \Delta a_\mu^{F=2} &= -\frac{N_C m_\mu^2}{8\pi^2 m_{S_B}^2} \left[(|y_L^{S_B \mu q_B}|^2 + |y_R^{S_B \mu q_B}|^2) (-\mathcal{Q}_{q_B} f_{LL}^q(x_B) + \mathcal{Q}_{S_B} f_{LL}^S(x_B)) \right. \\
 &\quad \left. + \frac{m_{q_B}}{m_\mu} (y_L^{S_B \mu q_B} (y_R^{S_B \mu q_B})^* + y_R^{S_B \mu q_B} (y_L^{S_B \mu q_B})^*) (-\mathcal{Q}_{q_B} f_{LR}^q(x_B) + \mathcal{Q}_{S_B} f_{LR}^S(x_B)) \right],
 \end{aligned} \tag{4}$$

where x_B is defined as $m_{q_B}^2/m_{S_B}^2$.

As we can see in Table I, only the R_2 and S_1 representations can give both the left-handed and right-handed (nonchiral) couplings to muons, which means that chiral enhancement is possible. The R_2 representation induces the $F=0$ -type interactions, and S_1 induces the $F=2$ -type interactions. Now, let us make the conventions. We use Q_L^i ,

¹There can be different definitions for the C operator, but this has no influence on the following discussions.

u_R^i , L_L^i , and e_R^i to label the SM quark and lepton fields, where i means the generation index and ranges from 1 to 3.

²Here, we only change the sign of the quark electric charge from the $F=0$ to $F=2$ case. But Kingman's paper [75] says that we need to change the signs of both the quark's electric charge and its mass. We think the sign of the quark mass should stay the same, because the mass term $\bar{\psi}\psi$ is invariant under the C transformation. The sign of the electric charge should be changed, however, because the electromagnetic current term $\bar{\psi}\gamma^\mu\psi$ changes the sign under the C transformation.

In particular, Q_L^3 , u_R^3 , L_L^2 , and e_R^2 are also marked as t_L , t_R , μ_L , and μ_R , respectively. For the doublet and following triplet fields, the labels a , b , and c are the weak isospin indices. The values τ^i ($i = 1, 2, 3$) are the Pauli matrices, and ϵ is defined as $i\tau^2$. Then, we have the following gauge eigenstate interactions [60]:

$$\begin{aligned}\mathcal{L}_{R_2} &\supset -x_{ij}\bar{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} + y_{ij}\bar{e}_R^i (R_2^a)^* Q_L^{j,a} + \text{H.c.}, \\ \mathcal{L}_{S_1} &\supset v_{ij}(\bar{Q}_L)^{C i,a} S_1 \epsilon^{ab} L_L^{j,b} + x_{ij}(\bar{u}_R)^{C i} S_1 e_R^j + \text{H.c.}\end{aligned}\quad (5)$$

In the mass eigenstates, they can be written as

$$\begin{aligned}\mathcal{L}_{R_2} &\supset \bar{\mu}(y_L^{R_2\mu t} \omega_- + y_R^{R_2\mu t} \omega_+) t (R_2^{5/3})^* + \text{H.c.}, \\ \mathcal{L}_{S_1} &\supset \bar{\mu}(y_L^{S_1\mu t} \omega_- + y_R^{S_1\mu t} \omega_+) t^C (S_1)^* + \text{H.c.}\end{aligned}\quad (6)$$

Before obtaining the explicit results of the R_2 and S_1 contributions, let us define the following four integrals:

$$\begin{aligned}f_{LL}^{R_2}(x) &\equiv -\frac{3-8x+x^2+4x^3+2x(2-5x)\log x}{4(1-x)^4}, \\ f_{LR}^{R_2}(x) &\equiv \frac{1-8x+7x^2+(4-10x)\log x}{4(1-x)^3}, \\ f_{LL}^{S_1}(x) &\equiv \frac{1+4x-5x^2+2x(2+x)\log x}{4(1-x)^4}, \\ f_{LR}^{S_1}(x) &\equiv -\frac{7-8x+x^2+(4+2x)\log x}{4(1-x)^3}.\end{aligned}\quad (7)$$

For the S_3 interactions in the following context, we have the results $f_{LL}^{S_3}(x) = f_{LL}^{S_1}(x)$ and $f_{LR}^{S_3}(x) = f_{LR}^{S_1}(x)$ because of the similar interaction types. By setting $Q_{q_A} = Q_t = 2/3$ and $Q_{S_A} = Q_{(R_2^{5/3})^*} = -5/3$ in Eq. (3), we have the following results for the R_2 case:

$$\Delta a_\mu^{R_2} = \frac{m_\mu^2}{8\pi^2 m_{R_2}^2} \left[(|y_L^{R_2\mu t}|^2 + |y_R^{R_2\mu t}|^2) f_{LL}^{R_2}(m_t^2/m_{R_2}^2) + \frac{m_t}{m_\mu} (y_L^{R_2\mu t} (y_R^{R_2\mu t})^* + y_R^{R_2\mu t} (y_L^{R_2\mu t})^*) f_{LR}^{R_2}(m_t^2/m_{R_2}^2) \right]. \quad (8)$$

Considering $m_t \ll m_{R_2}$, it can be approximated as

$$\Delta a_\mu^{R_2} \approx \frac{m_\mu^2}{32\pi^2 m_{R_2}^2} \left[-3(|y_L^{R_2\mu t}|^2 + |y_R^{R_2\mu t}|^2) + \frac{m_t}{m_\mu} \left(1 + 4 \log \frac{m_t^2}{m_{R_2}^2} \right) (y_L^{R_2\mu t} (y_R^{R_2\mu t})^* + y_R^{R_2\mu t} (y_L^{R_2\mu t})^*) \right]. \quad (9)$$

By setting $Q_{q_B} = Q_t = 2/3$ and $Q_{S_B} = Q_{(S_1)^*} = -1/3$ in Eq. (4), we have the following results for the S_1 case:

$$\Delta a_\mu^{S_1} = \frac{m_\mu^2}{8\pi^2 m_{S_1}^2} \left[(|y_L^{S_1\mu t}|^2 + |y_R^{S_1\mu t}|^2) f_{LL}^{S_1}(m_t^2/m_{S_1}^2) + \frac{m_t}{m_\mu} (y_L^{S_1\mu t} (y_R^{S_1\mu t})^* + y_R^{S_1\mu t} (y_L^{S_1\mu t})^*) f_{LR}^{S_1}(m_t^2/m_{S_1}^2) \right]. \quad (10)$$

Considering $m_t \ll m_{S_1}$, it can be approximated as

$$\Delta a_\mu^{S_1} \approx \frac{m_\mu^2}{32\pi^2 m_{S_1}^2} \left[(|y_L^{S_1\mu t}|^2 + |y_R^{S_1\mu t}|^2) - \frac{m_t}{m_\mu} \left(7 + 4 \log \frac{m_t^2}{m_{S_1}^2} \right) (y_L^{S_1\mu t} (y_R^{S_1\mu t})^* + y_R^{S_1\mu t} (y_L^{S_1\mu t})^*) \right]. \quad (11)$$

Considering the electric conservation condition and the notations, our results agree with those in Refs. [60,75,77]. For a LQ with mass above 1 TeV, we have the rough estimation $m_\mu^2/(8\pi^2 m_{LQ}^2) \lesssim 10^{-10}$. Because the LQ Yukawa couplings are bounded by the perturbative unitarity, the chiral enhancements are required to explain the $(g-2)_\mu$.

B. The one-LQ and one-VLQ extended models

There are seven types of VLQs in all, but only five of them contain the top partner T [78]. We will consider these five cases: one singlet, $T_{L,R}$; two doublets, $(X, T)_{L,R}$ and

$(T, B)_{L,R}$; and two triplets, $(X, T, B)_{L,R}$ and $(T, B, Y)_{L,R}$. Here, X , T , B , and Y carry electric charges of $5/3$, $2/3$, $-1/3$, and $-4/3$, respectively. With five scalar LQ types and five interesting VLQ types, we can have a total of 25 combinations if we consider one LQ and one VLQ at the same time, which will be dubbed as “LQ + VLQ” in the following context. After some attempts, we find that only the combinations with specific LQ representations can lead to the up-type quark chiral enhancements. In that case, we will study those combinations. For convenience, we only investigate the top- and $T_{L,R}$ -related LQ interactions in this paper.

- (1) For the $T_{L,R}$ singlet case, the LQ should be R_2 or S_1 .

(a) For the $R_2 + T_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{R_2+T_{L,R}} \supset -x_{ij}\overline{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} - x_{Tj}\overline{T}_R R_2^a \epsilon^{ab} L_L^{j,b} + y_{ij}\overline{e}_R^i (R_2^a)^* Q_L^{j,a} + \text{H.c.} \quad (12)$$

(b) For the $S_1 + T_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{S_1+T_{L,R}} \supset v_{ij}(\overline{Q}_L)^{C^i,a} S_1 \epsilon^{ab} L_L^{j,b} + x_{ij}(\overline{u}_R)^{C^i} S_1 e_R^j + x_{Tj}(\overline{T}_R)^C S_1 e_R^j + \text{H.c.} \quad (13)$$

- (2) For the $(X,T)_{L,R}$ doublet case, the LQ should be R_2 or S_1 .

(a) For the $R_2 + (X,T)_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{R_2+(X,T)_{L,R}} \supset -x_{ij}\overline{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} + y_{ij}\overline{e}_R^i (R_2^a)^* Q_L^{j,a} + \text{H.c.} \quad (14)$$

(b) For the $S_1 + (X,T)_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{S_1+(X,T)_{L,R}} \supset v_{ij}(\overline{Q}_L)^{C^i,a} S_1 \epsilon^{ab} L_L^{j,b} + x_{ij}(\overline{u}_R)^{C^i} S_1 e_R^j + \text{H.c.} \quad (15)$$

- (3) For the $(T,B)_{L,R}$ doublet case, the LQ should be R_2 or S_1 .

(a) For the $R_2 + (T,B)_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{R_2+(T,B)_{L,R}} \supset -x_{ij}\overline{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} + y_{ij}\overline{e}_R^i (R_2^a)^* Q_L^{j,a} + y_{iT}\overline{e}_R^i (R_2^a)^* \left(\frac{T_L}{B_L}\right)^a + \text{H.c.} \quad (16)$$

(b) For the $S_1 + (T,B)_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{S_1+(T,B)_{L,R}} \supset v_{ij}(\overline{Q}_L)^{C^i,a} S_1 \epsilon^{ab} L_L^{j,b} + v_{Tj}(\overline{(T_L)}^C, \overline{(B_L)}^C)^a S_1 \epsilon^{ab} L_L^{j,b} + x_{ij}(\overline{u}_R)^{C^i} S_1 e_R^j + \text{H.c.} \quad (17)$$

- (4) For the $(X,T,B)_{L,R}$ triplet case, the LQ can be R_2 , S_1 , or S_3 .

(a) For the $R_2 + (X,T,B)_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{R_2+(X,T,B)_{L,R}} \supset -x_{ij}\overline{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} - x_{iT}\overline{L}_L^{i,a} (\Psi_R)^{ab} \epsilon^{bc} (R_2^c)^* + y_{ij}\overline{e}_R^i (R_2^a)^* Q_L^{j,a} + \text{H.c.} \quad (18)$$

(b) For the $S_1 + (X,T,B)_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{S_1+(X,T,B)_{L,R}} \supset v_{ij}(\overline{Q}_L)^{C^i,a} S_1 \epsilon^{ab} L_L^{j,b} + x_{ij}(\overline{u}_R)^{C^i} S_1 e_R^j + \text{H.c.} \quad (19)$$

(c) For the $S_3 + (X,T,B)_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{S_3+(X,T,B)_{L,R}} \supset x_{ij}(\overline{Q}_L)^{C^i,a} \epsilon^{ab} (\tau^k S_3^k)^{bc} L_L^{j,c} + y_{Tj}\text{Tr}[(\overline{\Psi}_R)^C S_3] e_R^j + \text{H.c.} \quad (20)$$

In the above, we define the $(X,T,B)_{L,R}$ and S_3 triplets in the following bidoublet form:

$$\Psi_{L,R} \equiv \begin{pmatrix} T_{L,R} & \sqrt{2}X_{L,R} \\ \sqrt{2}B_{L,R} & -T_{L,R} \end{pmatrix}, \quad S_3 \equiv \begin{pmatrix} S_3^{1/3} & \sqrt{2}S_3^{4/3} \\ \sqrt{2}S_3^{-2/3} & -S_3^{1/3} \end{pmatrix}. \quad (21)$$

Then, the matrix elements $(\overline{(\Psi_R)^C})_{ij}$ are defined as $(\overline{(\Psi_R)_{ij}})^C$.

- (5) For the $(T,B,Y)_{L,R}$ triplet case, the LQ should be R_2 or S_1 .

(a) For the $R_2 + (T,B,Y)_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{R_2+(T,B,Y)_{L,R}} \supset -x_{ij}\overline{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} + y_{ij}\overline{e}_R^i (R_2^a)^* Q_L^{j,a} + \text{H.c.} \quad (22)$$

(b) For the $S_1 + (T, B, Y)_{L,R}$ model, there are the following gauge eigenstate interactions:

$$\mathcal{L}_{S_1+(T,B,Y)_{L,R}} \supset v_{ij}(\overline{Q_L})^{Ci,a} S_1 \epsilon^{ab} L_L^{j,b} + x_{ij}(\overline{u_R})^{Ci} S_1 e_R^j + \text{H.c.} \quad (23)$$

After the electroweak symmetry breaking (EWSB), we can obtain the $LQ\mu t$ and $LQ\mu T$ interactions. Then, we give the explicit forms in the following:

(1) For the $R_2 + T_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{R_2+T_{L,R}} \supset \bar{\mu}(y_L^{R_2\mu t} \omega_- + y_R^{R_2\mu t} \omega_+) t(R_2^{5/3})^* + y_R^{R_2\mu T} \bar{\mu} \omega_+ T(R_2^{5/3})^* + \text{H.c.} \quad (24)$$

(2) For the $S_1 + T_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{S_1+T_{L,R}} \supset \bar{\mu}(y_L^{S_1\mu t} \omega_- + y_R^{S_1\mu t} \omega_+) t^C(S_1)^* + y_L^{S_1\mu T} \bar{\mu} \omega_- T^C(S_1)^* + \text{H.c.} \quad (25)$$

(3) For the $R_2 + (X, T)_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{R_2+(X,T)_{L,R}} \supset \bar{\mu}(y_L^{R_2\mu t} \omega_- + y_R^{R_2\mu t} \omega_+) t(R_2^{5/3})^* + \text{H.c.} \quad (26)$$

(4) For the $S_1 + (X, T)_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{S_1+(X,T)_{L,R}} \supset \bar{\mu}(y_L^{S_1\mu t} \omega_- + y_R^{S_1\mu t} \omega_+) t^C(S_1)^* + \text{H.c.} \quad (27)$$

(5) For the $R_2 + (T, B)_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{R_2+(T,B)_{L,R}} \supset \bar{\mu}(y_L^{R_2\mu t} \omega_- + y_R^{R_2\mu t} \omega_+) t(R_2^{5/3})^* + y_L^{R_2\mu T} \bar{\mu} \omega_- T(R_2^{5/3})^* + \text{H.c.} \quad (28)$$

(6) For the $S_1 + (T, B)_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{S_1+(T,B)_{L,R}} \supset \bar{\mu}(y_L^{S_1\mu t} \omega_- + y_R^{S_1\mu t} \omega_+) t^C(S_1)^* + y_R^{S_1\mu T} \bar{\mu} \omega_+ T^C(S_1)^* + \text{H.c.} \quad (29)$$

(7) For the $R_2 + (X, T, B)_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{R_2+(X,T,B)_{L,R}} \supset \bar{\mu}(y_L^{R_2\mu t} \omega_- + y_R^{R_2\mu t} \omega_+) t(R_2^{5/3})^* + y_R^{R_2\mu T} \bar{\mu} \omega_+ T(R_2^{5/3})^* + \text{H.c.} \quad (30)$$

(8) For the $S_1 + (X, T, B)_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{S_1+(X,T,B)_{L,R}} \supset \bar{\mu}(y_L^{S_1\mu t} \omega_- + y_R^{S_1\mu t} \omega_+) t^C(S_1)^* + \text{H.c.} \quad (31)$$

(9) For the $S_3 + (X, T, B)_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{S_3+(X,T,B)_{L,R}} \supset y_L^{S_3\mu T} \bar{\mu} \omega_- T^C(S_3^{1/3})^* + y_R^{S_3\mu t} \bar{\mu} \omega_+ t^C(S_3^{1/3})^* + \text{H.c.} \quad (32)$$

(10) For the $R_2 + (T, B, Y)_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{R_2+(T,B,Y)_{L,R}} \supset \bar{\mu}(y_L^{R_2\mu t} \omega_- + y_R^{R_2\mu t} \omega_+) t(R_2^{5/3})^* + \text{H.c.} \quad (33)$$

(11) For the $S_1 + (T, B, Y)_{L,R}$ model, they can be parametrized as

$$\mathcal{L}_{S_1+(T,B,Y)_{L,R}} \supset \bar{\mu}(y_L^{S_1\mu t} \omega_- + y_R^{S_1\mu t} \omega_+) t^C(S_1)^* + \text{H.c.} \quad (34)$$

For convenience, we only consider the mixing between top quarks and T quarks [78–80]. Then, the t and T quarks can be rotated into mass eigenstates by the following transformations:

$$\begin{bmatrix} t_L \\ T_L \end{bmatrix} \rightarrow \begin{bmatrix} \cos \theta_L & \sin \theta_L \\ -\sin \theta_L & \cos \theta_L \end{bmatrix} \begin{bmatrix} t_L \\ T_L \end{bmatrix}, \quad \begin{bmatrix} t_R \\ T_R \end{bmatrix} \rightarrow \begin{bmatrix} \cos \theta_R & \sin \theta_R \\ -\sin \theta_R & \cos \theta_R \end{bmatrix} \begin{bmatrix} t_R \\ T_R \end{bmatrix}. \quad (35)$$

Hereafter, $\sin \theta_L$, $\cos \theta_L$, $\sin \theta_R$, and $\cos \theta_R$ will be abbreviated as s_L , c_L , s_R , and c_R , respectively. The left-handed and right-handed field mixing angles can be related through the relation $\tan \theta_{R(L)} = m_l \tan \theta_{L(R)} / m_T$ depending on the VLQ representations [78], in which m_T is the mass of $T_{L,R}$. Then, we collect the mass eigenstate interactions in the following.

(1) For the $R_2 + T_{L,R}$ model, they can be written as

$$\begin{aligned} \mathcal{L}_{R_2+T_{L,R}} \supset & \bar{\mu} [y_L^{R_2\mu t} c_L \omega_- + (y_R^{R_2\mu t} c_R - y_R^{R_2\mu T} s_R) \omega_+] t(R_2^{5/3})^* \\ & + \bar{\mu} [y_L^{R_2\mu t} s_L \omega_- + (y_R^{R_2\mu t} s_R + y_R^{R_2\mu T} c_R) \omega_+] T(R_2^{5/3})^* + \text{H.c.} \end{aligned} \quad (36)$$

(2) For the $S_1 + T_{L,R}$ model, they can be written as

$$\begin{aligned} \mathcal{L}_{S_1+T_{L,R}} \supset & \bar{\mu} [(y_L^{S_1\mu t} c_R - y_L^{S_1\mu T} s_R) \omega_- + y_R^{S_1\mu t} c_L \omega_+] t^C(S_1)^* \\ & + \bar{\mu} [(y_L^{S_1\mu t} s_R + y_L^{S_1\mu T} c_R) \omega_- + y_R^{S_1\mu t} s_L \omega_+] T^C(S_1)^* + \text{H.c.} \end{aligned} \quad (37)$$

(3) For the $R_2 + (X, T)_{L,R}$ model, they can be written as

$$\mathcal{L}_{R_2+(X,T)_{L,R}} \supset \bar{\mu} (y_L^{R_2\mu t} c_L \omega_- + y_R^{R_2\mu t} c_R \omega_+) t(R_2^{5/3})^* + \bar{\mu} (y_L^{R_2\mu t} s_L \omega_- + y_R^{R_2\mu t} s_R \omega_+) T(R_2^{5/3})^* + \text{H.c.} \quad (38)$$

(4) For the $S_1 + (X, T)_{L,R}$ model, they can be written as

$$\mathcal{L}_{S_1+(X,T)_{L,R}} \supset \bar{\mu} (y_L^{S_1\mu t} c_R \omega_- + y_R^{S_1\mu t} c_L \omega_+) t^C(S_1)^* + \bar{\mu} (y_L^{S_1\mu t} s_R \omega_- + y_R^{S_1\mu t} s_L \omega_+) T^C(S_1)^* + \text{H.c.} \quad (39)$$

(5) For the $R_2 + (T, B)_{L,R}$ model, they can be written as

$$\begin{aligned} \mathcal{L}_{R_2+(T,B)_{L,R}} \supset & \bar{\mu} [(y_L^{R_2\mu t} c_L - y_L^{R_2\mu T} s_L) \omega_- + y_R^{R_2\mu t} c_R \omega_+] t(R_2^{5/3})^* \\ & + \bar{\mu} [(y_L^{R_2\mu t} s_L + y_L^{R_2\mu T} c_L) \omega_- + y_R^{R_2\mu t} s_R \omega_+] T(R_2^{5/3})^* + \text{H.c.} \end{aligned} \quad (40)$$

(6) For the $S_1 + (T, B)_{L,R}$ model, they can be written as

$$\begin{aligned} \mathcal{L}_{S_1+(T,B)_{L,R}} \supset & \bar{\mu} [y_L^{S_1\mu t} c_R \omega_- + (y_R^{S_1\mu t} c_L - y_R^{S_1\mu T} s_L) \omega_+] t^C(S_1)^* \\ & + \bar{\mu} [y_L^{S_1\mu t} s_R \omega_- + (y_R^{S_1\mu t} s_L + y_R^{S_1\mu T} c_L) \omega_+] T^C(S_1)^* + \text{H.c.} \end{aligned} \quad (41)$$

(7) For the $R_2 + (X, T, B)_{L,R}$ model, they can be written as

$$\begin{aligned} \mathcal{L}_{R_2+(X,T,B)_{L,R}} \supset & \bar{\mu} [y_L^{R_2\mu t} c_L \omega_- + (y_R^{R_2\mu t} c_R - y_R^{R_2\mu T} s_R) \omega_+] t(R_2^{5/3})^* \\ & + \bar{\mu} [y_L^{R_2\mu t} s_L \omega_- + (y_R^{R_2\mu t} c_R + y_R^{R_2\mu T} s_R) \omega_+] T(R_2^{5/3})^* + \text{H.c.} \end{aligned} \quad (42)$$

(8) For the $S_1 + (X, T, B)_{L,R}$ model, they can be written as

$$\mathcal{L}_{S_1+(X,T,B)_{L,R}} \supset \bar{\mu} (y_L^{S_1\mu t} c_R \omega_- + y_R^{S_1\mu t} c_L \omega_+) t^C(S_1)^* + \bar{\mu} (y_L^{S_1\mu t} s_R \omega_- + y_R^{S_1\mu t} s_L \omega_+) T^C(S_1)^* + \text{H.c.} \quad (43)$$

(9) For the $S_3 + (X, T, B)_{L,R}$ model, they can be written as

$$\begin{aligned} \mathcal{L}_{S_3+(X,T,B)_{L,R}} \supset & \bar{\mu} (-y_L^{S_3\mu T} s_R \omega_- + y_R^{S_3\mu t} c_L \omega_+) t^C(S_3^{1/3})^* \\ & + \bar{\mu} (y_L^{S_3\mu T} c_R \omega_- + y_R^{S_3\mu t} s_L \omega_+) T^C(S_3^{1/3})^* + \text{H.c.} \end{aligned} \quad (44)$$

(10) For the $R_2 + (T, B, Y)_{L,R}$ model, they can be written as

$$\mathcal{L}_{R_2+(T,B,Y)_{L,R}} \supset \bar{\mu}(y_L^{R_2\mu t} c_L \omega_- + y_R^{R_2\mu t} c_R \omega_+) t(R_2^{5/3})^* + \bar{\mu}(y_L^{R_2\mu t} s_L \omega_- + y_R^{R_2\mu t} s_R \omega_+) T(R_2^{5/3})^* + \text{H.c.} \quad (45)$$

(11) For the $S_1 + (T, B, Y)_{L,R}$ model, they can be written as

$$\mathcal{L}_{S_1+(T,B,Y)_{L,R}} \supset \bar{\mu}(y_L^{S_1\mu t} c_R \omega_- + y_R^{S_1\mu t} c_L \omega_+) t^C(S_1)^* + \bar{\mu}(y_L^{S_1\mu t} s_R \omega_- + y_R^{S_1\mu t} s_L \omega_+) T^C(S_1)^* + \text{H.c.} \quad (46)$$

From the above mass eigenstate interactions, we can obtain the top and T contributions to $(g-2)_\mu$. For the $R_2 + \text{VLQ}$ models, we can derive the contributions from Eq. (8). For the $S_1/S_3 + \text{VLQ}$ models, we can also derive the contributions from Eq. (10). In the following, we only show the chirally enhanced parts for simplicity.

(1) For the $R_2 + T_{L,R}$ model, they are calculated as

$$\begin{aligned} \Delta a_\mu^{R_2+T_{L,R}} \approx & \frac{m_\mu m_t}{4\pi^2 m_{R_2}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[(y_R^{R_2\mu t} s_R + y_R^{R_2\mu T} c_R)(y_L^{R_2\mu t})^*] s_L \right. \\ & \left. + \left(\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2} \right) \text{Re}[(y_R^{R_2\mu t} c_R - y_R^{R_2\mu T} s_R)(y_L^{R_2\mu t})^*] c_L \right\}. \end{aligned} \quad (47)$$

(2) For the $S_1 + T_{L,R}$ model, they are calculated as

$$\begin{aligned} \Delta a_\mu^{S_1+T_{L,R}} \approx & \frac{m_\mu m_t}{4\pi^2 m_{S_1}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[(y_L^{S_1\mu T} c_R + y_L^{S_1\mu t} s_R)(y_R^{S_1\mu t})^*] s_L \right. \\ & \left. - \left(\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2} \right) \text{Re}[(y_L^{S_1\mu t} c_R - y_L^{S_1\mu T} s_R)(y_R^{S_1\mu t})^*] c_L \right\}. \end{aligned} \quad (48)$$

(3) For the $R_2 + (X, T)_{L,R}$ model, they are calculated as

$$\begin{aligned} \Delta a_\mu^{R_2+(X,T)_{L,R}} \approx & \frac{m_\mu m_t}{4\pi^2 m_{R_2}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*] s_L s_R \right. \\ & \left. + \left(\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2} \right) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*] c_L c_R \right\}. \end{aligned} \quad (49)$$

(4) For the $S_1 + (X, T)_{L,R}$ model, they are calculated as

$$\begin{aligned} \Delta a_\mu^{S_1+(X,T)_{L,R}} \approx & \frac{m_\mu m_t}{4\pi^2 m_{S_1}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] s_L s_R \right. \\ & \left. - \left(\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2} \right) \text{Re}[(y_L^{S_1\mu t} (y_R^{S_1\mu t})^*)] c_L c_R \right\}. \end{aligned} \quad (50)$$

(5) For the $R_2 + (T, B)_{L,R}$ model, they are calculated as

$$\begin{aligned} \Delta a_\mu^{R_2+(T,B)_{L,R}} \approx & \frac{m_\mu m_t}{4\pi^2 m_{R_2}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[(y_L^{R_2\mu t} s_L + y_L^{R_2\mu T} c_L)(y_R^{R_2\mu t})^*] s_R \right. \\ & \left. + \left(\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2} \right) \text{Re}[(y_L^{R_2\mu t} c_L - y_L^{R_2\mu T} s_L)(y_R^{R_2\mu t})^*] c_R \right\}. \end{aligned} \quad (51)$$

(6) For the $S_1 + (T, B)_{L,R}$ model, they are calculated as

$$\Delta a_\mu^{S_1+(T,B)_{L,R}} \approx \frac{m_\mu m_t}{4\pi^2 m_{S_1}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[(y_R^{S_1\mu T} c_L + y_R^{S_1\mu t} s_L)(y_L^{S_1\mu t})^*] s_R \right. \\ \left. - \left(\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2} \right) \text{Re}[(y_R^{S_1\mu t} c_L - y_R^{S_1\mu T} s_L)(y_L^{S_1\mu t})^*] c_R \right\}. \quad (52)$$

(7) For the $R_2 + (X, T, B)_{L,R}$ model, they are calculated as

$$\Delta a_\mu^{R_2+(X,T,B)_{L,R}} \approx \frac{m_\mu m_t}{4\pi^2 m_{R_2}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[(y_R^{R_2\mu t} s_R + y_R^{R_2\mu T} c_R)(y_L^{R_2\mu t})^*] s_L \right. \\ \left. + \left(\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2} \right) \text{Re}[(y_R^{R_2\mu t} c_R - y_R^{R_2\mu T} s_R)(y_L^{R_2\mu t})^*] c_L \right\}. \quad (53)$$

(8) For the $S_1 + (X, T, B)_{L,R}$ model, they are calculated as

$$\Delta a_\mu^{S_1+(X,T,B)_{L,R}} \approx \frac{m_\mu m_t}{4\pi^2 m_{S_1}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] s_L s_R \right. \\ \left. - \left(\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2} \right) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] c_L c_R \right\}. \quad (54)$$

(9) For the $S_3 + (X, T, B)_{L,R}$ model, they are calculated as

$$\Delta a_\mu^{S_3+(X,T,B)_{L,R}} \approx \frac{m_\mu m_t}{4\pi^2 m_{S_3}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{S_3} (m_T^2/m_{S_3}^2) \text{Re}[y_L^{S_3\mu T} (y_R^{S_3\mu t})^*] s_L c_R \right. \\ \left. + \left(\frac{7}{4} + \log \frac{m_t^2}{m_{S_3}^2} \right) \text{Re}[y_L^{S_3\mu T} (y_R^{S_3\mu t})^*] c_L s_R \right\}. \quad (55)$$

(10) For the $R_2 + (T, B, Y)_{L,R}$ model, they are calculated as

$$\Delta a_\mu^{R_2+(T,B,Y)_{L,R}} \approx \frac{m_\mu m_t}{4\pi^2 m_{R_2}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*] s_L s_R \right. \\ \left. + \left(\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2} \right) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*] c_L c_R \right\}. \quad (56)$$

(11) For the $S_1 + (T, B, Y)_{L,R}$ model, they are calculated as

$$\Delta a_\mu^{S_1+(T,B,Y)_{L,R}} \approx \frac{m_\mu m_t}{4\pi^2 m_{S_1}^2} \left\{ \frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] s_L s_R \right. \\ \left. - \left(\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2} \right) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] c_L c_R \right\}. \quad (57)$$

Now, let us make a summary for the one-LQ and one-VLQ models. In Table II, we list the related input parameters and mixing angle identities in the previously mentioned one-LQ and one-VLQ models. For the singlet and triplet VLQs, we choose s_L as the input mixing angle. For the doublet VLQs, we choose s_R as the input mixing angle [78]. In Table III, we list the coupling patterns of $LQ\mu t$ and $LQ\mu T$ interactions.

In Table IV, we show the approximate formulas of Δa_μ in different LQ + VLQ models. Assuming $y_{L,R}^{LQ\mu t(T)}$ to be of the same order and $s_{L,R} \ll 1$, we also show the ratio order of the product of left-handed and right-handed $LQ\mu T$ couplings to the product of top-quark couplings. From the behaviors in Table IV, we find that the T contributions are strongly suppressed by the factor $m_t s_{L,R}^2/m_T$ for

TABLE II. The input parameters and mixing angle relations in the LQ + VLQ models for different representations.

LQ	VLQ	Related input parameters	Mixing angle relation
R_2	$T_{L,R}$	$y_L^{R_2\mu t}, y_R^{R_2\mu t}, y_R^{R_2\mu T}, \theta_L, m_T$	$\tan \theta_R = \frac{m_L}{m_T} \tan \theta_L$
	$(X, T)_{L,R}$	$y_L^{R_2\mu t}, y_R^{R_2\mu t}, \theta_R, m_T$	$\tan \theta_L = \frac{m_L}{m_T} \tan \theta_R$
	$(T, B)_{L,R}$	$y_L^{R_2\mu t}, y_R^{R_2\mu t}, y_L^{R_2\mu T}, \theta_R, m_T$	$\tan \theta_L = \frac{m_L}{m_T} \tan \theta_R$
	$(X, T, B)_{L,R}$	$y_L^{R_2\mu t}, y_R^{R_2\mu t}, y_R^{R_2\mu T}, \theta_L, m_T$	$\tan \theta_R = \frac{m_L}{m_T} \tan \theta_L$
	$(T, B, Y)_{L,R}$	$y_L^{R_2\mu t}, y_R^{R_2\mu t}, \theta_L, m_T$	$\tan \theta_R = \frac{m_L}{m_T} \tan \theta_L$
S_1	$T_{L,R}$	$y_L^{S_1\mu t}, y_R^{S_1\mu t}, y_L^{S_1\mu T}, \theta_L, m_T$	$\tan \theta_R = \frac{m_L}{m_T} \tan \theta_L$
	$(X, T)_{L,R}$	$y_L^{S_1\mu t}, y_R^{S_1\mu t}, \theta_R, m_T$	$\tan \theta_L = \frac{m_L}{m_T} \tan \theta_R$
	$(T, B)_{L,R}$	$y_L^{S_1\mu t}, y_R^{S_1\mu t}, y_R^{S_1\mu T}, \theta_R, m_T$	$\tan \theta_L = \frac{m_L}{m_T} \tan \theta_R$
	$(X, T, B)_{L,R}$	$y_L^{S_1\mu t}, y_R^{S_1\mu t}, \theta_L, m_T$	$\tan \theta_R = \frac{m_L}{m_T} \tan \theta_L$
	$(T, B, Y)_{L,R}$	$y_L^{S_1\mu t}, y_R^{S_1\mu t}, \theta_L, m_T$	$\tan \theta_R = \frac{m_L}{m_T} \tan \theta_L$
S_3	$(X, T, B)_{L,R}$	$y_L^{S_3\mu T}, y_R^{S_3\mu t}, \theta_L, m_T$	$\tan \theta_R = \frac{m_L}{m_T} \tan \theta_L$

the $R_2 + (X, T)_{L,R}/(T, B, Y)_{L,R}$ and $S_1 + (X, T)_{L,R}/(X, T, B)_{L,R}/(T, B, Y)_{L,R}$ models. For the $R_2 + T_{L,R}/(T, B)_{L,R}/(X, T, B)_{L,R}$ and $S_1 + T_{L,R}/(T, B)_{L,R}$ models, the T contributions are suppressed by the factor $s_{L,R}$. Also, the T contributions are dominated for the $S_3 + (X, T, B)_{L,R}$ model because of the $s_L/s_R \approx m_T/m_t$ factor. As s_L and s_R go to zero, there will be no chiral enhancements from the T quark in all the one-LQ and one-VLQ extended models.

C. The one-LQ and two-VLQ extended models

Here, we will not enumerate all the possible combinations. Instead, we just take the singlet $T_{1L,R}$ and the doublet

$(T_2, B_2)_{L,R}$ extended models as the examples, which will be named LQ + $T_{L,R} + (T, B)_{L,R}$, with LQ as R_2 or S_1 . For simplicity, we just consider the mixing between two VLQs. Then, we have the following mass term and Yukawa interactions:

$$\begin{aligned} \mathcal{L} \supset & -M_1 \overline{T_{1L}} T_{1R} - M_2 \overline{(T_2, B_2)_L} \begin{pmatrix} T_2 \\ B_2 \end{pmatrix}_R \\ & - y_L^{TB} \overline{(T_2, B_2)_L} T_{1R} \tilde{\phi} + y_R^{TB} \phi^T \epsilon \overline{T_{1L}} \begin{pmatrix} T_2 \\ B_2 \end{pmatrix}_R + \text{H.c.}, \end{aligned} \quad (58)$$

where $\tilde{\phi} \equiv \epsilon \phi^*$, and ϕ is the SM Higgs doublet. Thus, the mass terms should be

TABLE III. Coupling patterns of the Yukawa couplings between muons and t/T quarks in the LQ + VLQ models for different representations.

LQ	VLQ	$\overline{\mu_R} t_L$	$\overline{\mu_L} t_R$	$\overline{\mu_R} T_L$	$\overline{\mu_L} T_R$
R_2	$T_{L,R}$	$y_L^{R_2\mu t} c_L$	$y_R^{R_2\mu t} c_R - y_R^{R_2\mu T} s_R$	$y_L^{R_2\mu t} s_L$	$y_R^{R_2\mu t} s_R + y_R^{R_2\mu T} c_R$
	$(X, T)_{L,R}$	$y_L^{R_2\mu t} c_L$	$y_R^{R_2\mu t} c_R$	$y_L^{R_2\mu t} s_L$	$y_R^{R_2\mu t} s_R$
	$(T, B)_{L,R}$	$y_L^{R_2\mu t} c_L - y_L^{R_2\mu T} s_L$	$y_R^{R_2\mu t} c_R$	$y_L^{R_2\mu t} s_L + y_L^{R_2\mu T} c_L$	$y_R^{R_2\mu t} s_R$
	$(X, T, B)_{L,R}$	$y_L^{R_2\mu t} c_L$	$y_R^{R_2\mu t} c_R - y_R^{R_2\mu T} s_R$	$y_L^{R_2\mu t} s_L$	$y_R^{R_2\mu t} s_R + y_R^{R_2\mu T} c_R$
	$(T, B, Y)_{L,R}$	$y_L^{R_2\mu t} c_L$	$y_R^{R_2\mu t} c_R$	$y_L^{R_2\mu t} s_L$	$y_R^{R_2\mu t} s_R$
LQ	VLQ	$\overline{\mu_R} (t_R)^C$	$\overline{\mu_L} (t_L)^C$	$\overline{\mu_R} (T_R)^C$	$\overline{\mu_L} (T_L)^C$
S_1	$T_{L,R}$	$y_L^{S_1\mu t} c_R - y_L^{S_1\mu T} s_R$	$y_R^{S_1\mu t} c_L$	$y_L^{S_1\mu t} s_R + y_L^{S_1\mu T} c_R$	$y_R^{S_1\mu t} s_L$
	$(X, T)_{L,R}$	$y_L^{S_1\mu t} c_R$	$y_R^{S_1\mu t} c_L$	$y_L^{S_1\mu t} s_R$	$y_R^{S_1\mu t} s_L$
	$(T, B)_{L,R}$	$y_L^{S_1\mu t} c_R$	$y_R^{S_1\mu t} c_L - y_R^{S_1\mu T} s_L$	$y_L^{S_1\mu t} s_R$	$y_R^{S_1\mu t} s_L + y_R^{S_1\mu T} c_L$
	$(X, T, B)_{L,R}$	$y_L^{S_1\mu t} c_R$	$y_R^{S_1\mu t} c_L$	$y_L^{S_1\mu t} s_R$	$y_R^{S_1\mu t} s_L$
	$(T, B, Y)_{L,R}$	$y_L^{S_1\mu t} c_R$	$y_R^{S_1\mu t} c_L$	$y_L^{S_1\mu t} s_R$	$y_R^{S_1\mu t} s_L$
S_3	$(X, T, B)_{L,R}$	$-y_L^{S_3\mu T} s_R$	$y_R^{S_3\mu t} c_L$	$y_L^{S_3\mu T} c_R$	$y_R^{S_3\mu t} s_L$

TABLE IV. The third column shows the approximate formulas of the Δa_μ in the LQ + VLQ models for different representations, and the fourth column shows the multiplication order of T left-handed and right-handed LQ Yukawa couplings with respect to the top-quark ones. In the above, we have extracted the common factor, which means Δa_μ is redefined as $m_\mu m_t \Delta \bar{a}_\mu / (4\pi^2 m_{LQ}^2)$.

LQ	VLQ	The approximate expressions of $\Delta \bar{a}_\mu$	Coupling product order of T compared to t
R_2	$T_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu T} (y_L^{R_2\mu t})^*] s_L + (\frac{1}{4} + \log \frac{m_T^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu T})^*]$	s_L
	$(X, T)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*] s_L s_R + (\frac{1}{4} + \log \frac{m_T^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*]$	$m_t s_R^2 / m_T$
	$(T, B)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu T})^*] s_R + (\frac{1}{4} + \log \frac{m_T^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*]$	s_R
	$(X, T, B)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu T} (y_L^{R_2\mu t})^*] s_L + (\frac{1}{4} + \log \frac{m_T^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*]$	s_L
	$(T, B, Y)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{R_2} (m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*] s_L s_R + (\frac{1}{4} + \log \frac{m_T^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*]$	$m_t s_L^2 / m_T$
S_1	$T_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu T} (y_R^{S_1\mu t})^*] s_L - (\frac{7}{4} + \log \frac{m_T^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu T})^*]$	s_L
	$(X, T)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] s_L s_R - (\frac{7}{4} + \log \frac{m_T^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*]$	$m_t s_R^2 / m_T$
	$(T, B)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu T})^*] s_R - (\frac{7}{4} + \log \frac{m_T^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*]$	s_R
	$(X, T, B)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu T} (y_R^{S_1\mu t})^*] s_L s_R - (\frac{7}{4} + \log \frac{m_T^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*]$	$m_t s_L^2 / m_T$
	$(T, B, Y)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] s_L s_R - (\frac{7}{4} + \log \frac{m_T^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*]$	$m_t s_L^2 / m_T$
S_3	$(X, T, B)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1} (m_T^2/m_{S_3}^2) \text{Re}[y_L^{S_3\mu T} (y_R^{S_3\mu t})^*] s_L + (\frac{7}{4} + \log \frac{m_T^2}{m_{S_3}^2}) \text{Re}[y_L^{S_3\mu T} (y_R^{S_3\mu t})^*] s_R$	m_T / m_t

$$-\overline{(T_1, T_2)_L} \begin{pmatrix} M_1 & \frac{1}{\sqrt{2}} y_R^{TB} v \\ \frac{1}{\sqrt{2}} y_L^{TB} v & M_2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \end{pmatrix}_R + \text{H.c.}, \quad (59)$$

which can be diagonalized through the following transformations:

$$\begin{aligned} \begin{bmatrix} T_{1L} \\ T_{2L} \end{bmatrix} &\rightarrow \begin{bmatrix} \cos \theta_L^T & \sin \theta_L^T \\ -\sin \theta_L^T & \cos \theta_L^T \end{bmatrix} \begin{bmatrix} T_L \\ T'_L \end{bmatrix}, \\ \begin{bmatrix} T_{1R} \\ T_{2R} \end{bmatrix} &\rightarrow \begin{bmatrix} \cos \theta_R^T & \sin \theta_R^T \\ -\sin \theta_R^T & \cos \theta_R^T \end{bmatrix} \begin{bmatrix} T_R \\ T'_R \end{bmatrix}. \end{aligned} \quad (60)$$

In the following, $\sin \theta_L^T$, $\cos \theta_L^T$, $\sin \theta_R^T$, and $\cos \theta_R^T$ will be abbreviated as s_L^T , c_L^T , s_R^T , and c_R^T , respectively. Besides these, we have the following relations:

$$M_1 = m_T c_L^T c_R^T + m_{T'} s_L^T s_R^T, \quad M_2 = m_{T'} c_L^T c_R^T + m_T s_L^T s_R^T \quad (61)$$

and

$$c_L^T s_R^T = \frac{v(y_R^{TB} m_T + y_L^{TB} m_{T'})}{\sqrt{2}(m_{T'}^2 - m_T^2)}, \quad s_L^T c_R^T = \frac{v(y_L^{TB} m_T + y_R^{TB} m_{T'})}{\sqrt{2}(m_{T'}^2 - m_T^2)}. \quad (62)$$

In the above, m_T and $m_{T'}$ are the physical masses of $T_{L,R}$ and $T'_{L,R}$. Differently from the single-VLQ cases, the left-handed and right-handed field mixing angles cannot be related with each other.

Let us consider the case where only the VLQs can couple to muons through one LQ, and we have the following gauge eigenstate LQ interactions:

$$\begin{aligned} \mathcal{L}_{R_2+T_{L,R}+(T,B)_{L,R}} &\supset -x_{Ti} \overline{T_{1R}} R_2^a \epsilon^{ab} L_L^{i,b} + y_{iT} \overline{e_R^i} (R_2^a)^* \begin{pmatrix} T_2 \\ B_2 \end{pmatrix}_L^a + \text{H.c.}, \\ \mathcal{L}_{S_1+T_{L,R}+(T,B)_{L,R}} &\supset x_{Ti} \overline{(T_{1R})^C} S_1 e_R^i + v_{Ti} \overline{((T_{2L})^C, (B_{2L})^C)^a} S_1 \epsilon^{ab} L_L^{i,b} + \text{H.c.} \end{aligned} \quad (63)$$

After the EWSB, they can be written as

$$\begin{aligned} \mathcal{L}_{R_2+T_{L,R}+(T,B)_{L,R}} &\supset y_L^{R_2\mu T_2} \bar{\mu} \omega_- T_{2L} (R_2^{\frac{5}{2}})^* + y_R^{R_2\mu T_1} \bar{\mu} \omega_+ T_{1R} (R_2^{\frac{5}{2}})^* + \text{H.c.}, \\ \mathcal{L}_{S_1+T_{L,R}+(T,B)_{L,R}} &\supset y_L^{S_1\mu T_1} \bar{\mu} \omega_- (T_{1R})^C (S_1)^* + y_R^{S_1\mu T_2} \bar{\mu} \omega_+ (T_{2L})^C (S_1)^* + \text{H.c.} \end{aligned} \quad (64)$$

When transforming the fields through the rotation in Eq. (60), we can get the following mass eigenstate interactions:

$$\begin{aligned} \mathcal{L}_{R_2+T_{L,R}+(T,B)_{L,R}} \supset & \bar{\mu}(-y_L^{R_2\mu T_2} s_L^T \omega_- + y_R^{R_2\mu T_1} c_R^T \omega_+) T(R_2^{\frac{5}{2}})^* \\ & + \bar{\mu}(y_L^{R_2\mu T_2} c_L^T \omega_- + y_R^{R_2\mu T_1} s_R^T \omega_+) T'(R_2^{\frac{5}{2}})^* + \text{H.c.} \end{aligned} \quad (65)$$

and

$$\begin{aligned} \mathcal{L}_{S_1+T_{L,R}+(T,B)_{L,R}} \supset & \bar{\mu}(y_L^{S_1\mu T_1} c_R^T \omega_- - y_R^{S_1\mu T_2} s_L^T \omega_+) T^C(S_1)^* \\ & + \bar{\mu}(y_L^{S_1\mu T_1} s_R^T \omega_- + y_R^{S_1\mu T_2} c_L^T \omega_+) T'^C(S_1)^* + \text{H.c.} \end{aligned} \quad (66)$$

For the R_2 leptoquark, the contributions can be approximated as

$$\Delta a_\mu^{R_2+T_{L,R}+(T,B)_{L,R}} \approx \frac{m_\mu^2}{4\pi^2 m_{R_2}^2} \text{Re}[y_R^{R_2\mu T_1} (y_L^{R_2\mu T_2})^*] \left[-\frac{m_T}{m_\mu} f_{LR}^{R_2}(m_T^2/m_{R_2}^2) s_L^T c_R^T + \frac{m_{T'}}{m_\mu} f_{LR}^{R_2}(m_{T'}^2/m_{R_2}^2) s_R^T c_L^T \right]. \quad (67)$$

For the S_1 leptoquark, the contributions can be approximated as

$$\Delta a_\mu^{S_1+T_{L,R}+(T,B)_{L,R}} \approx \frac{m_\mu^2}{4\pi^2 m_{S_1}^2} \text{Re}[y_L^{S_1\mu T_1} (y_R^{S_1\mu T_2})^*] \left[-\frac{m_T}{m_\mu} f_{LR}^{S_1}(m_T^2/m_{S_1}^2) s_L^T c_R^T + \frac{m_{T'}}{m_\mu} f_{LR}^{S_1}(m_{T'}^2/m_{S_1}^2) s_R^T c_L^T \right]. \quad (68)$$

We find that there can be cancellation for both R_2 and S_1 in the case of $m_T = m_{T'}$ and $\theta_L^T = \theta_R^T$, while there is constructive interference for the case of $\theta_L^T = -\theta_R^T$.

III. NUMERICAL ANALYSIS

The input parameters are chosen as $m_\mu = 105.66$ MeV and $m_t = 172.5$ GeV [81]. The VLQs can be mainly constrained by the direct search [82–85] and electroweak precision observables (EWPO) [78,86]. To satisfy these bounds, we choose the VLQ mass to be $\mathcal{O}(\text{TeV})$ and the input $t-T$ mixing angle to be $\mathcal{O}(0.1)$. According to the direct search results, the LQ mass is typically required to be above 1 TeV [87–90].

A. Numerical analysis in the one-LQ and one-VLQ extended models

To estimate the effects of the T -quark contribution, let us define the following two functions from Eq. (7):

$$\begin{aligned} R_{T/t}^{R_2}(m_T, m_{R_2}) &\equiv \frac{m_T f_{LR}^{R_2}(m_T^2/m_{R_2}^2)}{m_t f_{LR}^{R_2}(m_t^2/m_{R_2}^2)}, \\ R_{T/t}^{S_1}(m_T, m_{S_1}) &\equiv \frac{m_T f_{LR}^{S_1}(m_T^2/m_{S_1}^2)}{m_t f_{LR}^{S_1}(m_t^2/m_{S_1}^2)}. \end{aligned} \quad (69)$$

In Fig. 2, we show the plots of $R_{T/t}^{R_2}(m_T, m_{R_2}) - m_T$ and $R_{T/t}^{S_1}(m_T, m_{S_1}) - m_T$ for fixed LQ masses. According to the

FIG. 2. The $R_{T/t}^{R_2}$ (left) and $R_{T/t}^{S_1}$ (right) curves as a function of m_T for the LQ masses of 2 TeV (blue) and 5 TeV (yellow).

TABLE V. Leading-order numerical expressions of Δa_μ in different LQ + VLQ models. Here, we have chosen the parameters to be $m_T = 1$ TeV and $m_{LQ} = 2$ TeV. The input mixing angle is set as $s_L = 0.05$ (singlet and triplet VLQ) and $s_R = 0.05$ (doublet VLQ).

LQ	VLQ	The leading-order expressions of $\Delta a_\mu \times 10^7$
R_2	$T_{L,R}$	$-0.5238\text{Re}[y_R^{R_2\mu T}(y_L^{R_2\mu t})^*] - 5.393\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
	$(X, T)_{L,R}$	$-5.397\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
	$(T, B)_{L,R}$	$-0.5238\text{Re}[y_L^{R_2\mu T}(y_R^{R_2\mu t})^*] - 5.393\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
	$(X, T, B)_{L,R}$	$-0.5238\text{Re}[y_R^{R_2\mu T}(y_L^{R_2\mu t})^*] - 5.393\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
	$(T, B, Y)_{L,R}$	$-5.397\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
S_1	$T_{L,R}$	$0.2331\text{Re}[y_L^{S_1\mu T}(y_R^{S_1\mu t})^*] + 3.754\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$
	$(X, T)_{L,R}$	$3.756\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$
	$(T, B)_{L,R}$	$0.2331\text{Re}[y_R^{S_1\mu T}(y_L^{S_1\mu t})^*] + 3.754\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$
	$(X, T, B)_{L,R}$	$3.756\text{Re}[y_R^{S_1\mu t}(y_L^{S_1\mu t})^*]$
	$(T, B, Y)_{L,R}$	$3.756\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$
S_3	$(X, T, B)_{L,R}$	$0.2007\text{Re}[y_L^{S_3\mu T}(y_R^{S_3\mu t})^*]$

plots, we find that the T -quark loop integral can be comparable to that of the top quark. However, the contributions are suppressed by the mixing angle in most of the models.

In Table V, we give the approximate numerical expressions of Δa_μ in different LQ + VLQ models. We adopt the mass parameters to be $m_T = 1$ TeV and $m_{LQ} = 2$ TeV by default. The input mixing angle in this section is set as $s_L = 0.05$ for the singlet and triplet VLQ cases, while it is $s_R = 0.05$ for the doublet VLQ cases. Based on the numerical results in Table V, we show the allowed regions from $(g-2)_\mu$ in Fig. 3. In the $R_2 + (X, T)_{L,R}/(T, B, Y)_{L,R}$ models, $\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$ is bounded to be within the

ranges $(-5.74, -3.56) \times 10^{-3}$ and $(-6.84, -2.46) \times 10^{-3}$ at 1σ and 2σ C.L., respectively. In the $S_1 + (X, T)_{L,R}/(X, T, B)_{L,R}/(T, B, Y)_{L,R}$ models, $\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$ is bounded to be within the ranges $(5.11, 8.25) \times 10^{-3}$ and $(3.54, 9.82) \times 10^{-3}$ at 1σ and 2σ C.L., respectively. In the $R_2 + T_{L,R}/(T, B)_{L,R}/(X, T, B)_{L,R}$ models, $\text{Re}[y_R^{R_2\mu T}(y_L^{R_2\mu t})^*]$ can be constrained in the ranges $(-0.059, -0.037)$ and $(-0.07, -0.025)$ with $\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$ turned off at 1σ and 2σ C.L., respectively. In the $S_1 + T_{L,R}/(T, B)_{L,R}$ models, $\text{Re}[y_L^{S_1\mu T}(y_R^{S_1\mu t})^*]$ can be constrained in the ranges $(0.082, 0.133)$ and $(0.057, 0.158)$ with $\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$ turned off at 1σ and 2σ C.L., respectively. In the

FIG. 3. The allowed region in the plane of $\text{Re}[y_R^{R_2\mu T}(y_L^{R_2\mu t})^*] - \text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$ (left, R_2 + VLQ models) and $\text{Re}[y_L^{S_1\mu T}(y_R^{S_1\mu t})^*] - \text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$ (middle, S_1 + VLQ models). Here, the green and yellow areas are allowed at 1σ and 2σ C.L., respectively. Note that there are no $y_{L/R}^{R_2\mu T}$ or $y_{L/R}^{S_1\mu T}$ couplings involved in the $R_2 + (X, T)_{L,R}/(T, B, Y)_{L,R}$ or $S_1 + (X, T)_{L,R}/(X, T, B)_{L,R}/(T, B, Y)_{L,R}$ models, but we show them just for comparison. Besides this, the subscripts L and R of the vertical axis title are reversed between the doublet $(T, B)_{L,R}$ and singlet $T_{L,R}$ cases. The right plot is for the $S_3 + (X, T, B)_{L,R}$ model.

FIG. 4. The regions allowed by $(g-2)_\mu$ at 1σ (green area) and 2σ (yellow area) C.L. For the $R_2 + T_{L,R}/(T,B)_{L,R}/(X,T,B)_{L,R}$ models, we consider the $y_L^{R_2 \mu T} = y_R^{R_2 \mu T}$ (upper left) and $y_L^{R_2 \mu T} = -y_R^{R_2 \mu T}$ (upper middle) cases in the plane of $y_{L/R}^{R_2 \mu T}$ and $y_{L/R}^{R_2 \mu T}$. For the $R_2 + (X,T)_{L,R}/(T,B,Y)_{L,R}$ models, we show the regions in the plane of $y_{L/R}^{R_2 \mu T}$ and $y_{R/L}^{R_2 \mu T}$ (upper right). For the $S_1 + T_{L,R}/(T,B)_{L,R}$ models, we consider the $y_L^{S_1 \mu T} = y_R^{S_1 \mu T}$ (central left) and $y_L^{S_1 \mu T} = -y_R^{S_1 \mu T}$ (central middle) cases in the plane of $y_{L/R}^{S_1 \mu T}$ and $y_{R/L}^{S_1 \mu T}$. For the $S_1 + (X,T)_{L,R}/(X,T,B)_{L,R}/(T,B,Y)_{L,R}$ models, we show the regions in the plane of $y_{R/L}^{S_1 \mu T}$ and $y_{L/R}^{S_1 \mu T}$ (central right). Here, the subscripts in front of the symbol “/” are for the singlet and triplet VLQs, and the subscripts after the symbol “/” are for the doublet VLQs. The lower plot is for the $S_3 + (X,T,B)_{L,R}$ model.

$S_3 + (X,T,B)_{L,R}$ model, $\text{Re}[y_L^{S_3 \mu T} (y_R^{S_3 \mu T})^*]$ is bounded to be within the ranges (0.096, 0.154) and (0.066, 0.184) at 1σ and 2σ C.L., respectively. Although we only keep the chirally enhanced contributions in Table V, they are good approximations. According to the numerical estimation, the

dropped contributions are always $m_\mu/m_t(m_T) \leq 10^{-3}$ -order suppressed.

In Fig. 4, we show the allowed regions from $(g-2)_\mu$ in the plane of y_L and y_R . Here, all the LQ couplings are set as real for convenience. For the $R_2 + T_{L,R}/(T,B)_{L,R}/(X,T,B)_{L,R}$

TABLE VI. The constraints on $\text{Re}[y_R^{R_2\mu T_1}(y_L^{R_2\mu T_2})^*]$ and $\text{Re}[y_L^{S_1\mu T_1}(y_R^{S_1\mu T_2})^*]$ at 1σ and 2σ C.L. Here, we consider the two scenarios $s_L^T = 0.05$, $s_R^T = 0.1$ and $-s_L^T = s_R^T = 0.1$ in the $LQ + T_{L,R} + (T, B)_{L,R}$ model. The VLQ and LQ masses are chosen as $m_T = m_{T'} = 1$ TeV and $m_{LQ} = 2$ TeV.

$(m_T, m_{T'}, m_{LQ})/\text{TeV}$	(s_L^T, s_R^T)	$\text{Re}[y_R^{R_2\mu T_1}(y_L^{R_2\mu T_2})^*]$		$\text{Re}[y_L^{S_1\mu T_1}(y_R^{S_1\mu T_2})^*]$	
		1σ	2σ	1σ	2σ
(1, 1, 2)	(0.05, 0.1)	(-0.059, -0.037)	(-0.07, -0.025)	(0.082, 0.133)	(0.057, 0.158)
(1, 1, 2)	(-0.1, 0.1)	(-0.015, -0.009)	(-0.018, -0.006)	(0.021, 0.033)	(0.014, 0.04)

models, there are three LQ Yukawa parameters: $y_{R/L}^{R_2\mu T}$, $y_L^{R_2\mu t}$, $y_R^{R_2\mu t}$. Then, we consider the two cases $y_L^{R_2\mu t} = y_R^{R_2\mu t}$ and $y_L^{R_2\mu t} = -y_R^{R_2\mu t}$ to eliminate one parameter. For the $R_2 + (X, T)_{L,R}/(T, B, Y)_{L,R}$ models, there are two LQ Yukawa parameters: $y_L^{R_2\mu t}$ and $y_R^{R_2\mu t}$. For the $S_1 + T_{L,R}/(T, B)_{L,R}$ models, there are three LQ Yukawa parameters: $y_{L/R}^{S_1\mu T}$, $y_L^{S_1\mu t}$, $y_R^{S_1\mu t}$. Then, we also consider the two cases $y_L^{S_1\mu t} = y_R^{S_1\mu t}$ and $y_L^{S_1\mu t} = -y_R^{S_1\mu t}$. For the $S_1 + (X, T)_{L,R}/(X, T, B)_{L,R}/(T, B, Y)_{L,R}$ models, there are two LQ Yukawa parameters: $y_L^{S_1\mu t}$ and $y_R^{S_1\mu t}$. For the $S_3 + (X, T, B)_{L,R}$ model, there are

two LQ Yukawa parameters: $y_L^{S_3\mu T}$ and $y_R^{S_3\mu t}$. In these plots, we include the full contributions. Obviously, the $y_L^{R_2\mu t} = -y_R^{R_2\mu t}$ case is favored, considering the perturbative unitarity for the $R_2 + T_{L,R}/(T, B)_{L,R}/(X, T, B)_{L,R}$ models. For the $S_1 + T_{L,R}/(T, B)_{L,R}$ models, the $y_L^{S_1\mu t} = y_R^{S_1\mu t}$ case is favored, considering the perturbative unitarity.

B. Numerical analysis in the one-LQ and two-VLQ extended models

Because we have turned off the mixing with top quarks in the two-VLQ extended models, the constraints from

FIG. 5. For the $R_2 + T_{L,R} + (T, B)_{L,R}$ model, we consider the two scenarios with $s_L^T = 0.05$, $s_R^T = 0.1$ (upper left) and $-s_L^T = s_R^T = 0.1$ (upper right) in the plane of $y_L^{R_2\mu T_2}$ and $y_R^{R_2\mu T_1}$. For the $S_1 + T_{L,R} + (T, B)_{L,R}$ model, we also consider the two scenarios with $s_L^T = 0.05$, $s_R^T = 0.1$ (lower left) and $-s_L^T = s_R^T = 0.1$ (lower right) in the plane of $y_R^{S_1\mu T_2}$ and $y_L^{S_1\mu T_1}$.

EWPOs can be loose. The free parameters are m_T , $m_{T'}$, m_{LQ} , s_L^T , s_R^T , $y_{L/R}^{LQ\mu T_1}$, and $y_{L/R}^{LQ\mu T_2}$ with LQs to be R_2 or S_1 . Here, s_L^T and s_R^T are independent parameters. Then, we consider the two scenarios $s_L^T = 0.05$, $s_R^T = 0.1$ and $-s_L^T = s_R^T = 0.1$. The VLQ and LQ masses are chosen as $m_T = m_{T'} = 1$ TeV and $m_{LQ} = 2$ TeV in this section.

In the absence of $LQ\mu t$ couplings, the muon anomaly can also be explained by the VLQ contributions. In Table VI, we give the $(g-2)_\mu$ constraints on $\text{Re}[y_R^{R_2\mu T_1}(y_L^{R_2\mu T_2})^*]$ and $\text{Re}[y_L^{S_1\mu T_1}(y_R^{S_1\mu T_2})^*]$ at 1σ and 2σ C.L. Assuming $m_T \approx m_{T'}$ and considering $s_{L,R}^T \ll 1$ in Eqs. (67) and (68), the contributions to $(g-2)_\mu$ can be approximated as

$$\begin{aligned}\Delta a_\mu^{R_2+T_{1L,R}+(T_2,B_2)_{L,R}} &\approx \frac{m_\mu m_T}{4\pi^2 m_{R_2}^2} f_{LR}^{R_2}(m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu T_1}(y_L^{R_2\mu T_2})^*](s_R^T - s_L^T), \\ \Delta a_\mu^{S_1+T_{1L,R}+(T_2,B_2)_{L,R}} &\approx \frac{m_\mu m_T}{4\pi^2 m_{S_1}^2} f_{LR}^{S_1}(m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu T_1}(y_R^{S_1\mu T_2})^*](s_R^T - s_L^T),\end{aligned}\quad (70)$$

which means that the contributions are determined by the difference of the mixing angles.

In Fig. 5, we show the allowed regions from $(g-2)_\mu$. Again, all the LQ Yukawa couplings are set as real for simplicity. For the $LQ + T_{L,R} + (T, B)_{L,R}$ models, we consider two scenarios with different mixing angles. In these plots, we also include the full contributions rather than those only with chiral enhancements.

IV. LQ PHENOMENOLOGY AT HADRON COLLIDERS

A. LQ decays

The traditional LQ decay channels are the SM quark and lepton, but LQs can possess exotic decay channels in our

models. Using them, we can propose new LQ search channels. The general LQ decay formulas are given in the Appendix.

In the LQ + VLQ models, the final states can be $t\mu$ and $T\mu$. It is reasonable to ignore the masses of the top quark and muon compared to VLQ and LQ masses. Assuming $y_{L,R}^{LQ\mu T}$ and $y_{L,R}^{LQ\mu t}$ to be of the same order, we list the approximate formulas of width ratio $\Gamma(LQ \rightarrow T\mu)/\Gamma(LQ \rightarrow t\mu)$ in Table VII. From the observation, we find that the $t\mu$ decay is dominated in the $R_2 + (X, T)_{L,R}/(T, B, Y)_{L,R}$ and $S_1 + (X, T)_{L,R}/(X, T, B)_{L,R}/(T, B, Y)_{L,R}$ models because of the mixing angle suppression. At the same time, both the $T\mu$ and $t\mu$ decay channels are important in the $R_2 + T_{L,R}/(T, B)_{L,R}/(X, T, B)_{L,R}$ and $S_1 + T_{L,R}/(T, B)_{L,R}, S_3 + (X, T, B)_{L,R}$ models, which

TABLE VII. The approximate formulas of the partial decay width ratio in the LQ + VLQ models for different representations (third column). In the fourth column, we show the $\Gamma(LQ \rightarrow T\mu)$ partial decay width suppression factors compared to those of $\Gamma(LQ \rightarrow t\mu)$.

LQ	VLQ	The approximate expressions of $\frac{\Gamma(LQ \rightarrow T\mu)}{\Gamma(LQ \rightarrow t\mu)}$	Suppress or not
R_2	$T_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2 y_R^{R_2\mu T} ^2 / (y_L^{R_2\mu t} ^2 + y_R^{R_2\mu t} ^2)$	No
	$(X, T)_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2 y_R^{R_2\mu t} ^2 s_R^2 / (y_L^{R_2\mu t} ^2 + y_R^{R_2\mu t} ^2)$	s_R^2
	$(T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2 y_L^{R_2\mu T} ^2 / (y_L^{R_2\mu t} ^2 + y_R^{R_2\mu t} ^2)$	No
	$(X, T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2 y_R^{R_2\mu T} ^2 / (y_L^{R_2\mu t} ^2 + y_R^{R_2\mu t} ^2)$	No
	$(T, B, Y)_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2 y_L^{R_2\mu t} ^2 s_L^2 / (y_L^{R_2\mu t} ^2 + y_R^{R_2\mu t} ^2)$	s_L^2
S_1	$T_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2 y_L^{S_1\mu T} ^2 / (y_L^{S_1\mu t} ^2 + y_R^{S_1\mu t} ^2)$	No
	$(X, T)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2 y_L^{S_1\mu t} ^2 s_R^2 / (y_L^{S_1\mu t} ^2 + y_R^{S_1\mu t} ^2)$	s_R^2
	$(T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2 y_R^{S_1\mu T} ^2 / (y_L^{S_1\mu t} ^2 + y_R^{S_1\mu t} ^2)$	No
	$(X, T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2 y_R^{S_1\mu t} ^2 s_L^2 / (y_L^{S_1\mu t} ^2 + y_R^{S_1\mu t} ^2)$	s_L^2
	$(T, B, Y)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2 y_R^{S_1\mu t} ^2 s_L^2 / (y_L^{S_1\mu t} ^2 + y_R^{S_1\mu t} ^2)$	s_L^2
S_3	$(X, T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_3}^2})^2 y_L^{S_3\mu T} ^2 / y_R^{S_3\mu t} ^2$	No

can lead to interesting collider signals. In Fig. 6, we show the contour plots of $\Gamma(\text{LQ} \rightarrow T\mu)/\Gamma(\text{LQ} \rightarrow t\mu)$ under the consideration of $(g-2)_\mu$ constraints. In these plots, we include the full contributions rather than those with only chiral enhancements. For the $R_2 + T_{L,R}/(T, B)_{L,R}/(X, T, B)_{L,R}$ models, we consider the $y_L^{R_2\mu t} = -y_R^{R_2\mu t}$ case. For the $S_1 + T_{L,R}/(T, B)_{L,R}$ models, we consider the

$y_L^{S_1\mu t} = y_R^{S_1\mu t}$ case. Then, we find that the $T\mu$ partial decay width can be comparable to and even larger than the $t\mu$ for some parameter space in these models.

In the $\text{LQ} + T_{L,R} + (T, B)_{L,R}$ model, we have turned off the $\text{LQ}\mu t$ couplings. Thus, the main decay channels are $\text{LQ} \rightarrow T\mu, T'\mu$. According to Eqs. (65), (66), and (A1), we have the following results:

FIG. 6. The contour plots of $\log_{10}[\Gamma(\text{LQ} \rightarrow T\mu)/\Gamma(\text{LQ} \rightarrow t\mu)]$, where the colored regions are allowed by $(g-2)_\mu$ at a 2σ C.L. Here, we consider the $R_2 + T_{L,R}/(T, B)_{L,R}/(X, T, B)_{L,R}$ models with $y_L^{R_2\mu t} = -y_R^{R_2\mu t}$ (upper left), the $R_2 + (X, T)_{L,R}/(T, B, Y)_{L,R}$ models (upper right), the $S_1 + T_{L,R}/(T, B)_{L,R}$ models with $y_L^{S_1\mu t} = y_R^{S_1\mu t}$ (central left), the $S_1 + (X, T)_{L,R}/(X, T, B)_{L,R}/(T, B, Y)_{L,R}$ models (central right), and the $S_3 + (X, T, B)_{L,R}$ model (lower).

FIG. 7. The contour plots of $\log_{10}[\Gamma(\text{LQ} \rightarrow T'\mu)/\Gamma(\text{LQ} \rightarrow T\mu)]$, where the colored regions are allowed by $(g-2)_\mu$ at a 2σ C.L. For the $R_2 + T_{L,R} + (T,B)_{L,R}$ model, we consider the $s_L^T = 0.05, s_R^T = 0.1$ (upper left) and $-s_L^T = s_R^T = 0.1$ (upper right) scenarios in the plane of $y_L^{R_2\mu T_2}$ and $y_R^{R_2\mu T_1}$. For the $S_1 + T_{L,R} + (T,B)_{L,R}$ model, we also consider the $s_L^T = 0.05, s_R^T = 0.1$ (lower left) and $-s_L^T = s_R^T = 0.1$ (lower right) scenarios in the plane of $y_R^{S_1\mu T_2}$ and $y_L^{S_1\mu T_1}$.

$$\begin{aligned} \frac{\Gamma(R_2^{5/3} \rightarrow T'\mu^+)}{\Gamma(R_2^{5/3} \rightarrow T\mu^+)} &\approx \frac{(m_{R_2}^2 - m_{T'}^2)^2}{(m_{R_2}^2 - m_T^2)^2} \cdot \frac{|y_L^{R_2\mu T_2}|^2}{|y_R^{R_2\mu T_1}|^2}, \\ \frac{\Gamma(S_1 \rightarrow \bar{T}'\mu^+)}{\Gamma(S_1 \rightarrow \bar{T}\mu^+)} &\approx \frac{(m_{S_1}^2 - m_{T'}^2)^2}{(m_{S_1}^2 - m_T^2)^2} \cdot \frac{|y_R^{S_1\mu T_2}|^2}{|y_L^{S_1\mu T_1}|^2}. \end{aligned} \quad (71)$$

Again, we consider $y_{L/R}^{\text{LQ}\mu T_1}$ and $y_{L/R}^{\text{LQ}\mu T_2}$ to be of the same order and $s_{L,R}^T \ll 1$ in the above results. In Fig. 7, we show the contour plots of $\Gamma(\text{LQ} \rightarrow T'\mu)/\Gamma(\text{LQ} \rightarrow T\mu)$. For the $R_2 + T_{L,R} + (T,B)_{L,R}$ and $S_1 + T_{L,R} + (T,B)_{L,R}$ models, we consider two scenarios: $s_L^T = 0.05, s_R^T = 0.1$ and $-s_L^T = s_R^T = 0.1$. In these plots, we adopt the full expressions of $(g-2)_\mu$ and LQ decay width rather than the approximate ones.

B. LQ production at hadron colliders

Generally speaking, there are pair- and single-LQ production channels [59,91–95]. In Figs. 8 and 9, we show the typical Feynman diagrams. Then, the LQ can decay into $t\mu$ or $T(T')\mu$, which has been discussed in previous sections. Usually speaking, the $T(T')$ quark can further decay into the bW, tZ, th final states. Thus, it will exhibit characteristic multitop and multimueon signals. In fact, we can distinguish different models through the LQ production and decay channels. Besides these, there are also off-shell LQ channels [58,96]. In Fig. 10, we show the Feynman diagrams for $t(T)\bar{t}(\bar{T})\mu^+\mu^-$ associated production. In this section, we only show the possible collider signals; the detailed phenomenology studies are beyond the scope of this work.

FIG. 8. The typical Feynman diagrams contributing to the pair production of LQs.

FIG. 9. The typical Feynman diagrams contributing to single-LQ production.

FIG. 10. The Feynman diagram with the virtual LQ.

V. SUMMARY AND CONCLUSIONS

We consider the models containing the LQ and VLQ simultaneously, which can be the solution to the $(g-2)_\mu$ anomaly because of the chiral enhancement. For the one-LQ and one-VLQ extended models, there are contributions from the $LQ\mu t$ and $LQ\mu T$ interactions. For the $R_2 + (X, T)_{L,R}/(T, B, Y)_{L,R}$ and $S_1 + (X, T)_{L,R}/(X, T, B)_{L,R}/(T, B, Y)_{L,R}$ models, the top-quark contributions are dominated. For the $R_2 + T_{L,R}/(T, B)_{L,R}/(X, T, B)_{L,R}$ and $S_1 + T_{L,R}/(T, B)_{L,R}$ models, we need to consider both the top and T -quark contributions. In addition to the traditional R_2 and S_1 choices, we find the new S_3 solution to the anomaly in the presence of the $(X, T, B)_{L,R}$ triplet, which is dominated by the T -quark contributions. For the $LQ + T_{L,R} + (T, B)_{L,R}$ models, the anomaly can be explained even in the absence of $LQ\mu t$ interactions. Based on the constraints from $(g-2)_\mu$, we propose new LQ search channels. Besides the traditional $t\mu$ decay channel, the LQ can also decay into $T\mu$ final states. It will lead to the characteristic multitop and multimuon signals in these models, which can be tested at hadron colliders.

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Note added.—We recently had a discussion with Chen Zhang. Thereby, we noticed that there are already some works to explain the flavor anomalies in the LQ and VLQ extended models. In Ref. [97], the authors considered the VLQ representation $(3, 2, -5/6)$. The VLQs in their model are mainly for the B physics anomalies and neutrino mass, while the contributions to the $(g-2)_\mu$ are the same as those in the minimal LQ models. In Ref. [98], the authors studied the VLQ with $(3, 2, 1/6)$ representation, where no chiral enhancements are involved because of the Z_2 symmetry.

APPENDIX: THE DECAY WIDTH FORMULA

Let us consider the interaction $\mathcal{L} \supset S \bar{f}_1 (y_L \omega_- + y_R \omega_+) f_2$, where the masses of S , f_1 , and f_2 are labeled as m_S , m_1 , and m_2 . If $m_S > m_1 + m_2$, we can obtain the following decay width formula:

$$\Gamma(S \rightarrow \bar{f}_2 f_1) = \frac{1}{16\pi m_S} \sqrt{\left(1 - \frac{m_1^2 + m_2^2}{m_S^2}\right)^2 - \frac{4m_1^2 m_2^2}{m_S^4}} \cdot [(m_S^2 - m_1^2 - m_2^2)(|y_L|^2 + |y_R|^2) - 2m_1 m_2 (y_L (y_R)^* + y_R (y_L)^*)]. \quad (\text{A1})$$

If $m_S, m_1 \gg m_2$, it can be approximated as

$$\Gamma(S \rightarrow \bar{f}_2 f_1) \approx \frac{m_S}{16\pi} \left(1 - \frac{m_1^2}{m_S^2}\right)^2 (|y_L|^2 + |y_R|^2). \quad (\text{A2})$$

If $m_S \gg m_1, m_2$, it can be approximated as

$$\Gamma(S \rightarrow \bar{f}_2 f_1) \approx \frac{m_S}{16\pi} (|y_L|^2 + |y_R|^2). \quad (\text{A3})$$

Different from the $\text{Re}[y_L (y_R)^*]$ -dominated $(g-2)_\mu$ case, the main contributions are the $(|y_L|^2 + |y_R|^2)$ parts in the decay width formula.

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